Two puzzles about requirements

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Abstract. Modal expressions like *have to, require, need, must* etc. are standardly considered to be universal quantifiers over possible worlds. In this extended abstract, I point out that this is problematic for cases in which these expressions interact with scalar operators. In particular, I argue that such modals appear to be able to express existential modal force.

1 First puzzle: Minimum requirements

My friend and me are having a bet in which I claim to be able to score at least 300 points in the game of scrabble we are about to start. The following would be an accurate paraphrase for the bet in question.

(1) The minimum number of points I need to score to win the bet is 300.

That is, the bet involves a *minimum requirement*: If I score 300, I win. If I score more, I also win. But if I score less, then I lose. To be sure, at first sight it seems obvious why (1) is interpreted as such, for I do *need* to score 300, and 300 is the *minimum* score that makes me win the bet. Yet, when we make things precise, and given common assumptions on the semantics of modal auxiliaries, it turns out that it is rather mysterious why (1) means what it means.

The common assumptions I am alluding to are, first of all, that *need* is a universal quantifier over possible worlds and, second, that the *to*-phrase in (1) restricts quantification over possible worlds (von Fintel and Iatridou 2005). In other words, "*to p, need to q*" is true if and only if all the p-worlds are q-worlds; i.e. if p entails q. At first sight, this view appears to make good predictions. For example, a case like (2) is now interpreted as saying that you went to the Twijnstraat in all the worlds where you got good cheese.

(2) To get good cheese, you need to go to the Twijnstraat.

However, when we apply the above assumptions to (1), the outcome is very puzzling. Note first the following: in the scenario I sketched about the scrabble bet, there are no worlds in which I win the bet while scoring fewer than 300 points. Furthermore, the worlds where I do win come in many variations: in some (but not all) of them I score 300 points, in some (but not all) my score is 301, in some (but not all) it is 302, 310, or even 550. The problem is that

$\vdots$

$\Downarrow$

I scored 303 points

$\Downarrow$

I scored 302 points

$\Downarrow$

I scored 301 points

$\Downarrow$

I scored 300 points

$\Downarrow$

$\vdots$

Fig. 1. The *at least* interpretation for numerals

the most obvious referent for *the minimum number of points I need to score* is the smallest number x such that *I scored x points* is true in all relevant worlds. However, for no value for x is this open sentence true in all bet-winning worlds. It would seem then that the definite description *the minimum number of points I need to score* fails to refer in the described situation.

It might appear that there is an obvious solution. If we assume that numerals are interpreted as lower-bounded only (usually, this is dubbed the *at least* interpretation for numerals), then we get an entailment scale as in figure 1.

Given this scale, it is now true that in all bet-winning worlds I scored 300 points, for the worlds where my exact score was higher will be worlds in which I scored at least 300 points. Unfortunately, if we assume that such an entailment scale is appropriate then, by entailment, it is also true that in all bet-winning worlds I scored 200 points. In fact, it is entailed that I score a single point in every bet-winning world. Consequently, *the minimum number of points I need to score to win the bet* is now predicted to be 1, not 300. Yet, (3), obviously, seems an unacceptable way of paraphrasing the bet between my friend and me.

(3) The minimum number of points I need to score to win the bet is 1.

In sum, independent of how we interpret numerals, it appears far from straightforward how to come to a compositional interpretation of (1).

2 Second puzzle: Maximum requirements

If we assume the entailment scale in figure 1, then there is a further puzzle. The proposition *I scored 1 point* is true in all bet-winning worlds, and so is the proposition *I scored 300 points*. The proposition *I scored 301 points*, however, is the first proposition in the scale that is not true in all bet-winning worlds. (It's only true in some.) This makes 300 the highest value for x such that *I scored x points* is true in all victorious worlds and so we predict that instead of

(1), the correct way of paraphrasing the bet is, in fact, (4). Clearly, this is an unwelcome prediction, for the scenario I sketched was a prime example of what we call *minimum requirements*, not of *maximum* ones.

(4) The maximum number of points I need to score to win the bet is 300.

In the given scenario there was no upper limit on my score. That is, the bet is only about me scoring 300 or more. No matter how much higher than 300 I score, I will keep on winning. In certain other scenarios maximal requirements do make sense, however. Take the following example.

(5) The maximum number of sets needed to decide a men's tennis match is 5.

In men's tennis, the first player to win three sets wins, hence (5). At the same time, this means that there can be no men's tennis match which lasts for fewer than 3 sets. In other words:

(6) The minimum number of sets needed to decide a men's tennis match is 3.

Once again, we derive the wrong interpretation for such examples under the standard assumptions discussed above. Let us first assume that numerals create entailment scales, as in figure 1. That is, if 4 sets were played in a match, then this entails that 3 sets were played, as well as that 2 sets were played, etc. The minimum number n of sets such that n sets are played in every world in which the match is decided is now 1. The maximum number n of sets such that n sets are played in every match-deciding world is 3, since all such worlds contain (at least) 3 sets, while only some contain a 4th or a 5th one.

Things are no better if we interpret numerals as doubly-bound. In that case the definite descriptions in (5) and (6) will fail to refer, for there exists no n such that in all possible ways in which a tennis match could be decided the match counted exactly n sets.

3 Modal Force

The two puzzles above point out a problem with the interaction between universal modals and scalar operations. Abstracting away from the scenarios above, the puzzles can be generalised as follows. Let $\mathcal{P}$ be a scale of propositions, as in (7).

(7) $p_1 < p_2 < p_3 < \ldots < p_i < p_{i+1} < \ldots$ $\mathcal{P}$

Following standard assumptions, a proposition p_k is a *minimum requirement* for q if it is the case that p_k is entailed by q, while for no $l < k$, p_k is entailed by q. If (7) is an entailment scale, however, then for any p_k it holds that $p_k \models p_{k-1}$ and, in fact, $p_k \models p_1$. So, if q entails p_k, then it also entails p_1. In other words, the minimum requirement for q could only ever be p_1, which renders the notion of minimum requirement quite useless. If, however, $\mathcal{P}$ is not an entailment scale, then it is easy to imagine that p_k is entailed by q while this is not the case for any proposition that is lower on the scale. Unfortunately, this is not how we normally understand the notion of *minimum requirement.* In fact, part of the problem with our assumed definition of minimum requirement is that if p_k is a minimum requirement for q, then q need not entail p_k. For instance, if my bet is that I will score 300 or more in the game of scrabble, then me winning the bet does not entail me scoring exactly 300 points.

There is a surprising way out: minimal and maximal requirements are not about necessity, but rather about possibility. Assume $\mathcal{P}$ to be a non-entailment scale: p_k is a minimal (maximal) requirement for q iff p_k is compatible with q and there is no $l < k$ ($l > k$) such that p_l is compatible with q. In the scrabble bet scenario, the proposition that I scored exactly 300 points is the lowest ranked proposition such that there *exists* a bet-winning world in which that proposition is true. Furthermore, 3 is the smallest number n such that there is a world in which the tennis match is decided in such a way that the number of played sets is exactly n. Also, 5 is the highest number n such that there is a world in which the match is decided such that the number of played sets is exactly n. Thus, under the assumption that the modal force of *need* is existential, the analysis of examples like (1), (5) and (6) appears straightforward.

There is however clear evidence that the modal force of *need* and it's kin is not existential. For instance, if it were, we would predict (8) to be true. The intuition, however, is that it is false.

(8) To decide a men's tennis match, you have to play exactly 3 sets.

Furthermore, with an existential semantics one would expect (9-a) to entail (9-b), a rather unwelcome prediction.

(9) a. In order to win the bet, I need to score more than 300 points.
b. In order to win the bet, I need to score more than 400 points.

In fact, the intuition is that (9-b) entails (9-a). This intuition is captured under the assumptions I have considered to be standard.[1]

[1] Such entailments are discussed in von Fintel and Iatridou 2005. There are further predictions, however, that at first sight are slightly counter-intuitive. For instance, von Fintel and Iatridou discuss an example like (i):

(i) a. To get good cheese, you have to go to the Twijnstraat.
b. ⇒ To get good cheese, you have to breathe.

If you go to the Twijnstraat in all worlds in which you get good cheese, then since you breathe in all the worlds in which you go to the Twijnstraat, it follows that you breathe in all worlds in which you get good cheese. Von Fintel and Iatridou judge (i-b) true, yet unhelpful in the context of (i-a), an intuition I agree with. However, the truth of the following example, suggested to me by David Beaver, is a further prediction of the theory, but it is not clear that it is a welcome one.

4 Interlude: data

Central to the puzzles that I presented above is a rather specific family of noun phrases, namely definite descriptions that contain some sort of minimality or maximality operator (*minimum, smallest, maximum, highest* etc.) and a necessity modal like *need,* or *require,* or *have to*, etc. Since these noun phrases play a crucial role in my arguments, I would like to take away any skeptical reader's impression that such constructions are somewhat artificial. To this end, I will give some (natural) examples. (Below, I moreover argue that the puzzle is part of a larger set of phenomena that includes, for instance, certain modified numerals.)

Examples like (10) are typical and common cases where operators expressing minimality (*smallest* in this case) interact with modality.

(10) **Question:**
One-half of a road construction project was completed by 6 workers in 12 days. Working at the same rate, what is the smallest number of workers needed to finish the rest of the project in exactly 4 days?
Answer:
The smallest number of workers needed to finish the project in 4 days is 18.

The answer A in (10) spells out a minimal requirement: 18 workers allow you to finish the project in 4 days, fewer than 18 workers won't allow you to do so.

Explicit minimal requirement formulations are quite common even outside the realm of maths problems, as is illustrated by (11). (Here, (11-b) and (11-c) are naturally occurring examples.)

(11) a. The smallest amount of butter you need for a nice and tasty cake is 250 grams.
b. The minimum number of partitions you need to install linux is 3.
c. The minimum number of credits you need to graduate is 85.

It should be pointed out that minimal requirement statements are not limited to the modal *to need*. In fact, *must, require, should* and *have to* allow for similar constructions, witness the following naturally occurring examples.

(12) a. Determine the smallest number of digits that must be removed from x so that the remaining digits can be rearranged to form a palindrome.[2]
b. REM level is the minimum number of BYTES you require to continue.[3]

(ii) To climb Everest you need 3 to equal 2+1.

[2] `http://cemc.uwaterloo.ca/Contests/past_contests/2008/2008FryerContest.pdf`
[3] `http://www.cramsession.com/articles/files/checking-free-space-9262003-1044.asp`

c. What is the minimum number of karanga I should know before I can say that I can karanga?[4]
d. We are usually interested in knowing the smallest number of colors that have to be used to color a graph.[5]

5 Modified Numerals

So far, I have been assuming that, in statements of minimum or maximum requirement, the scope of *minimum* and *maximum* is wider than that of the modal. So, I have been analysing (1) as (13-a), rather than (13-b).[6]

(13) a. $\min_n(\Box[\text{I score } n \text{ points}])=300$
b. $\Box[\min_n(\text{I score } n \text{ points}) = 300]$

Note that an analysis along the lines of (13-b), however, does not solve our puzzles. On an 'exactly' reading for *n points*, there is just the single value for *n* which makes *I score n points* true. The use of *minimally* would then be vacuous. Worse, we would expect that there is no difference between (14) and (15).

(14) The minimum number of points I need to score to win the bet is 300.

(15) The maximum number of points I need to score to win the bet is 300.

On an at least perspective for *n points*, (13-b) will be a contradiction. Since for any *n*, *I score n points* entails that I scored a single point, (13-b) ends up stating that $\Box[1 = 300]$.

Interestingly, there is a variation on (13-b) that yields the correct truth-conditions without the need for a change in modal force for *need*.

(16) $\Box[\iota_n(\text{I score } n \text{ points}) \geq 300]$

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[4] `http://www.maori.org.nz/faq/showquestion.asp?faq=3&fldAuto=99&MenuID=3`

[5] `http://www.math.lsa.umich.edu/mmss/coursesONLINE/graph/graph6/index.html`

[6] It is difficult to extend the above puzzles of minimal and maximal requirement to cases of epistemic modality. This might actually be expected if the analysis of a wide scope minimality operator is on the right track, given the generalisation that epistemic modals tend to take wide scope (von Fintel and Iatridou 2003).

Consider the following example. Say, you have seen me put 10 marbles in a box, but you do not know how many marbles there were in the box to begin with. Structurally, your knowledge state now resembles that of a minimal requirement scenario: in all compatible worlds, there are (at least) 10 marbles in the box, while in no compatible worlds there are fewer than 10 marbles in the box. Yet, in contrast to the examples given above, we cannot express this knowledge state as (i).

(i) #The minimum number of marbles that must be in the box is 10.

This analysis is not as far fetched as it might seem at first sight. As a numeral modifier, *minimally* shares its semantics with *at least.* In other words, the proper treatment of (14) could be thought to be whatever works for (17) or (18).

(17) To win the bet, I need to score minimally 300 points.

(18) To win the bet, I need to score at least 300 points.

Unfortunately, there are reasons to believe that (16) is too simplistic as an analysis for (17) or (18). As Geurts and Nouwen (2007) argue in detail, *at least* does not correspond to the $\geq$-relation. Moreover, Nouwen (2010) shows that both *minimally* and *at least* are part of a class of numeral modifiers that is incompatible with specific amounts. That is, whereas (19) is felicitous and true, (20) is unacceptable.

(19) A hectagon has more than 2 sides.

(20) A hectagon has { at least / minimally } 2 sides.

A further property of numeral modifiers like *minimally* is that they trigger readings of speaker uncertainty (Geurts and Nouwen 2007; Krifka 2007; Nouwen 2010). For instance, (21) is interpreted as being about the minimum number of people John *might* have invited (according to the speaker).

(21) John invited { minimally / at least } 30 people to his party.
(#To be precise, he invited 43.)

Such speaker uncertainty readings carry over to adjectives like *minimum.*[7]

(22) The { minimum / smallest } number of people John invited to the party is 30.
(#To be precise, it's 43.)

Apart from understanding (22) as a case of speaker uncertainty, one might also understand it as saying that 30 is the smallest number of people that John at *some* time in the past invited to the party. Crucially, all available readings somehow involve existential quantification.

The point I want to make is that it seems to me that there is a general puzzle underlying the interaction of universal modals and scalar operators, be they adjectives like *minimum*, *smallest*, *highest* etc. or numeral modifiers like *minimally* and *at least.*[8] What such expressions appear to have in common is that they operate on existential structures.

[7] I am grateful to an anonymous Amsterdam Colloquium reviewer for urging me to attend to the relevance of such data.

[8] In fact, an anonymous reviewer suggests that the data extends to cases where *minimum* is used as a noun, as in (i).

(i) I need a minimum of 300 points to win the bet.

6 Conclusion: towards an account of existential needs

I will conclude by suggesting a way forward. In her 2005 AC paper, Schwager argues that imperatives and modals verbs like *need* cannot always be interpreted as universal operators. For instance, (23) has a paraphrase: having a lot of money is an example of something you *could* do to get into a good university.

(23) To get into a good university, you must for example have a lot of money.

Schwager proposes that necessity modals are essentially exhaustified possibility modals, where $exh(\Diamond) = \Box$. (See Schwager's paper for details.) Expressions like *for example* are de-exhaustifiers, which can reveal the existential nature of the modal.

Schwager's proposal helps to solve the two puzzles of minimal and maximal requirement. The above suggests that scalar operators like *minimum/maximum* can intervene with exhaustification. If this idea is on the right track, then we might expect to find that the interaction between necessity modals and scalar operators is generally mystifying.

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