Paul Dekker

PROCEEDINGS OF THE

Seventh Amsterdam Colloquium

DECEMBER 19-22, 1989

PART 1

EDITED BY
MARTIN STOKHOF
& LEEN TORENVLIET

ITLI, Institute for Language, Logic and Information

Universiteit van Amsterdam, 1990

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Preface

From December 19-22, 1989, the Seventh Amsterdam Colloquium was held. It was organized by the Institute for Language, Logic, and Information (ITLI), which is founded by the Departments of Philosophy and Mathematics of the University of Amsterdam.

Financial support was received from the Royal Dutch Academy of Science (KNAW), which is gratefully acknowledged.

Published here are 32 of the 42 contributed talks that were delivered at the Colloquium. They appear as the authors prepared them.

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Contents of Part 2

Three Problems for the Semantics of Gerundive Constructions

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The semantics of gerundive constructions in English has several interesting problems whose solution requires a careful investigation of the interaction between semantics and syntax. This paper addresses three basic difficulties within the framework of discourse representation theory (DRT).

1. The Problems

First Problem: Sentence Meanings and Gerundive Nominals

A first criterion of adequacy for a semantics will be to predict the synonymy of the a-b pairs in (1) - (4) while avoiding that prediction for the pair in (5) and (2.a-b).

(1.a) Fred's shooting of Bill led to his imprisonment.
(1.b) Fred shot Bill, and that led to his imprisonment.

(2.a) Bill's hitting of that policeman will land him in prison.
(2.b) Bill hit that policeman. It will land him in prison.

(3.a) John's hitting Bill caused Bill to yell.
(3.b) John hit Bill. That caused Bill to yell.

(4.a) John sprinting past Bill was a welcome sight.
(4.b) John sprinted past Bill. It was a welcome sight.

(5.a) Mary doubted that John was unhappy.
(5.b) John was unhappy. Mary doubted that.

Second Problem: Spectra of Nouniness and World-Immanence.

Derived nominal and gerundive constructions exhibit a varied range of syntactic forms and a correlated range among their semantic denotations. Ross, Quirk and others have spoken of a spectrum of "nouniness." Derived nominal constructions are very very much like true noun phrases; they may contain determiners, be pluralized and require adjectival modification. At the other end of the spectrum of nouniness are *that* clauses; they do not take determiners, cannot be pluralized and require adverbial modification. In the middle are various gerundive constructions. Gerundive constructions may come with or without determiners; some take adjectival modification, while others take adverbial modification. The nouniness of a nominal also affects the argument structure it inherits from the verb from which it derives. The argument structure of very "nouny" result nominals may may drift substantially from the argument structure of the verb.[1] All gerundive constructions, on the other hand, have an argument structure very close to that of the verb from which they are derived.

The syntactic spectrum of nouniness correlates with a spectrum of *world immanence* among the semantic denotations of sentential nominals. At one end of this spectrum lie concrete physical objects, while at the other end are abstract objects like propositions or the objects of thought. Intermediate between these two poles lie events and states as more concrete sorts of objects while toward the abstract end of the spectrum like sorts of objects like facts, possibilities, situations and the like.

One way to distinguish between such denotations is to look at certain predicates as Vendler (1967) suggested. Vendler distinguished between predicates assigning roughly a

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[1]I owe this observation to Lebeaux (1986).

property presupposing a locational, or temporal property for their arguments and predicates and those whose arguments cannot have temporal or locational properties. Those predicates which do assign such properties can be plausibly interpreted as taking events or states as semantic arguments. Derived nominals and *of ing* gerundive constructions denote events and states according to this test.

(6.a) John´s kissing of Mary took place in the park,

or

(6.b) The destruction of the city took less than an hour.

They may also, as Vendler (1967) also sometimes have a fact reading, as in, *the collapse of the Germans*. This is a complication that I will only briefly touch on here.[1]

Other gerundive constructions, like the ACC-ing gerundive of English, do not denote events and states by Vendler's test:

(7.a) *John singing the Marseillaise happened in the park.

(7.b) *Fred shooting Bill was bloody.

(7.c) *She cried after him singing the Marseillaise.

POSS-ing gerundive constructions also often don't denote events or states according to Vendler's test. The following sentences are not generally considered felicitous.[2]

(8.a) ?? Fred's hitting Bill happened yesterday.

(8.b) ?? Fred's having shot Bill is something that took place behind the bar.

(8.c) *?She cried after his singing the Marseillaise.

ACC-ing and POSS-ing constructions appear as (syntactic) arguments to many of those predicates which take *that* clauses as their syntactic arguments, thus indicating that they denote some sort of abstract object like a fact or a possibility

(8.d) John's having shot Bill is a fact.

(8.e) John shooting Bill was a possibility too horrible to contemplate.

Nevertheless, POSS-ing gerundive constructions may also denote events or states, especially when their subject place arguments are missing:[3]

(9.a) Building the Panama Canal took longer than expected.

(9.b) Gathering pecans in Central Texas starts in September.

(9.c) Before proving the theorem, Robyn spent all day thinking about it.

Finally we should distinguish at least between propositions and facts. ACC-*ing* and POSS-*ing* gerundive constructions do not go with certain containers that are paradigmatic predicates of propositions, like for instance truth and falsity.

(10.a) *? John's going to the store is true.

(10.b) *? John kissing Mary is true.

All this data raises several questions. How do we account for these different semantic denotations? What is the relation between syntactic form, DRS structures and the typology of natural language metaphysics? Should and if so how can we integrate an account of the different semantic denotations with an account of the syntactic spectrum of nouniness? Why do the POSS-ing constructions have the flexibility to denote different types of abstract entities, whereas others like derived nominal phrases, and ACC-ing gerundive phrases do not? An account of the semantics of derived nominal and gerundive constructions must answer these questions.

<u>Third Problem: Event Negation.</u>

Corresponding to sentences containing a negation operator, there exist all the gerundive forms.

(11.a) John´s not kissing Mary

(11.b) Fred not seeing Mary

There is a difficulty with interpretating such nominals-- the problem of *event negation*. It is best appreciated by looking at a negated sentence first. Consider the sentence

(12) John ran

Following Davidson (1967), (10) would have the analysis:

(12') $\exists e\ run(e, j)$

where e is a variable ranging over events and 'j' denotes John. How does negation affect this analysis? We appear to get the right truth conditions for (11) if we simply append a negation to the front of (10') as in (11').

(13) John did not run.

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[1]See Zucci this volume for a fuller discussion

[2](7.c) is due to Vendler.

[3](9.a) and (9.b) are due to Weir (1986).

(13') $\neg\exists e$ run(e, j)
(13') tells us that (13) will be true just in case there is no event of John's running.

The difficulty appears now when we turn to sentential nominals. The gerundive construction,

(14.a) John's running

plausibly denotes the event or activity, to be more precise, of John's running. But then what does the gerundive construction,

(14.b) John's not running

denote? As we have seen the sentence correlated with (13), (11), denies the existence of an event of John's running, and we have some reason (given problem 1) to believe that nominals and their correlated sentences have the same content.

For metaphysical and logical reasons, it seems absurd to postulate an event of John's not running. The domain of events is not closed under negation; i.e., if φ denotes an event, then $\neg\varphi$ does not necessarily also denote an event. To suppose otherwise is to countenance objects like the event of no event's ever happening in the universe.

Nevertheless nominals such as (14.b) denote something with temporal duration, as the felicity of (15) indicates.

(15) John's not running lasted only until his foot healed.

Given a typology of abstract objects in natural language metaphysics which distinguishes between states and events, the effect of negation on the denotation of sentential nominals is this: negation transforms an event denoting nominal into a state denoting nominal. Negation seems to be unique in this role among the various operators that can occur within sentential nominals.

There are in fact three ways we might test to see whether a negated event statement (a sentence introducing an event discourse referent) introduces a state. The first of these concerns durative, temporal and spatial sentential modifiers like *for over two hours* and *in the park*. According to most linguists and philosophers who countenance events and states, states take durative temporal modifiers (whereas many events don't) and do not take spatial modifiers very well. A second test for a state is to see whether one can anaphorically refer back to a state introduced by a negated event description. Again durative, temporal predicates will be good indicators of whether the object anaphorically referred to is a state or not. A third test for statehood is to look at the associated sentential nominals themselves, which is what we have been doing. These tests do not all agree in this case, so we must interpret their results judiciously. Our semantics must predict which constructions containing negation will denote states and which will not.

2. The Framework for the Analysis.

The basic semantic framework I will use is that of discourse representation theory (DRT). I do so for two reasons. First, the solution to the first problem requires a theory of intersentential anaphora, and DRT's approach to discourse anaphora can be easily extended to account for anaphoric reference to events and abstract objects. Second, I will exploit in my analysis the two stage semantics of DRT-- viz., the DRS construction procedure and the correctness definition. The typology of abstract entities in natural language metaphysics which distinguishes acitivities, achievements, accomplishments, and states, as well as facts and propositions, and such distinctions are needed to make sense of natural language discourse. But as well known the very same real event may be described without falsehood as an activity or as an accomplishment. Similarly, the same real eventuality may be described from a world-immanent perspective as some sort of event or state or from a more abstract perspective as a fact. A complex interaction between abstract and world immanent entities is apparent in a problem like the the problem of event negation. This interaction can be much more simply described, if one exploits a two stage semantic analysis like DRT. The flexible relation between the two levels of semantic analysis in DR theory that allows us to make these distinctions between abstract objects at the level of natural language metaphysics without being committed to the same differences and distinctions in the model theory. In particular, the flexibility of the notion of an embedding of a DRS in a Tarskian model enables one to claim that propositions, facts and states of affairs or, alternatively, activities, accomplishments, achievements and states, postulated as distinct types at the level of the DRS each with a particular domain structure, need not exist as separate entities or have the same domain structure at the level of the Tarskian model.

In using the DRT framework, I will add to the extant construction procedures for DRSs[1] to get the appropriate conditions and DR-theoretic structures for sentential nominals and their constituents. Standard DRT treatments already distinguish between discourse referents for states and for events. I'll expand this list of different types of discourse referents here. A useful distinction from our natural language metaphysics divides the realm of events into accomplishments, achievements (accomplishments and achievements often being classed together as "telic"), processes and activities (the atelic events). I have already distinguished between states and events at the level of discourse referents, and I will now also distinguish between event discourse referents that stand for accomplishments or achievements and those that stand for activities and processes. I'll use a_1, a_2 and so on for achievements and accomplishments, and p_1, p_2, etc. for processes and activities. I'll continue to use e_1, e_2, etc. to stand for event discourse referents of either kind, when it doesn't matter or it's unclear which sort of event is involved. Other abstract object denoting expressions may introduce discourse referents of other types, for example fact denoting expressions give rise to a distinct set of discourse referents f_1, f_2, etc. In addition to atomic predicates on discourse referents of various types, identity '=', and the usual logical operations on DRSs, I will also include the logical operations of lambda abstraction, λ, and application, [].[2] These operations defined on DRS structures are used in dealing with "bare" nominals like *hitting*. Bare gerunds are nominals in which all the arguments are lexically suppressed.

The model theory of the expanded DRT fragment expands on the intensional semantics for DRT of Asher (1986) (1987) and incorporates a domain **K** of abstract objects which form an explicitly closed family of functionals in the sense of Aczel (1980). This domain furnishes a model of the lambda calculus. These abstract objects are, however, none other than DR-theoretic structures themselves. Besides a domain of abstract objects, however, is also a domain of events and a domain of individuals and a domain of possible worlds. Relative to these domains, I define embedding functions and the satisfaction of DRS conditions in a familiar way. All event-like discourse referents map under embedding functions to a domain of events in the models. Since only facts inhabit the realm of more abstract entities for the purposes of this paper, there are a variety of choices about what to map such discourse referents onto; one could map them onto subsets of the set of worlds or equivalence classes of elements of **K**. I will remain agnostic here.[3]

The DRS construction procedure translates the lexical items and follows the syntactic structure "from the bottom up" to put these translations together via a form of lambda conversion into a DRS for a sentence. Such an approach demands a specification of the structure that various constituents of sentential nominals contribute to DRS construction. The DRS construction algorithm is robust, in the sense that it appears to be compatible with a wide variety of syntactic theories. The syntax of nominals, however, requires more syntactic assumptions than is usual for a DRT analysis. Here are the assumptions I will use.

Recently, Abney (1987) has put forward some interesting syntactic arguments for a parallel treatment of inflection and determiners in his so called "DP analysis." DP is a theoretical reconstruction of the intuitive notion of noun phrase. I will follow Abney's syntactic, DP analysis. DP is a theoretical reconstruction of the intuitive notion of noun phrase. Like the inflection position or I, the determiner is the location of agreement features in the noun phrase, and like I it is a head of a maximal projection. The maximal projection of I is IP (standing for S); the maximal projection of D is DP. The inflectional elements in I take a projection of V as complement, while the inflectional elements in D take a projection of N as complement. The agreement features in I determine the tense and number markings on the verb, while the agreement features in D determine the genitive case marking on the subject of the noun phrase, the possessive modifying noun phrase in English. I will also argue that they assign accusative case to the subject of ACC-*ing* accusatives.

I will also assume an analysis of the nominalization affix *-ing*, on which it may move and take wider scope at some level than their position at the phonological level or surface string implies. This analysis was proposed by Jackendoff (1977) and followed for the most part in Baker (1985), Lebeaux (1986) and Abney (1987). This syntactic assumption greatly simplifies the interpretation of various gerundive constructions and yields an "almost" uniform syntactic analysis of all sentential nominals.

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[1] See for instance Kamp (1983), Asher (1986) or Kamp and Reyle (forthcoming) for basic DRS construction procedures. Concerning the bottom up algorithm for putting together DRSs, see for instance Frey (1984), Reyle (1985), Wada and Asher (1986), Asher (forthcoming).

[2] For details see Klein (1986).

[3] This interpretation is explained in detail in Asher (forthcoming).

Finally, I will need to assume some notion of argument structure for the verbs which occur within gerundive constructions. The apppropriate notion of argument structure might be explained through the use of theta roles with rules for linking these roles to grammatical functions like subject, object, etc. I will not discuss these complicated notions, however. I will often refer to the grammatical argument structure only, though that is certainly insufficient to explain the intricacies of the data presented by Roeper, Grimshaw and others.[1] The nominalizing affix in the nominal construction does not affect any of the arguments of the verbal component it dominates; it carries those arguments up the structure as suggested by Roeper. With Roeper (1988) (1989) I will distinguish between various sorts of affixes. One sort of affix preserves the argument structure of verbs (+A affixes); another sort does not (-A affixes). I will also assume with Roeper that when the affix preserves the argument structure, the argument structure is preserved and percolates up the syntactic tree to the nearest NP or IP containing the affix. If such an argument structure C-commands a PP, then the PP may furnish an argument in the structure, not otherwise. Arguments outside of the scope of the nominalization affix may undergo change and even be deleted altogether. Some positions may also be "suppressed" or unfilled at some levels of the syntax. I will follow the general lines of analysis of Government and Binding Theory on these matters and assume that at some level of analysis the unfilled argument positions are filled with a phonologically empty element, PRO.[2] Semantically, PRO behaves similarly to an anaphoric pronoun, in that it introduces a discourse referent that may be identified with some other accessible discourse referent already introduced in the discourse. On the other hand, in certain syntactic configurations, the discourse referent introduced by PRO must be identified, on syntactic grounds, with the discourse referent introduced by a particular noun phrase.

3. Summary of the Analysis

I distinguish three classes of gerundive constructions. *Of-Ing* gerundive constructions or phrases are phrases like*Fred's shooting of Bill* and *his loving of Mary*. This form of gerundive phrase is distinguished syntactically from two others-- POSS-*ing* and ACC-*ing* gerundive phrases (e.g., *Fred's shooting Bill, Fred shooting Bill*)-- in a number of ways. I also here consider "bare gerunds" like *flying*. I analyze these along the lines of *of-ing* gerundive constructions. The analysis of all these forms contains the two general features: the general, semantic consequences of the DP analsysis and the aspectual contributions of -ing and negation. I discuss each in turn.

3.1 Semantic Consequences of the DP Analysis

The DP analysis of noun phrases and inflection supports the view that the construction procedure should interpret inflectional elements and determiners in a similar way. Both inflectional elements and determiners will introduce discourse referents whose type will be determined by the predicates assigned to this discourse referent. Roughly, the scope of the nominalizing element (at the appropriate) level of syntactic analysis together with the proposed semantic interpretation of I will determine whether a nominal introduces a semi-concrete object like an event or state or an abstract object like a proposition or a fact into the discourse. The presence of a negation may force the scope of the nominalizing affix higher than it would otherwise be with a particular construction, thus transforming an achievement or accomplishment denoting construction into an abstract entity denoting nominal.[3]

Another aspect of the DP analysis which affects DRT interpretation is the structure of the noun phrase itself. In order to understand the semantics of gerundive nominals, we must give a semantics for the possessive modifier that makes general sense in the grammar. The DP analysis also forces us to make certain assumptions about the noun phrase marked by the possessive-- *Fred's* for example in (1.a). I'll call such noun phrases *DPs in possessive case*. It seems desirable to treat the DP in possessive case according to its normal semantic translation. The possessive is a case marking that indicates something important about argument structure,

[1] See for instance Roeper (1988), (1989).

[2] Actually this analysis was already suggested for gerunds in Roeper and Wasow (1972)

[3] The distinction between abstractand semi-concrete objects is strongly marked by syntactic distribution, practices of anaphoric reference, and semantic intuitions about the nature of these entities. My assumptions already reflect this in the syntactic structure. But there is no 1-1 or even a functional correspondence between syntactic constructions and various types of abstract and semi-concrete objects in natural language metaphysics. The presence of gerundive constructions, which may denote several different types of objects of natural language metaphysics, makes such a thesis highly dubious.

though this argument may have a quite different function depending on the head noun and its argument structure.[1] But it does not,contra Chierchia (1984), indicate a semantically signification relation like the application relation or that a different translation for the noun phrase itself or a particular translation of this instance of possessive case marking is called for. A difficult problem for the construction procedure is to come up with a compositional interpretation of gerundive nominals in which the noun phrase in possessive case is given its usual interpretation.The possessive will have a uniform translation in the DRS construction procedure; the difference in meaning will depend upon the N' which is the head of the DP and its argument structure.

Following Abney (1987) I will take the head determiner of a DP to assign the possessive case to the DP in possessive case, should the latter exist. Often, however, as in (1.a) - (5.a), there is no lexically expressed determiner. The DP in nominal subject position is assigned case (the possessive). According to the DP analysis there must then be a head determiner of the whole DP which assigns case to the DP in possessive case. Since there is no lexically expressed determiner in the DP, I infer the existence of a null determiner in the D slot. What sort of determiner should this be, however? One thing to notice is that the noun phrase,

(16) John's friends

denotes the group of all of John's friends. A DP containing a DP in possessive case thus appears to be have an existential force and have a maximality effect built into its semantics. The appropriate sort of null determiner then in the D slot then should be a definite determiner similar in translation to *the.* I'll call it *sthe* for "silent *the.*" Here I will just assume the existential quantificational form and assume definiteness or maximality effect is supplied by the context or the form of nominal.

The assumption of null determiners follows the assumptions of the syntactic analysis. Null determiners are needed to assign case to the nominal subject in constructions with DPs in possessive case. If we further assume that overt determiners do not assign case, then we predict that there can be no other determiner realized in the D slot of a DP in possessive case, thus predicting the following:[2]

(17.a) *The army's the destruction of the city
(17.b) *John's most moves
(17.c) *The friend's no claim
(17.d) *A man's all dreams
(17.e) *The army's a destruction of the city

The most important reason for having null determiners, however, is this. The assumption of a null determiner here is essential in giving the DP with possessive case marking its normal semantic analysis and the semantics of the entire noun phrase in a compositional way.[3] The

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[1] Consider for instance a derived nominal like
John's honesty
The DP in possessive case here denotes the bearer of the property of being honest. This is a quite different relation from that borne by the possessive to the head noun in a noun phrase like
John's mother.

[2] One might reply to this observation by noting that at least some lexically overt determiners, including all the cardinality determiners, appear to go with DPs in possessive case:
(1.a) John's every move
(1.b) a man's many dreams
(1.c) Cantor's uncountably many sets
(1.d) Everyone's three wishes
But note that each one of the determiners in (1) may be read as a modifier of the set introduced by the N'-- i.e., as measure expressions. There is no measure reading for *the*; it must always introduce a partial DRS structure and be treated like a full determiner. So the examples in (1) do not constitute a counterexample to the use of null determiners as essential case assigners. We can combine the use of the determiner *the* with a lexically realized subject with the following construction:
(2) The destruction of the city by the army
But here the lexically realized nominal subject is assigned the appropriate case by the preposition, so this too fails to generate a counterargument against the hypothesis that null determiners exist and are essential case assigners.

[3] Another complexity about the interpretation of possessives is the relation between the quantificational force of the null determiner and the DP in nominal subject position. In almost all cases, the quantifier that results from translating the DP in nominal subject position has wide scope over the quantifier formed by combining the null determiner with the head of the main DP. Thus,
every car's steering wheel
will yield the following partial DRS, if we use the condition of(x, y) to indicate the appropriate relation between the denotations of the nominal subject and the head noun.[3]

presence of such a determiner in constructions with DPs in possessive case is essential to DRS interpretation. If possessives are present and contribute one argument to the head common noun, typically one needs another discourse referent for the head common noun to saturate it. I will assume that this is given by a lexically unrealized determiner, which occupies the D position whenever there is no overt determiner.[1]

The DP analysis together with the assumption that there are null determiners filling in the D position when DPs in nominal subject position are present accounts for the possessive construction in general and for their use in conjunction with derived nominals in particular. The argument about possessives is perfectly general, but a key feature of my analysis of gerundive nominals is to exploit the analysis of possessives proposed here.

3.2 Aspects about *-Ing*

In gerundive constructions, the nominalizing affix makes a semantic contribution to the information content of the nominal. This contribution is aspectual and this affects the denotation of gerundive nominals. States can disappear under the process of nominalization. Suppose first that the verb or verb constellation (i.e., the verb + its arguments)[2] introduces either an achievement, accomplishment, process or activity type, and suppose further that there is no derived nominal form of the verb. Then the function introduced by the *ing* affix, ⟦-ing⟧ may either leave the aspectual character of the event type unchanged or convert the particular event type into an activity or process type. Suppose that there is a derived nominal form of the verb

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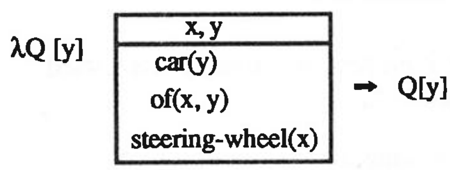
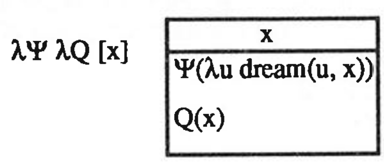


This seems to get just the right results in most of the cases.

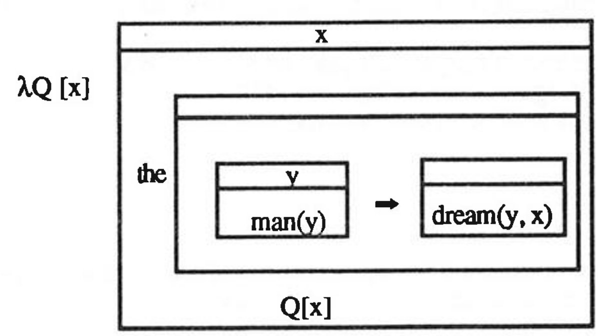
But there are examples in which in which it appears that the generalized quantifier formed from the null determiner and the head noun may take wide scope, as pointed out to me by Hans Kamp:

(A) Martin Luther King voiced every good man's dream. This dream has liberty and justice for all the people, not just those with money or with a light skin.

Here it does seem as though the context can force a wide scope for the embedded quantifier. One can easily get the appropriate translation for (A) if one countenances type shifted meanings for quantifiers including null quantifiers. If we allow the null determiner to have a type shifted translation of $\lambda\Phi\lambda Q\,[x]\,[\{x\},\{\Phi(x), Q(x)\}]$, which I will discuss presently, then we can get the wide scope reading of the null determiner. Φ is a variable ranging over partial DRSs. Let us look at the derivation of *every good man's dream.* First assume that *dream* yields a predicative DRS with two argument places. This combines with the new type raised translation of *sthe* to yield the following sturcture:



We now combine the translation of *every man* with this structure, putting the partial DRS in for Φ and then converting:



One may object to using type raising within the DRS construction procedure. But it seems dubious that we will be able to steer clear of it entirely. Type raising is a general tool in many semantic frameworks, and it is not clear why DRT should be able to avoid its use.

[1] This assumption is not novel; already many grammars which deal with bare plurals and mass terms in a theoretically rigourous way make use of null determiners. See Schubert and Pelletier (1988) for a survey.

[2] I take this term from Carlota Smith.

(viz. *arrival*). Then ⟦-ing⟧ must convert the particular event type into an activity or process type. Suppose now that the verb or verb constellation introduces a state type. Again ⟦-ing⟧ must convert this into an activity or process type, whenever a possible, correlated activity or process type exists. Otherwise, the nominal has an undefined denotation.[1] Thus, we can think of the affix *ing°* as introducing a partial function on eventuality types.

Negation also has an aspectual force, as many have recognized: it has been claimed to be a "stativizer," a function that converts eventuality types into state-types. But this description is somewhat misleading: whether negation has this function or not will depend on its scope. Negation demands a wide scope over the inflectional components that introduce eventuality discourse referents, unless adverbials are present. In the absence of such adverbials or other scope affecting operators, the presence of a negation does not introduce a state type. The interaction between negation and *-ing* is also complex. First of all *-ing* is one of those operators that appears to induce a reading in which negation takes narrow scope with respect to the event introducing inflectional elements. Further, *-ing* is sensitive to the fact that in its scope falls the negation of a particular event type. The negation of a particular event type might be thought to yield a state-type of a particular kind; it is such state-types to which the aspect transforming function introduced by *-ing* is sensitive. Typically, the function is total on negations of activity or process types; there always appears to be an associated activity. For the other eventuality types, the function is truly partial. If there is a single lexical item *ing* as suggested by Milsark (1988), this aspectual force of *-ing* is somewhat expected, since stative verbs do not combine with the progressive *-ing*.

The general theory of the aspectual force of *ing* predicts the peculiar status of nominals like

(18) ?John's knowing of Mary

It is difficult to interpret this nominal, for it is not clear what process or activity is associated with the state of knowledge.[2] Similarly, on my view nominals like

(19.a) John's loving of Mary

(19.b) John's believing that Mary stayed out after midnight

do not denote states, but should denote some corresponding activities if there are any. In these cases the corresponding activities are difficult to imagine and so sentences like (20.a) and (20.b) are anomalous.

(20.a) ??John's loving of Mary has gone on many years.

(20.b) ??John's believing that Mary stayed out after midnight lasted for hours.

This is not to say that we cannot refer to the states in natural language. But instead of *-ing°* phrases, we use derived nominals:

(21.a) John's infatuation with Mary has lasted for many years.

(21.b) John's belief that Mary stayed out after midnight lasted for many hours.

Of course (19.b) may also denote something other than an eventuality as in

(22) John's believing that Mary stayed out after midnight resulted in his being angry.

In (22) the gerundive nominal appears to refer to a fact. Facts like events have causal efficacy but they do not have spatio temporal locations.

The other facets of the aspectual force of *-ing* predict that

(23.a) ??John's arriving for dinner occurred at seven.

is bad, because we have a derived nominal form *arrival*, while

(23.b) John's arriving for dinner took forever.

should be OK. Finally, in the case where there is no corresponding derived nominal form, the theory claims that ⟦-ing⟧ has no aspectual effect. This seems to be borne out by the examples in (24):

(24.a) The government's cancelling of elections happened after the riots.

(24.b) The Space Shuttle's landing occurred at 7:34 am PST.

the nominal appears to denote just an event of achievement type.

4. *Of-Ing* Gerundive Phrases

4.1 The Basic Analysis.

Of-ing gerundive phrases count traditionally as true NPs. The denotations of *of-ing* gerundive phrases appear to meet all the Vendlerian tests of eventhood. In fact the event

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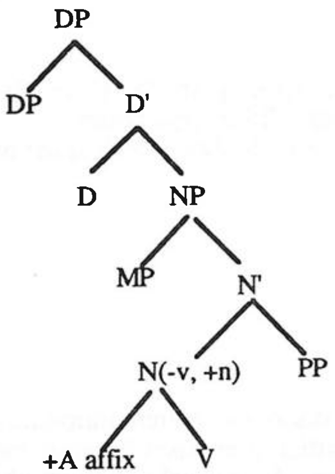
[1] See Bartsch (1986) for some related observations for German and Dutch.

[2] The most salient interpretation of (31) concerns the "Biblical" sense of knowledge, which is quite different from the usual stative interpretation of *know* and is not at issue here.

introduced by the sentence *Fred shot Bill* is just the same event introduced by *Fred's shooting of Bill*. This observation should follow from any DRS construction rules for gerundive phrases.

The affix *-ing* has the syntactic function of turning a verbal form into a nominal form. *-ing* preserves the argument structure of the embedded verb. So we must think about how the arguments are realized. PPs typically realize object and indirect object arguments of derived nominals verbs as in *the giving of a math problem to Alexis.* Given my assumptions, these PPs must be C-commanded by a node encoding the verb's argument structure. The highest node encoding an argument structure will be NP, for NP is the maximal projection of the nominalizing affix. I will thus assign an *of ing* gerundive the following sort of syntactic structure.[1]



The translation of the gerund will be that of the verb, and it will have the same argument structure. This argument structure will be preserved by the affix and so will percolate up the tree so that it can determine the semantic effects of various PPs that it C-commands. The noun phrase in nominal subject position (with possessive case) will typically be the subject of the verb in V,[2] while *of* PPs will yield object or theme arguments.[3] Gerundive phrases formed from transitive verbs realize their object argument with a particular form of prepositional phrase. The structure for gerund phrases formed from ditransitive verbs is an obvious extension of the structure for gerunds formed from transitive verbs. As a final remark about the syntax of *of-ing* gerundives, one can account for the adjectival modification of *of -ing gerundives* with the postulated syntactic structure (they go in MP or "modifying phrase").

With these general remarks in mind, let us consider the translation of (2.a) in some detail. Our syntactic structure for *of-ing* gerundives suggests that the translation of the gerund will have the same argument structure as the verb ordinarily has-- to wit the property or predicative DRS:

λyλeλx

x, y, e
e-shoot(x,y)

Now combining a (simplified) translation of *Bill* with the above we get the following property ascription:

---

[1]One might make a stronger case that *of-ing* gerunds are deverbalizations of V' as would Lebeaux (1986), assuming again that PPs headed by *of* furnish internal arguments. One cannot say for instance,

*the destroying by the army of the city

However, other insertions of material between members of a putative constituent are fine:

(a) The unveiling by the President of his new budget was greeted with guffaws by Congressmen from both sides of the aisle.

(b) The flogging by the guards of the prisoners was overlooked by the committee.

[2]However, there appear to be some counterexamples to this rule, however. For instance, consider

John's shooting led to his death.

It appears that people will say this when they mean John to be the one who is shot. I would much prefer myself,

John's being shot led to his death.

[3]The *of* in *of ing* gerundive phrases is inserted as at PF after the appropriate placement of the affix. There are no exceptions to this generalization of which I am aware.

λeλx

x, y, e
e-shoot(x,y)
Bill(y)

By my assumptions there must be a lexically unrealized determiner which assigns case. It yields a partial DRS of a kind familiar to those who have studied bottom up DRS construction:[1]

λP λQ [u]

u
P [u]
Q[u]

The partial DRS above looks very similar to the IL higher order property λPλQ ∃x (Px & Q(x)). The differences between them come out at the level of DRS interpretation.

The partial DRS above combines with the predicative DRS above to yield the hybrid "predicative partial DRS" structure:

λx λQ [e]

x, y, e
e-shoot(x,y)
Bill(y)
Q[e]

Notice that the predicative DRS dictates that the type of the discourse referent introduced by the null determiner must be an event. The hybrid predicative partial form arises because one of the argument places of the translation of the N' is not yet filled in. As is usual in bottom up DRS construction, the order of the λ-operators is fixed by the way the argument structures and rules linking arguments to syntactic realizations interact.[2] To fill in the argument structure of *shoot*, we must find the syntactic realization of the agent argument place. This comes from the DP in possessive case.

The DP in possessive case *Fred's* yields on translation a partial DRS that is just the translation of *Fred*:

λQ [x]

x
Fred(x)
Q[x]

The discourse referent introduced by *Fred* now fills in the subject argument place of the gerund's translation to yield the expected partial DRS structure for the gerundive construction as a whole.

λQ [e]

x, y, e
Fred(x)
e-shoot(x,y)
Bill(y)
Q[e]

It is now a routine matter to combine this structure with the VP to get a DRS for (1.a):

---

[1]I will here avoid complications involving definites as opposed to indefinites. For a discussion of how this issue affects the translation of nominals, see Asher (forthcoming). For a discussion of bottom up DRS construction, see Frey (1985), Reyle (1985), Wada and Asher (1986), Asher (forthcoming).

[2]For one view on this see Reyle (1988) for a discussion. For a view which develops the order of predicates in terms of syntactic realizations, see also Asher (forthcoming).

x, y, e, z, u, e'
Fred(x) e-shoot(x,y) Bill(y) z's imprisonment(u) z = x e'-lead to(e,u)

The theory predicts the trivial equivalence in meaning of (1.a) and (1.b). The construction procedure yields the essentially same DRS from (1.a) as from (1.b). Here is the DRS for (1.b).

x, y, e, z, u, e'
Fred(x) e-shoot(x,y) Bill(y) z's imprisonment(u) z = x e'-lead to(z',u) z' = e

The DRS for (1.b) differs from that for (1.a) only because of the anaphoric equation introduced by the pronoun. They are truth conditionally equivalent.

(2.a) and (2.b), on the other hand, do not appear to be truth conditionally equivalent, though (2.b) entails (2.a). (2.a) appears to have two readings-- one in which The reason for this is clear, once we look at the *temporal information* conveyed by each discourse. (2.b) is true only if the event of hitting the policeman takes place later than the speech time. The DRS construction procedure for tenses would make this clear.[1] The analysis of the nominal (2.a) on the other hand carries with it no temporal information. The nominal itself does not yield any information about when the event it introduces must take place. Such information is contained in the inflection node of a sentence and *of ing* constructions do not contain inflectional elements. Often the context makes clear when the event takes place (relative to speech time), but not in this case.

An issue to be addressed is the relation between the quantificational force of the null determiner and the DP in nominal subject position. The construction procedure delineated so far entails that the DP in nominal subject position prefers wide scope over the quantifier formed by combining the null determiner with the head of the main DP. But this is not always a desirable interpretation. Consider for instance,

(25) Everyone's gathering at his place amazed John.

Here one would intuitively interpret (25) as meaning that the event of everyone's gathering amazed John-- thus in effect attributing wide scope to the null determiner. One can easily get the appropriate translation for (25) if one countenances type shifted meanings for quantifiers including null quantifiers.[2] If we allow the null determiner to have a type shifted translation

λPλΦ λQ [x]

x
Φ(P(x)) Q(x)

where Φ is a variable over partial DRSs. Now we can get the wide scope reading of the null determiner. Let us look at the derivation of the gerund in (). First assume that *gathering at his place* yields a predicative DRS with two argument places. This combines with the new type raised translation of *sthe* to yield the following sturcture:

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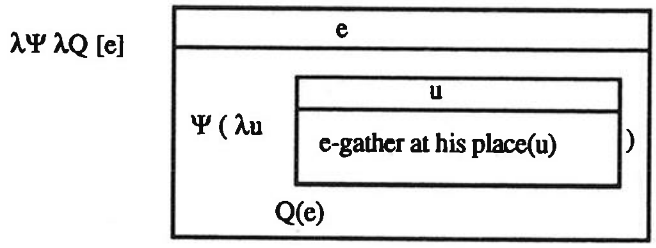
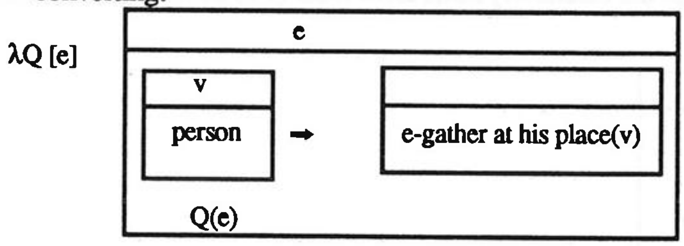
[1]See Kamp and Reyle (forthcoming).

[2]This is a perfectly general point about the interpretation of DPs, but I am interested here only in its application to gerundive constructions



We now combine the translation of *everyone* with this structure, putting the partial DRS in for Ψ and then converting:



Such type raising is restricted when the discourse referent the determiner translation introduces gets typed as an eventuality discourse referent. Such type raised translations for all determiners are permissible, as long as the partial DRS to be put in for Ψ is negation free or the discourse referent introduced by the translation of the determiner is not typed in the sentence as an event discourse referent. This constraint arises because we know that the domain of events is not closed under negation. Not every negated event description yields an event. Since unrestricted type raising could in principle yield any such negated event description, it must be constrained.

4.2 Bare Gerunds.

I turn now to bare gerunds. Consider:

(26.a) Unsafe flying caused the aircraft to crash.

(26.b) Unsafe flying is foolish.

(26.a-b) have no lexically expressed arguments or determiners at all-- hence the name of "bare gerund." Bare gerunds apparently have several readings. These readings are similar (though not altogether identical) with those obtained for bare plurals, as many have remarked. (25.a) for instance may have an existential reading-- i.e. someone's unsafe flying of aircraft x caused x to crash.[1] It may also have a generic reading: unsafe flying typically was the cause of the aircraft crashes. (26.b) has a prominent generic reading: typical acts of unsafe flying are wicked. It may also have a universal reading: all acts of unsafe flying are wicked.

If (26.b) had an ordinary quantificational reading like the one just suggested, however, the mechanisms for plural anaphora in DRT would predict that plural anaphora exploiting a principle of abstraction, should be possible.[2] But it is not. Compare (26.a) with (26.b) and (26.c).

(26.c) *Unsafe flying is foolish. They$_i$ are usually also unncessary.

(26.d) All acts$_i$ of unsafe flying are foolish.. They$_i$ are also almost always unnecessary.

(26.e) Unsafe flying$_i$ is foolish. It$_i$ is usually also unncessary.

Thus, the bare nominals in (26.a-b) for this and other reasons[3] most likely denote individual objects of some kind-- either properties, kinds or other abstract objects-- and do not have straightforward quantificational readings.

Let us see how my approach fares with these examples. I take (26.a-b) to be examples of *of -ing* gerundive constructions and not POSS-*ing* or ACC-*ing* gerundives, because of the adjectival modification present. Bare gerunds will thus have the same syntactic analysis as *of -ing* gerundives. By my syntactic assumptions, since the arguments of bare gerunds are not lexically realized, they are syntactically realized by instances of PRO. Under translation, these instances of PRO introduce discourse referents that are either anaphorically identified with previously introduced accessible discourse referents or end up being bound by the quantificational force of the DRS context in which they are introduced. That they can be anaphorically bound is clear from examples like

---

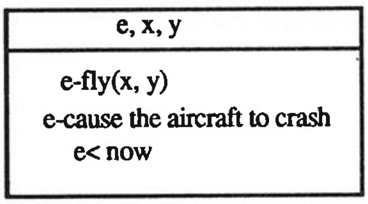
[1]This observation is due to Weir (1986)

[2]That event anaphora does obey such a principle of abstraction over right hand side DRSs of quantificational conditions is discussed in Asher (forthcoming).

[3]Many of the arguments in Carlson (1977) are applicable here.

(27) Flying was the general's favorite pastime.
(27) has a reading on which the general is the agent of *flying*. This is because the discourse referent introduced by the PRO that fills in the Agent slot of *flying* is identified with the discourse referent introduced by *the general*. Note also that (27) gives us an additional argument for considering the denotation of *flying* to be some sort of abstract object. *Method* identifies a way of acting or an event type, and presumably this forces *slaughtering* to denote some sort of abstract entity.

By exploiting the lexically null determiner *sthe* and the same translation procedure as for *of-ing* gerundive constructions, I predict the quantificational force of the DRS context to be existential in the DRS derived from (26.a). The DRS for (25.a) is



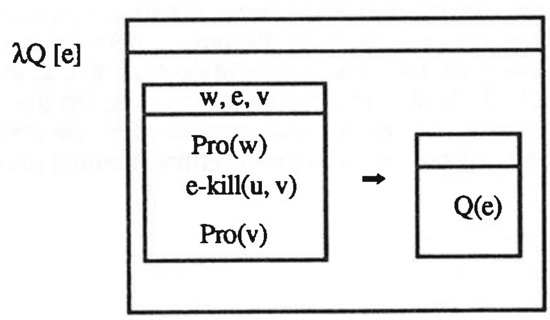
and this gives us the right truth conditions for (26.a).

This analysis and the principles of DRT anaphora predict that we can anaphorically refer to the unsafe flying event in subsequent discourse:

(28) Unsafe flying caused the aircraft to crash. It was a typical fighter jock stunt.
Another consequence of the analysis is that overt quantifiers may occur in in the determiner position of a *of-ing* gerundive construction as long as no lexical element needs to be assigned case. This happens of course with bare gerunds. The analysis predicts that the quantificational force of the DRS context will affect the interpretation of the gerundive.[1] Consider for instance:

(29.a) Every killing is an offense against God.

In (29.a) the truth conditional effect of the DRS construction procedure is a universal quantification over the discourse referent introduced by PRO in subject position as well as a universal quantification over the event introduced by *killing*. In order to get a proper treatment of (29.a), we must use a type raised translation for the main determiner, *every*. The quantifier introduced by the abstract determiner is always given a wide scope interpretation with respect to the translation of the arguments of the nominal when those arguments are lexically null. This does not reflect the configurational structure. With verbal or imperfect nominals, however, argument structure has as much effect on the semantics as the configurational structure. The type raised translation allows us to combine the partial DRS that derives from the PRO directly with the predicative DRS derived from *killing*. Here is the result of having combined the type raised translation of *every* with the translation of *killing* and its PRO arguments:



My strategy of handling lexically suppressed arguments with PRO appears not to yield the right results when we try to obtain the property or "abstract" reading that is the dominant one for (25.b). The key to a successful interpretation of (25.b) lies in what the determiner slot provides. I have up to now assumed the existence of only one type of lexically null determiner. But if you have one lexically null determiner, why not more than one? I shall accordingly introduce another determiner, the *abstract* determiner. It identifies a discourse referent with a DRS structure derived from the nominal. It introduces a partial DRS of the form:

---

[1]I should add that not only the quantificational force but also e.g., the modal force of the context will affect the interpretation of the gerundive. But I will not go into that here.

$$\lambda P \lambda Q\ [x] \quad \begin{array}{|c|}\hline x \\ \hline x \approx P \\ Q[x] \\ \hline \end{array}$$

Let us loosely understand x ≈ K as meaning that x is characterized by K. The abstract determiner and *sthe* are quite similar. They only differ in what sort of object they introduce into the discourse. *sthe* introduces a semi-concrete eventuality, while the abstract determiner introduces an abstract object like a property, a fact or a proposition.

Let us now see how this translation combines with the translation of the rest of the nominal. I have inserted instances of PRO both for the subject and object positions. According to our construction procedure, we get as a translation of the N',

$$\lambda e \lambda x \quad \begin{array}{|c|}\hline x, y, e \\ \hline \text{e-fly}(x, y) \\ \text{Pro}(y) \\ \text{unsafe}(e) \\ \hline \end{array}$$

One might want to assume here that the abstract determiner takes widest scope in the interpretation of the nominal. We should then use a type raised translation of the abstract determiner

$$\lambda P \lambda \Phi\ \lambda Q\ [k] \quad \begin{array}{|c|}\hline k \\ \hline k \approx \Phi(P) \\ Q(k) \\ \hline \end{array}$$

By a translation procedure completely analogous to the one given for (29.a) we now complete the nominal's translation.

One thing that is very different from what we have seen so far is that in this construction nothing fills in the event argument in the nominal. The translation of the PRO in the modifying position of the DP now fills in argument slot for the subject of *fly.*[1] So we have after combination a structure of this form:

$$\lambda e \quad \begin{array}{|c|}\hline x, y, e \\ \hline \text{Pro}(x) \\ \text{e-fly}(x, y) \\ \text{Pro}(y) \\ \text{unsafe}(e) \\ \hline \end{array}$$

This predicative DRS denotes an event type-- the event type of someone's flying something unsafely. The predicative DRS as a whole is identified with the discourse referent introduced by the abstract determiner; thus, the λ-operator and its event argument do not come to the outside of the partial DRS derived for the whole nominal but rather stay attached to the predicative DRS derived above. This types the discourse referent introduced by the abstract determiner as a concept. So we get the following partial DRS for the entire nominal (the discourse referent c is of concept type):

$$\lambda P\ \ [c] \quad \begin{array}{|c|}\hline c \\ \hline c \approx \ \lambda e \begin{array}{|c|}\hline x, y, e \\ \hline \text{Pro}(x) \\ \text{e-fly}(x, y) \\ \text{Pro}(y) \\ \text{unsafe}(e) \\ \hline \end{array} \\ P[c] \\ \hline \end{array}$$

[1]Recall that under DRS conversion, one may insert an argument anywhere in the list of λ abstracts.

The semantic effect of my proposal is to give the discourse referent introduced by PRO an existential reading within an intensional context; according to my analysis, (25.b) says that the event-type of anyone's flying anything unsafely is foolish.[1]

With the abstract determiner one can generate property readings for many event nominals. If an event nominal supplies an argument to an event predicate, however, then the property reading will be predicted to be unavailable, because one cannot apply an eventuality predicate to an abstract object. By supposing two sorts of null determiners, the translation procedure makes sense not only of the truth conditional readings and anaphoric properties of bare gerunds. The translation also allows a for uniform, syntactical treatment of all *of/ing* gerundive constructions and treats bare gerunds in the same principled way in sentences like

(30.a) All unsafe flying is foolish

and

(30.b) Most unsafe flying is foolish.

In (30.a-b) the determiners *all* and *most* occur in the D position of the syntax. The event type derived from the gerund would combine with a translation of these determiners in the standard way.[2]

The typology of semi-concrete entities is also important in the analysis of the examples in (30). Bare *of-ing* gerundive phrases that type their event arguments as activities or states do not combine well with singular determiners like *a* or *every*. They do combine well with the mass-noun determiners *much, most* and *all*. The aspectual analysis of *-ing* then predicts the asymmetry between (31.a) and (31.b):

(31.a)??Every arriving was greeted with enthusiasm.

(31.b) Every arrival was greeted with enthusiasm.

According to our analysis of *-ing*, (31.a) must be understood, like (30.a-b), as involving an activity. Following Krifka (1987), the structure of the domain of activities and processes is analogous to that of the domain of quantities of matter; it is a lattice without atoms. Mass term determiners operate on those structures, and so naturally these such nominals would select mass term determiners rather than the count determiners *each, every* and *a*. On the other hand, ccomplishments and achievements have, rather the structure of individuals and their plural sums. Their domain structure is that of a lattice with atoms. Thus, they should accept singular count determiners and mass determiners, given that one refers to the sum of events in the singular:

(32.a) Every crossing of the street was fraught with danger.

(32.b) All crossing of the street was fraught with danger.

The analysis of bare gerunds and *of-ing* gerundives carries over to the nominalizations involving abstract nouns like *honesty, goodness* and the like. Such nouns traditionally have been thought to pick out universals or properties. To the noun *honesty* then should correspond something like the property λx honest(x). But *honesty* does not just denote this property, since sometimes the argument of the property may be filled in-- e.g., by the denotation of a DP in possessive case as in (33).

(33.a) John's honesty lasted only until he realized he would never be caught cheating the customers.

(33.b) John's honesty is well-known.

(33.a) indicates that *John'shonesty* may denote a state. But then what do these noun phrases contribute to fill in the argument structure of the rest of the matrix clause in which they occur? That is, what in (33.a) is the argument of *lasted*? The analysis of bare gerunds gives us an answer. The translation of *John* is what one would expect, and the translation of *honesty* is a predicative DRS with two argument places-- a state argument place and an individual argument place.

---

[1] I am not sure that these are absolutely the right truth conditions for (33.b). In particular these truth conditions might better be reformulated in terms of a generic quantifier on events and aircraft. It does not seem to me that in these cases the correctness definition must remain absolutely faithful to the structure of the DRS; certain transformations are surely possible, and perhaps the transformation from statements about properties to truth conditionals involving generic quantification over events and individuals is one of these.[1] But these transformations are not my concern here.

[2] The DRS translation of such determiners, I assume, is similar to that for the singular determiners. Notice that one can easily say,

Most water is salty. It is not good to drink.

What one is able to pick up anaphorically is the water that is salty. It is difficult to pick up all the water, however. So it looks like the mass of all water is not itself accessible to the pronoun. Such facts can be explained by the machinery for plural anaphora in DRT, if we assume that mass term determiners have a similar DRS translation to those for count determiners.

λsλu

s-honest(u)

But there must be a lexically null, abstract determiner to assign case to the nominal subject. This could be either *sthe* or the abstract determiner. The use of*sthe* will give the entirely predictable DRS for (33.a).

u, s
John(u) s-honest(u) [s lasted only until...]

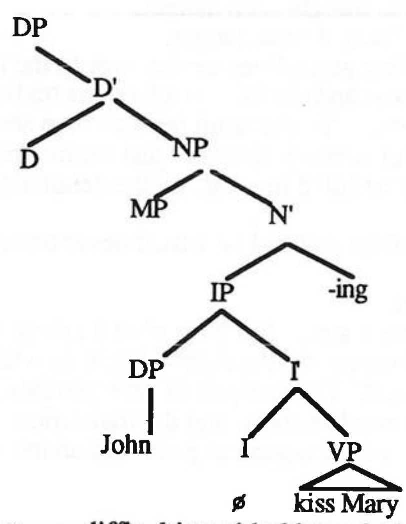
(33.b) indicates that such a nominal may also denote an abstract object. If we use the abstract determiner, we get an abstract entity as the denotation of the nominal, as we did with bare gerunds. This abstract object is an event type. If we further assume an operation that allows us to transform event types into the appropriate saturated object, fact or proposition, we have a plausible analysis of (33.a) and of propositional interpretations of nominals in general. On the other hand, the analysis of bare gerunds also gives us an analysis of (34):

(34) Honesty is the best policy.

My treatment of *of-ing* gerundive phrases reveals a characteristic ambiguity of many sentential nominals. They have a "chameleon-like" character, in that they may denote different sorts of abstract objects. Gerunds and derived nominals most of the time denote states or events, but sometimes they appear to denote event- or state-types and their saturated completions, facts.[1]

5. ACC-*ing* Constructions

The subjects of ACC-*ing* constructions have accusative case. This signals an important difference for the syntactic structure of these constructions in contrast to *of ing* gerundive constructions. One analysis of *John kissing Mary* that one might suggest would follow the lines of our previous analysis:



There are, however, two difficulties with this analysis. The first is that it predicts that sentential adverbs in argument position are admissable in ACC-*ing* gerundive constructions. They are not.[2]

(35.a) *Probably John going to the store was the result of their discussion.

We can have sentential adverbs in second position, though:

(35.b) John probably being a spy, many thought it was best to deny him the security clearance.

---

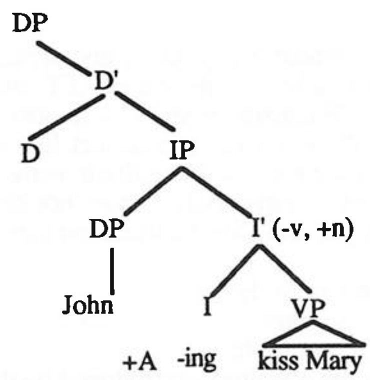
[1]This is of a piece, I believe, with the more widespread ambiguity in natural language concerning events and event-types, on which verbs like *happen* and frequency adverbials may take either event-types or events and states as arguments. However, in my treatment here I have always attempted to give the lowest type in my analysis.

[2]Again this observation is made by Abney (1987).

(35.c) The teacher's comment about Fred possibly having to repeat second grade came as no surprise to his parents.

The second difficulty is that ACC-*ing* and POSS-*ing* gerundives do not admit adjectival modification of the nominal (though in other languages such as Italian and Dutch there are nominalizations containing VPs that do admit adjectival as well as adverbial modification).[1] The structure above, however, contains an NP node, which was used to explain the possibility of adjectival modification with *of- ing* gerundive phrases and derived nominals.

Let us first address the problem of adjectival modification. X' theory requires that a D' have as daughters D and some maximal projection. But it also requires that any XP dominate an X. This is not the case with the tree above. The maximal projection under NP could be IP itself, and the *-ing* affix could naturally go in I. This would make I and its projections (-v, +n). This leads to the following structure that is consonant with the principles of X' theory.



In this structure adjectival modification of the nominal is no longer possible. Adverbial modification is, since the structure contains a VP. But there are other benefits to this change. Since I is (-v, +n), there is no agreement marking in I. Consequently, I does not assign *John* nominative case. Since D and IP are sister nodes, however, it is possible in the above structure for the null determiner in D to assign case to *John*. If we assume a lexically null determiner that assigns case in D position, it can assign *John* accusative case in this configuration.[2]

Turning now to the data about distribution of sentential adverbs, if we allow sentential adverbs in sentence initial position, then presumably they must attach to IP. I hypothesize that this would break the appropriate case assigning relation between the null determiner in the main D slot and *John*. This relation is similar to that in infinitival CP constructions. There sentential adverbs in front position are not allowed either:

(36.a) *For possibly Fred to be a spy was unthinkable.

(36.b) *John thought probably Fred to be a spy.

If this is right, then we should not expect sentential adverbs at the front of the IP. We should expect them in other positions, and this is what the data indicates.

The semantic interpretation of the ACC-*ing* construction is relatively straightforward. Given my hypothesis about case assignment, the nominal is to be interpreted as containing an entire IP. This yields under translation an entire subDRS. If this is so, then we should, as Abney reports, get typical scope ambiguities (de re/ de dicto) for the quantifiers in subject position of ACC-*ing* constructions, if we appeal to the usual scoping mechanisms used within DRS construction. Because we have a completed subDRS that results from the IP, the only null determiner that can occur in the main D slot is the abstract determiner. The abstract determiner introduces a discourse referent that will be characterized by the subDRS derived from the translation of the IP. The discourse referent must have the type appropriate for a saturated abstract entity. Both the syntax and the semantics of I gerundives contribute to determining the appropriate type of discourse referent. With an ACC-*ing* or POSS-*ing* gerundive construction, we must have a fact discourse referent. Fact discourse referents range over facts and other closely related entities like possibilities or "possible facts". A fact discourse referent in a factive context conveys a factive presupposition; the presupposition is that the subDRS characterizing the fact-like discourse referent is true. The presupposition is handled in the DRS as in Asher (1987), with the contents of the subDRS copied out into the main DRS. In some contexts, this factive presupposition may be blocked or not arise-- as in

(37) He prevented John kissing Mary.

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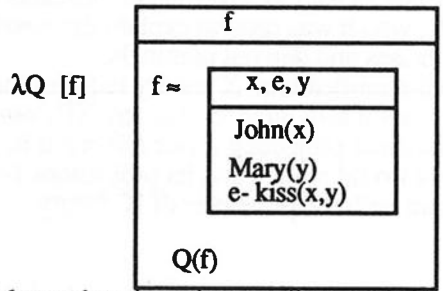
[1] I became of this from reading Zucci (1989).

[2] That is D functions somewhat like a preposition here, I am endebted to H. Haieder for this suggestion.

In this case the fact discourse referent does not represent a fact in the ordinary sense of this term but rather a closely related entity like a possibility or possible fact.

As an example of a translation of an ACC-*ing* gerundive construction, *John kissing Mary* yields the following partial DRS structure:



The null determiner introduces a discourse referent which the syntactic and semantic structure and content of the nominalized IP determine to be fact-like. This makes ACC-*ing* gerundives semantically different from *that* clauses and other CP constructions, which introduce proposition discourse referents. The mandatory typing of the discourse referent as fact-like accounts for the unacceptablility of sentences like (10.b). On the other hand, many attitude verbs and other complented constructions accept facts as arguments. So my analysis does not block an explanation of why ACC-*ing* gerundive constructions and CP constructions may occur in many of the same linguistic contexts.

The analysis accounts easily for the synonymy of the pair,

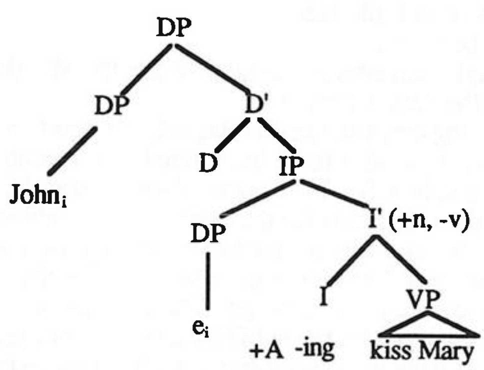
(4.a) John sprinting past Bill was a welcome sight.

(4.b) John sprinted past Bill. It was a welcome sight.

The difference between the two DRSs is only that the conditions introduced by the gerundive phrase are presupposed, not stated in the principal DRS. In (4.a) nothing blocks the factive presupposition. So assuming the DRT account of presupposition like the one just mentioned, the sentences (4.a-b) are truth conditionally equivalent.

6. POSS-*ing* Constructions

I now turn to POSS-*ing* gerundives. In virtue of the semantic closeness between POSS--*ing* and ACC-*ing* constructions, one might take a syntactic structure for POSS-*ing* constructions to be very simlar to that for ACC-*ing* constructions. Here is the syntactic structure I have chosen.



I assume that *John* has moved before S-structure from its position within the IP to the specifying position of the whole DP, where it gets the usual possessive case that all DPs have in that position. Since the head of the DP dominates an I projection, the lexically null, abstract determiner must occur in the D position. A DP in possessive case is the subject argument of the nominalized IP in POSS-*ing* gerunds, with case again being assigned by the lexically null determiner.

This syntactic analysis explains the syntactic and semantic similarities between ACC-*ing* and POSS-*ing* gerundives. ACC-*ing* and POSS-*ing* gerundives are predicted not to take lexically overt determiners, because the determiner position is already filled with the lexically null determiner, which is needed to assign the appropriate case to the nominal subject. ACC-*ing* and POSS-*ing* gerundives are also both predicted to accept VP adverbial modification but not adjectival modification. But all this could also be explained on a different analysis. My principal syntactic reason for the my analysis is this. POSS-*ing* gerundive phrases sometimes take some temporal and aspectual modifications; for instance in such phrases as *John's having kissed Anna* the *having* is plausibly an auxillary with both temporal and aspectual information that should be located in I. So some component of I must occur in the syntactic analysis of the POSS-*ing* construction.[1] Our X' theory requires that there be a maximal projection of I under the DP-- hence the DP over IP analysis given. Semantically, the DP over IP analysis also gives us the correct results for the factive readings of these constructions.

There is some evidence that POSS-*ing* gerundives are noun phrases which do not contain a full IP. Let me just mention one piece of evidence that I find more persuasive than others.[2] Several authors have claimed[3] that POSS-ing gerundives do not combine with sentential adverbs or sentential adverbial PPs in the way that ACC-*ing* expressions do:

(37.a) ? John's probably being a spy made the committee reject his application for a security clearance.

(37.b) ?? John's, to our delight, leaving so early distressed Susan.

I do not find (37.a-b) so bad or so different from (35.b-c). In any case, one can have such adverbials in a different position at least for some speakers:

(37.c) John's being probably a spy made the committee reject his application for a security clearance.

(37.d) John's leaving, to our delight, so early distressed Susan.

These last two sentences may in fact show that such sentential adverbials may attach to VP-- which vitiates them as a test for the presence of an IP.

The translations of POSS-*ing* gerundives are similar to the fact introducing ACC-*ing* constructions. Consider the phrase *John's kissing Mary.* Proceeding bottom up, the translation of *kissing Mary-*, the nominalization of an IP, should be a DRS when the trace is interpreted as introducing a discourse referent. I'll treat the trace like an anaphoric pronoun, though the syntax determines what the anaphoric antecedent is.

(K1)

x, e, y
x = ? Mary(y) e- kiss(x, y)

The partial DRS (K1) now combines with the partial DRS given by the abstract determiner.

λQ [k]

k
k ≈ [x, e, y | x = ?, Mary(y), e- kiss(x, y)] Q(k)

We now combine the above with the translation of *John* 's to get the following structure:

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[1]One could also give virtually the same semantic analysis by giving up the thesis that with POSS-*ing* gerundives the D' dominates a maximal projection. But that would be an exception to our X' theory rules and also it would not explain the presence of sentential adverbs in other positions.

[2]Horn (1975), Chierchia (1984) and Abney (1987) have several other purported differences. I find them either not persuasive on empirical grounds or amenable to treatment by the syntactic theory here or due to non-syntactic effects. But to argue that here would take too long.

[3]See Abney (1987), Zucci (1989) for details.

λQ [f] | f, u
John(u)
f ≈ | x, e, y
x = ?
Mary(y)
e- kiss(x, y)
Q(f)

The syntactic information encoded for x and u insure that x will be identified with u by the anaphora resolution routine. This yields an appropriate translation of the whole gerundive phrase, *John's kissing Mary.*

λQ [f] | f, u
John(u)
f ≈ | x, e, y
x = u
Mary(y)
e- kiss(x, y)
Q(f)

Once again with a fact discourse referent introduced by the POSS-*ing* gerundive, we would expect that sentences like (10.a) to be unacceptable. I also expect that in certain contexts the factive presuppositions of fact-like discourse referents do not obtain. Consider for instance,

(38.a) Fred prevented John's kissing Mary.

(38.b) John denied (his) having kissed Mary.

It appears that our construction procedure assigns always a wide scope to the DP that is the nominal subject. But this would not be correct. One can very easily interpret the quantifier in subject position as being contained within the semantic content of either form of the I-gerundive:

(39) John's puerile behavior precluded anyone's having a good time at the party.

In (39) *anyone* is a negative polarity item, and so it must be included within the complement of the verb. However, we must account then for (39). Further, speakers find little difference between the *de re* - *de dicto* possible readings for this pair of sentences.

(40.a) Someone's coming to her party pleased Mary.

(40.b) Someone coming to her party pleased Mary.

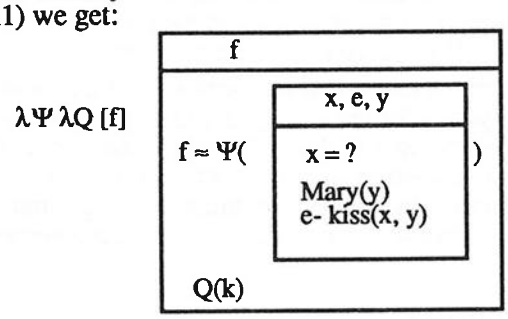
The *de dicto* reading is the one in which the DP in possessive or accusative case is read as contributing its content to the subDRS characterizing the fact-like entity denoted by the nominal. The *de re* reading is the one in which the DP in possessive or accusative case contributes its content to the main DRS. With some sentences, the *de re* reading of the DP in possessive case seems impossible. This is shown by the fact that the following intended anaphoric links are impossible. If the *de re* reading were permissible, then we should be able to get the intended anaphoric reading in (41).

(41) *I won't permit a dog's$_i$ licking Shirley's hand. She is terrified of it$_i$.

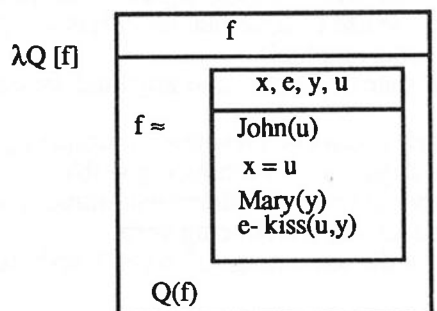
Permit is a verb that is non-factive and takes ACC-*ing* and POSS-*ing* gerundives as complements. The indefinite *a dog* prefers a narrow scope reading in the examples below, so the intended anaphoric links in (41) bad to me. The first sentence of (41) with the preferred narrow scope reading could be much more easily continued with *She is terrified of them.* That is what DRT would predict if the DP in possessive case contributes its content to the DRS characterizing the fact discourse referent introduced by the null determiner. So it appears that at least sometimes the DP in possessive case semantically constributes information to the subDRS characterizing the fact discourse referent.[1]

[1]This data also indicates that both POSS-ing and ACC-ing gerundives denote a saturated object like a proposition or fact, not a property as Chierchia (1984) suggests. As far as I can tell, the property analysis is committed to construing the subject of the gerundive construction in a *de re* manner. The data concerning arguments that Chierchia produces to support his position is intriguing, but I believe that it can be explained by appealing to a more intricate

How can we give the DP in possessive case a narrow scope, *de dicto* reading? If we give the abstract determiner a type shifted translation, which we already have supposed to exist, we get the *de dicto* interpretation. The appropriate translation for the determiner is a partial DRS of the sort already introduced in the discussion of bare gerunds. Combining such a partial DRS with (K1) we get:



(K2) now combines with the translation of *John's* to yield the desired conclusion, when anaphora resolution is performed:



There are no constraints on type shifting the meaning of the abstract determiner in this way, since the abstract determiner never introduces an event discourse referent. This derivation makes the differences between POSS-*ing* and ACC-*ing* constructions semantically insignificant. The construction procedure, with its flexible routines for combining partial and predicative DRSs, then allows both interpretations of *John's loving Mary.*[1]

This analysis of POSS-ing gerundives has several consequences. First, it explains why POSS-ing gerundives in general do not go well with event predicates: POSS-ing gerundives don't introduce event discourse referents to fill the argument places in those event predicate. They introduce subDRSs which characterize some more abstract object like a fact. The lexical realization of some inflectional element using the auxillary *have* in English within the gerundive construction requires a syntactic analysis on which a DP dominates some component of I.[2] This predicts that POSS-ing gerundives with past tense or perfective gerunds like in (8.b) are always infelicitous with event predicates; such POSS-ing gerundives must have the analysis given above and thus denote abstract entities like facts, not world-immanent objects like events or states. This leaves open the possibility that POSS-ing gerundives without lexically realized

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view of how logical inferences proceed in natural language when complex constructions exploiting control phenomena are used. But I cannot go into that here. For a discussion see Asher (forthcoming).

[1] Abney (1987) reports that the subject of a POSS-*ing* gerundive construction must always be read as having wide scope over negation. He claims that (74.b) is ambiguous, but (74.a) only has a wide scope reading for the nominal subject.

(A) Everyone's not coming to her party devastated Mary.

(B Everyone not coming to her party devasted Mary.

Abney's data appears to depend on complicated scope effects between negation and quantificational elements. (C) is an example in which the negation has narrow scope with respect to the DP in possessive case, but in which the DP contributes its content to the characterizing subDRS. Since the matrix verb is not factive, then DRT would predict that the indicated anaphoric link in (78) is bad:

(C) *A dog's not being potty trained drives my mother crazy. It_i is so messy.

The first sentence (C) could be continued much more plausibly with *They are so messy*, again buttressing the point about the *de dicto* interpretation of the DP in possessive case.

[2] Incorporating inflectional elements within POSS-ing gerundive yields appropriate translations for the past and present forms of the gerundive. The past gerundive introduces an event discourse referent that is prior to the reference point set by the matrix clause verb, as is evident from this sentence:

His having lost his driver's license once made John an especially careful driver when the cops were around.

The present gerundive introduces a discourse referent that overlaps the temporal perspective point set by the reference point of the matrix verb.

inflectional elements might have a different syntactic and semantic analysis. In fact this is what I would propose to account for the examples in (8).[1]

Second, my analysis explains semantically why gerundive constructions don't combine with standard count or mass determiners. The translation of standard determiners introduce discourse referents for which there is no appropriate argument place in the translation of the gerund. Third, I can now also explain why POSS-ing gerundive nominals semantically can't admit pluralization.[2] Pluralization of NPs is an operation that converts λ-abstracts (translations of N') of one type into λ-abstracts of another type. For instance, pluralization converts *of -ing* gerundives from a λ-abstract over individual events to a λ-abstract over groups of events. But in the case of POSS-ing gerundives, there is no λ-abstract over events to convert into a λ-abstract over groups of events. One could pluralize the subject argument of the gerund, but this would not yield a group of objects but only a different, singular denotation for the gerund phrase.

7. The Problem of Event Negation Revisited.

The problem of event negation is complex. There are three tests for the way negation might interact with event descriptions. The tests do not give us a clear signal about the effects of negation on nominal denotations. The tests involve the use of adverbial modifiers, anaphora, and examining the truth conditions of negated sentential nominals.

According to the first test, negated event or state descriptions of any kind are capable of introducing states.

(42.a) No one died in the hospital for over two hours (achievement introducing verb).
(42.b) No one knew the answer for several days. (state introducing verb)
(42.c) No one crossed the picket line for several hours. (achievement introducing verb)
(42.d) No one laughed for over two hours.(activity introducing verb)

Compare now the outcomes of the two other tests for the same range of event descriptions. Here are the outcomes of the anaphora test.

(43.a) *No one died in the hospital. This lasted for several days.
(43.b) *No one knew the answer. This lasted for several days.
(43.c) ?? No one crossed the picket line. This lasted for several hours until the police arrived.
(43.d) ??No one laughed. This lasted for only a few seconds, however.

From this data, it appears as though the Davidsonian logical form for sentences like those in (43)

$$\neg\exists x\exists e\ (\varphi\ \&\ \psi)\ \&\ \theta(e)$$

is correct.

Consider now the results of the third test.

(44.a) *No one's dying in the hospital lasted for several days. (achievement iintroducing verb).
(44.b) *John's not dying in the hospital lasted for several days. (achievement iintroducing verb).
(44.c) *No one's knowing the answer lasted for several days. (state introducing verb)
(44.d) ? John's not knowing the answer lasted for several days. (state introducing verb)
(44.e) *?No one's crossing the picket line lasted for several hours. (accomplishment introducing verb)
(44.f) John's not crossing the picket line lasted for several hours. (accomplishment introducing verb)
(44.g) *No one's sleeping lasted for hours. (activity introducing verb)
(44.h) Sheila's not sleeping lasted for hours. (activity introducing verb)

To my ears (44.d) is much more felicitous than (44.b), but still is perhaps questionable.

Part of the explanation of the data concerns the scope of negation. When negation combines with the subject quantifier in sentential nominals, no state nominals appear. When negation is present somewhere within the VP, states sometimes appear. Let us first settle on the place of negation in syntactic structure. Negation may either combine with a verb to give us essentially a new sort of predicate or it must go in I. I assume that negation prefers wide scope over the discourse referent(s) introduced by the translation of I, especially when when the negation occurs combined with a subject determiner for instance. Thus, the sentences in (43)

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[1]For a discussion see Asher (forthcoming).

[2]Of course syntacticians rule out pluralization here by speaking of a verbal form. But the appeal to verbal form here does not in itself explain much.

have the translation that Davidson said they should have. In the presence of sentential adverbials like the durative phrases in (42), however, we may introduce a state discourse referent outside the scope of the negation. The negation thus takes narrow scope in these examples.[1]

But determining what is the scope of negation in a nominal cannot be the whole story. Our typology of "natural language metaphysics" distinguishes events into activities, states, accomplishments and achievements. Each of these entity types may be thought of as having a structured domain. Events, as opposed to states, are *not* closed under the following complementation principle: $\forall \bar{x}$ if $\neg\exists e\ \varphi(\bar{x}, e)$, then $\exists e\ \neg\varphi(\bar{x}, e)$. This, together with the principles of scope for negation, account for the impossibility of the anaphoric connection to the event discourse referent introduced by the translation of I in all of (43) but (43.b). If we think of the domain of states as obeying the complementation principle above-- namely, $\forall \bar{x}$ if $\neg\exists e\ \varphi(\bar{x}, e)$, then $\exists s\ \neg\varphi(\bar{x}, s)$-- then we can explain the acceptability of the intended anaphoric connection in (43.b).[2]

We must also consider the aspectual effects of [[ing]] on event types. Nominals that can be interpreted as denoting activities or accomplishments may be interpred as denoting states when negated, as (42) and (44) indicate. But achievement and state denoting nominals may not denote states by the same tests:[3]

(45.a) *The train's not-arriving lasted over two hours.

(45.b) *Tom's not knowing the answer lasted for over two hours.

When combined with negation, nominals formed from state or achievement verbs seem to only have a fact reading, which simply denies that an event or state of a particular type occurred. Thus, one would predict that (46.a & c) are synonymous with (46.b & d):

(46.a) The non-arrival of the train put everyone in a bad mood.

(46.b) The fact that the train did not arrive put everyone in a bad mood.

(46.c) Tom's not knowing the answer resulted in his being embarrassed.

(46.d) The fact that Tom did not know the answer resulted in his being embarrassed.

The fact paraphrases of the gerundive constructions in (47) containing nominals formed from accomplishment and activity verbs also have fact readings. This suggests that such nominals may either be read as fact or eventuality denoting nominals.

(47.a) Tom's not crossing the street indicated that he was cautious.

(47.b) The fact that Tom did not cross the street indicated that he was cautious.

(47.c) Tom's not smoking made everyone else on the trip much happier.

(47.d) The fact that Tom did not smoke made everyone else on the trip much happier.

Crucial to explaining this data will be the analysis of the aspectual force of *-ing.*

7.1 Negation and Wide Scope Adverbials

The solution to the treatment of adverbial modifiers and the translation of sentences like (42.a-d) and (83) is available to us from the study of plurals in DRT. Eventualities may sum to form new eventualities, just as groups may sum together to form new groups. Whenever a sentential adverb takes wide scope, the translation procedure calls on a summation procedure used for plurals to form the sum of the eventualities described within the scope of the logical operator.[4] The procedure gives negation narrow scope when a sentential adverbial is present. Negation, as our first test concerning adverbial modification might suggest, is a "stativizer" in at least this sense: it turns a description of an eventuality into a state description. Within the DRS formalism developed here, if a verb (or other construction) introduces (under DRS translation) an event type whose event argument place is of type μ, then the result of applying a negation operator to μ is to yield a state *type*. In symbols,

Suppose we have the eventuality type $\lambda e \lambda \bar{x}\ e\text{-}\varphi(\bar{x})$. The translation of "verbal negation" or more generally speaking "property negation" is:

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[1]This wide scope reading of the adverbial in these examples is preferred. There are other sentences, however, where a narrow scope reading is much more plausible. Consider for instance,

No one laughs for five days.

A similar phenomenon is true also of sentences with other logical structures like the quantificational ones introduced in:

(**) Everyone played happily for over two hours.

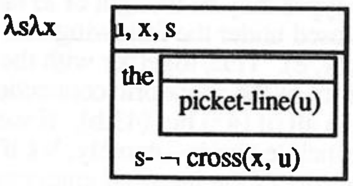
Sentences like (**) have a reading on which the adverbial has wide scope over the quantifier .

[2]I defend the view that states are closed under such a complementation principle and the view that events are not in Asher (forthcoming).

[3]This observation is due originally, I believe, to Cresswell. For a discussion see Baeuerle (1987).

[4]For details, see chapter 3 of Asher (forthcoming). Kamp suggested something like these principles in a talk on plurals at Texas, Spring 1989.

$\lambda P \lambda s \lambda \bar{x}\ \neg P(s, \bar{x})$,
where P´ is just like P except that the event argument place in P is reassigned a state type.
Thus, for example, the result of interpreting the negation verbally in the VP
not cross the picket line
yields the state type,



Natural language metaphysics indicates that any event type φ yields under negation a state-type of the form ¬φ, as long as the bearers of the state type exist, whenever the negation is taken to have narrow scope with respect to the inflectional elements introducing eventuality discourse referents. Wide scope adverbials are one sort of construction that force the negation to take narrow scope. One might argue that there is no appropriate state corresponding to the negation of an activity, etc. But the criticism seems to imply that there are clear, general criteria by which we can individuate states when they fall under positive descriptions. There aren't any so far as I can tell for either case; we can always assume the existence of a state as long as the bearer exists.

I now describe the translation procedure formally within the DRT formalism:
Suppose that

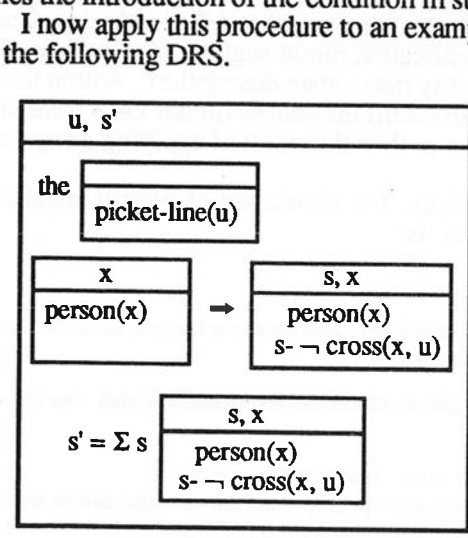
(i) we have an adverbial modifier τ whose translation is an event type λe (φ(e) and the adverbial modifier is attached to an IP.
(ii) the translation of the IP yields a DRS containing a complex condition of the form ¬K' , where K' contains the condition derived from the matrix verb and has an event discourse referent **e** declared within UK'.

Then the construction procedure will

(1) Convert ¬K' to the logically equivalent "∀¬" statement, $K_1 \Rightarrow K_2$, where K_2 contains a complex condition of the form $\neg K_3$ and $\mathbf{e} \in U_{K_3}$ and K_3 contains only those conditions which have **e** as an argument.
(2) Introduce a state discourse referent **s** into U_{K_2} and rewrite K_3 replacing **e** with **s** and deleting **e** from U_{K_3}.
(3) Introduce the condition
s' = Σ **s** K_2 into K_0 and the discourse referent **s'** in U_{K_0}.
(4) Apply λeφ(e) to **s'**.

The negation has narrow scope with respect to the bearer of the state, and so the negation of the eventuality description yields a state by our closure principle for states. States are also closed under summation of states that all fall under a particular state description. This principle underlies the introduction of the condition in step 3.

I now apply this procedure to an example. The translation for (42.c), for instance, yields the following DRS.

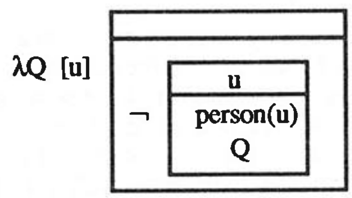


7.2 Negation Without Wide Scope Adverbials

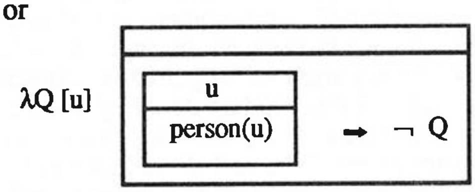
When a sentence adverbial with wide scope is not present, negation prefers wide scope over the event discourse referents introduced in I. Negation converts eventuality *types* to state *types*-- not, in general, eventualities into states. When negation has wide scope over the inflectional elements introducing event discourse referents, we do not automatically postulate the existence of states outside the negation sign, unless we add certain closure principles to the domains of eventualities. Events are not closed under complementation. States are closed under the sort of complementation principle discussed above. This implies that sentences introducing event discourse referents under negation will not in general yield suitable antecedents for event anaphora. States on the other hand are closed under negation, modulo the existence of the bearer of the state. So this predicts state anaphora to be better than event anaphora where we have negated event descriptions furnishing antecedents.[1]

7.3 Negation and Event Nominals

We now have left only the data of test #3 to account for. Given what I have said about the structure of sentential nominals, the construction procedure immediately predicts that no state should be available from sentences where the negation is combined with the subject DP, as in (44.a,c, e and g). The only plausible translations of *no one's* -- viz.,



or



will force the negation to have wide scope over the event discourse referent and the predicate in the VPs of the nominals in (44). But this will then not allow us to get the wide scope readings for the properties of lasting hours or occurring in New York. To get those readings we would need to use the type raised translation for *sthe*. But the limits on the use of type raised translations make this strategy unavailable. The determiner under translation ends up introducing an event discourse referent, and one cannot use the type raised translation when the DP in nominal subject place contains a negation. To do so would violate the closure principles on eventualities considered in the last section.

Now what about the sentences in (44) in which the negation arguably has scope only over the VP-- i.e., sentences (44.b, d, f, h)? The interaction of negation and gerundive nominalizations is curious. When possible, verbal negation (the narrow scope reading for negation) occurs with activity, but not with achievement or state introducing verbs on the whole, though perhaps in some uses of stative verbs one can get a verbal negation reading. Some nominalizations containing negated accomplishment verbs appear to denote some sort of eventuality such as (44.f). But on the other hand,

(44.h) ??John's not building a house lasted several years (began after his wife died).

is quite bad. It appears that only nominalized activity verbs under negation denote eventualities without fail. It should also be mentioned that these effects are sensitive to the wider context; some nominals with negation may denote coherent eventualities in a particular context while not doing so in another. (44.h) for example would become much better if uttered in a context in which John is a developer who decides to leave off building houses (perhaps because of what he takes to be poor market conditions).

What causes this distinction between verbs introducing state, accomplishment and achievement types on the one hand and activities on the other hand? It is the interaction of the aspectual forces of negation, *-ing*, and the basic eventuality types. Aspectual information indeed does affect the denotations of sentential nominals, as we saw earlier. States, for instance, can disappear under nominalization. On an event reading of a gerundive construction, the *-ing*

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[1]There seems to be a difference however in the negation properties of the various types of eventualities. Achievements are definitely not closed under negation. Activities and accomplishments are a little more lenient.

nominalizing affix makes an aspectual contribution to the DRS translation, and transforms a state type into an activity type. The affix changes the aspectual character of the event type given by the VP, if that event type is a state type. This transformation is only partially defined. For many states there may be no corresponding activity. In particular, it appears that the negations of achievement types are especially fragile, and often fail to have corresponding activities. Negations of activities on the other hand typically have associated activities. For instance, there are lots of activities that might constitute not sleeping-- tossing fitfully, lying awake and trying to count sheep, etc. The interpretability of negated accomplishment and state verbs under *-ing* varies, according to how easy we find it to associate activities with them. The state in which one is when does not cross a picket line, for instance, may have lots of associated activities-- standing on one side, discussing matters with the police or the picketers, etc. On the other hand, there may be no well defined activities associated with not building a house outside of a very determinate context. At least some states that are negations of other states also have corresponding activities. A state in which one does not know the answer, for instance, has natural, correlated activities-- trying to figure out what the answer is for instance. On the other hand, not owning a car may not have any natural corresponding activities-- hence the discrepancy to my ears between

(48.a) ?*John's not owning a car began after he crashed his volvo.

(48.b) John's carlessness began after he crashed his volvo.

The combination of aspectual transformations generated by both the negation and *-ing* nominalizing affix explain the data-- in particular the bizarre behavior of achievement verbs in negated nominals. On a verbal interpretation of negation, the interpretation of the nominal yields a state type of not having achieved that achievement. A state discourse referent is then supplied by the appropriate inflectional elements. But when the gerundive contains the affix -*ing*, we must convert our state type into an activity type. Typically there is no well defined activity that corresponds to not having achieved the achievement. What activity for instance corresponds to or could correspond to the train's not arriving? Consequently, the denotation of the nominal is undefined. The only other possibility then is to interpret the negation as having wide scope; i.e., the negation must occur in I. Then by my analysis for gerundive constructions in which the nominalization affix dominates I, the nominal must end up denoting a fact or some other abstract entity-- not an event or state. On the other hand, if a discourse referent of activity or accomplishment type has been introduced as an argument of the verbal predicate in a gerundive nominal's translation, then the interpretation of the negation operator as verbal negation is usually possible.[1]

This solution to the problem of event negation depends on the two stage analysis of DRT. It cannot be implemented in a single stage semantic theory, in which sentences and nominals denote events, states, facts or propositions and on which the truth functional int. In fact, I do not see any solution to the problem of event negation in a single stage semantic theory for the following reason. One cannot simply exploit complementation principles at the levels of denotations to get a sensible theory. To capture the data, one must suppose that negation is a partial function from eventuality denoting nominals into fact denoting nominals. But if facts are closed under complementation-- as seems entirely reasonable, then one quickly gets difficulties by considering, for instance, double negations. If φ denotes an event and $\neg\varphi$ a fact, then $\neg\neg\varphi$ most likely denotes a fact. But the fact that $\neg\neg\varphi$ is just the fact that φ. So then it appears that $\neg\neg\varphi$ should denote what φ denotes, but this would end up equating facts and events, which is no solution to the problem at all. To solve the difficulty, one must define negation as an operator on types. If one wants to keep the ordinary truth functional meaning of negation, then one must construe the aspectual and transformational character of negation as affecting a level of semantic interpretation other than denotations. Negation is a function from event types to state types at the DRS level of interpretation on my analysis; the truth functional meaning of negation is made evident in the mapping from DRSs to intensional models. Similarly, the aspectual force of *-ing* is analyzed at the DRS level. By exploiting DRT's two stage semantics, one can keep an intuitive, truth conditional meaning for negation and solve the problem of event negation.

Conclusions

I have used a bottom up construction procedure for building DRSs to give a uniform treatment of gerundive constructions. The analysis has been extended to treat other forms of nominals-- derived nominals, *that* clauses and infinitivals in Asher (forthcoming). Gerundive constructions may denote either eventualities or facts; gerundive constructions must denote facts

[1] I have talked about negation with gerundive nominals. But what about negation with derived nominals like *arrival*? I would claim for them a very similar analysis. See Asher (forthcoming).

if they contain inflectional elements. I have shown how the analysis solves three problems: the problem of the relation between sentence meanings and the meanings of gerundive nominals, the problem of the spectra of nouniness and world immanence, and finally the problem of event negation.

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A Dynamic Predicate Logic for Plurals☆

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Introduction

The main subject of this paper is the problem of negation in dynamic logic. It is a claim of most approaches to the logic of discourse that negation closes off formulas and hides any quantifiers that might occur in the syntactic scope of that negation. This explains why

No man walks in the park. He whistles.

is deviant. We think that it is not at all the case that a negated sentence is is always closed off in this sense. In fact, in most cases it can quite easily bind pronouns in sentences that follow it. This is certainly true in the case of the universal quantification, which is usually defined in terms of the negation of an existential quantifier, but should not inherit the closure properties of that negation, as is shown by the successful binding of pronouns in an example like

Every woman gave every man a present. She put it on his desk.

Even bigger problems arise when plurals are concerned, as is shown by

It is not the case that there is a man walking in the park. They are all home watching tv.

This paper is set up as follows. In the first three sections we discuss what we expect a dynamic logic to be capable of in the treatment of natural language. We do this with Dynamic Predicate Logic (DPL) [Groenendijk and Stokhof, 1989a] as example. One of the matters we discuss is the nature of variables in dynamic logic. We are so used to interpreting variables in logic in a certain way, that we tend to forget the original intuitions behind them and here we try to retrieve those. The next three sections discuss plural reference and how discourse phenomena influence our particular choice of formalism for treating plurals. Finally in the last three sections the previous sections are put together in a dynamic logic that treats plurals and a number of examples are discussed.

☆ This paper could not have been written without the help of the members of the discourse group at my department. Special thanks should go to Jeroen Groenendijk, who first mentioned the idea to me of taking sets of assignments as building blocks for a theory instead of assignments, to Paul Dekker, for pointing out a flaw in an earlier attempt to formalisation, and to Hub Prüst, for some fruitful discussions.

Dynamic Semantics

You know the meaning of a sentence if you know the change it brings about in the information state of anyone who wants to incorporate the news conveyed by it.
[Veltman, 1990]

1 sentence conjunction

Text is read (or heard) in a sequential way from beginning to end, and it is interpreted in this same sequential way by interpreting the units of the discourse one at a time. At any moment during this process some initial segment of the text is already interpreted. This initial segment is part of the context at that moment. No sentence of a text is interpreted in a vacuum, it is always interpreted in a context to which the sentences preceding it have contributed. Consider for example *He sees her run* in the following text

1.1 *There is a woman in the park. She is jogging. A man is sitting on a bench. He sees her run. A girl is sitting on a bench opposite him. She is wearing a red sweater. I also see some boys. They are taking their dogs for a walk. They are in a hurry because they want to be home with them before dark.*

The discourse preceding it gives us the information that *He* gets its interpretation from *a man* and *her* from *a woman.*
We take it that the interpretation of a sentence takes place in two stages. The first stage we call the *coreferencing* of the sentence, the second is the conjunction of the resulting translation of the sentence with the context. In the first stage suitable indices are given to all anaphors and quantifiers in the sentence. This process takes place in tandem with the translation of the sentence in a formal language and should be regarded a part of this translation rather than of the semantics. That the semantics is not the decisive factor in determining the indices is also illustrated by (1.1). In the sentence *She is wearing a red sweater* the pronoun can only be coindexed with the girl sitting on the bench, even if that girl is wearing a blue t-shirt whereas the running woman is wearing a red sweater. Although some lexical semantics may play a role, sentence semantics does not.
In the second stage, the translation of a sentence resulting from the first stage is conjoined with the translation of the preceding discourse, which gives an interpretation of the discourse that results from incorporating that sentence. We call a logic a *dynamic logic* iff it formalises the way in which quantifiers in one formula can bind variables in another, to achieve cross-sentential binding, even if the variable is not syntactically in the scope of that quantifier. It is finding a logic that correctly implements this dynamic binding that this paper aims at — we are not concerned with the coreferencing process that this

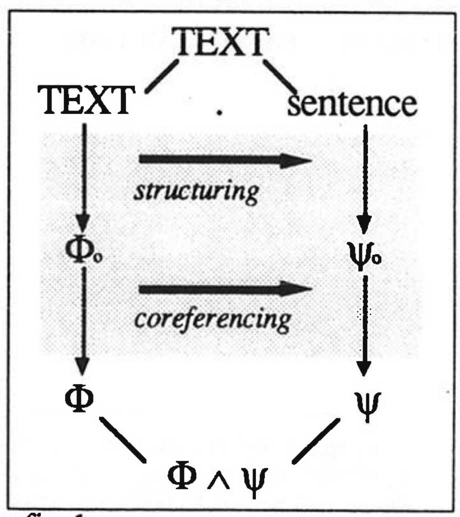

fig. 1

logic presupposes. We will assume some resolution mechanism that assigns "old" indices to every anaphor in the sentence, to link it with some appropriate quantifier in the preceding discourse, and assigns new indices to all quantifiers in the sentence.
(Cf. [Prüst, 1990] for a discussion of a resolution mechanism for anaphora, in particular VP anaphora, which also addresses the problem of disambiguating the structure on the basis of discourse structure and semantics, e.g. quantifier scope and the other components that hide in the shaded part of *fig. 1*).

That part of the context that is relevant for the value assignation to anaphors in any possible continuation of a discourse we call the *(binding-) state* of the discourse at that point. It is finding a sufficiently rich notion of state, which is not so rich as to become trivial, that takes most of the work. States should be defined in such a way that they give us sufficient information to determine the binding of any pronoun or other anaphor that could occur in the next discourse unit. In fact, most properties of a dynamic logic are fixed as soon as an appropriate choice of the notion of state that we are going to use has been made.
This holds in particular for the definition of *conjunction* of two formulas of dynamic logic. Suppose at a given point in a discourse we are in state s_A (*fig.2*). If we now interpret the next unit, incorporating the new information added by this unit will change the state to s_B. This means that we can regard discourse units as functions from states to states: given any old state as input the discourse unit gives us a new output state. If we want to take this idea seriously, we are committed to interpreting conjunction as function composition:

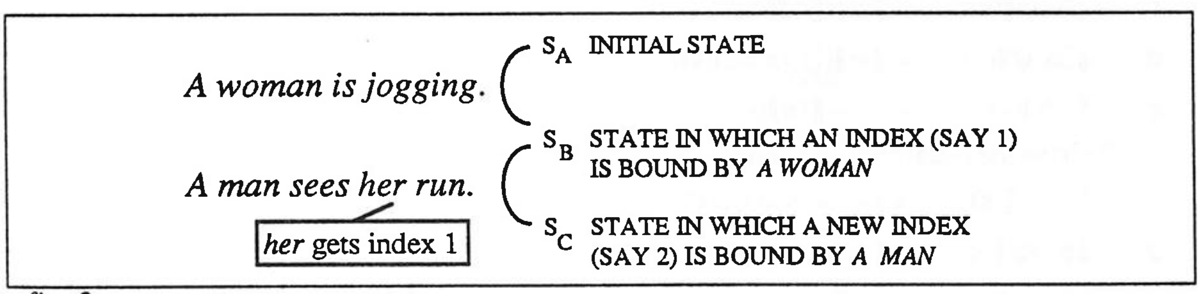

fig. 2

The interpretation of the concatenation of two sentences 's.t' is done by first interpreting s in the old state (s_A), followed by the interpretation of t in the resulting new state (s_B) to give the final state (s_C).

Giving a dynamic logic in our sense will always consist of:

1) giving its syntax.

2) giving the states.

3) giving the action of formulas on these states, where conjunction is always interpreted as function composition.

2 dynamic predicate logic

A good example of a logic that interprets formulas as state transformations and conjunction as function composition is Dynamic Predicate Logic [Groenendijk and Stokhof, 1989a]. Strictly speaking, Groenendijk and Stokhof do not formulate DPL in this way, but it can easily be accommodated to have this form.

2.1 DEFINITION (syntax of DPL)

The *syntax* of DPL is that of ordinary predicate logic with identity.

2.2 DEFINITION (states of DPL)

The *states* of DPL are sets of assignments.

the *interpretation* $[\![\phi]\!]^{\mathfrak{M}}$ of a formula ϕ as transformations of states in a given standard first order model $\mathfrak{M} = \langle D, \mathfrak{I} \rangle$, with D a domain of entities and $\mathfrak{I}$ a function that assigns to any n-place predicate a set of n-tuples in the standard way, is defined as follows (we will suppress the model $\mathfrak{M}$ where possible):

2.3 DEFINITION (semantics of DPL)

a. $[\![P(x_1...x_n)]\!](s) = \{g \in s \,.\, \langle g(x_1)...g(x_n)\rangle \in \mathfrak{I}(P)\}$

b. $[\![x=y]\!](s) = \{g \in s \,.\, g(x)=g(y)\}$

c. $[\![\phi \wedge \psi]\!](s) = [\![\psi]\!]([\![\phi]\!](s))$

d. $[\![\exists x.\psi]\!](s) = [\![\psi]\!](\bigcup_{d \in D}[x:=d](s))$

e. $[\![\neg\phi]\!](s) = s - [\![!\phi]\!](s)$

We have the usual abbreviations:

f. $[\![\phi \vee \psi]\!](s) = [\![\neg(\neg\phi \wedge \neg\psi)]\!](s)$

g. $[\![\phi \rightarrow \psi]\!](s) = [\![\neg(\phi \wedge \neg\psi)]\!](s)$

h. $[\![\forall x.\phi]\!](s) = [\![\neg\exists x.\neg\phi]\!](s)$,

where we used the definitions

i. $[x:=d](s) := \{h \,.\, \exists g \in s \,.\, h \approx_x g \wedge h(x)=d\}$

j. $h \approx_x g$ iff for all variables z not equal to x: $h(z)=g(z)$

k. $[\![!\phi]\!](s) := \{g \in s \,.\, [\![\phi]\!](\{g\}) \neq \varnothing\}$ (the *closure* of ϕ)

In DPL *prime-formulas* are interpreted as *tests* (also called *filters*) on states, the resulting state is the set of those assignments in the input state that make the prime-formula true in the old (static) sense. The same holds for *identity*. As required, *conjunction* is interpreted as function composition, as is, maybe a bit more surprising, *existential quantification*. Applying $(\exists x.\psi)$ to a state consists of first forgetting everything that is known of the possible values of the variable x and then applying ψ to the resulting state: $[\![\psi]\!](\mathbf{Forget}_x(s)) = [\![\psi]\!](\bigcup_{d \in D}[x:=d](s))$. The fact that existential quantification is defined in terms of function composition makes that the associativity of function composition gives this logic its dynamic properties, as can be seen from example (2.6) below.

Forgetting about the possible values of the variable x is achieved by adding to the state all assignments which are equal to an assignment in the input state with the possible exception of their value for x. A prime-formula is a real update function, an existential quantifier a "downdate" function, it makes the state larger instead of smaller and causes the logic to be non-monotonic, or, as it is called in this paper, non-eliminative. A direct consequence of this is that *negation* can not be defined as simple complementation relative to the input state. The most natural interpretation for the action of $(\neg\phi)$ on s is that part of s that does not satisfy ϕ, i.e. the complement relative to s. But this does not make much sense if ϕ can also make the state larger. In that case you can not write $[\![\neg\phi]\!](s) = s - [\![\phi]\!](s)$ because that would make you loose all bindings added to the state by existential quantifiers inside ϕ.
The solution taken by Groenendijk and Stokhof is to follow Discourse Representation Theory (DRT) [Kamp, 1981] in interpreting negated formulas as tests, which causes negations to close off the formula they negate. Some consequences of this are discussed below.

2.4 DEFINITION (truth and validity)

A formula ϕ is *true* with respect to an assignment g (in some model $\mathfrak{M}$) iff $[\![\phi]\!]_{\mathfrak{M}}(\{g\})\neq\varnothing$
A formula ϕ is *valid* iff $[\![\phi]\!]_{\mathfrak{M}}(\{g\})\neq\varnothing$ for all models $\mathfrak{M}$ and all assignments g.

It was shown by Groenendijk and Stokhof [1989a] that the resulting logic can be seen as a reformulation of DRT.

That associativity gives this logic its dynamic properties is illustrated by the following example:

2.5 *A man walks in the park. He whistles.*
$\exists x.\ (\ man(x) \wedge walk\text{-}in\text{-}the\text{-}park(x)\ ) \wedge whistle(x)$

This formula gets the right interpretation in DPL, i.e. the pronoun *He* in the second sentence is bound by *a man* in the first sentence, because of the equivalence:

2.6 $\exists x.\ (\ man(x) \wedge walk(x)\ ) \wedge whistle(x) \Leftrightarrow \exists x.\ (man(x) \wedge walk(x) \wedge whistle(x))$

which follows by the associativity of function composition:

$$[\![\exists x.(\ man(x) \wedge walk(x)\ ) \wedge whistle(x)]\!]$$
$$= [\![whistle(x)]\!]\circ[\![walk(x)]\!]\circ[\![\exists x.man(x)]\!]$$
$$= [\![whistle(x)]\!]\circ[\![walk(x)]\!]\circ[\![man(x)]\!]\circ Forget(x)$$
$$= [\![man(x) \wedge walk(x) \wedge whistle(x)]\!]\circ Forget(x)$$
$$= [\![\exists x.(\ man(x) \wedge walk(x) \wedge whistle(x)\ )]\!]$$

However, the equivalence of DPL with DRT means that with its virtues DPL also inherits the vices of DRT. The major cause for problems is the definition of negation. It is defined by first taking the closure of the formula it is applied to (i.e. $!\phi$), which hides all quantifiers inside the scope of the negation, and then taking the complement of this closed formula (which is a test) relative to the input state. This is not an coincidence. It is claimed to be an empirical fact of the language-fragment described by DRT that a quantifier cannot bind from within the scope of a negation. An example[1] of this is a sentence pair like:

2.6 *It is not the case that a man walks in the park. *He whistles.*

Obviously, *He* cannot refer to *a man*, so the variable that is used to interpret *He* should not be bound by the quantifying phrase *a man.* And indeed in DPL it is not, because the quantifier is blocked by the closure of the negation. Quite rightly so, you might think. But as Groenendijk and Stokhof themselves notice, this is not always what one would like to have. Take for instance the following case of a double negation:

2.7 *It is not the case that Mary has no car. It is right outside.*

Here we would like *It* to be bound by *car* even if this would mean binding from within a negation. But this cannot be done in DPL because double negation is just a double closure of the formula.

If we allow for plural pronouns, things get really problematic. Almost no context is closed for plural coreference, even the quantifier *a man* in example (2.6) can be refered to, as the following example shows:

2.8 *It is not the case that a man walks in the park. They stayed home to watch tv.*

It seems to us that the apparent closure property of the negation is only an accidental property of the originally chosen fragment. The presumed closure property seems not inherent to the underlying logic but only to the actual semantic content of the sentences — it has to do with world knowledge rather than logical structure. This feeling is strengthened if we consider the case of the universal quantifier.

Following (standard) predicate logic the universal quantifier is defined using the existential quantifier and negation (cf. definition 2.3.h). This means that universal quantification blocks reference in just the same way as negation does. An example that corroborates this is given by:

2.9 *Every man walks in the park. *He whistles.*

But what about:

2.10 *Every player chooses a pawn. He puts it on square one.*

One could argue that a prescription like this is a very exceptional context. But that doesn't explain way the following is correct:

2.11 *Every woman gave every man a present. She put it on his desk.*

and again, if you admit plurals it is even easier[2]:

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1 cf. the next footnote.

2 What is or is not accepted as a correct example is, as always, a big question. This example and the other ones in this section are deliberately chosen simple, to exemplify one aspect and one aspect only. This means that they are not the most natural sounding ones. In particular, you might argue that *They* is not correct here. Parallelism effects (cf. [Prüst, 1990]) probably demand that in sentences like these the same syntactic number on the same position is used. We think of these and the preceding examples as simplified versions of more realistic examples like: ***Every girl was playing in the park. After Mary called them, they came in to eat.*** Which is different from: ***Every girl was playing in the park. After Mary called her, she came in to eat.*** Remarks about the examples above should be regarded as remarks about more realistic examples like these.

2.12 *Every man walks in the park. They whistle.*

A general way of solving these problems seems to be needed. The singular counterexamples could arguably be seen as marked cases, where the syntactic singular is used to force a distributive reading of a semantic plural. (This feeling is strengthened by the fact that one seems to need a lot of "processing" to interpret sentences like 2.8 or 2.10, although less for 2.11. We will come back to this below.) However, in a lot of cases the plurals seem natural enough. For that reason we will concentrate on plurals and discuss the singular as a derived case. We will show that a formalism can be set up in such a way that the possibility or impossibility of reference depends on the actual model, i.e. the world-knowledge, and not on the structural properties of the formalism.

3 more about states

Now that we have some idea about what we regard as a dynamic logic and have seen a simple example of one, we will discuss in this section some properties that we expect the states of any dynamic logic to have. We will illustrate these requirements by showing how these properties are implemented in DPL. We start off by reconstructing the role that variables play in logic.

A. from pronouns and noun-phrases to variables and quantifiers
In natural language it is not always clear to which noun-phrase a given pronoun refers. To make the reference explicit in the analysis we could follow Quine in using braces like in:

3.1 *A man walks in the park with his brother. He whistles.*

(A MAN) WALKS IN THE PARK WITH (( )S BROTHER). ( ) WHISTLES.

or in a one-dimensional notation:

3.2 *(A MAN)$_1$ WALKS IN THE PARK WITH ((HE)$_1$S BROTHER)$_2$. (HE)$_1$ WHISTLES.*

Pronouns refer to some quantifier and only the quantifier refers to the "real" world (i.e. the model). Because of this we do not have to worry about plural or singular entities, let the quantifier worry about those things. All pronouns refer to the same kind of thing, a quantifier. In particular (3.3) can be analyzed in exactly the same way as (3.1)

3.3 *Every player puts a pawn on his board. He puts it on square one.*

(EVERY PLAYER)$_1$ PUT (A PAWN)$_2$ ON ((HE)$_1$S BOARD)$_3$. (HE)$_1$ PUT (IT)$_2$ ON SQUARE ONE.

It is common practice to write such interpretations using variables and existential or universal quantifiers. This means that we can write the interpretation of (3.1) and (3.3) something like

3.4 $\exists x.(\ \mathbf{man}(x) \wedge \exists y.\ (\mathbf{brother\text{-}of}(x,y) \wedge \mathbf{walk\text{-}in\text{-}the\text{-}park\text{-}with}(x,y) \wedge \mathbf{whistle}(x))\ )$

3.5 $\forall x.\ \mathbf{man}(x) \rightarrow \exists y.(\mathbf{pawn}(y) \wedge \mathbf{put\text{-}on\text{-}board\text{-}of}(x,y) \wedge \mathbf{put\text{-}on\text{-}square\text{-}one}(x,y))$

It must be stressed that the variables are only intended as an alternative to the use of indices. Like a smile can not exist without a face, except in the case of that of the Cheshire cat, variables do not have an existence of there own. Nor can they be bound twice: Even the Cheshire cat can not have somebody else's grin. A variable is just a way of remembering which quantifier binds the location it fills.

B. relations between possible bindings

States provide a means to express, at any moment in the text, the way possible anaphors in the subsequent part of the text could be bound by the preceding fragment of it. In most cases what a pronoun is bound to is not completely determined by the preceding discourse and in most cases this is intentional. Consider an example of the previous section:

3.6 *A man walks in the park. He whistles.*

He is bound by whatever the quantifying phrase *A man* binds. This still leaves every MAN THAT IS WALKING IN THE PARK as a possibly intended man.

This is one of the most important observations about the role of states: they express the potential, not the actual binding – They express the bindings that are still possible according to the preceding discourse (i.e. that successfully resolve the bindings in it and make it true). Bindings can depend on each other as in

3.7 *A man walks in the park with his wife. She talks to him.*

What *She* is bound to depends on what *him* is bound to. The different possible bindings of *him* caused by *A man*, and the different possible bindings of *She* caused by *his wife* are related. You can not just choose one possible binding for *him* and another for *She*, independently of each other. The referents have to be man and wife! The state should preserve not only the information of what bindings are still possible for different pronouns, but also all the relations that exist between these pronouns.

In DPL this is done quite simply by the assignments that are the elements of the state. Any one of these assignments corresponds to one possible way the bindings to the variables can be. An assignment g could assign to x the value g(x)=JOHN and to y as value john's wife: g(y)=MARY. Another assignment h can assign quite different values h(x)=BILL, h(y)=ANNE. After interpreting the first sentence, the only assignments that are preserved as elements of the state are those assignments g that assign to x a value g(x)=A MAN THAT WALKS IN THE PARK and to y a value g(y)=THE-WIFE-OF(g(x)). So these are the only assignments that the second sentence gets as input and therefore only the right pronoun bindings are obtained.

Although bindings of *different* pronouns can depend on each other, the different possible bindings of *one and the same* pronoun can not interfere with each other. For instance, whether JOHN and JOHNS WIFE are possible bindings of *him* and *she* caused by *A man* and *his wife* is independent of the question whether BILL and BILLS WIFE are. This means that, in as far as states express the different possible bindings to the pronouns, they can be regarded as sets of such bindings, just like the sets of assignments are in DPL. We can also formulate this the other way around. If formulas are defined in such a way that way can determine its action on a state by determining what the formula does on the elements of that state, *independence* is said to hold for that formula, and an action of this form is called *pointwise* (because it acts on the points = elements of the set).

C. negations

If we think of states as sets of possible bindings, the interpretation of the negation of a sentence is fairly straightforward. Suppose we are at a point where a text has been interpreted up to but not including some sentence ϕ and the state at that point is s. After interpreting ϕ in state s, the result will be a new state $s'=[\![\phi]\!](s)$, where $[\![\phi]\!]$ is the interpretation of ϕ and s' is what is still possible after the interpretation of ϕ in state s. In general there will be less possible in s' than there is in s because ϕ will have added some restrictions on the old bindings (cf. example (2.5), where *he whistles* restricts the original binding possible from all men in the park to only those that whistle) and ϕ might also have added new bindings. So, again viewing the state as a set of possibilities, s' is a subset[3] of s.

Now what if, being in state s, we interpret $\neg\phi$ instead of ϕ in s? The resulting state would be a different state $s''=[\![\neg\phi]\!](s)$. We know that ϕ and $\neg\phi$ contradict each other, so something that is possible after incorporating ϕ (i.e. makes it true) should not be possible after incorporating $\neg\phi$, and vice versa:

3.8 $[\![\phi]\!](s) \cap [\![\neg\phi]\!](s) = \varnothing$. (non-contradiction)

If we furthermore assume[4] that for every formula ψ something that is impossible in s remains impossible after the interpretation of ψ in that s, i.e. that *elimination*, $[\![\psi]\!](s) \subseteq s$, holds for all formulas ψ, then

3.9 $[\![\phi]\!](s) \cup [\![\neg\phi]\!](s) \subseteq s$. (elimination)

It is not unusual to choose the weakest possible negation ([Groenendijk & Stokhof, 1989], [Veltman, 1990]), i.e. a negation that preserves everything that is not acceptable for the non-negated formula, which amounts to the identity: $[\![\neg\phi]\!](s) = s - [\![\phi]\!](s)$. But we do not have to make this choice and in our logic we won't. However, by (3.9) we always have:

3.10 $[\![\neg\phi]\!](s) \subseteq s - [\![\phi]\!](s)$.

D. scope

In static logic the *syntactic scope* of a quantifier is important, because a quantifier over some variable x can only bind occurrences of the variable x inside the syntactic scope of that quantifier, outside of the scope a variable is free. In dynamic logic variables can get bound even if they are outside the syntactic scope of a quantifier. At first sight one might think that this means that the distinction between being inside and outside the scope of a quantifier is not necessary in dynamic logic. However, also in dynamic logic scope has an effect, be it a more subtle one. Consider the following examples

3.11 *Exactly one man walks in the park. He whistles.*

3.12 *Exactly one man who walks in the park whistles.*

The first example, with *he* outside the scope of the quantifying phrase *Exactly one man*, is true if there is exactly one man in the park, and that man whistles. The second sentence, with *he* inside the scope of the quantifying phrase *Exactly one man*, is true for any number of men in the park, as long as only one of them whistles. So the dynamic binding we are after is not just an extension of the scope of the quantifier from its syntactic scope to the whole of the subsequent text.

---

3 This assumption is not valid for DPL because in DPL the existential quantifier may cause a formula to downdate instead of update. We already saw in the comment on the form of DPL that this is the reason why the negation causes technical problems in DPL: for DPL the argument of this section does not hold.

4 but remember the previous footnote.

Plurals

4 Link's plurals and beyond

In the next three sections we will present a theory of plurals which can easily be extended to a dynamic logic. We will assume that the reader has at least some knowledge of the lattice-theoretical approach to plurality, as developed by Link [1983/*forthcoming*] and given a set theoretic form in [Landman, 1989]. In this section we will loosely follow Landman's version, which we will refer to as L/L. We will define an extension of L/L, to deal with quantifiers and predicates with more than one argument. We leave out all details that are not directly relevant to the present paper. The second section of these three shows that if we would try to base a logic of discourse on L/L, we would get the wrong interpretation for plural pronouns. In the last of the current three sections an alternative logic for plurals is defined, that can easily be extended to a dynamic logic.

Plurals are implemented by changing to a domain of entities that not only incorporates the usual individuals like JOHN, but also plural entities like JOHN AND MARY , SAM AND BILL, (JOHN AND MARY) AND (SAM AND BILL) , etc. The simplest way to do this is to follow Landman in building the whole hierarchy of sets over some given set of individuals.
We start with a set U of Ur-elements (for which we take the original, singular entities). Then we define an entity to be any set you can get by applying the set-theoretic operations to U (*fig.3*). With Landman we will assume for practical purposes that this is not iterated beyond ω. Note that the Ur-elements do not themselves form part of the domain, they are only used to generate it.

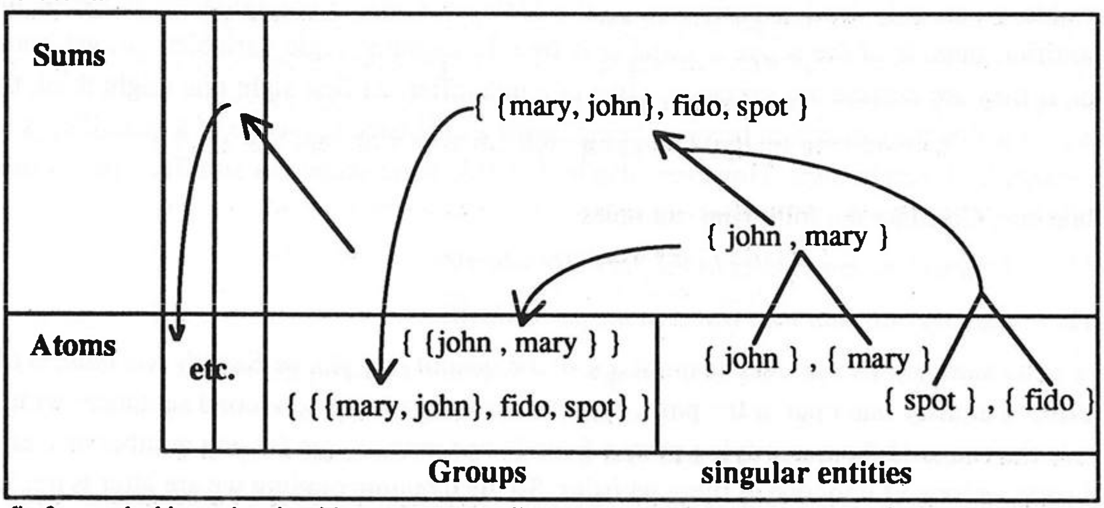

fig.3 the hierarchy of entities generated by {john, mary, spot, fido}.

These sets form a lattice structure with a join-operation induced by the sub-set function. If ‖ - ‖ is the interpretation function we define for two terms of the language:

4.1 $\|\alpha \sqsubseteq \beta\| = 1$ iff $\|\alpha\| \subseteq \|\beta\|$

Singleton sets are called *atoms,* non-singleton sets *sums.*
We distinguish two kinds of atoms: *singular entities* and *groups.* A singular entity is a singleton set containing an element of U. A group is any other singleton set. The set of singular entities is of course equivalent to the original set U of entities.

4.2 DEFINITION (atomic part)

α is an *atomic part* of β iff $\| \alpha \sqsubset \beta \|$ and $\|\alpha\|$ is a singleton. We write this as $\| \alpha \sqsubset_a \beta \|$.

Essential to the Link/Landman approach is that most of the work is done by the prime-formulas and operations on them. Predicates only apply to atoms of a certain level. For instance, **walk** can only apply to singular entities like {JOHN}, {MARY}, **gather** only to groups like { {JOHN, MARY} }, etc. If we want to apply these predicates to plural entities (sums) we have to transform them to do so. This is done by an operator[5] (which Landman only gives for one place predicates):

4.3 DEFINITION (distributive extension)

The *distributive extension* *P of a 1-place predicate P is defined by $*P(x) \Leftrightarrow \forall y \sqsubset_a x.P(y)$

This says that *P holds of a sum iff P distributively holds of the atomic parts of that sum. We would like to extend this definition to n-place predicates. There are two ways to do this. One is to let the * work on one of the positions of the predicate, independent of what fills that position. Another is to let it work on a specific variable, independent of the place it fills. The former choice is the one made in [Link, *forthcoming*]. We choose the latter option because it can be generalised more easily, as we shall see below.
Let x be a variable that occurs in $x_1 \ldots x_n$ and let $({}^{[x/y]}P(x_1 \ldots x_n))$ be the expression that results from replacing x in $x_1 \ldots x_n$ by y, then we define:

4.4 DEFINITION (atomic distribution, first attempt)

The *atomic distribution* of x over a prime-formula $P(x_1 \ldots x_n)$ is defined by:
$\Sigma^x P(x_1 \ldots x_n) \Leftrightarrow \forall y \sqsubset_a x.\ ({}^{[x/y]}P(x_1 \ldots x_n))$

Let's consider a simple example to explain the part of the formalism we will need:

4.5 *Some boys are taking a dog for a walk.*

We will first show that taking a Σ operator that works on prime-formulas only is inadequate. In that case we would get the following translation for (4.5):

4.6 $\exists x.(\ \Sigma^x \mathbf{boy}(x) \wedge \exists y.\ (\mathbf{dog}(y) \wedge \Sigma^x \mathbf{walk}(x,y))\ )$

This would only be satisfied if all boys took the same dog for a walk, as can easily be seen from writing out the definition for Σ :

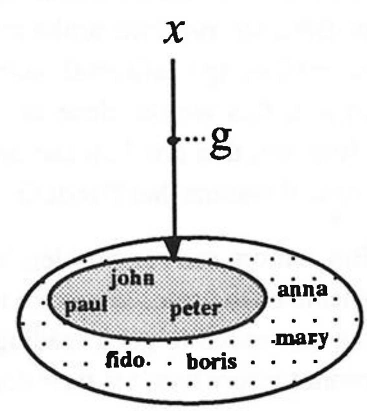

fig.4. standard plurals: variables are assigned sets as values.

4.6' $\exists x.\ (\forall z \sqsubset_a x.\mathbf{boy}(z) \wedge \exists y.\ (\mathbf{dog}(y) \wedge \forall z \sqsubset_a x.\ \mathbf{walk}(z,y)\ ))$

---

5 Landman [1989] gives all kinds of other operators that transform atoms into sums, sums into groups, groups into sums again etc. We refer to his article for the details.

The quantifier $\exists y$ is not in the scope of $\forall z$, so it selects the same dog for all atomic parts of x. This is not the correct interpretation[6]. You need to be able to associate different dogs with different boys. This can be done if we extend the application of Σ to all formulas.
For ϕ any formula, let $([x/y]\phi)$ be the formula you get by replacing all free occurrences of x in it by y, under the usual conditions on substitution

4.7 DEFINITION (atomicdistribution)

The *atomicdistribution* of x over ϕ is defined by: $\Sigma^x(\phi) :\Leftrightarrow \forall y \sqsubset_a x.\ ([x/y]\phi)$

Using this we can give the required interpretation of (4.5):

4.8 $\exists x.(\ \Sigma^x(\ \mathbf{boy}(x)\ ) \wedge \Sigma^x(\ \exists y.\ (\mathbf{dog}(y) \wedge \mathbf{walk}(x,y)\ )\ )\ )$

In words: the atomic elements of x are boys and for everyone of them there is a dog which he is taking for a walk in the park. Translation (4.8) not only has the right interpretation, but it also puts the distribution operator in a very natural position, giving the sentence a translation of the form QUANTIFIER(RESTRICTION-ARGUMENT)(ARGUMENT). We can now read *some boys* as a generalised quantifier that, for a distributive predicate like **walk**, is distributive in both arguments. In the case of **gather** we would expect the second argument to be non-distributive, to enable the predicate to get the group as a whole as an argument (cf.footnote 6):

4.9 $\exists x.(\ \Sigma_x \mathbf{boy}(x) \wedge \mathbf{gather}(x)\ ).$

An alternative translation for (4.5) that also has the right interpretation is:

4.10 $\exists x.(\ \Sigma^x(\ \mathbf{boy}(x) \wedge \exists y.(\mathbf{dog}(y) \wedge \mathbf{walk}(x,y)\ )\ )\ )$

But that would blur the distinction between subject and predicate, so we prefer (4.8).

5 taking a hint from discourse

If we try to make the plural logic of the preceding section into a dynamic logic along the lines of DPL, we run into problems. First of all, there is a non-essential problem, because Σ is defined using a universal quantifier. Given the standard definition of universal quantification in DPL this would close off the formula and hide any existential quantifier inside of it. However, this problem can be solved by choosing a somewhat different definition for Σ and we will assume that Σ is defined in this way to make it transparent for existential quantifiers[7].

But a more serious problem still arises. The treatment of plurals presented above does not enable us to preserve enough information in the context to exclude absurd readings. Although we succeeded to get a reading of (4.5) that assigns different dogs to different boys, we still cannot assure that it is *their* dog. Consider the following "text":

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6 At least not for the most natural reading of this sentence. Even if the boys are walking a dog together, this would not be a natural translation, in that case you would rather expect **walk** to get as argument the group of boys itself, not the individual boys that make up that group. The translation that would correspond to that interpretation is something like: $\exists x.(\ \Sigma_x \mathbf{boy}(x) \wedge \exists y.\ (\mathbf{dog}(y) \wedge \mathbf{walk}(x,y))\ ).$

7 A suitable definition for Σ is the following:
Assume for the moment that ϕ does not bind x. $\|\Sigma^x(\phi)\|(\{g\})$ = the set s of assignments created by first "splitting" g with respect to x in atom-related parts: $s_x=\{h{\sim_x}g\ .\ h(x) \sqsubset_a g(x)\ \}$, then giving ϕ these h as input one at a time: $t = \{\ \|\phi\|(\{h\})\ .\ h \in s_x\ \}$. Finally defining $k \in s$ iff k is generated by taking from every $X \in t$ exactly one $m \in X$. In other words, using the definition $\oplus_{h\in Y} h=g$, iff $\forall \tau.g(\tau)=\bigcup_{h\in Y} h(\tau)$:
$\|\Sigma^x(\phi)\|(\{g\}) = \{\ \oplus_{h\in s_x} \sigma(h).\ \sigma{:}h \in s_x \mapsto \sigma(h) \in \|\phi\|(\{h\})\ \}$. If ϕ does bind x this would give a strange result, but then, if you do not want to be bitten by a snake, you shouldn't stroke it.

5.1 *Some boys are taking their dog for a walk. They want to be home with it before dark, so they walk very fast.*

Even if we assume that the translation (4.8), interpreting it as a dynamic formula, gives us a state that assigns to x a set of boys and to y the set of dogs they are taking for a walk, and if we further assume that *They* and *it* in the second sentence refer to these x and y, this still gets the wrong interpretation. The second sentence would be true irrespectively of which dog which boy gets home with. This surely is not what you mean with (5.1).

This problem is related to the one mentioned in section 3A, where the following example was given to show that the different possible bindings to different variables, according to a state of a dynamic logic, might have some relation:

5.2 *A man walks in the park with his wife. She talks to him.*

5.3 **∃x. man(x) ∧ ∃y.wife-of(y,x) ∧ walk-itp-with(x,y) ∧ talk-to(y,x)**

We saw that DPL represents the relation between different possible bindings correctly just because states are sets of assignments, and every assignment g relates a specific choice for x, namely g(x), to a specific, related, choice of y, namely g(y). But in plural logic the assignment assigns a *set* of entities to x and a related *set* of entities to y. The elements of these sets are not related, only the sets themselves are.

We can learn something from the way the states of DPL are sets of different possible bindings, if we look at a plural as the set of different possible elements of some group. For example *Some boys* denotes the set of individual boys of some group of boys. The sentence *Some boys are taking their dog for a walk* can then be understood as giving every boy in the group a related dog. The interpretation of *They want to be home with it before dark...* would then be true if every boy wants to be home with *the it* (i.e. the *dog*) related to that specific boy by the preceding sentence. The formal similarity between this problem and that of section 3A suggest that we can find a similar solution. Instead of interpreting a formula relative to assignments that assign sets of atoms to a variable, it works better to use sets of assignments, each of which assigns one of the atoms of that set of atoms to the variable. In the next section we will define a logic for plurals that does exactly that. It defines an interpretation of formulas relative to sets of assignments instead of assignments.

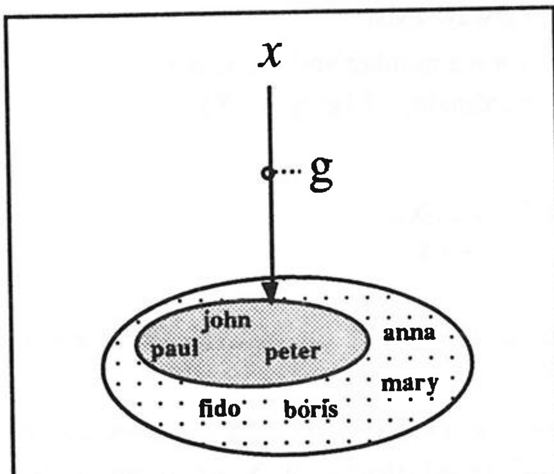

fig. 5. Standard plurals refer in one step to plural entities (sums).

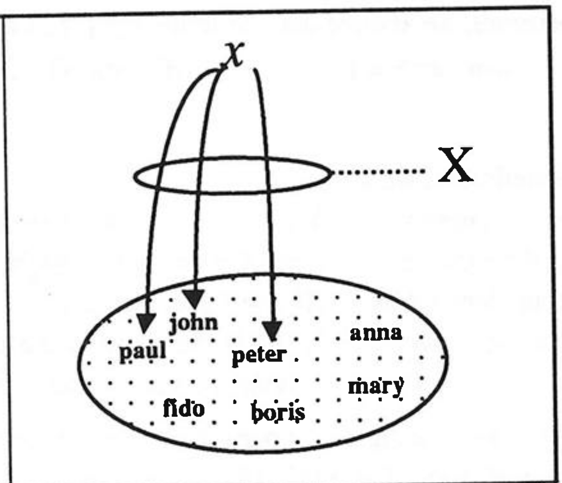

fig. 6. Referential plurals refer in a plural way to singular entities.

6 a logic for plurals

Let's follow the suggestion made in the previous section, and interpret a formula relative to a (non-empty) set of assignments instead of one assignment. This gives us an alternative way of interpreting plurals which does not need sums anymore. We can take as our domain of entities not the whole *singular entity/group/sum* universe of [Landman, 1989], but only the atoms, only what is refered to by a singular. We throw the sums out of the domain, but keep the singular entities and the groups. The resulting domain consists of singletons only, so we can strip the outer braces of all objects (Note that because every sum {...} also occurs as a singleton {{...}}, this leaves us with more or less the same domain, only the "real" singulars, like JOHN, are added). To repeat the point: this puts the plurals somewhere else, in the interpretation of the formulas rather than in the domain the formulas get interpreted in.

A typical Predicate Logic for Plurals along these lines can be defined as follows:

6.1 DEFINITION (syntax of PLP)

the syntax of PLP is the same as that of standard predicate logic with restricted quantification and with the following extensions:

a. if ϕ is a formula, then so is $\Sigma_x(\phi)$, the *distribution* of x over ϕ.

b. if x, y variable and ϕ a formula, then $(\Gamma^x y.\phi)$ is a formula, the *group*, y, of entities, x, satisfying ϕ.

6.2 DEFINITION (semantics of PLP)

a. $\| P(x_1,...,x_n) \|^X = 1$ iff $\{<g(x_1),...,g(x_n)> . g\in X\} \in \mathfrak{I}(P)$

b. $\| \phi \wedge \psi \|^X = 1$ iff $\| \phi \|^X = 1$ and $\| \psi \|^X = 1$

c. $\| \neg\phi \|^X = 1$ iff $\| \phi \|^X = 0$

d. $\| \exists\phi x.\psi \|^X = 1$ iff $\exists Y \approx_x X.\ \| \phi\wedge\psi \|^Y = 1$

e. $\| \Sigma_x(\phi) \|^X = 1$ iff $\forall Y \in \delta_x(X).\ \| \phi \|^Y = 1$.

f. $\| \Gamma^x y.\phi \|^X = 1$ iff $\exists Y \approx_y X.\ \forall g\in Y.\ g(y)=Y(x)$ and $\| \phi \|^Y = 1$.

Furthermore, we assume that the following predicates always exist:

g. $\| \#(n)(x) \|^X = 1$ iff $\#X(x) = n$, for n a number and $\#X(x)$ the cardinality of $\{g(x) . g\in X\}$.

In this definition we used:

equivalence upto x: $Y \approx_x X$ iff $\forall d.\ ([x:=d]Y = [x:=d]X)$,

setting the variable: $[x:=d]X = \{h.\ \exists g\in X.\ g \approx_x h \wedge h(x)=d\}$,

the group denoted by x: $X(x) = \{g(x) . g\in X\}$.

partitioning X over x: $\delta_x(X)$ is the partitioning of X induced by the equivalence relation "giving the same value to x".

A set Y is an element of the partitioning of X relative to x, $Y\in \delta_x(X)$, iff $Y\subseteq X$ and there exists an entity d, such that $Y(x)=\{d\}$ and for all $g\in X$, if $g(x)=d$, then $g\in Y$. Y are all and only assignments in X that assign d to x. Such an Y we call a *proper part* of X relative to x.

We will use the usual notation for numerals and write **#(1)(x)** as **one(x)** or **sng(x)** (for singular), and the same for other numeric predicates: **#(30)(x_{137})** as **thirty(x_{137})**, etc.

This interpretation is defined in such a way as to make it as close as possible to standard predicate logic (PL). But of course, some differences arise because the interpretation is relative to sets of assignments instead of a single assignment. The main difference lies in the interpretation of *prime-formulas*. The interpretation of a predicate is defined as a set of sets of n-tuples. This makes that a set of n-tuples can be in the interpretation whereas its subsets are not, making real plural predicates like **gather** possible. Distributive predicates are the special case where if a set is in the interpretation, all its subsets are. Or if we look at it from the other way, a distributive predicate has as its interpretation the powerset of its interpretation in PL.

6.3 DEFINITION (distribution)

a. a formula ϕ is said to be *distributive in x*
iff for all sets of assignments X: $\|\Sigma_x\phi\|^X=\|\phi\|^X$.

b. an n-place predicate P is said to be *distributive in the i^{th} place*, for $0 \leq i \leq n$,
iff for all sets of assignments X: $\|\Sigma_{x_i}P(x_1,\ldots x_n)\|^X=\|P(x_1,\ldots x_n)\|^X$.

c. a predicate P is said to be *distributive* if it is distributive in all its argument places
a formula ϕ is said to be *distributive* if it is distributive in all variables.

Conjunction and *negation* are defined in exactly the same way as in standard predicate logic.
Existential quantification is defined in a restricted form, both for consistency with the dynamic formulation in the next section, and because such a generalised quantifier format seems better suited to natural language (cf. 4.8). The way $\exists x$ changes one set of assignments into an other directly mirrors the way assignments are changed by $\exists x$ in PL.
The *distribution*, $\Sigma_x(\phi)$, of a formula ϕ is true in X iff ϕ is true for every element Y of the partitioning of X relative to x.
The *group* operator, $\Gamma^x y.\phi$, which is only added for consistency with [Landman, 1989], is an operator that gives to y as value that group g(y) that are exactly the entities assigned to x. So if X(x) = {JOHN, MARY, SAM, BILL }, then Γ will assign to all g$\in$Y the value
g(y) = {JOHN, MARY, SAM, BILL }, and Y(y) = { {JOHN, MARY, SAM, BILL } }.
Using Γ, all the operators of the Link/Landman logic can be translated in PLP[8].

6.4 DEFINITION (truth and validity)

A formula ϕ is *true* with respect to X (in some model $\mathfrak{M}$) iff $[\![\phi]\!]^X_{\mathfrak{M}}=1$.
A formula ϕ is *valid* iff in all models $\mathfrak{M}$ and for all sets X of assignments $[\![\phi]\!]^X_{\mathfrak{M}}=1$.

---

8 The following is a translation of the most important operators of the Link/Landman logic, which is the starting point for a conservative translation of L/L into PLP:

L/L		PLP	L/L		PLP
$\phi(\downarrow\alpha)$	$\rightsquigarrow$	$\exists y.\exists x.(y=\alpha \wedge \Gamma^x y.\phi'(x))$	$\phi(\uparrow\alpha)$	$\rightsquigarrow$	$\exists y.\exists x.(x=\alpha \wedge \Gamma^x y.\phi'(y))$
$\Sigma^x(\phi)$	$\rightsquigarrow$	$\Sigma_x(\phi)$	$\phi(\sigma x.\psi(x))$	$\rightsquigarrow$	$\forall x.(\psi(x) \wedge \exists y.\Gamma^x y.\phi'(y))$

but a lot of work still has to be done, not in the least finding a reasonable definition of identity ('=').

Example: Some remarks on scope ambiguity
As an illustration we consider the case of what is generally called scope ambiguity. One sentence has plagued Montague grammar more than any other

6.5 *Every man loves a woman.*

Just to remind you, it is claimed that this sentence is ambiguous between two readings, expressed in the translation by the scope of the quantifiers. In our notation:

6.6 $\forall^{man(x)}x.\exists^{woman(y)}y.love(x,y)$

6.7 $\exists^{woman(y)}y.\forall^{man(x)}x.love(x,y)$

A problem with this (cf. [Scha, 1981]) is that it does not explain why in

6.8 *A woman is loved by every man*

the wide scope reading of the existential quantifier, i.e. (6.7), is strongly prefered over the narrow scope one. Besides, other examples seem to suggest to be ambiguous in the same way, but cannot be analysed using a quantifier scope ambiguity, consider:

6.9 *five men love three women.*

This has three readings[9] (cf. *fig. 7*), two of which are parallel to the two readings of *Every man loves a woman*

a) Every one of these five men loves three women, but not necessarily the same. This is called the distributive reading.
b) Every one of the five men loves exactly the same three women. This is called the collective reading.
c) The five men together love the three women together, and every men loves at least one. This is called the cumulative reading.

Note that in (6.5) the readings (b) and (c) are the same, because if they love at least one, and the men love this one woman together, then they love exactly the same woman.
Observe that (6.8) does not have the reading
d) every one of three women loves five men, but not necessarily the same,
A reading you would get if the quantifiers would be exchangeable, as they are in the standard analysis of (6.5).

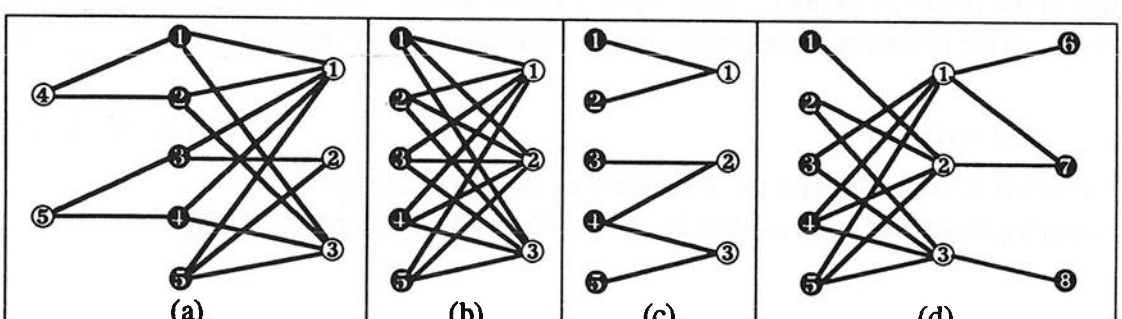

fig 7. three possible and one impossible state of affairs, corresponding to five men love three women. (❶-❽ are men, ①-⑤ are women).

9 We ignore the fact that part of the content of distributivity and cumulativity of numeric quantifiers like five is, according to [Scha, 1981], the fact that there are *exactly* five, not more. We will return to this in the conclusion and restrict ourselves here to the weaker, *at least* five, reading of numerals.

With the plural logic defined above we have the machinery to give an alternative analysis of (6.5), that gives the same two interpretations to (6.5), but makes quantifiers ambiguous between being distributive or not, instead of putting the source of the ambiguity in the scope of the quantifiers. This alternative analysis will also makes (6.9) ambiguous in the right way.

Because we did not give a full Montague grammar, we will have to do the translation in an ad hoc fashion. All quantifiers are taken to be ambiguous between a distributive and a non-distributive reading. As a general rule this is much too simplistic, but for the purpose of this paper it will do, as long as we take care of certain restrictions:

Every$_x$ man ... is ambiguous between $\forall^{\mathbf{man}(x)}x.\Sigma_x(\phi)$
and (provided ϕ is distributive) $\forall^{\mathbf{man}(x)}x.(\phi)$.

n_x woman ... is ambiguous between $\exists^{\mathbf{woman}(x)}x.(\#(n)(x)\wedge\phi)$ and
$\exists^{\mathbf{woman}(x)}x.(\#(n)(x)\wedge\Sigma_x(\phi))$,

A_x woman... is interpreted as $\exists^{\mathbf{woman}(x)}x.(\#(n)(x)\wedge\phi)$
where ϕ is the translation of the rest of the sentence.

This gives as possible translation of (6.4)

6.10 $\forall^{\mathbf{man}(x)}x.\Sigma_x(\exists^{\mathbf{woman}(y)}y.\ (\mathbf{sng}(y)\wedge\mathbf{love}(x,y)))$

6.11 $\forall^{\mathbf{man}(x)}x.\exists^{\mathbf{woman}(y)}y.(\mathbf{sng}(y)\wedge\mathbf{love}(x,y))$

In words, the meaning of (6.10) is: For the group of all the men X(x), there is a group of women X(y), such that the group of pairs $\{<g(x),g(y)>.\ g\in X\}$ is in the interpretation of **love**. Because **love** is distributive in both argument places, the latter just means that every g(x) loves g(y). This shows that the first translation gives the same truth conditions as (6.6). The interpretation of (6.11) is that of (6.10), with the extra condition #X(y)=1. This is exactly the interpretation of (6.7).

Furthermore, it also gives the right readings of (5.15):

6.12 $\exists^{\mathbf{man}(x)}x.(\mathbf{five}(x)\wedge\Sigma_x(\ \exists^{\mathbf{woman}(y)}y.(\mathbf{three}(y)\wedge\mathbf{love}(x,y)))\ )$

6.13 $\exists^{\mathbf{man}(x)}x.(\mathbf{five}(x)\wedge\exists^{\mathbf{woman}(y)}y.(\mathbf{three}(y)\wedge\mathbf{love}(x,y)))$

The interpretation corresponding to (6.12) is the cumulative one, the one corresponding to (6.13) is the distributive one. Note that there is no explicit collective reading. If all the (5) men love the same (3) women, both the cumulative and distributive interpretation hold. The collective reading is a border case of both and not a reading on its own.

Dynamic Logic and Plurals

> *When reading a piece of text, [many people] assume that some unique world is being build up – and it always takes some time to persuade students in a logic class to shift to the standard technical perspective of huge classes of different models for sentences.* [van Benthem, 1989]

7 why dynamic logic is more difficult for plurals than for singulars.

The most simple logic that you might think of that is both dynamic in our sense and is capable of treating plurals would be constructed by following DPL directly and reason as follows:

- predicate logic (PL) interprets formulas relative to assignments, DPL is made out of PL by interpreting formulas as transformations of sets of assignments.
- PLP is interpreted relative to sets of assignments, so what would be more obvious then to interpret formulas of dynamic plural logic (DPLP) as transformations of sets of sets of assignments and copy the definitions from DPL for the semantics of the logical terms?

This does not work however. The problems that are only minor inconveniences in DPL would blow up in our face. In DPL the "downdate" property of the existential quantifier makes that we have to choose a definition for negation that closes off the formula, and this in its turn makes it impossible to refer to quantifiers inside the scope of a negation. Because DPL only makes claims about singular examples, exceptions to the impossibility of this reference can more or less be reasoned away. But plurals can almost always refer back into the scope of a negation (cf. the last part of section 2), so no amount of arguments can save us here. It seems that we need something new.

What we need is an different approach to existential quantification. It causes the problems, so it will have to go. The question is, can we define a non-"down-dating" existential quantifier[10]? We would like to define the existential quantifier in such a way, that it makes it possible to define a notion of negation with $[\![\neg\phi]\!](s) \subseteq s - [\![\phi]\!](s)$, where states are sets of sets of assignments: $X \in s$.

For such a definition to be possible at all we have to have for *all* ϕ that $[\![\phi]\!](s) \subseteq s$. This property we have called *elimination* in section 3C. It is not easy to define an existential quantifier that satisfies this property and still has the properties of an existential quantifier, for example: $\forall x.\phi = \neg\exists x.\neg\phi$. The problem is that if we believe that every interpretation of a formula of the logic is an eliminative function from states to states, and also that states are sets of sets of assignments, such that the elements of the states are independent possible ways for variables to be bound (we argued for both elimination and independence in section 3A), we run into problems when we want to decide what the relation between ϕ and $\exists x.\phi$ should be. Suppose that states are such sets. Because of independence we know what a formula does as a function on such states, by looking at what it does on the singletons: $[\![\phi]\!](\{X\})$. Let's compare ϕ and $\exists x.\phi$. Any set X of assignments that makes ϕ true, i.e. $[\![\phi]\!](\{X\})=\{X\}$, will also make

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10 In fact something like that has already been done in an other compositionalisation of DRT, namely in [Zeevat, 1989]. We will define a logic that in that respect is more like Zeevat's than like DPL.

$\exists x.\phi$ true, because if X gives values to the variable x that make ϕ true, it is also gives values to x that show that there are x that make $\exists\phi$ true. By similar reasoning the converse also holds. Consequently, if the states are sets of assignments, we have for all states s that $[\![\phi]\!](s) = [\![\exists x.\phi]\!](s)$, hence $[\![\neg\phi]\!](s) = [\![\neg\exists x.\phi]\!](s)$, which gives by substituting $\neg\psi$ for ϕ, $[\![\neg\neg\psi]\!](s) = [\![\neg\exists x.\neg\psi]\!](s)$. In other words, $\neg\exists x.\neg\psi$ is made true if there is some X that satisfies $\neg\neg\psi$. But the latter is not at all the same as demanding that ψ holds for all x.
So we have to proceed somewhat differently.
To solve this problem we define a somewhat richer notion of state. After that we will use it in the next section to define the Dynamic Predicate Logic for Plurals (DPLP).

As states we will not take sets of sets of assignments, but *pairs* of sets of sets of assignments:

7.1 DEFINITION (states of DPLP)

A state of DPLP is a pair $s = \langle u_v, u_f\rangle \in \wp(\wp(\mathcal{A}) - \emptyset) \times \wp(\wp(\mathcal{A}) - \emptyset)$, for $\mathcal{A}$ the set of assignments. The first component we call the *verifier*, the second component the *falsifier*.

The interpretation of these two components is as follows. The first component, the *verifier*, contains information that would make the function of which it is the output *true*. This corresponds to the output states of DPL.
The second component, the *falsifier*, contains information that would make the function of which it is the output *false*. In DPL this would correspond to an assignment not-being in the output state[11].
A few examples will probably make this more clear. In the following we mean with 'set' non-empty sets of assignments.

7.2 **man(x) $\wedge$ walk(x)**

this is *verified* by any set X of which the elements g assign to x those values g(x), such that g(x) is a man and g(x) walks.

this is *falsified* by any set X that is not a verifier.

7.3 $\exists^{man(x)}x$. **walk(x)**

this is *verified* by any set X of which the elements g assign to x those values g(x), such that g(x) is a man and g(x) walks (the same as in the previous case).

this is *falsified* by any set X that has as elements all g such that the g(x) are all the men (i.e. X(x)=THE MEN) and all of them do not walk[12].

Given the interpretation of states as tuples, a number of properties of the states can be written down directly. Let $\langle u_v,u_f\rangle$ denote a state as a pair of sets of sets of assignments, with u_v the set of verifiers and u_f the set of falsifiers. The following facts are direct consequences of this interpretation of the states. Let $\langle w_v,w_f\rangle = [\![\phi]\!](\langle u_v,u_f\rangle)$:

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11 States in DPLP are not intended to represent proofs of the truth or falsehood of the formula they are the output of, but only of the possibility of there being an interpretation that makes them true or false. However, this is still a close relative to the role that proofs play in the propositions as types semantics. Just as for *Bill walks*, BILL can be considered a proof of WALK, and for *there is a man that walks*, BILL can be seen as a proof for the existence, provided the "type" of BILL is MAN THAT WALKS, so would in DPLP a set X assigning the value BILL to x verify **walk(x)** and $\exists^{man(x)}x$.**walk(x)**. Cf. [Ranta, 1990].

12 A problem occurs if there are no men. In that case we will make the arbitrary choice to let all sets falsify (7.3). An alternative solution would be to only accept sets that are maximal for x, i.e. that assign all entities as values to x.

dat heb ik ook

7.4 FACT

If a set falsifies ϕ, it will also falsify $(\phi\wedge\psi)$, which means that $u_f \subseteq w_f$ and if a set verifies $(\phi\wedge\psi)$, it will also verify ϕ. which means that $w_v \subseteq u_v$.

7.5 FACT

If a set falsifies ϕ, it verifies $\neg\phi$ and if a set verifies ϕ it falsifies $\neg\phi$. This gives:
if $<w_v,w_f> = [\![\phi]\!](<u_v,u_f>)$ then $<w_f,w_v> = [\![\neg\phi]\!](<u_v,u_f>)$.

Combining these two facts gives the elimination property for states, which was discussed in section 3C:

7.6 FACT (elimination)

for all ϕ, u_v, u_f, w_v, w_f if $<w_v,w_f> = [\![\phi]\!](<u_v,u_f>)$, then $w_v \cup w_f \subseteq u_v \cup u_f$

Further we assume, following the remarks in section 3B, that the different possible (sets of) bindings are independent:

7.7 FACT (independence)

for all ϕ, u $\quad [\![\phi]\!](<u, \varnothing>) = \bigcup_{X\in u} [\![\phi]\!](<\{X\}, \varnothing>)$

where we extended the set operations to pairs, in this case for the union:
$<u, v> \cup <w,r> := <u\cup w, v\cup r>$.
Now we are ready to define the logic DPLP.

8 a dynamic logic for plurals

The logic DPLP is defined as follows

8.1 DEFINITION (syntax of DPLP)

the syntax of DPLP is the same as that of PLP.

8.2 DEFINITION (states of DPLP)

A state of DPLP is a pair $<u_v,u_f> \in \wp(\wp(\mathcal{A})-\varnothing) \pm \wp(\wp(\mathcal{A})-\varnothing)$, of a falsifier and a verifier, as discussed in the previous section.

Because of elimination and independence we only need to define what the result of interpreting a formula is with respect to $<\{X\}, \varnothing>$, using the notation:
$\mathfrak{M}, X \Vdash \phi :\Leftrightarrow [\![\phi]\!]_{\mathfrak{M}}<\{X\}, \varnothing> = <\{X\}, \varnothing>$ (*X verifies ϕ in $\mathfrak{M}$*) and
$\mathfrak{M}, X \dashv\!\!\!\mid \phi :\Leftrightarrow [\![\phi]\!]_{\mathfrak{M}}<\{X\}, \varnothing> = <\varnothing, \{X\}>$ (*X falsifies ϕ in $\mathfrak{M}$*)
(We will suppress the model $\mathfrak{M}$ where possible).

In terms of this we define the interpretation of ϕ as a function on states as

$$[\![\phi]\!]<u,v> := \left(\bigcup_{X\in u}[\![\phi]\!]<\{X\}, \varnothing>\right) \cup <\varnothing, v>$$
$$= <\{ Y\in u \,.\, Y \Vdash \phi \}, \{ Y\in u \,.\, Y \dashv\!\!\!\mid \phi \} \cup v>.$$

Hoezo 'because', die twee zaken volgen gewoon uit de definitie eronder!

8.3 DEFINITION (semantics of DPLP)

a.	$X \Vdash P(x_1,\dots,x_n)$	iff	$\{\langle g(x_1),\dots,g(x_n)\rangle \,.\, g \in X\} \in \Im(P)$,
	$X \dashv\!\!\!\!\mid P(x_1,\dots,x_n)$	otherwise	
b.	$X \Vdash \phi \wedge \psi$	iff	$X \Vdash \phi$ and $X \Vdash \psi$
	$X \dashv\!\!\!\!\mid \phi \wedge \psi$	iff	$X \dashv\!\!\!\!\mid \phi$ or ($X \Vdash \phi$ and $X \dashv\!\!\!\!\mid \psi$)
c.	$X \Vdash \neg\phi$	iff	$X \dashv\!\!\!\!\mid \phi$
	$X \dashv\!\!\!\!\mid \neg\phi$	iff	$X \Vdash \phi$
d.	$X \Vdash \exists^{\phi}x.\psi$	iff	$X \Vdash \phi \wedge \psi$
	$X \dashv\!\!\!\!\mid \exists^{\phi}x.\psi$	iff	$X \dashv\!\!\!\!\mid \phi \wedge \psi$ and either $\neg\exists\, Y.\, Y \Vdash \phi$, or $X \Vdash \phi$ and $\forall Y \approx_x X.\ Y \Vdash \phi \rightarrow Y(x) \subseteq X(x)$
e.	$X \Vdash \Sigma_x(\phi)$	iff	for all $Y \in \delta_x(X)$, $Y \Vdash \phi$
	$X \dashv\!\!\!\!\mid \Sigma_x(\phi)$	iff	$X \not\Vdash \Sigma_x(\phi)$ and for all $Y \in \delta_x(X)$, $Y \dashv\!\!\!\!\mid \phi$ or $Y \Vdash \phi$.
f.	$X \Vdash \Gamma^x y.\phi$	iff	$X \Vdash \phi$ and $\forall g \in X.g(y)=X(x)$
	$X \dashv\!\!\!\!\mid \Gamma^x y.\phi$	iff	$X \dashv\!\!\!\!\mid \phi$ or ($X \Vdash \phi$ and $\exists g \in X.g(y) \neq X(x)$)

(We have included (f) only to keep the syntax the same as that of PLP. We will not use $\Gamma^x y.\phi$ in the examples below.).

Prime-formulas are interpreted as test, just like they are in DPL. Prime-formulas are verified by sets that satisfy them in PLP, and falsified by sets that do not.
Conjunction is interpreted as function composition, which, because of the elimination property of the logic, looks quite classical here.
Negation exchanges the two components of a state as required by FACT 7.5 .
Just as in DPL the *existential quantifier* does most of the work. First, $\exists^{\phi}x.\psi$ is defined in terms $\phi \wedge \psi$ because, as was argued for in the previous section and as was exemplified by examples 7.2 and 7.3, $\exists^{\phi}x.\psi$ and $\phi \wedge \psi$ are verified by the same sets of assignments. As falsifiers of $\exists^{\phi}x.\psi$, however, we only take a *subset* of the falsifiers of $\phi \wedge \psi$: only those sets X that falsify $\phi \wedge \psi$ are said to falsify $\exists^{\phi}x.\psi$ that are maximal verifiers of ϕ. Note that existential quantification is defined in such a way that if no x satisfies ϕ, and therefore no set X verifies ϕ, all sets are regarded to be falsifiers for $\exists^{\phi}x.\psi$.
One peculiar consequence of the fact that the notion of existential quantification used does not "down-date" the state is that, if the same variable is used over twice, the restrictions are, more or less, added up. The following equality holds for all ψ and χ, such that ψ does not contain a quantifier binding a variable in χ, nor χ a quantifier binding a variable in ψ (this restriction is the reason for the 'more or less' in the previous sentence):

8.4 $\quad \exists^{\phi}x.\psi \wedge \exists^{\chi}x.\rho \Leftrightarrow \exists^{\phi\wedge\chi}x.(\psi\wedge\rho)$

This has some nice consequences (one of which is that it makes the Bach-Peters sentences relatively simple to interpret).
The *distribution* operator Σ is defined in such a way that a set X verifies $\Sigma_x(\phi)$ iff all elements (proper parts) of the partitioning relative to x, $Y \in \delta_x(X)$, verify ϕ. A set X falsifies $\Sigma_x(\phi)$ iff it does not verify it and every proper part $Y \in \delta_x(X)$ either verifies or falsifies ϕ.
The *group* quantifier Γ is defined to mirror the definition it has in PLP.

8.5 DEFINITION (truth and validity)

In some model $\mathfrak{M}$

a formula ϕ is *true* with respect to X iff $\mathfrak{M},X \Vdash \phi$,

a formula ϕ is *false* with respect to X iff $\mathfrak{M},X \dashv\!\mid \phi$,

a formula ϕ is *undefined* with respect to X iff $\mathfrak{M},X \nVdash \phi$ and $\mathfrak{M},X \not\dashv\!\mid \phi$.

a formula ϕ is *contingent* iff ϕ is false for some X in $\mathfrak{M}$ and true for some Y in $\mathfrak{M}$.

a formula ϕ is *valid* iff in all models $\mathfrak{M}$, ϕ is true for some X and false for no X,

a formula ϕ is *contradictory* iff in all models $\mathfrak{M}$, ϕ is false for some X and true for no X and

welke? → a formula ϕ is *problematic* iff in some models $\mathfrak{M}$, ϕ is false for no X and true for no X.

kan een formule overdefined zijn wrt X? Ik denk van niet.

The difference between validity in DPL (and also with PL) and in DPLP is that in DPL free variables in the formula are bound by a universal quantifier (using the clause: "for all assignments g, ... holds") whereas in DPLP they are bound by an existential quantifer (" for some X, ... holds"). Therefore a formula in DPLP can have both verifiers and falsifiers at the same time. Some formulas are made true by one value of x and false by another. Such formulas are called contingent. Note that just as in standard logics, contingency can only occur if the variable x is free. In fact we have the following proposition that we state without proof:

8.6 PROPOSITION

Call a formula ϕ *normal* iff for all variables x either (a) there does not occur a quantifier over x in ϕ, or (b) there occurs exactly one quantifier over x in ϕ, and all occurrences of x are in the syntactic scope of that quantifier. Then the following holds for all normal formulas ϕ:
ϕ is valid (contradictory) in PLP iff ϕ is valid (contradictory) in DPLP.

In order to show how this formalism works we will discuss some examples from section 2, some of these could be dealt with in DPL and some of these couldn't. Finally we discuss the problematic example introduced in section 5. The translations are straightforward extensions of the ones of DPL and PLP above:

Mary$_x$	gets translated as $\exists^{\mathbf{mary}(x)}x.(\ldots)$. Where **mary** binds exactly one entity.
it$_x$,	is ambiguous between $\exists^{\mathrm{it}(x)}x.(\ldots)$ and $\exists^{\mathrm{it}(x)}x.\Sigma_x(\ldots)$,
he$_x$,	is ambiguous between $\exists^{\mathbf{male}(x)}x.(\ldots)$ and $\exists^{\mathbf{male}(x)}x.\Sigma_x(\ldots)$
she$_x$,	is ambiguous between $\exists^{\mathbf{female}(x)}x.(\ldots)$ and $\exists^{\mathbf{female}(x)}x.\Sigma_x(\ldots)$
they$_x$ ...	gets translated as $\exists^{\mathrm{it}(x)}x.(\ldots)$

For pronouns the variable x that is quantified over is to be an old variable, and $\mathrm{it}(x) := \mathsf{T}$, the tautological map, $\mathsf{T}:\langle u_v,u_f\rangle \mapsto \langle u_v,u_f\rangle$. Using re-quantification over an old variable gives a pronoun a translation as a quantifier, and at the same time a way of refering back to an earlier quantifier (cf. 8.11)[13]. Σ is added to implement the idea that singulars force a distributive reading when they are used to refer back to a plural quantifier. The cause of this can be found in some (syntactic) parallelism between the sentence the pronoun occurs in and the sentence

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[13] One interesting related use of this mechanism is described by Ranta [1990], where definites are just very specified pronouns. So *the man* would translate as $\exists^{\mathrm{man}(x)}x.(\ldots)$, *the car* as $\exists^{\mathrm{car}(x)}x.(\ldots)$, etc., using old variables in the same way as the pronouns do.

the quantifier occurs in. We already saw an example of this in section 2: *Every woman gave every man a present. She put it on his desk.*

first the prototype example of DPL:

8.7 *A man walks in the park. He whistles.*

The translation of this example is $\exists^{\mathbf{man}(x)}x.\mathbf{walks}(x) \wedge \exists^{\mathbf{male}(x)}x.\Sigma_x(\mathbf{whistle}(x))$, which is equivalent to $\exists^{\mathbf{man}(x) \wedge \mathbf{male}(x)}x.(\mathbf{walks}(x) \wedge \Sigma_x(\mathbf{whistle}(x)))$. This translation is equivalent to:

8.8 $\exists^{\mathbf{man}(x)}x.(\mathbf{walks}(x) \wedge \mathbf{whistle}(x))$

In this equivalence, we used the fact that **whistle** is a distributive predicate, i.e. $\Sigma_x(\mathbf{whistle}(x)) \equiv \mathbf{whistle}(x)$, to simplify the formula. In the following examples we will leave out the Σ_x's that are part of the interpretation of singular pronouns over distributive predicates, because the effect of such occurrences Σ_x would be void anyhow.

An example that could not be treated by DPL is the following:

8.9 *It is not the case that Mary has no car. It is right outside.*

This gets the translation $\neg(\exists^{\mathbf{Mary}(z)}z.\neg\exists^{\mathbf{car}(x)}x.\mathbf{have}(z,x)) \wedge (\exists^{\mathbf{It}(x)}x.\mathbf{outside}(x))$. A set X verifies this formula (i.e. $X \Vdash \ldots$) if it verifies both parts.
$X \Vdash \neg(\exists^{\mathbf{Mary}(z)}z.\exists^{\mathbf{car}(x)}x.\neg\mathbf{have}(z,x))$ iff $X \dashv\vdash (\exists^{\mathbf{Mary}(z)}z.\neg\exists^{\mathbf{car}(x)}x.\mathbf{have}(z,x))$. The latter holds if X is a maximal verifier of **Mary**(x) and X falsifies $\neg\exists^{\mathbf{car}(x)}x.\mathbf{have}(z,x)$. This in its turn means that $X \Vdash \exists^{\mathbf{car}(x)}x.\mathbf{have}(z,x)$.
Putting it all together: $X \Vdash \neg(\exists^{\mathbf{Mary}(z)}z.\neg\exists^{\mathbf{car}(x)}x.\mathbf{have}(z,x)) \wedge (\exists^{\mathbf{It}(x)}x.\mathbf{outside}(x))$ iff for all $g \in X$, g(z) is Mary, g(x) is a car that Mary has and g(x) stands outside.
Which again is as it should be.

Universal quantification is also dealt with in a satisfactory manner:

8.10 *Every man walks in the park. They whistle.*

The translation of this is $\forall^{\mathbf{man}(x)}x.\ \Sigma_x(\mathbf{walk}(x)) \wedge \exists^{\mathbf{It}(x)}x.\mathbf{whistle}(x)$, which by definition can be written as $\neg\exists^{\mathbf{man}(x)}x.\ \neg\Sigma_x(\mathbf{walk}(x)) \wedge \exists^{\mathbf{It}(x)}x.\mathbf{whistle}(x)$. By the same kind of reasoning as before, this formula is verified by sets X such that X(x) are all the men, where each of the men walks and whistles. A problem arises if there are no men. In that case the first of the sentences would be verified by any set X. So the translation would be verified by any set X that verifies **whistle**(x). (This is obviously wrong, see below.)

Finally, we show that the example of section 5, which caused problems for the L/L plurals and which gave rise to our new way of interpreting formulas, can indeed be given a satisfactory translation in DPLP:

8.11 *Some boys are taking their dog for a walk. They want to be home with it before dark, so they walk very fast.*

Which, ignoring subtleties, has as its translation

8.12 $\exists^{boy(x)}x.\Sigma_x(\exists^{dog(y)}y.(\text{ sng}(x) \wedge \textbf{walk}(x,y)\ )$
$\wedge\ \exists^{it(x)}x.\Sigma_x(\exists^{it(y)}y.(\text{ sng}(x) \wedge \textbf{home}(x,y))$

The first part is verified by a set X such that X(x) are boys, and for every $Y\in\delta_x(X)$, Y(y) is (exactly) one dog, and the (one boy) Y(x) is taking (the one corresponding dog) Y(y) for a walk. The second part is verified by a set X that gives every $Y\in\delta_x(X)$ one Y(y) that is taken home by (one) Y(x). They do come home with the right dog!

9 concluding remarks

The logic presented in this paper is a simplified version of the logic presented at the 7th Amsterdam Colloquium. This simplification causes problem.

One problem we already encountered when we discussed example (8.11). Universal quantification restricted by a condition that is never satisfied is always verified.
A way out of this is to observe, that although if there are no men, all sets will verify $\forall^{man(x)}x.\Sigma_x(\textbf{walk-itp}(x))$, none of these assign to x the right kind of values. They do not assign MEN to x, something that should be demanded of a variable introduced by **man**(x). A solution to this problem is suggested by the 'propositions as types' approach to semantics [Ranta, 1990]. With every variable a "type" is associated, which is the set it takes its values from, or, if you are not too fuzzy about (intuitionistic) subtleties, the predicate that a value assigned to a variable has to satisfy in order to be acceptable as value for that variable. If we add some formalisation of this to the logic and a way of checking of a value is of the right type for a variable, we can let pronouns check the types and only accept the right ones.

Another problem, one we encountered when discussing quantifier scope in section 6, is that there is no way to express that a certain group is maximal in the model. This means that the use of the terminology of [Scha, 1981], distributive, collective and cumulative was not fully correct. For example, Scha's translation of the distributive reading of *Five boys love three girls*, would say that the number of boys in the model, that love exactly three girls, is exactly five.
One way to express this is by introducing a strict numeric quantifier for every number n:

g'. $\|\exists_n\phi x.\psi\|^X = 1$ iff $\|\exists\phi x.\psi\|^X = 1$ and for all Y such that $\|\exists\phi x.\psi\|^Y = 1$, it holds that $Y(x) \subseteq X(x)$.

This is still a bit too simplistic for dynamic logic, where it is better to test the maximality of X only relative to sets that are still possible as verifiers.
A problem related to this is due to Heim: *John has some sheep. Peter shears them.* Where also some maximality of the set of sheep seems to play a role.

As a final concluding remark. [Groenendijk and Stokhof, 1989a] concludes with the sentence *Our claim is that the kind of dynamic semantics that DPL is an instance of, naturally suggests itself as a first step on the right track.* I hope that this paper shows that there are wonderful things to be found on that track and that it gives another incentive to continue exploring this particular part of the linguistic forest.

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Towards the Semantics of Open Sentences: *Wh*-phrases and Indefinites

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0. Introduction

I propose to pursue the idea that certain natural language constructions are logically open sentences, that is, their logical translation contains an essential free variable. Such an analysis of indefinite NPs is due to Lewis (1975), Kamp (1981), and Heim (1982). A basic empirical motivation for this treatment is the fact that indefinites display variable quantificational force; the sentences in (1), the first two being from Lewis, illustrate this:

(1)a. A quadratic equation usually has two different solutions.
b. Riders on the Thirteenth Avenue line seldom find seats.
c. If a farmer owns a donkey, he feeds it.

(1.a) has to do with most quadratic equations, (1.b) with few riders on the Thirteenth Avenue line, (1.c) with all pairs of a farmer and a donkey owned by the farmer. Such quantificational variability suggests that indefinites are inherently unquantified, in contrast to the classical analysis of them as existentially quantified. This idea is captured by the following respective logical translations of these sentences (simplified to confine the discussion to aspects that I will be concerned with throughout the paper):

(2)a. MOST(x) [quadratic-equation'(x)] [have-two-different-solutions'(x)]
b. FEW(x) [rider-on-the-13th-Ave-line'(x)] [seat'(y) & find'(x,y)]
c. ALL(x,y) [farmer'(x) & donkey'(y) & own'(x,y)] [feed'(x,y)]

The quantifiers are the logical translations of the adverbs of quantification in (1); they are restricted quantifiers, binding the free variables in their restrictive term, the first bracketed clause. The translation of (1.c) shows that, as Lewis observed, when then is no explicit adverb, a universal reading nevertheless obtains; to account for this, a default universal quantifier is supplied in the logical translation.

The general form of this kind of analysis may be summarized as follows. Let Q stand for the translation of an adverb of quantification, R for that of the restrictive term, and N for that of the rest of the sentence, what Heim calls the nuclear scope of the quantifier. Q quantifies over sequences of free variables (used in the translations of indefinite NPs); let Σ be such a sequence. Further, let Σ^+ be a supersequence of Σ. This is to allow the nuclear scope to contain free variables that do no appear in the restrictive term, necessary in light of sentences such as (1.b). In relation to modeltheoretic interpretation, this means that the assignments satisfying the nuclear scope are basically extensions in the model of those satisfying the restrictive term (as Kamp (1981) proposed). The following relation will guarantee this in the truth definition: where g and g' are partial assignment functions, let $g' \geq g$ mean that for all x in the domains of g and g', $g'(x) = g(x)$. A final piece of notation: $g'|x|g$ means that $g' = g$ except possibly on the value assigned to x. Schematically, then, we have the logical form in (3.a), which receives an (extensional) evaluation relative to a model M roughly as in (3.b).[1]

(3)a. $\alpha = Q(\Sigma)\ [R(\Sigma)]\ [N(\Sigma^{+})]$
   b. For all assignments g, $\|\alpha\|^{M,g}$ is true iff for Q assignments g' of values to each free variable $x_i \in \Sigma$ such that $\|R(\Sigma)\|^{M,g'}$ is true, there is an assignment g'' such that $\|N(\Sigma^{+})\|^{M,g''}$ is true, where $g'|x_i|g$ and $g'' \geq g'$.

Note in particular that a consequence of this truth definition is that indefinites in the nuclear scope, such as the translation of *seats* in (2.a), receive an existential interpretation. This is the semantic effect of what Heim called existential closure of the nuclear scope.

I want to argue that the same analysis may be given for sentences containing *wh*-phrases. The presentation is in two parts. In the first part, I examine parallels in the quantificational behavior of *wh*-phrases and indefinites. I will show that there are certain environments where neither type of phrase can be quantified and argue that a determining role in their quantifiability is played by presupposition accommodation, as a result of which the appropriate logical forms are derived. In the second part I examine asymmetries in the quantifiability of these phrases, and argue for an account based on the interaction of *wh*-movement and presupposition accommodation.

1. Parallels between *wh*-phrases and indefinites

1.1. Quantificational variability

The view that *wh*-phrases are inherently unquantified runs contrary to almost all recent work on the semantics of questions, although it can be inferred from the grammatical treatment of Jespersen (1924) and the analysis of Cohen (1929). There are two things I want to distinguish here. One is the quantificational force of the *wh*-phrase, the other is what we might call the 'quantificational force' of the containing *wh*-clause. As to the former, in those analyses where the *wh*-phrase is treated as an autonomous category, it

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[1]In the appendix I consider alternative formulations of the logical form and evaluation in light of examples that do not play an essential role in my basic analysis.

has commonly been translated as existentially quantified, for example by Karttunen (1977) and Boër (1978). As to the 'quantificational force' of the *wh*-clause, here the leading idea has been what Baker (1968) called *exhaustiveness*, which he characterized as amounting to giving "not just the truth, but the whole truth" (p.34). Baker observed that this notion accounts for the anomalous--as Karttunen (1977) put it, contradictory--nature of the sentence in (4) (assuming John knows who George is).

(4) John knows who is running, but he doesn't know that George is running.

In effect, exhaustiveness amounts to universal quantificational force of the *wh*-clause; this is illustrated by the argument in (5).

(5) John found out who came to the party.
Mary came to the party.
John found out that Mary came to the party.

For this argument to be valid, it must be that for each person who came to the party, John found out that she came to the party.

Most recent analyses have followed Baker in assuming that *wh*-clauses are exhaustive. Nevertheless, I want to argue that this is not an inherent property of *wh*-clauses, but results from the presence in logical form of a universal quantifier.[2] One argument against inherent exhaustiveness of *wh*-clauses is that it does not obtain under certain predicates. For example, the sentence in (6) is not anomalous, in contrast to (4).

(6) John pictured to himself who would run, but he didn't picture to himself that George would run.

The applicability of such a test to demonstrate nonexhaustiveness was pointed out by Belnap (1982), who uses it to argue against analyses such as Karttunen's. Another argument is brought by Hintikka (1983), who gives the sentence in (7) as an example of his observation that sometimes *wh*-clauses aren't exhaustive even where the embedding predicate is one under which they are usually taken to be exhaustive.

(7) Janet knows how one can get from Heathrow to Oxford.

This may be true even if Janet does not know every way to get from Heathrow to Oxford. According to Hintikka's analysis, *wh*-clauses are in general ambiguous between a universal and an existential reading.

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[2]Groenendijk and Stokhof (1982) argue for a stronger notion of exhaustiveness; if my arguments against the weaker notion are valid, they hold a fortiori of the stronger one.

In fact, both Karttunen and Groenendijk and Stokhof are aware of such counterexamples to exhaustiveness, and attribute them to the pragmatics of the question-answer relation. What I want to argue is that, at least in the case of embedded *wh*-clauses, there are positive semantic grounds for denying inherent exhaustiveness. My evidence comes from the interaction with adverbs of quantification. Consider the sentences in (8):

(8)a. Prof Jones usually finds out which students cheat on the exam.
b. Sue mostly remembers what gifts she got for her birthday last year.
c. Bill seldom acknowledges which colleagues he gets his examples from.
d. With few exceptions, Mary knows which students submitted which abstracts to which conferences.
e. John discovered which books were stolen.

The *wh*-phrases here can all be understood as having the quantificational force of the adverb or adverbial phrase in the sentence. For example, (8.a) can mean that for most students who cheat on the exam, Prof Jones finds out of them that they cheat on the exam. In addition, just like with indefinites, a single adverb can quantify more than one *wh*-phrase in its scope, as (8.d) illustrates, where triples of variables representing a student, an abstract, and a conference are quantified over. Also parallelling the behavior of indefinites, when there is no explicit adverb of quantification, as in (8.e), the *wh*-phrase is understood to have universal force--that is, for all books that were stolen, John discovered that they were stolen. I assume, following Lewis, that in such a case there is an implicit universal quantifier in logical form. In fact, I take this to be the source of the exhaustive reading of the *wh*-clauses. Thus, exhaustiveness is simply a special case of the general quantificational variability of *wh*-phrases.[3]

I draw the suggestive conclusion from the data in (8) that *wh*-phrases, like indefinites on the Lewis/Kamp/Heim analysis, contain an essential free variable.[4] I further propose to logically analyze sentences containing a *wh*-clause in terms of the tripartite structure of restricted quantification in (3.a); for this, it is necessary to determine the restrictive term and the nuclear scope of the quantifier. In section 1.2.

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[3]The fact that *wh*-phrases may be associated with a range of quantificational forces is explicitly recognized by Belnap (cf Belnap and Steel (1976), Belnap (1982)), but his analysis doesn't make any connections to work on indefinites, nor to adverbs of quantification, the issues I'm interested in, so I won't try to make a comparison to it here.

[4]This idea is anticipated in Nishigauchi (1986), though his analysis differs substantially from mine in its details, as does the range of data he treats; cf also Comorovski (1989).

I will examine a general way of deriving the restrictive term, and in section 2.1. I will take up the relation to the syntactic structure; but for the time being, let us consider the question intuitively. The interpretation of (8.a) is, again: for most students who cheat on the exam, Prof Jones finds out of them that they cheat on the exam. The paraphrase indicates that it is the *wh*-clause itself that serves to restrict the quantifier, and the nuclear scope is the entire sentence, except for the adverb. Thus the translation of (8.a), on the pattern of (3.a), will be roughly as in (9):

(9) MOST(*x*) [student'(*x*) & cheat-on-the-exam'(*x*)]
[find-out'(pj, [student'(*x*) & cheat-on-the-exam'(*x*)])]

I would like to propose that, quite generally, a semantic function of the *wh*-clause is to restrict a quantifier. If this generalization sounds familiar, it is because the same thing has been said about *if*-clauses by Lewis, and subsequently defended by Kratzer (1978) and Heim (1982). The idea that *wh*-clauses and *if*-clauses are restrictive terms is strikingly supported by the sentence in (10).

(10) If students cheat on the exam, Prof Jones usually finds out about it.

This has the same interpretation (on one reading) as the sentence in (8.a), and receives the same logical translation, that in (9) (plausibly assuming that the pronoun *it* is anaphoric for the whole *if*-clause). This is just the translation it would receive in any case on the Lewis/Kamp/Heim analysis. Moreover, *if*-clauses can also be complements, and in this case too they serve as restrictive terms, as illustrated by the sentence in (11).

(11) Prof Jones usually finds out if students cheat on the exam.

Again, this sentence (on one reading) has the same interpretation as (8.a) and (10), and also has the logical translation in (9).

Excursus: What do adverbs of quantification quantify over?
In formulating my analysis of *wh*-phrase quantificational variability, I have followed the original Kamp/Heim treatment of adverbs of quantification as quantifiers over (sequences of) individuals. This is the simplest treatment, and seems to suffice for the examples we have been considering. But as Lewis observed in his original presentation, there are examples that apparently demand a richer ontology; he used the cover term 'case' to include, besides individuals, temporal intervals, events, states of affairs, and so on, and proposed that these are what adverbs of quantification quantifier over. The question has empirical, not just ontological, significance, in view of the so-called 'proportion problem', i.e., the existence of readings where the adverb of

quantification appears to quantify some but not all variables in its restrictive term. If they are uniformly quantifiers over individuals, this behavior is unexpected and unaccounted for.

This is not the place for me to enter into the debate over the best analysis of the proportion problem, as it would take us far afield (vide Kratzer (1988) and Chierchia (1990) for detailed discussion and references); but I would like to make one or two relevant observations. On a certain view of adverbs of quantification as quantifiers over cases, or situations or occasions, it could be argued that *wh*-clauses are, after all, exhaustive. Thus (8.a), for example, if the quantification is taken to be over exam-taking situations, could mean that for most exams, Prof. Jones knows of each student who does well on that exam that she does well on that exam. This could then be equivalent to the interpretation discussed above, where the quantification was over individuals. However, consider (8.a) uttered in the following situation: for each exam (ie, exam-taking situation), John knows, of 90% of the students who do well on it, that they do well on it, though for none of the exams does he know this of 100% of the students who do well. I believe that (8.a) is true in this situation, though exhaustiveness of the *wh*-clause is excluded. In this case quantifying over individuals yields the correct interpretation, while quantifying over situations, simply conceived, may not.[5]

Another type of case where an adverb of quantification may be straightforwardly treated as quantifying over individuals is exemplified by (8.b). This is an episodic sentence, holding of a single identifiable interval of time. Now, many adverbs cannot be used as adverbs of quantification in episodic sentences, among them *always*, *usually*, *often*, *seldom*; thus the following sentence cannot have the same interpretation as (8.b), and is in fact unacceptable unless the adverb can be given a strictly temporal interpretation.

(12) #Sue usually remembers what gifts she got for her birthday last year.

It appears, then, that there is some kind of sortal division among adverbs with respect to whether or not they can quantify in episodic sentences. Those that can seem best treated as quantifying simply over (sequences of) individuals, while in nonepisodic sentences the possibility of quantifying over something like situations may also be available.

In view of the evidence discussed here, I will in the remainder of this paper maintain the simple position that

[5]There are more sophisticated conceptions, which may do the trick: for example, the notion of a minimal situation employed in Berman (1987) is probably flexible enough to work for the example just described; cf Heim (1989), Kratzer (1988).

adverbs of quantification quantify over sequences of individuals, while acknowledging that, especially with regard to the 'proportion problem', something more complex, such as (suitably conceived) situations, may also be in the domain of quantification. (I accept the view of Kratzer (1988) that the proportion problem is perhaps best viewed, not as a uniform 'problem' with a unified 'solution', but rather as a body of data that must be explained; the explanation may be drawn from several independent sources and involve distinct analytical machinery. This does not mean that the account is less desirable than a uniform one--it is an empirical issue; cf eg the heterogeneous treatment of 'passive' constructions in Chomsky (1981).)

1.2. Presuppositions and the restrictive term

Notwithstanding the above evidence, it turns out that there are environments in which a *wh*-phrase fails to display quantificational variability. For example, in each of the sentences in (13) the *wh*-phrase does not have the force of the matrix adverb.

(13) a. Prof Jones usually wonders which students cheat on the exam.
b. Sue seldom asks her parents what gifts she is getting for her birthday.
c. Bill almost never inquires as to which colleagues agree with his examples.
d. Mary often imagines which abstracts are accepted for which conference.

(13.a), for instance, cannot mean that most students who cheat on the exam are such that Prof Jones wonders whether they cheat on the exam. Instead, the adverb has only a frequency interpretation, so the sentence means that most of the time, Prof Jones wonders which students cheat on the exam, and the *wh*-phrase can be regarded as denoting the set of values satisfying the predication made by the *wh*-clause (cf section 2.4. for details of this). This behavior of the *wh*-phrase contrasts markedly with that in the sentences in (8). I will propose an account of this difference that relates presuppositional behavior and logical form.

What I propose is that the presuppositions of the nuclear scope in a quantified structure become part of the restrictive term. This provides an immediate account of the distinction between the sentences in (8), where the *wh*-phrases are quantified, and those in (13), where they are not quantified. Notice that the matrix predicates in (8) are all factive, while those in (13) are all nonfactive. A property of factive predicates, recognized since Kiparsky and Kiparsky (1971), is that they presuppose their complement.[6] Now, according to my

[6] I do not say "presuppose their truth", because I am dealing with open sentences, which by themselves have no truthvalue. Below I will be more explicit about what I mean by presupposing.

analysis, the nuclear scope of a quantified sentence containing an embedded *wh*-clause is just the sentence minus the quantifier (adverb of quantification), with the *wh*-word replaced by a free variable. Thus, by the present proposal, in the logical forms of the sentences in (8) the complement, being a presupposition of the nuclear scope (because of the factivity of the matrix predicate), becomes part of the restrictive term. This results in translations such as (9), which, as we have seen, yields the correct interpretation for the sentence in (8.a). On the other hand, in the nuclear scopes of the sentences in (13) the complements are not presupposed (because the matrix predicates are nonfactive), so do not become part of the restrictive term, accounting for the lack of quantificational variability in these sentences.

The difference in quantifiability observed between the sentences in (8) and those in (13) obtains not just with *wh*-phrases but also, as might be expected, with indefinites in an *if*-clause; compare the sentence in (11), where the indefinite has the force of the matrix adverb, with those in (14), in which the indefinites cannot be so understood.

(14) a. Prof Jones usually wonders if students cheat on the exam.
b. Sue seldom asks her parents if she is getting gifts for her birthday.
c. Bill almost never inquires if colleagues agree with his examples.

Instead, the indefinites here have either universal/generic force (in (14.a,c)) or existential force (in (14.b)). (These interpretations fall out of the account given in section 2.2.; cf fn (12).)

While I have stipulated that the presuppositions of the nuclear scope become part of the restrictive term, there is in fact an independent process that, in many cases, will yield this result. I am referring to a familiar process by which the felicity of a discourse is maintained, to which Lewis (1979) has given the name of *presupposition accommodation*. The basic idea is that, if an assertion is made in a context whose conversational background is incompatible with the assertion--that is, if the assertion presupposes something not already in the conversational background--, then as a rule the context is automatically incremented so as to be compatible with the assertion, in other words, the presupposition becomes part of the conversational background.[7] In the context of deriving the logical form for quantified sentences, what this amounts to is the claim that the presuppositions of the nuclear scope are accommodated into the

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[7]This makes it clear, if it wasn't already, that I am employing the pragmatic notion of presupposition, not the logical, or semantic, notion. See Soames (1989) for a lucid discussion of this distinction, and related issues, including presupposition accommodation.

restrictive term.[8] (This does not mean that they survive as a presupposition of the utterance--that is a matter of presupposition projection; cf Soames (1989).)

An example adapted from Schubert and Pelletier (1989) clearly shows the effect of accommodation in a quantified sentence (I have added the adverb and used a singular noun phrase to facilitate the discussion):

(15) A cat always lands on its feet.

Schubert and Pelletier observe that the quantified sentence in (15) "is *not* evaluated as if it said that at all or most times cats are landing on their feet, but rather a certain class of 'cases' or 'situations' is set up--such as all those cases where cats drop to the ground--and the sentence is evaluated with respect to those cases (p.194)." They call such situations an "ensemble of cases" and add: "We think that the relevant ensemble is determined in part or entirely by context and presuppositions, or in part or entirely by restrictive clauses and adverbials (ibid.)." This is supported by sentences such as (16), with an explicit *if*-clause, which, as we have seen, is a canonical restrictive term.

(16) If a cat drops to the ground, it always lands on its feet.

The interpretation of (16) is the same as that of (15) evaluated with respect to the class of situations given by the *if*-clause. In terms of the Kamp/Heim analysis, its logical translation is roughly as in (17):

(17) ALL(x) [cat'(x) & drop-to-the-ground'(x)]
[land-on-its-feet'(x)]

This is also just what the translation of (15) will be if it is evaluated with respect to the given class of situations. What has happened in this case, as the discussion of Schubert and Pelletier makes clear, is that the (contextually determined) presupposition of the nuclear scope has been accommodated into the restrictive term. (The NP *a cat* becomes part of the restrictive term by different means; see section 2.1.)

Precisely the same process can be seen at work in the translation in (9) of the sentence in (8.a). Since the matrix predicate here, *find out*, is factive, its complement must be part of the context for the utterance to be felicitous. This discourse requirement is satisfied by having the complement

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[8]This is in fact a consequence of the theory of Heim (1983), though it is not stated there in so many words; I discuss this briefly below. Kratzer (1988) is the immediate antecedent of my adoption of this idea, which she puts in the form of an observation: "Quite generally, presuppositions of the nuclear scope can be accommodated into the restrictive clause (p.36)". Note that her formulation is more cautious than mine.

be the restrictive term. On the other hand, in the sentence in (13.a) the matrix predicate, *wonder*, is nonfactive, so its complement must not be part of the context of utterance. Consequently, it cannot serve as the restrictive term of the quantifier, and this is why the *wh*-phrase here does not get quantified by the adverb of quantification.[9] (I return in section 2.4. to the interpretation of such sentences.)

Presupposition accommodation interacts with the theory of quantification I am employing, as discussed by Heim (1983). According to Heim (1982), quantification is a dynamic process, in which, first, the restrictive term tentatively operates on the context, establishing the domain of quantification, then the nuclear scope does, and finally, after these 'testing out' moves, the quantification operates definitively on the context. When the nuclear scope contains presuppositions, there is thus a choice of contexts to be checked: either the total prior context, or the context determined by the restrictive term. Accommodation to the total context is what Heim (1983) terms 'global'; accommodation to the context of the restrictive term is 'local'. Local accommodation "is rather like adjusting the context only for the immediate purpose of evaluating the constituent sentence(p.120)"--ie, the restrictive term, given the procedural view of quantification. Now, since, as I have proposed in section 1.1., a semantic function of *wh*- and *if*- clauses is to restrict a quantifier, the effect of accommodation of these clauses is local, in Heim's sense. (A plausible case of 'global' accommodation will be presented in section 2.3.)

While accommodation may be taken as the default process by which the presuppositions of the nuclear scope become part of the restrictive term, it cannot be the only means by which this happens. Accommodation, by definition, operates to repair an otherwise incompatible context, thereby circumventing conversational infelicity. But sometimes the context is in no need of amendment; still the presuppositions of the nuclear scope must be in the restrictive term in order for the translation to be correct. This is illustrated by the following discourse (Gennaro Chierchia, p.c.):

(18) There are some students who cheat on the exam. But Prof. Jones usually finds out which students cheat.

By the time the second sentence is processed, the presupposition of its nuclear scope, ie., [student'(*x*) & cheat-on-the-exam'(*x*)], is already part of the context of evaluation, because of the first sentence. Therefore, no accommodation

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[9]Asher (1987), working within the framework of Kamp (1981), appeals to presuppsotion accommodation to distinguish between the representations of factives and nonfactives, though he is concerned with anaphoric relations, not with quantifiability, and thus he draws no conclusions regarding the relation between presuppostion accommodation and the restrictive term.

takes place. Nevertheless, for the quantified sentence to receive its proper interpretation, this presupposition must be in the restrictive term of its translation. This is further evidence that *wh*-clauses are semantically restrictors. (Whether this function is realized depends on the presuppositional properties of the utterance it is embedded in; eg, under a nonfactive predicate it does not become part of the restrictive terms.)

2. Asymmetries between *wh*-phrases and indefinites

Despite the similarities we have seen between *wh*-phrases and indefinites, there are distributional differences in the quantifiability of the two types of phrases. These have to do with the structural relation between the quantifier (adverb of quantification) and the phrase to be quantified. In addition, there are differences in quantifiability within the indefinites; this depends on the kind of clause they are in. Let's begin examining these differences by considering the paradigm in (19) and (20).

(19) a. The maître d' at Maxim's seldom remembers which regular customers tip big.
b. The maître d' at Maxim's seldom remembers if regular customers tip big.
c. The maître d' at Maxim's seldom remembers that regular customers tip big.

(20) a. The maître d' at Maxim's remembers which regular customers seldom tip big.
b. The maître d' at Maxim's remembers if regular customers seldom tip big.
c. The maître d' at Maxim's remembers that regular customers seldom tip big.

The quantificational behavior of the *wh*-phrases and indefinites in these sentences is as follows: the matrix adverb quantifies the *wh*-phrase in (19.a) and the indefinite in (19.b), but not the indefinite in (19.c), which instead has universal or generic force; the embedded adverb does not quantify the *wh*-phrase in (20.a), which has universal force, but does quantify the indefinites in both of (20.b-c). Where the adverb does not quantify over individuals it is a frequency quantifier; thus (19.c) may be paraphrased as saying that all (or, as a rule) well-dressed customers tip big and it is seldom that the maître d' remembers this, and (20.a), that the maître d' at Maxim's remembers, of all regular customers who seldom tip big, that they seldom tip big. The generalization that emerges from the data here is that the (variable translating the) *wh*-phrase cannot be bound from within its clause by the adverb, while the (variable translating the) indefinite can be bound from within its clause--must be, in the case of a *that*-clause and may be, in the case of an *if*-clause. This generalization is further supported by the sentences in (21).

(21)a. Which regular customers seldom tip big?
  b. Regular customers seldom tip big.

The indefinite in (21.b) has the quantificational force of the adverb, but the *wh*-phrase in (21.a) does not have any particular quantificational force: the sentence is asking for the set of values for the NP *which regular customers* that have the property of tipping big most of the time. This behavior is identical to that of the *wh*-phrases in the sentences in (13), in which the *wh*-clause is not presupposed.

My account of these facts exploits the interaction of two processes, *wh*-movement and presupposition accommodation. Both of these contribute in particular ways to the logical representations of the sentences in question. We have seen how presupposition accommodation can contribute to the restrictive term in a quantified structure; I will argue below that this isn't always its effect. The effect of *wh*-movement is tied to certain assumptions about syntactic structure, and how this structure is related to the logical representation. I therefore turn next to these considerations.

2.1. Syntactic assumptions and the mapping to logical form

For the mapping from the syntactic structure to the logical form (LF) of sentences like those in (19)-(21), I adopt proposals of Diesing (1988). The sentence category is taken to be a maximal projection of the inflection node, IP, which is a sister of the complementizer node C. Two base-generated positions for the subject are hypothesized, [Spec,I'] and [Spec,V'] (ie, daughters of IP and VP, respectively, in the notation of Chomsky (1970)). *Wh*-phrases move to [Spec,C'] as a result of *wh*-movement.[10] These structural assumptions are graphically represented in (22).

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(22)            CP  (≈ S')
               / \
      [Spec,C']   C'
          |      / \
       (wh-)   C    IP  (≈ S)
                   / \
           [Spec,I']  I'
               |     / \
            (subj)  I   VP
                       / \
               [Spec,V']  V'
                   |     / \
                (subj)  V  (complement)
```

One of the benefits of this structure is that it permits

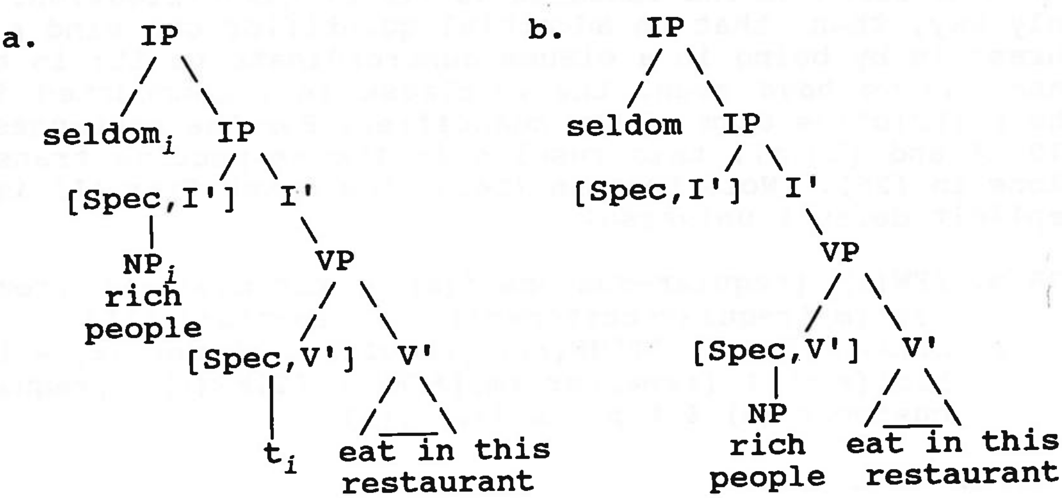
[10]This may happen in the surface in some languages, such as English, but it has been argued that it must happen in the LF of languages where it does not happen in the syntax, such as Chinese, Japanese, and Korean; cf Huang (1982). We will see in section 2.4. arguable cases of LF *wh*-movement in English.

a syntactically straightforward account of the observation that an indefinite subject is sometimes ambiguous between a reading where it has variable quantificational force, and one where it has existential force (cf Milsark (1974), Carlson (1977)). An example is the sentence in (23):

(23) Rich people seldom eat in this restaurant.

On the variable quantificational reading, (23) means that few rich people are in the habit of eating in this restaurant; that is, the adverb is quantifying rich people. On the existential reading it means that it is seldom that there are rich people eating in this restaurant; here the adverb is a frequency quantifier, say, over temporal intervals. In terms of the Kamp/Heim analysis, as shown by Wilkinson (1986), this ambiguity means that on the first reading the indefinite is in the restrictive term of the quantifier at LF, where it gets bound by the adverb of quantification; while on the second reading it is in the nuclear scope, where, by the truth definition in (3.b), it gets existentially evaluated (in other words, the nuclear scope is existentially closed). As Diesing observed, this difference in logical form falls out of the structural representation in (22) if the VP subtree is mapped to the nuclear scope and the superordinate structure to the restrictive term. I assume an adverb of quantification, being a sentential operator, is adjoined to IP. I assume for concreteness that the subject uniformly starts out in [Spec,V'] and then raises to [Spec,I'] at S-structure, at least in languages like English, leaving behind a coindexed trace. Whether it stays in [Spec,I'] at LF or reconstructs back into [Spec,V'], that is, whether in a given sentence an indefinite has a variable or an existential reading, depends on properties of the predicate and the sentence as a whole that I can't go into here (vide Kratzer (1988) for discussion). As an illustration, the LFs for the two readings of the sentence in (23) are given in (24) and the logical translations are given in (25).

(24)



(25)a. FEW(*x*) [rich-person'(*x*)] [eat-in-this-restaurant'(*x*)]
b. FEW(*t*) [TIME(*t*)] [rich-person'(*x*) & eat-in-this-restaurant'(*x*,*t*)]

In (25.b), the frequency reading of *seldom* is represented in the form of restricted quantification, where TIME is a sortal predicate over temporal intervals.[11]

Diesing's proposal adds to the analytical means by which, at least in part, the restrictive term in a quantified structure is determined. This is a syntactic correlation, based on a refined notion of the syntactic representation of the subject-predicate relation. I have also proposed above that certain syntactic constructions are inherently restrictive terms, namely, *wh*- and *if*-clauses, although, as we have seen, whether this function is realized in a particular case depends on the presuppositional structure. Thus, the semantic process of presupposition accommodation interacts with the syntax in determining the restrictive term. In addition, Rooth (1985) argues that focus also contributes to determining the restrictive term, and aspects of his analysis strongly indicate a relation to presupposition accommodation, though it goes beyond the scope of this paper to examine this connection. In short, there are a number of linguistic means, both syntactic and semantic, and sometimes a combination of these, by which the restrictive term is determined.

2.2. The effect of *wh*-movement

Now let's return to the quantificational contrasts between *wh*-phrases and indefinites. The first thing to notice is that a *wh*-phrase undergoes *wh*-movement, which puts it in [Spec,C'], that is, outside the domain of the subordinate IP. Since an adverb of quantification is adjoined to IP, an immediate consequence of *wh*-movement is the impossibility of a *wh*-phrase getting directly bound from within its own clause; this accounts for the fact that the *wh*-phrase in (20.a) does not have the force of the embedded adverb of quantification. The only way, then, that an adverbial quantifier can bind a *wh*-phrase is by being in a clause superordinate to it; in this case, as we have seen, the *wh*-clause is accommodated into the restrictive term of the quantifier. For the sentences in (19.a) and (20.a), this results in the respective translations in (26). (Note that in (26.b) the quantifier ALL is an implicit default universal.)

(26)a. FEW(*x*) [regular-customer'(*x*) & tip-big'(*x*)] [remember'(m,[regular-customer'(*x*) & tip-big'(*x*)])]
b. ALL(*x*) [FEW(*t*) [TIME(*t*)] [regular-customer'(*x*) & tip-big'(*x*,*t*)]] [remember'(m,[FEW(*t*) [TIME(*t*)] [regular-customer'(*x*) & tip-big'(*x*,*t*)]])]

[11] I do not intend this to be a serious analysis of the frequency use of adverbs (vide eg Stump (1985)), but employing it considerably simplifies the presentation and allows us to concentrate on the quantificational use, which is my principal concern.

Evidence that this effect of *wh*-movement is indeed at the level of logical form comes from Japanese, where there is no *wh*-movement in the syntax. Nevertheless, binding of the *wh*-phrase can only be from an adverbial quantifier in a superordinate clause, as in English. The following sentences are from Kawasaki (1990):

(27)a. John-wa taitei [dare-ga ukar-u-ka] shit-te i-ru
TOP mostly who-NOM pass-PRES-Q know PRES
'John mostly knows who will pass'

b. John-wa [dare-ga taitei ukar-u-ka] shit-te i-ru
'John knows who usually passes'

Kawasaki gives evidence that *taitei*, like its English counterpart, has IP as its scope. She observes that the *wh*-phrase is quantified only in (27.a), not in (27.b), and concludes that *dare* moves to [Spec,C'] in LF. For (27.b), this has the result that the *wh*-phrase is outside the scope of *taitei*, hence not bound by it.

In contrast to *wh*-phrases, indefinites do not undergo *wh*-movement; this means they can be bound within their clause. Here there are two possibilities: either they end up in the restrictive term of adverb of quantification and get bound by it, or they end up in the nuclear scope, where they existentially evaluated.[12] The first possibility is exemplified by (19.c), where the quantificational force is supplied by an implicit default universal or generic quantifier; (20.b,c); (21.b); and the reading of (23) according to (25.a). The second possibility is exemplified by the reading of (23) according to (25.b), and also by the sentences in (28).

(28)a. The maître d' at Maxim's remembers that regular customers tipped big yesterday.
b. The maître d' at Maxim's doesn't remember if regular customers tipped big yesterday.
c. Regular customers tipped big yesterday.

These sentences are episodic, which favors the existential reading, in contrast to the habitual sentences we have looked at, which generally favors the quantificationally variable reading of indefinites.

Note that, since by *wh*-movement a *wh*-phrase moves out of the domain of its IP, it follows a fortiori that it cannot get bound within the nuclear scope of its clause. Moreover, as a result of presupposition accommodation, it cannot get bound within the superordinate nuclear scope. This accounts

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[12]This accounts for the intepretations of the sentences in (14) in section 2.2.: the subject indefinites end up in [Spec,I'] and thus get bound by an implicit default universal or generic quantifier; while the object indefinite remains within VP, ie, the nuclear scope, hence getting existentially interpreted.

for the contrast between the following sentences and those in (28) (I've changed the adverb of quantification in (29.a) to one compatible with an episodic reading).

(29)a. The maître d' at Maxim's hardly remembers which regular customers tipped big yesterday.
b. Which regular customers tipped big yesterday?

In (29.a), notwithstanding the episodicity of the sentence, the *wh*-phrase is quantified by the matrix adverb, and has about the same interpretation as (19.a) (aside from the habitual/episodic distinction). And the *wh*-phrase in (29.b) has the same set-of-values reading as (21.b).

2.3. Presupposition accommodation of *that*-clauses

Having shown how *wh*-movement affects the quantifiability of *wh*-phrases, I turn now to the differences between indefinites in *if*-clauses and in *that*-clauses. We need to account for why in the latter case an indefinite can only be quantified from within its clause, while in the former case it can in addition be quantified from outside its clause. We have already seen that an embedded indefinite can be bound from within its clause because it remains within the IP, that is, within the scope of any embedded quantifier. And we have also seen why an indefinite in an *if*-clause can be quantified from outside, namely, for the same reason a *wh*-phrase is: by presupposition accommodation, the clause becomes part of the restrictive term of the matrix adverb. Evidently, the same thing does not happen with *that*-clauses.

We have seen in section 1.1. that a semantic function of *wh*- and *if*-clauses is to restrict a quantifier; when these clauses are presupposed, they are accommodated into the restrictive term of the quantifier. Apparently, *that*-clauses do not have this restrictive function. Instead, I suggest that, when a *that*-clause is presupposed, it is accommodated independently of the restrictive term, in syntactic terms, above it and hence outside the scope of the matrix quantifier. In other words, I propose that presupposition accommodation of *that*-clauses is syntactically as high up in the sentence as possible. On this account, the sentences in (19.c) and (20.c) receive the translations in (30), where G represents the generic operator.

(30)a. [G(*x*) [regular-customer'(*x*)] [tip-big'(*x*)]] &
[FEW(*t*) [TIME(*t*)] [remember'(m,[G(*x*) [regular-customer'(*x*)] [tip-big'(*x*)]],*t*)]
b. [FEW(*x*) [regular-customer'(*x*)] [tip-big'(*x*)]] &
[remember'(m,[FEW(*x*) [regular-customer'(*x*)] [tip-big'(*x*)]])]

Additional evidence for this effect of presupposition accommodation of *that*-clauses comes from the following observation by Angelika Kratzer (p.c.). She notes that the sentence in (31.a) does not have the interpretation in (31.b), in which the *that*-clause appears in the restrictive term, but rather that in (31.c), in which it has been accommodated outside of the restrictive term:

(31)a. If Galileo claims that the earth is round, he knows that the earth is round.
b. If Galileo claims that the earth is round and the earth is round, he knows that the earth is round.
c. The earth is round and if Galileo claims that the earth is round, he knows that the earth is round.

In terms of Heim's presupposition theory, briefly discussed in section 1.2., accommodation of a *that*-clause seems to be of the 'global' kind. That is, the effect of accommodation of a *that*-clause p on a context c is "like pretending that c&p [ie, c as changed by the *that*-clause] obtained instead of c all along" (1983, 120). The accommodation is global because the semantics of the *that*-clause is such that it stands independently, unlike *wh*- and *if*-clauses, which have a restrictive function and thus are locally accommodated.

2.4. Nonquantifiable *wh*-phrases

I have shown how a *wh*-phrase gets bound from outside its clause, namely, as a result both of *wh*-movement, which puts it beyond the scope of a quantifier adjoined to its IP, and of presupposition accommodation of the *wh*-clause, which puts it in the restrictive term of the matrix quantifier. It remains to deal with direct questions, as in (21.a), and sentences in which the *wh*-clause is not presupposed, as in (13). In the former, there is no higher clause from which the *wh*-phrase can be bound, and in the latter, there is no presupposition accommodation, which yields the same result. In such cases, I argue, the *wh*-phrase is not quantified at all, but is evaluated as a set of appropriate values, along the lines of Hamblin's analysis of questions. Hamblin (1973) proposes to treat *wh*-phrases as denoting sets of individuals and, correspondingly, *wh*-clauses as denoting "sets of propositions, namely, those propositions that count as answers to" them (1976, 254). He calls such sets *denotation-sets*.

Hamblin intends that every *wh*-clause should be analyzed as a denotation-set; in contrast, I want to restrict this treatment to those cases where a *wh*-phrase does not display quantificational variability. I will do this by adapting a proposal going back to the analysis of questions by Katz and Postal (1964) and further developed by Baker (1970). Katz and Postal introduce an abstract morpheme *Q*, which they say "indicates semantically... that the sentence is a question" (p.89). Structures containing the Q-morpheme are mostly matrix clauses, but in addition Katz and Postal assume that the complements of *wonder* and a few other verbs contain Q. With all other *wh*-embedding verbs, the complement clause is not headed by Q (this differs from Baker's analysis; vide infra). What I propose is that Q heads all matrix *wh*-clauses and also those *wh*-clauses that are complements of a nonfactive predicate. This idea may be implemented along the following lines. Suppose that Q is a base-generated operator in [Spec,C'] of matrix *wh*-clauses. Similarly, suppose that nonfactive predicates that subcategorize for *wh*-clauses also semantically select Q (which is base-generated in the embedded [Spec,C']).[13] Now Q, though not a quantifier, is a variable-binding operator: it binds free *wh*-variables in its scope. Such variables consequently cannot get bound by a higher quantifier; this accounts for the lack of *wh*-phrase quantificational variability in matrix clauses and clauses embedded under a nonfactive predicate.[14]

The result of evaluating a Q-bound structure will be a set of propositions, as in Hamblin's interpretation of questions; the members of such a set are determined by assigning an appropriate value to each of the variables bound by Q, which translate the *wh*-phrases within the clause. This interpretation algorithm is formalized in (32) (where each x_i represents a variable translating a *wh*-phrase):

(32) $\|Q(x_1...x_n)S(x_1...x_n)\|^{M,g} = \{p : E(a_1...a_n)[p = \|S(x_1...x_n)\|^{M,g'}]\}$, where g' is just like g with the possible difference that $g'(x_i) = a_i$, $1 \leq i \leq n$.

In (33) are illustrated two Q-bound LFs, for (13.a) and (21.a).

```
(33)a. VP                                    b.  CP
       / \                                       / \
      /   \                                     /   \
     V     CP                           [Spec,C']     C'
     |     |\                              / \        | \
   wonder  | \                            /   \       C  IP
           |  \                         Qi   which       | \
     [Spec,C']  C'                           regular     |  \
       / \      |\                        customersi  seldom IP
      /   \     C IP                                         / \
    Qi   which    |\                                        /   \
      studentsi   | \                                      /_____\
                  |  \                                   ti tip big
                  |___\
                 ti cheat
               on the exam
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[13]This follows the familiar account of Grimshaw's (1979), but only part way, because she has all *wh*-embedding predicates s-selecting Q.

[14]Nishigauchi (1986) employs a variable-binding Q-operator, though it differs both syntactically and semantically from mine, being more like an adverb of quantification; this seems to be problematical, cf Kawasaki (1989). Asher (1987) also uses a Q-operator, though it is not a variable-binder but is used to block anaphoric relations between the matrix clause and the indirect question; this opacity inducing function is conceptually similar to the effect of my Q-operator in making the embedded clause opaque to quantification the Q-bound variables from outside.

My analysis follows Baker's in treating Q as a variable-binding operator. Baker also uses Q as a clausal scope-marker for *wh*-phrases; further, he proposes that every *wh*-clause, direct and indirect, contains Q. I have argued against the latter[15] (see also Munsat (1986)); however, the idea that Q can mark scope turns out to interact with my analysis of *wh*-phrase quantifiability in an interesting way. It appears that the only cases where it is necessary to distinguish the clausal scope of a *wh*-phrase is in sentences combining a direct *wh*-question and in situ *wh*-phrases. This was first pointed out by Baker (1968), who observes that the sentence in (34) has two interpretations, depending on which clause the in situ *wh*-phrase, *which book*, is associated with.

(34) Who remembers where we bought which book?

If this NP is interpreted in the lower clause, an appropriate answer to (34) would be: *Mary remembers where we bought which book*. If *which book* is interpreted in the higher clause, an appropriate answer would be: *Mary remembers where we bought the math book and John remembers where we bought the physics book*. In terms of my Q-operator analysis, on the first reading only *who* is bound by Q, while on the second reading both *who* and *which book* are bound by Q. It seems that, in general, an in situ *wh*-phrase may have the scope of any *wh*-phrase in [Spec,C'].[16] Now, it is interesting that this scope ambiguity appears to obtain in the presence of an adverb of quantification; consider the following sentence:

(35) Which professors usually find out which students cheat on which exams?

I believe that the phrase *which exams* in this sentence can be interpreted either in the lower or the higher clause. In the first case, the sentence is asking for the set of professors for each of whom it is true that, for most pairs of a student and an exam the student cheats, s/he find out that the student cheats on the exam. On the second reading the sentence is asking for the set of pairs of a professor and an exam such that, for most students who cheat on the exam, the professor finds out that the student cheats on the exam. Now, on the assumptions I have been making, *wh*-in-situ is a case of *wh*-movement at LF in English. On one reading, the in situ phrase moves to the embedded [Spec,C'], on the other reading, to the matrix [Spec,C']. These two readings correspond, on my

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[15]Such a move is syntactically unnecessary, given the designated landing site for *wh*-movement, [Spec,C'], as well as subcategorization for [+*wh*] CP; Baker's framework lacks these features.

[16]Delimiting the distribution and interpretion of *wh*-in-situ is actually quite complicated, though even an adequate superficial discussion of it goes beyond the scope of this paper; cf Pesetsky (1987), Comorovski (1989), and references there.

analysis, to the respective translations in (36):[17]

(36)a. Q(x) [[professor'(x)] MOST(y,z) [student'(y) & exam'(z) & cheat-on'(y,z)] [find-out'(x,[student'(y) & exam'(z) & cheat-on'(y,z)])]]

b. Q(x,z) [[professor'(x) & exam'(z)] MOST(y) [student'(y) & cheat-on'(y,z)] [find-out'(x,[student'(y) & exam'(z) & cheat-on'(y,z)])]]

The reader may check that successive application of the evaluation algorithms in (32) and (3.b) to these translations yields the two interpretations for (35) I have paraphrased.

3. Conclusion

We have seen that there are a number of similarities between *wh*-phrases and indefinites with respect to quantification, in particular, both exhibit quantificational variability under adverbs of quantification, and for both types of phrases this quantifiability depends on presupposition accommodation. Perhaps such parallel behavior shouldn't be surprising; after all, *wh*-phrases have commonly been analysed as existentially quantified, the classical treatment of indefinites. So if indefinites are reanalysed as open sentences, we might expect, ceteris paribus, that *wh*-phrases are also amenable to such a treatment. What I have shown is that this expectation is borne out. Further support for parallel treatment of both phrase types comes from the fact that in many languages the same lexical item, or regular morphophonological variants of the same word, has both a quantificational (either existential or variable force) function (ie, indefinites) and an interrogative one (ie, *wh*-phrases), with (syntactic, semantic, or pragmatic) context determining the function in a particular case.

While both *wh*-phrases and indefinites are analyzed logically as open sentences, there are nevertheless differences between them; one of most important being that the former undergo *wh*-movement (at least in LF), which puts them outside the scope of a same-clause sentential quantifier. This means that *wh*-phrases never get bound from within their clause, whereas indefinites can. Another difference is that, when a *wh*-phrase is not quantified, it is interpreted as a set of appropriate values; this yields the meaning of a question. Finally, I have argued for a difference in the effect of

[17] Pesetsky (1987) argues that these are not cases of LF *wh*-movement, proposing instead (essentially following Baker), that the in situ phrase gets its scope by being bound by a Q-operator. While I cannot go into Pesetsky's arguments here, his position does allow, as he observes, having Subjacency apply uniformly at LF as well as S-structure--though it has been argued, eg by Huang (1982), that this applies only at the latter level, precisely in view of such examples as (35) on the reading in (36.b).

presupposition accommodation of *wh*-clauses and *if*-clauses on the one hand, and *that*-clauses on the other: with the former, accommodation is local, into the restrictive term of a quantifer; with the latter, it is global, beyond the quantifier, hence *that*-clauses are evaluated independently of the quantified structure.

Appendix

I would like here to briefly consider some kinds of examples that necessitate some technical modifications to my basic analysis. My remarks will be rather programmatic; I address these matters at greater in work in progress. The examples have to do with the presence of free variables in the restrictive term that are *not* quantified by Q. Lewis (1975) pointed out the existence of such cases, and one type constitutes a primary motivation for Heim's (1982) Novelty/ Familiarity Condition (henceforth, NFC). This is a principle intended to distinguish between indefinite and definite NPs. For Heim, both types of phrases are translated as open sentences. Yet their behavior under an adverb of quantification is very different, as the following sentences illustrate:

(37)a. Prof. Jones usually finds out if a student cheats.
b. Prof. Jones usually finds out if the student cheats.

Here, *a student* is quantified by the adverb, but *the student* is not (it refers, instead, to a specific student). Heim's account of this is that *the student*, qua definite, must be represented in the context of utterance prior to the processing of (37.b), while *a student*, qua indefinite, must not be represented in the context of utterance prior to the processing of (37.a)--this is the essence of the NFC, that is, indefinites are novel to utterance, definites are familiar to it. This is a procedural wellformedness condition, checking each minimal step in the processing of a logical form. To be explicit, a set-theoretic construct, Dom(c(u)), is introduced to represent the domain of the context of utterance, which is the set of referential indices of NPs that are in the context prior to the utterance of u. In terms of this object, the NFC says that, where i is the index of an indefinite, i must not be in Dom(c(u)) when u is processed, and where j is the index of a definite, j must be in Dom(c(u)) when u is processed. It is assumed that, among the features specified in the representation of noun phrases is one for definiteness, [±def], and that the NFC refers to this.

As Heim observed, the NFC allows eliminating selection in the logical form of the variables that are quantified by substituting reference to Dom(c(u)) in the evaluation algorithm. Hence we can revise (3) accordingly, as in (38); here $g' \approx_{Dom(c(u))} g$ means that for each $i \in Dom(c(u))$, $g'(i) = g(i)$.

(38)a. α = Q[R][N]
b. For all assignments g, $\|\alpha\|^{M,g}$ is true iff for Q assignments g' $\approx$Dom(c(α)) g such that $\|R\|^{M,g'}$ is true, there is an assignment g'' $\geq$ g' such that $\|N\|^{M,g''}$ is true.

Now, adopting Heim's treatment of definites and using the analysis I have presented, the two sentences in (37) have the same basic logical translation, as in (39):

(39) MOST [student'(*x*) & cheat'(*x*)]
[find-out'(pj,[student'(*x*) & cheat'(*x*)])]

Where student'(*x*) translates *the student*, *x* must be in Dom(c(α)); otherwise, the NFC would be violated. Therefore the value of g(*x*) = g'(*x*) is the student in question, ie, the value does not depend on the quantifier. On the other hand, where student'(*x*) translates *a student*, *x* must not be in Dom(c(α)); otherwise, again, the NFC would be violated. Then, g(*x*) $\neq$ g'(*x*) in general, ie, the value does depend on the quantifier.

There is, however, a problem with using the NFC to filter out incorrectly quantified LFs. Since it is based on the definiteness or familiarity of the NP, it prevents any definite NP from being quantified. However, we have already seen many counterexamples to this, namely, the sentences in which a *which*-phrase is quantified, such as those in (8). These *wh*-phrases have long been classified as definite (eg Katz and Postal (1964)) or familiar (eg Quirk et al (1972)). There is, however, a restriction on the quantifiability of *which*-phrases that I have till now ignored, namely, they must be plural. Thus, contrast the following sentence with (8.a):

(40) Prof Jones usually finds out which student cheats on the exam.

The *wh*-phrase here, unlike that in (8.a), is not quantified by the adverb; instead, it refers to a specific student (the one satisfying the predication). Interestingly, the same dichotomy holds for non-*wh* definite NPs; contrast the following sentence with (37.b):

(41) Prof. Jones usually finds out if the students cheat.

This sentence has a reading in which *the students* has the quantificational force of the adverb.

There is yet another kind of example where a definite NP can be quantified. In (40), although the *wh*-phrase can't be quantified by the adverb, *the exam* can be. That is, the sentence can mean that, for most instantiations of a contextually specified exam that a certain student cheats on, Prof Jones finds out who the student is. This reading is more or less equivalent to quantifying over exam-situations, discussed in the excursus in section 1.1. In general, nonrigid singular definites seem to be quantifiable in this way. Other examples include 'role nouns': *The president is often a Methodist* (Carlson (1977, 455)) and generic definites, including natural kind terms: *The grey wolf is seldom found in the lower forty-eight states.*

In short, the NFC seems to cast too wide a net, correctly excluding some kinds of definites from being quantified, but wrongly excluding others. What is needed is a way of grouping together both plural definites (both *wh* and non-*wh*) and nonrigid singular definites, as well as singular and plural indefinites, on the one hand, and (rigid) definites (both *wh* and non-*wh*), on the other hand. I would like to suggest that the distinction between these two classes is exactly the same as that between *wh*- and *if*- clauses and *that*-clauses, ie, local vs. global accommodation (vide sections 1.2. and 2.3.). This will immediately yield the difference in quantifiability, since local accommodation puts the NP in the restrictive term of a quantifier, while global accommodation puts the NP beyond the scope of any quantifier. On this view, the translation of (37.b) is not (39) but the following:

(42) [professor'(x)] MOST [question'(y) & dislike'(x,y)]
[ignore'(x,y)]

In addition, we can retain the NFC as a filter on logical forms, though it evidently cannot refer to the nominal feature [±def], given that both indefinites and definites can be quantified. It appears that no such grammatical feature will do the job; instead, I will tentatively suggest a semantic feature, [±ref], indicating whether an NP is directly referential or not. For an open sentence to be directly referential, [+ref], means that its variable receives its values directly from the context (ie, from the model as determined by the context); this can only happen if it is not bound, and global accommodation ensures this. If an open sentence is [-ref] the variable in its translation receives its values by assignment as specified, eg, by a quantifier; this will be the case, for example, if it is locally accommodated.

To summarize these sketchy remarks: I have tried to show that there is no simple division between definite and indefinite NPs with respect to quantifiability, but that sometimes definites, too, can be quantified. In such cases, they seem to be amenable to my general analysis. I believe that this is also the key to the treatment of concealed questions, which are usually definite. Moreover, given two kinds of accommodation, nonquantified definites also fit into the analysis. Specific indefinites seem to be another class of NPs that undergo global accommodation; ie, they too are [+ref].

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Forward References in Natural Language

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SUMMARY

This paper deals with forward references (also called ***kataphoric*** references) in natural language. In order to calculate truth conditions for sentences that involve kataphoric references, an extension of Discourse Representation Theory, PATIENT DRT, is proposed, inspired by so-called backpatching techniques for the parsing of programming languages. The main idea is that a kataphoric element introduces an ***incomplete*** discourse entity, to be completed by subsequent material under certain conditions. This approach is applicable to pronominal as well as complex Noun Phrases, and has no special difficulties with crossing coreferences. The main virtue of this approach is that it allows parsing of kataphors from left to right, which makes it suitable for on-line language processing by computer and plausible as an element of a theory of human language processing as well. However, the approach suggests that a left-to-right treatment of kataphoric constructions is hard to reconcile with the requirements of compositionality.

1 Introduction

Recent approaches to anaphora such as Kamp's Discourse Representation Theory (DRT) ([Kamp-81]) and Heim's file-change semantics ([Heim-82]) adhere to the procedural principle of familiarity. According to this principle, expressions whose denotation is dependent on other material (i.e. anaphoric expressions) may only depend on previously processed, and therefore "familiar", expressions. A particularly interesting variety of familiarity is obtained if it is assumed that processing operates in the same "direction" as speech, that is — in the western tradition of writing — if it operates from left to right. Henceforth, this variety of the familiarity approach will be called the left-to-right, or briefly the l-t-r approach.[1] It is sometimes thought that theories such as Kamp's and Heim's are instances of the l-t-r approach, and this appearance may have added considerably to their intuitive appeal. However, this appearance is deceptive, as we will show.[2]

At least four parallel levels of text processing can be distinguished: speech recognition, syntactic analysis, the algorithm that constructs Discourse Representation Structures (DRSs)), and finally truth conditional interpretation:

```
Interpretation:    ---------->------>----------------->
DRS construction:  --->-->-->-->--->----------->----->
Syntax analysis:   ->->-->-->-->->->------>---->----->
Recognition:       >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
   TEXT:           blabla.......................blabla
```

Arrows depict steps in the processing of the TEXT: recognition may result in words, syntax analysis produces constituents, DRS construction results in partial DRSs, and interpretation results in truth conditions.

In its unrestricted form, the l-t-r hypothesis would require that *all four* processing stages proceed from left to right. Applied to **recognition**, for instance, this requirement is intuitively quite plausible. For **interpretation**, it would amount to the highly desirable property of incremental (i.e. on-line) interpretation. Note that there must be several nontrivial dependencies between the four processes. For instance, it is widely acknowledged that speech recognition needs "higher" linguistic cues (e.g. [Lea-80] on HEARSAY). Consequently, the higher processes must have the same direction as recognition and cannot be allowed to lag too far behind it. In short: the l-t-r principle is a highly attractive hypothesis for language processing in general.

For theories of anaphora, the most important stage of processing is **DRS construction**, where discourse entities are introduced and subsequently picked up by anaphors. Consequently, the processing direction of DRS construction is our central concern. We will largely leave aside the feasibility of l-t-r processing in the other areas of text processing, and the same holds for questions of synchronisation between the four processes.[3]

Note that especially the explanatory value of familiarity based theories of *anaphora* would be greatly enhanced by l-t-r DRS construction, since it would explain *why* certain material can be considered as "familiar" at a certain moment. Also, it would help to legitimize the often claimed role of recency in anaphora resolution ([Chafe-76], [Sanford and Garrod-81], more recently e.g. [Pinkal-86]). The problem addressed in this paper is, how can the hypothesis of l-t-r DRS construction be reconciled with the empirical phenomenon of *kataphoric* expressions (cf. [Buehler-34]) — anaphoric expressions whose denotation depends on material to their right? For an l-t-r analysis of kataphoric expressions would only be possible if an antecedent can already be familiar before it has been processed, and it is hard to see how this is possible.

After a brief exposition of the linguistic data (section 2), we shall discuss some previous treatments of kataphora (section 3). Then (section 4) we outline a modified, *patient* version of DRT which deals with kataphoric reference, after which (section 5) we will apply Patient DRT to some problematic sorts of kataphors. In the concluding section, we try to answer some hard questions prompted by the solutions proposed in the body of the paper. Throughout, the word 'anaphora' will — contrary to Greek grammar — be used to denote both forward and backward cases of anaphora, that is both kataphoric and strictly anaphoric reference. Similarly, an 'antecedent' may either preceed of follow the anaphor. No ambiguity will arise from this usage, I hope.

2 The phenomenon of kataphora

Kataphoric reference has sometimes been depicted as a rather marginal phenomenon. In some cases, however, kataphoric reference (1) is decidedly more felicitous than a "backward" anaphoric analogue (2):

(1) *Ever since her childhood, Dorit has been extremely lazy.*

(2) *Ever since Dorit's childhood, she has been extremely lazy.* [4]

Now one *might* argue that 'her' must be anaphoric rather than kataphoric: that it can only refer to Dorit if she was introduced *earlier*. However, compare:

(3) *Mary, Dorit and Bill are a strange lot. She is weird. The others ... ,*

(4) *Mary, Dorit and Bill are a strange lot. Ever since her childhood, Dorit has been extremely lazy. The others ... .*

'Her' in (4) can refer to Dorit, while 'she' in (3) cannot. The explanation must be that 'her' is linked to the second, rather than the first, occurrence of the proper name 'Dorit' in (4). Consequently, 'she' must be a kataphoric pronoun.[5]

Kataphoric constructions can be considerably more involved than than the examples provided so far. One complicated kind of kataphora is **"mutual anaphora"**, where two Noun Phrases (NPs) mutually depend on each other (or on an NP embedding the other) for their denotation (Cf. [Bach-70], [Karttunen-71]):

(5) *[A woman who works in his_2 department$]_1$ was interviewed by [the manager who hired $her_1]_2$*

(subscripts indicate coreference). Further, kataphors are **not restricted to pronouns**. For instance, if we label the relation between 'the parents' and 'these puppies' in:

(6) *These puppies were born this spring. The parents took good care of them*

as *relational* **anaphora**, then there is also such a thing as relational **kataphora**. For instance, in

(7) *Whenever the parents sleep, these puppies do not eat,*

'the parents' can be described as kataphoric to 'these puppies'. (For an analysis along the lines of DRT, see [van Deemter-89].) Cases of **extrasentential** kataphora are reported as well, but their grammaticality seems debatable. A case in point is

(8) *First he lost his wallet. Then his car got stolen. Fred was having a bad day*

in which, according to [Asher and Wada-89]), 'he' and 'his' can be kataphoric to 'Fred', althought one may doubt whether this can amount to a completely natural reading. Some literary texts provide interesting examples:

(9) *He was an old man who fished alone in the Gulf Stream ....* (Hemingway:"The Old Man and the Sea")

Note that this is the opening sentence of a book , so a backward anaphoric reading of the pronoun 'he' is excluded. In section 5 we will return to the topic of kataphors that cross coordination boundaries, as well as to the other phenomena just described. While dealing with kataphoric reference, it will not suffice to indicate possible antecedent-anaphor (kataphor) pairs, since the possibility of an anaphoric relationship will also depend upon analysis. For instance, in

(10) *Every farmer who admires her courts a widow,*

a kataphoric relationship is only possible if 'a widow' has wide scope over the universal quantifier in 'every farmer', as Kamp observed ([Kamp-81]). Scope phenomena motivate much of the complexity of the rules in the main section of this paper.

3 Kataphora in DRT and related approaches

In the introduction the l-t-r approach to anaphora was advertized. This approach may be considered to consist of the following three principles:

P1) Parsing operates from left to right on surface structure;

P2) Discourse entities are introduced in the context during parsing;

P3) Pronouns pick up existing discourse entities from the context.

Together, these three principles rule out kataphora. Consequently, actual proposals for the treatment of kataphora have departed from one or more of them. Thus, in [Heim-82], **P1** is amended to apply only to a level of *logical form*, to be obtained from surface structure by a preprocessing stage that puts all the definite NPs in front. Unfortunately, this means that parsing works in two "passes". As a result, on-line interpretation of a sentence is impossible, for the second pass cannot get started before the first pass has seen the very last NP of the sentence. Further, Heim's approach fails on *mutual* anaphora. For given that neither of two NPs in a mutual anaphora construction can be interpreted without the other, no level of analysis can have them in the "right" order.

Another departure from **P1** can be found in [Kamp-81]. Here, processing order is highly indeterministic. Although the numbering of processing steps in the boxes which depict Kamp's Discourse Representations (DRs) may suggest determinism, the numbering constitutes only one of several possible scenario's yielded by the processing rules. In particular, whenever a DR is split into two subordinate boxes – say b_1 and b_2 – to represent a universally quantified sentence or a conditional ([Kamp-81]), b_1 and b_2 can be processed concurrently, except when an NP from one box has to be used as antecedent for a pronoun in the other. In the idiom of parallel programming, the two processes entertain a producer/consumer relation (see e.g. [Ben-Ari-82]): when a pronoun in b_1 cannot be resolved, control may be shifted from b_1 to b_2; as soon as a suitable antecedent has been found there, control is allowed to return to b_1. It is due to these departures **P1** that kataphoric constructions can be treated appropriately in [Kamp-81].[6]

From our own point of view, however, **P1**, being the heart of the l-t-r hypothesis, deserves to be upheld, of course. On the other hand, principle **P2** is too central an assumption of DRT to give up. **P3**, to the contrary, must be given up anyway in order to account for incompletely perceived discourse: suppose someone overhears (11), hearing everything of it except the first sentence:

(11) *(John walks.) He talks. He keeps forgetting the time.*

Then obviously, the processor should not be precluded from making any sense of this story — as Kamp's theory would have it. This observation suggests that the pronoun 'he' should introduce a Reference Marker (RM) of its own. Given the interpretation rules of DRT, this means that (11) is true if and only if there is at least one person who talks and keeps forgetting the time. Kataphoric and incompletely perceived discourse should be interpreted along similar lines. Therefore, we propose to maintain **P1** and **P2** but to abandon **P3**.

Recently, some steps towards a solution along these lines were taken in [Asher and Wada-89]. There, pronouns introduce their own RMs. Resolution of the pronouns is postponed until there are no more reducible conditions left. Several rules constrain resolution, the most central one being that the antecedent must either be *accessible* for the anaphor, *or* be a definite NP, *or* a wide scope indefinite (as in (10)). However, note that the postponement of resolution is at odds with the requirements of on-line interpretation. In particular, it rules out that interpretation starts before DRS construction is finished. Moreover, there are empirical difficulties as well; not all the possible scopes of descriptions are covered. For instance, there is no way, in [Asher and Wada-89] — nor is there in [Kamp-81], see our note 6 — to interpret sentences such as (10):

(10) *Every farmer who admired her courts a widow,*

where the antecedent of a kataphoric pronoun would be introduced in the "wrong" position.[7] For, after the resolution decision $y = z$, the following box would result:

```
b1 --------------------          b2 ----------------
   |      . x    . y    |           |      . z        |
   |                    |           |                 |
   | farmer (x)         |   ===>    | widow (z)       |
   | admire (x,y)       |           | court (x,z)     |
   | y=z                |           |                 |
   ---------------------            -----------------
```

But this box can only be true relative to an embedding function g if, for each g' which differs from g at most in its values for x and y, if $g'(x)$ is a farmer who admires $g'(z)$, then $g'(z)$ is a widow who is courted by $g'(x)$. In other words, if someone is a farmer who admires $g'(z)$, then $g'(z)$ is a widow who is courted by him — which is not a viable reading for the sentence. There seems to be no simple way out of this inconvenient situation.[8] What is needed is the possibility to introduce the condition widow(z) in b_1, rather than b_2, due to its kataphoric link to the RM y which is introduced by the pronoun 'her'. But then this kataphoric link must be known by the time the widow NP is processed. Consequently, resolution cannot be postponed until the rest of DRS construction is finished.

Now we come to our own proposal, in which these lessons are taken to heart. We will not discuss structural constraints of the type proposed in the literature, although some of these are obviously relevant to kataphoric reference, since we have little to add to them. We will think of structural constraints as additional constraints on accessibility. Thus, although DRT on its own would allow coreference in

(12) SHE *thinks* MARY *is pretty,*

the addition of Reinhart's Non Coreference Rule to DRT forbids the two NPs to relate to the same RM, since 'she' c-commands 'Mary'. We will not choose between different versions of these constraints (e.g. [Reinhart-76], [Reinhart-83], [Bosch-83]), nor will we discuss strategies to integrate them with DRT (cf. [Asher and Wada-89] for an interesting proposal). Instead, we will assume a suitably enriched version of DRT and concentrate on the specific mechanisms needed to account for kataphoric reference in an l-t-r based approach.

4 A treatment of kataphora in patient DRT

We propose to allow that pronouns can introduce RMs. Although such RMs will not be complete as they stand, the idea is to be *patient* and to allow that the process is completed when the antecedent is reached. For instance, in order to arrive at one of the readings for the sentence

(13) *Whenever she was off duty, Mary spent her time in the swimming pool,*

'she' may introduce a reference marker x in a box b with property female(x), also written as she(x). Upon encountering the proper name, the condition x=Mary is added to b.

In computer science, a similar procedure for dealing with forward references is known as *backpatching* ([Aho,Sethi,Ullman-86]). Backpatching is a way to deal with forward references in programming languages which prevents that entire program having to be scanned more than once during parsing: a forward reference generates an incomplete translation that is completed later. For instance, forward references in GOTO statements are translated into machine code by first generating a "skeletal instruction" in which the target address of the GOTO statement is left open until the target instruction is reached, so that its address is known. Thus, no second "pass" of the program text is necessary. Our treatment of forward references in natural language will mirror this procedure.

Assume that an NP arises in a condition ϕ occurring in a DR m, that is part of a DRS K. Let $u^{\#}$ be u, with the additional information that u is incomplete. The notation $\phi\ [\ \alpha := u\ ]$ stands for ϕ, with α everywhere replaced by u. Con_m is the set of conditions in the box m. U_m is the set of RMs in m. V is the total set of variables available as RMs. $U_K \subseteq V$ is the set of variables used in K. Finally, $K^{\geq}(m)$ denotes the set of boxes that are accessible to m. The central rules for processing kataphoric NPs in m are the following two principles:

PATIENCE PRINCIPLE: A kataphoric pronoun introduces a new discourse entity into m (with appropriate number and gender features) that is marked as incomplete. *Formally:* a pronoun α in ϕ can be processed as follows: add to U_m a suitable element $u^{\#}$ from $V - U_K$, and add $\alpha = u$ and $\phi[\alpha := u]$ to Con_m.

COMPLETION PRINCIPLE: A nonpronominal NP β can pick up a reference marker $u^{\#}$ from a box m', introduced by an earlier occurring kataphor; the incompleteness mark is deleted; the conditions normally associated with β are added to m'. Other conditions on u are added as normal. *Formally:* When processing β, choose a "suitable" member $u^{\#}$ from an element m' from $K^{\geq}(m)$, add $\phi[\beta := u]$ to Con_m, and

— if β is an indefinite of the form 'a η' or a quantifying NP of the form 'every η', then add $\eta(u)$ to $\mathrm{Con}_{m'}$.

— if β is a proper name, then add $u = \beta$ to $\mathrm{Con}_{m'}$.

— if β is definite description, then add the content of the description to $\mathrm{Con}_{m'}$.

In the sequel, we shall assume that these rules belong to patient DRT, alongside Kamp's DRS construction rules. This time, of course, processing operates from left to right. In particular, by requiring that in a DR of the form $m' \Rightarrow m$, DRS construction processes m' before m, DRS construction is forced to proceed deterministically from left to right.

Before we actually illustrate the operation of the principles of Patience and Completion, we will add provisions for "deviant" scopes, not only to move proper names into their required wide scope position, but also to allow the scope of other NPs to diverge from their place in surface order.

> OPTIONAL RISE PRINCIPLE: Pronouns, definites and indefinites can introduce an RM in any existing DR higher up in K's accessibility hierarchy.
> *Formally:* An indefinite or pronoun α can introduce their RM $x \in V - U_K$ in any member m' of $K^{\geq}(m)$. If α is a pronoun, then an incompleteness mark is added. Further, add $\alpha = u$ to $\mathrm{Con}_{m'}$ and add $\phi\ [\ \alpha := u\ ]$ to Con_m.

This "Quantifier Raising" principle validates "wider than surface" scopes for all except quantifying NPs. To illustrate the rules so far, consider

(14) *Whenever she was off duty, the president spent her time in the swimming pool,*
where there is an ambiguity in the relative scopes of 'the president' and 'whenever'. The rules lead to the following representations:

DRS-1:

```
 -------------------------
 | . t           . x     |          -----------------
 |    she(x)             |  ====>   | swimming(x,t) |
 |    off duty(x,t)      |          -----------------
 |    the president(x,t) |
 -------------------------
```

DRS-2:

```
 -------------------------------------------------------
 |                . x  she(x)                           |
 |                     the president(x,t')              |
 |  --------------------            -----------------   |
 |  |       . t        |  ====>     | swimming(x,t) |   |
 |  |  off duty(x,t)   |            -----------------   |
 |  --------------------                                |
 -------------------------------------------------------
```

Explanation: The variable t ranges over time intervals. DRS-1 deals with all those x and t for which x is female and president at t. DRS-2, where t' is either utterance time or reference time, is verified if there is a female president x at t', such that, for all intervals t during which x is off duty, x is swimming at t.

DRS-1 is obtained via Patience and Completion only. DRS-2 results from Optional Rise. Our earlier sentence (10) would be analysed on the same pattern as DRS-2.

Yet, it seems that there is a fundamental problem with the treatment sketched: consider a kataphoric pronoun occurring outside the scope of a conditional. This pronoun will only have universal meaning (and thus belong at the left-hand side of a split box) if it will later be completed by a quantifying NP; but the future occurrence of a quantifying NP cannot be anticipated. This seems to imply that introduction of the RM for the kataphoric pronoun must be postponed — which would be at odds with the l-t-r approach to kataphora. Instead, we will assume that an incomplete RM may be introduced wherever the other rules allow it, but add the following constraint on completion:

> PROPER PLACE PRINCIPLE: (i) A RM in subordinate position cannot be completed by an indefinite NP[9] or by a proper name. (ii) A RM in the principal DR cannot be completed by a quantifying NP.

Clause (i) will block a reading with narrow scope for 'a president' in

(15) ***Whenever she was off duty, a president spent her time in the swimming pool.***

As a result, 'she' cannot have universal meaning. Now consider the sentence

(16) ***The widow he loves is courted by each farmer.***

Clause (ii) forbids that 'each farmer' completes the RM introduced by 'he' if this RM is part of the principal DR. Optional Rise allows a pronoun to be introduced into a "higher" DR, but only if this DR already exists. If we want to derive the — somewhat problematic — kataphoric reading of (16), we should also allow the introduction of the pronoun into a new box immediately to the left of the current box. This resembles the behaviour that is sometimes noted in quantifying NPs, namely that they have wide scope over a preceding indefinite. Therefore, if one also wants to derive a reading of

(17) ***A rich widow attracts the attention of each poor bachelor***

with wide scope for the desirous bachelors:

```
----------------          --------------------------
|  . y          |   ===>   |  . x  widow(x)          |
|  bachelor(y)  |          |  x attr attention of y  |
----------------          --------------------------
```

one can generalize over pronouns and quantifying NPs as follows:

> OPTIONAL LEFTWARD MOVEMENT: Assume that the current sentence has introduced at least one RM into m. Then a pronoun or quantifying NP α can introduce its RM into a new box m' to the left of the current box, which has to contain only the conditions normally associated with α.
> ***Formally:*** Replace m by $m' \Rightarrow m$. Let $y \in V - U_K$ be the RM newly introduced into m'. Now if α is a pronoun, then $\mathrm{Con}_{m'} := \{y = \alpha\}$. If α has the form 'every η', then $\mathrm{Con}_{m'} := \{\eta(y)\}$. Further, add $\phi[\alpha := y]$ to Con_m.

The kataphoric reading of (16) would be derived by applying leftward movement to the pronoun 'he' in that sentence.

5 Patient DRT put to Work

We have seen how Patient DRT deals with simple sentence-internal kataphors and how it manages to account for some of the difficult scope problems they raise. Now, we will briefly show how Patient DRT deals with some of the difficult cases noted in section 2, namely mutual anaphora, full NP kataphora, and extrasentential kataphora.

Mutual Anaphora. We claim that, in contrast to the approaches of Heim and Kamp[10], our own approach can deal with mutual anaphora. It may be instructive to compare this with the situation in computer science when a program contains two statements, x and y, whose bodies contain references to each other. For instance,

[x :] IF ... THEN GOTO y ...
[y :] IF ... THEN GOTO x ...

Due to backpatching, parsing is not troubled by the mutuality involved in this situation. Using primes for translations, translation of x will contain the "skeletal instruction" GOTO′ y as a part. At this stage an address, say x', is allocated for x in memory. As a result, the relevant part of y can be translated: GOTO′ x'. This enables the program to substitute the address y' for y in the skeletal instruction which translated x: GOTO′ y', which completes the translation.

The same holds for our analogue of backpatching: after one "pass" of a sentence with mutual anaphora, all the necessary information is collected. Our "mutual anaphora" sentence (5) leads to the following representation:

```
------------------------------------------------------------
| A woman who works in his department was interviewed by the |
| manager who hired her.                                     |
|                                                            |
|      . x  woman who works in his department (x)            |
|           woman(x)                                         |
|           x works in his department                        |
|      . y  male(y)                                          |
|           x works in y's department                        |
|           the manager who hired her (y)                    |
|           the manager who hired x (y)                      |
|           x was interviewed by y                           |
------------------------------------------------------------
```

The embedding conditions of DRT are satisfied if there are a man and a woman, x and y, where x works in y's department, y is the manager who hired x, and y interviewed x. The desired reading is obtained without difficulties.

Full NP kataphora. Until now we have only dealt with cases where the kataphor is a pronoun. To account for nonpronominal anaphora, assume that a full NP introduces a set-reference marker X along with a condition NP(X) (in the fashion of [van Eijck-83]); if this NP is anaphoric to another NP with reference marker Y, the *relativized* condition $NP^Y(X)$ is generated. In the case of a *relational* NP (such as 'the parents'), $NP^Y(X)$ holds if X contains the elements which stand in the required relation to the elements in the antecedent set Y ([van Deemter-89]). Now kataphoric full NPs can be covered if Patience is streched to cover full NPs. For in that case, an NP can give rise to the condition $NP^Y(X)$, even though Y was not introduced before. For example, the sentence

(18) *Whenever* THE PARENTS *sleep,* THE PUPPIES *do not eat,*
can, on its kataphoric reading (see section 2), be represented as follows:

```
----------------------------------------------------
|              . X                      . Y         |
|                                                   |
|                          Y .                      |
|              (the parents) (X)                    |
|              (the puppies) (Y)                    |
|    -------------           ---------------       |
|    | sleep (X) |    ===>   | not eat (Y) |       |
|    -------------           ---------------       |
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The relativized condition (the parents)$^Y(X)$ can be glossed as 'X contains the parents of all the elements in the antecedent set Y (and nothing more)'. Along these lines, an adequate kataphoric reading for the sentence results. All the rules for pronominal kataphora remain valid. For instance, the Proper Place Principle predicts, quite adequately, that a kataphoric reading of 'all customers' in

(19) *Whenever* ALL CUSTOMERS *were gone,* A TAXIDRIVER *fell asleep*
(in which 'all customers' means 'all customers of the taxidriver') is only possible if 'a taxidriver' gets wide scope over the tense-operator in 'whenever'. In other words, we claim that (19) is about one particular taxidriver, rather than about taxidrivers in general. Extra constraints on full NP kataphora may be needed. For instance, the facts suggest that definite NPs cannot so easily be equated to future antecedents.[11] This observation can be explained along the following lines: suppose we *would* allow identity kataphora by means of a definite description. Given that the distance between a definite NP and its antecedent can be very great ([Grosz and Sidner-86]), a kataphoric reading can never reliably be inferred from the absence of backwards-anaphoric antecedents. Therefore, the possibility of a *kataphoric* reading would complicate resolution considerably. This argument does not apply to pronouns, as *their* antecedents can much more often be found in the current or previous sentence ([Pinkal-86], [Ariel-85]).

Extrasentential kataphora. Considerations of computational complexity can also be brought to bear on extrasentential kataphora. As we have seen, there is some doubt about the acceptability of kataphors such as the one in

(20) *First* HE *lost his wallet. Then* HIS *car got stolen.* FRED *was having a bad day* ([Asher and Wada-89]). In order to account for the facts, we would propose to *rule out* extrasentential kataphora explicitly:

NON-COORDINATION CONSTRAINT: Kataphors cannot cross sentence boundaries (more generally: coordination boundaries).

Instead, we would explain (20) as a case of backwards anaphora. It has been argued (cf. [Weijters-89], [van Deemter-89]) that proper names are no exception to the rule that definite NPs can be anaphoric. An example in [Maes-90] runs as follows:

(21) *The inventor of dynamite had a profound influence on the nature of warfare. Alfred Nobel ...,*

where the description 'the inventor of dynamite' is assumed to be an antecedent to the proper name 'Alfred Nobel'. Now if proper names can be anaphoric, the proper name 'Fred' in (20) can be analysed as anaphoric to the RM introduced by 'he'. Note that, due to the Non-Coordination Constraint, (20) is not a case of kataphora and consequently, the RM introduced by 'he' remains *incomplete*. This explains why (20) may be less than felicitous (lacking an antecedent for 'he' in an earlier sentence), but nevertheless understandable, in the same way as incompletely perceived discourse (cf. (11)). Note that if 'Fred' is replaced by 'someone', the two NPs cannot corefer, which is explained by the assumption that indefinites are never anaphoric (familiarity hypothesis). Thus, the Non-Coordination Constraint precludes that the parser needs infinite patience: given any bound on sentence length, this constraint induces a bound on the maximal distance between kataphor and antecedent.

6 Conclusions

The Patient DRT treatment of kataphoric references shows that the l-t-r approach to anaphora — which, we have seen in the introduction, has much to commend itself on independent grounds — can provide adequate descriptions for most kataphoric constructions. In dealing with these constructions, we have only accounted for grammatically possible readings, disregarding the further question how to decide which of them is most likely to be intended. However, Patient DRT raises a number of questions we cannot avoid saying a few words about. We will briefly discuss three of these, dwelling somewhat longer on the last one than on the other two.

— *When and why is Patient DRT's backpatching method an appropriate strategy for dealing with indeterminism?* In this paper, a number of phenomena are described that seemed to resist l-t-r DRS construction and we have dealt with them by means of backpatching. But similar phenomena exist at other levels of parsing. For instance, at the level of speech recognition, phoneme pairs such as 'w' and 'u:' can only be told apart with the help of future phonetic material. Similarly, in syntactic analysis, only new syntactic material can decide whether, for example, 'flying' is a present participle (in 'Flying planes are dangerous') or an NP (in 'Flying is dangerous'). The same thing occurs at the level of semantic interpretation, since semantic ambiguities are often resolved by future context.

We do not mean to imply that all these different-level kataphoric phenomena are to be treated by means of backpatching. A prudent general rule seems to be the following: when the processing of the kataphoric element faces finitely many "resolution" candidates, then it is preferable to proceed by trial and error, backtracking over the different candidates; the above-mentioned examples belong to this category. When, to the contrary, there are infinitely — or otherwise inconveniently — many candidates, then backtracking has to give way to a back*patching* strategy such as outlined in the body of this paper. The kataphoric phenomena in the realm of DRS construction clearly belong to this category, since the number of possible referents for a kataphoric pronoun can, before the antecedent is processed, at best be limited to the universe of discourse as a whole.

— *What has Patient DRT gained us in terms of the prospects for incremental semantic*

interpretation? It is clear that, on the premisses of DRT, l-t-r DRS construction is a prerequisite for on-line interpretation. But it is still a long way from l-t-r DRS construction to on-line interpretation. What we do have — due to the Non-Cooordination Constraint — is interpretation per completed sentence. Incompletely perceived discourse (11) and sentences that purportedly contain extrasential kataphors are attributed interpretations in which unresolved pronouns are existentially quantified:

(22) *First he lost his wallet. (Then ...)*

is interpreted as 'at least one (male) person lost his wallet'. Interpretation of unfinished sentences is problematic, however. To illustrate, suppose the language fragment in [Kamp-81] is enlarged with conditional sentences of the form 'S_1 if S_2', then straightforward truth conditional interpretation of the first sentential part of

(23) *[John will succeed]$_S$ if he is lucky*

will, too optimistically, say that John will succeed. At this stage, it is unclear how serious these problems must be taken. Either they may be regarded as harmless semantic gardenpath phenomena — with sentence intonation, if and when it is available, coming in to provide extra information. Or, alternatively, they may be taken as arguments for the psychological reality of the level of Discourse Representations. For if a human interpreter of (23) has, after parsing the first clause, some degree of understanding of what is said, and if his understanding is not captured by truth conditions, then it might be hard to improve upon the DRS level as a reflection of this understanding.

— ***Can kataphors be dealt with in DPL?*** From our point of view, one of the most promising rivals to DRT as a semantic theory of anaphora is J. Groenendijk and M. Stokhof's theory of Dynamic Predicate Logic (DPL) (cf. [Groenendijk Stokhof-87], [Groenendijk Stokhof-88]). Their main motivation is to design an alternative to DRT that has the same descriptive power, while operating strictly compositional (cf. e.g. [Janssen-83]) in the construction of representations. Moreover, DPL adheres to the l-t-r principle, as we will shortly see.

Instead of the box-representations of DRT, DPL employs the syntax of normal predicate logic as a representation language. In order to explain the relevant phenomena (donkey sentences, discourse anaphora, etc.), the semantics of the logical language is changed in such a way that, most notably, existential quantifiers bind variables ***beyond*** their scopes. To illustrate, (24)(a) is translated as (24)(b):

(24) (a) *Somebody walks. He talks.*
(24) (b) $\exists x : Walk(x) \,\&\, Talk(x)$.

Given DPL's semantics, the existential quantifier in (24)(b) binds *all* the occurrences of x. Translation proceeds in two steps. First, 'Someone walks' is translated as $\exists x : Walk(x)$, then 'He talks' is translated as $Talk(x)$. (24)(b) is obtained, as it were, by simple *concatenation* of these formulas[12]. In DRT, by comparison, the addition of a new sentence to an existing DR takes place without a separate representation for the newly parsed sentence; instead, the existing representation is modified in one of several ways.[13] DPL avoids such inherently procedural doings: the semantics of the existential quantifier suffices to get the bindings right.[14]

In order to provide formulas such as (24)(b) with the appropriate meanings, DPL has them denote "state-changers": technically, formulas denote pairs $< g, h >$ of assignments, where g is an input assignment (input state) and h an output assignment. In other words, h may result if the formula is processed in g. Assume, for instance, that g is the input state; then the processing of

(25) $\exists x : Walk(x)$.

changes g into a state h that assigns an individual $h(x)$ to x such that $h(x)$ walks. Formally,

(26) $||\exists x : \phi(x)|| = \{< g, h > | \; \exists k : g[x]k \; \& \; < k, h > \epsilon ||\phi||\}$,

where $g[x]k$ holds if $g(y) = k(y)$ for each variable y such that $y \neq x$. The fact that processing operates from left to right can be seen from the definition of sentence conjunction:

(27) $||\phi \; and \; \psi|| = \{< g, h > | \; \exists k :< g, k > \; \epsilon \; ||\phi|| \; \& \; < k, h > \; \epsilon \; ||\psi||\}$.

It is due to this non-commutative conjunction that variables to the left of a quantifier cannot be bound by a quantifier. Now, given the attractiveness of the DPL perspective, one may try to accommodate kataphors in DPL. Processing order could be reversed from l-t-r into r-t-l (right-to-left) by the following move:

(28) $||\phi \; and \; \psi|| = \{< g, h > | \; \exists k :< g, k > \; \epsilon \; ||\psi|| \; \& \; < k, h > \; \epsilon \; ||\phi||\}$.

But if this is done, *only* kataphors are allowed, disallowing (backwards) anaphors. In order to make DPL suitable for both kataphors and anaphors, the following combination of (27) and (28) might be proposed, in which conjunction is stipulated to be commutative again:

(29) $||\phi \; and \; \psi|| = \{ < g, h > | \; \exists k : (< g, k > \epsilon \; ||\phi|| \; \& \; < k, h > \epsilon ||\psi||) \vee$
$(< g, k > \epsilon ||\psi|| \; \& \; < k, h > \; \epsilon \; ||\phi||) \}$.

As a result, a quantifier $\exists x$ can either bind or not bind a given occurrence of x, no matter whether x preceeds or follows the quantifier. Consequently, (30) (a) and (b)

(30)(a) He walks; Someone talks,
(30)(b) Someone talks; he walks,

become equivalent: given an assignment g, both are true iff *either* somebody walks and talks, *or* $g(x)$ walks and somebody (possibly somebody else) talks. Overgeneration would, as ever, have to be prohibited by a set of constraining rules. For instance, the kataphoric reading of (30)(a) could be ruled out by a the Non Coordination Constraint from section 5.

However, it will be clear that, from our perspective, the proposal contained in (29) is unattractive, since it would bereave DPL of its l-t-r orientation[15]. Of course, DPL's l-t-r perspective can be maintained if the "patient" approach we have described for DRT is adopted in DPL: an assignment g that is undefined for a variable x may process x "incompletely", to be completed by a subsequent quantifier under certain conditions .... Although this is, technically speaking, possible, it would be at odds with the philosophy of DPL to introduce such blatantly procedural elements into a neatly compositional framework. Thus, it seems that although kataphors can be reconciled with the principle of l-t-r processing, this can only happen at the expense of compositionality. In other words, it might be that noncompositionality of translation is an asset, rather than a disadvantage of DRT.

7 Notes

0. I thank Johan van Benthem, Jeroen Groenendijk and Martin Stokhof for helpful discussions on the subject of this paper. Robbert-Jan Beun and Sieb Nooteboom provided useful comments on an earlier draft. Any remaining errors ....

1. The first statement of an l-t-r principle I have been able to find is Hintikka and Carlson's *Progression Principle* ([Hintikka Carlson-79]). However, like in [Heim-82] (cf. section 3), this principle applies to a level of logical form rather than surface structure.

2. Cf. section 3, where Kamp's treatment of kataphors is briefly discussed.

3. For instance, DRS construction in [Kamp-81] requires syntactic information which can only be available if **syntactic analysis** can "look" quite far "ahead".

4. Note that the unacceptability of (2) is predicted by Reinhart's NonCoreference Rule, for in (2) the pronoun c-commands the proper name 'Dorit'.

5. An interesting situation in which kataphoric constructions (a) are preferable over their backwards-anaphoric counterparts (b) obtains when a discourse entity (here: Peter) is introduced in a part of the text that is marked as optional reading:

(a) *Joan — no doubt the person he admired most — kept Peter from going insane.*

(b) *Joan — no doubt the person Peter admired most — kept him from going insane.*

6. In principle, processing order within a "monolithic" DR is constrained by *scope*: if NP_1 and NP_2 do not belong to b_1, b_2, respectively, where b_1 and b_2 stand in the relation $b_1 \Rightarrow b_2$, and if NP_1 has wider scope than NP_2, then NP_1 is processed before NP_2. However, in [Kamp-81], scope order is simplified to coincide with surface order. As a result, as Kamp points out himself, Kamp's mechanism does not account for kataphors such as (10), where there is a difference between the surface position of the antecedent and its scope. An even more radical departure from **P1** than described in [Kamp-81] would be needed in order to get the right predictions in cases like this. See also our later remarks about Asher and Wada's proposals.

7. A possibly more natural example, where the "disposed" kataphoric antecedent is a definite description with an underlying temporal quantifier, runs *Whenever he threatened the position of the military,* THE KING *was toppled.* Here it is impossible, on the account of Asher and Wada, to get the reading where the tense operator in 'whenever' has wide scope over the one in 'the king'.

8. In particular, exactly the same result is obtained if the verification clause for boxes of the form $b_1 \Rightarrow b_2$ is redefined in such a way that *all* the new variables in b_1 are quantified, rather than only those which have been introduced in b_1 (i.e. introduced in U_{b1}).

9. Note, however, that NPs of the form 'a(n) ...' have to be excluded from the first clause of the PROPER PLACE PRINCIPLE when they are used in a *generic* sense — to be distinguished from a strictly universal sense. (An example would be a present-tense version of (15): *Whenever she is off-duty, a president spends her time in the swimming-pool*). While earlier versions of DRT have, I think, not dealt with generic NPs, we will simplify matters somewhat and assume that genericity is not a structural phenomenon and that any present-tense subject NP of the form 'a(n) ...' can be used generically. Such generic NPs will be considered as *quantifying*, rather than *indefinite* NPs, and consequently, they will not be affected by the restriction in (i).

10. In [Kamp-81], the desired reading of two mutual anaphors that do not belong to different boxes (as in (5)) cannot be obtained, since processing within such a monolithic DRS does not allow any departures from l-t-r processing order (see note 6), while DRT also, of course, does

not have provisions for backpatching.

11. Some cases were brought to my attention where a full NP definite appears to be used as a kataphor. A typical example is *Even if the bastard ends up bringing in* FIFTY *million, I don't think Berkeley should hire Ronald Reagan to teach political science.* However, we doubt whether this statement can be made without *previous* occurrence of the proper name Ronald Reagan. (Compare our reasoning in relation to sentences (3) and (4)). The situation is not entirely clear but a prohibition of the kind indicated in the text may be in force.

12. What really makes the treatment compositional is the fact that the calculation of (24)(b) can be viewed as a strictly semantical operation, performed on the *meanings* — rather than the logical translations — of the constituent sentences (cf. [Janssen-83]).

13. Updating existing DRs is done in Kamp's rules **CR1** - **CR5**. In Patient DRT, rules such as the Completion Principle can change an existing DR in even more ways.

14. In this respect, DPL is in line with the claim in [Chierchia and Rooth-84] to the effect that DRT's embeddability definitions make a definition of accessibility redundant.

15. Actually, as Stokhof pointed out to me, the new definition of conjunction (29) would make DPL virtually indistinguishable from H. Zeevat's system of Static Semantics (cf. [Zeevat-90]), where directionality is abandoned completely. From our perspective, of course, Zeevat's proposal has the same drawback as the bidirectional version of DPL (cf. (29)), namely that it fails to observe the l-t-r principle.

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DYNAMIC INTERPRETATION FLEXIBILITY AND MONOTONICITY*

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Abstract
This paper discusses some basic techniques in a theory of dynamic interpretation, a theory that gives a compositional account of intersentential anaphoric relationships. Following Janssen and Groenendijk and Stokhof a variant of intensional logic is used in which a quantifier may bind, semantically, pronominal anaphors that are not in its syntactic scope. The adoption of Hendriks' flexible account of scope phenomena next suffices for a treatment of intersentential anaphoric relationships in the framework of a Montague grammar. This, powerful, flexible approach brings along with it two further questions. The first is that of controlling the number of derivable readings. The second one concerns the apparent conflict between the need to give quantifiers of downward monotonicity intersentential scope, and the desire to retain upward monotonicity of information update. A solution is offered to both issues.

1 Introduction

Our purpose is to develop a compositional mechanism of interpretation in a Montagovian framework, that can adequately deal with sentential and intersentential anaphoric relationships such as those exhibited in the following examples:

(1) *A_i man walks in the park. He_i whistles.*
(2) *$Every_i$ chessbox comes with a_j spare pawn. It_j is taped on top of it_i.*
(3) *$Every_i$ farmer who owns a_j donkey beats it_j. He_i is never nice to it_j.*
(4) *If a_i farmer owns a_j donkey he_i beats it_j.*
(5) *It is not true that Mary has no_i car. It_i is red and it_i is standing in front of the house.*
(6) *No_i farmer beats a_j donkey he_i owns. He_i doesn't kick it_j either.*
(7) *Either there is no_i bathroom in the house or it_i is in a funny place. It_i is not on the first floor though.*

The problems that these examples pose for a Montague grammar which deals with anaphoric relationships by means of quantification rules are quite familiar by now. In the first place, we find anaphoric relationships holding between noun phrases (NP's) that occur in *different* sentences (or sentential clauses). If, as we take it, the principle of compositionality dictates that the meanings of the constituent sentences, together with their mode of combination, determine the meaning of a composed discourse, then a

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treatment of these relationships with quantification rules is bound to be inadequate. On this approach, an antecedent NP in a sentence S should be quantified into a *sequence* of sentences in order to bind anaphoric pronouns occurring after S. So, in that case, the meaning of the sequence would *not* be determined by the meaning of S. The sentence S, with the antecedent NP as a genuine part of it, just wouldn't show up as a constituent in the analysis.

A second problem with the treatment of anaphoric relations by means of rules of quantification is that it is empirically unsatisfactory. We often get the wrong interpretation if, as happens when a rule of quantification is used, an NP gets scope over continuations of discourse. Except for (1) and (2), none of the examples above allows for a truth-preserving rephrasing in which antecedent NP's are fronted and range over the part of discourse that they are drawn from. Example (7), for instance, is not equivalent to *No bathroom is in the house or in a funny place and not on the first floor*, which amounts to the statement that every bathroom in the house, and every bathroom in a funny place, is on the first floor.

In order to cope with examples like these discourse representation theories have been developed (Kamp (1981), Heim (1982), Seuren (1985)). However, the alternatives proposed involve a heavy shift in paradigm, a shift that complicates comparison and unification with traditional Montague grammar. Notably, the principle of compositionality is abandoned (but see Zeevat (1989)). Furthermore, discourse representation theories provide for a (uniform) interpretation procedure that deals, for the most part, with indefinite antecedent NP's. A general procedure that also deals with other problematic antecedent NP's, like in (5) to (7), is lacking.

In this paper we will proceed upon the assumption that the relation between antecedent noun phrases and anaphoric pronouns is a scope relation, and elaborate this in a Montagovian framework. For this, two (or better, three) hurdles have to be cleared. First, in order to give extrasentential scope to quantifiers in a compositional way, we need an alternative for Montague's rules of quantification. A suitable alternative is Hendriks' system of type change. In Hendriks' flexible Montague (sentence-)grammar, all different quantifier scope configurations are derivable with the help of three type changing operations (Hendriks (1989)). It appears that this system of type change is easily exploited also at the intersentential level of discourse.

Hendriks' system of type change is an alternative for Montague's quantification rules for as far as quantifier scope is involved. However, another function of quantification rules, the interpretation (and binding) of anaphoric pronouns, falls beyond this treatment. The type changes determine what parts of text are in the scope of what quantifiers, and the quantifiers and the parts of text in their scope are given their place in such a configuration by functional application. But, of course, no variables come to get bound in the process of functional application. A quantifier in a functional expression cannot bind a free variable in an argument expression to which it is applied: the variable is outside its scope.

So, whereas the adoption of Hendriks' system of type change gives us the (extrasentential) *scopings* that we want, we still dispose of the *binding* of pronouns we want, and this makes up the second hurdle we have to take. It is here that dynamic intensional logic (DIL) comes in (Janssen (1986), Groenendijk and Stokhof (1989)). In DIL, a variant of intensional logic, intensions abstract over variable assignments (more precisely: over discourse marker assignments). Precisely this feature of DIL allows for the possibility that pronouns -which are translated as discourse markers- can get bound by quantifying expressions in the process of intensional functional application (whence the attribute 'dynamic intensional').

We may immediately note a difference between the version of dynamic Montague grammar developed by Groenendijk and Stokhof and the one that we present here. In Groenendijk and Stokhof's proposal all sentences are translated as generalized quantifiers over states, type ((s,t),t). This is the lowest type of sentence translations that allows one to capture the basic results of discourse representation theory in a Montague grammar that uses DIL. In the system that we present, higher types of sentence translations are used in order to account for the dynamics of natural language, the lowest one of which is that of generalized quantifiers over *sets* of states. However, the types that we use are derivable from a basic sentential type t by means of independently motivated type changing rules. Furthermore, if one uses these higher types of sentence translations more intricate anaphoric relationships can be accounted for. For some discussion on the issues of dynamic flexibility and of adopting higher dynamic types, we refer to Dekker (1989).

Our presentation of a flexible dynamic Montague grammar in this paper will proceed as follows. First, in section (2), we briefly discuss the system of dynamic intensional logic as a system that generates the mere possibility of dynamic binding. DIL will take the place of intensional logic in a dynamic Montague grammar. In the same section, we introduce Hendriks' flexible account of scope phenomena. This type flexibility is added to (a simplified version of) a dynamic Montague grammar.

In section (3) we show that such a basic flexible dynamic Montague grammar already gives way to a dynamic interpretation of natural language. Moreover, the combination of dynamics and flexibility appears to be an explosive mixture. Whereas, on the one hand, puzzling complex sequences of sentences turn out to be interpretable to our satisfaction, on the other hand, overgeneration results. So, after sketching the basics of a flexible dynamic Montague grammar, we show, in the same section, one way of structurally restricting the set of derivable readings. The restriction we propose (one that is to meet the intuition that discourse processing is directional, proceeding from left to right) thus may serve as an example of how to keep type flexibility under control.

All of this still leaves us with a third hurdle, hinted at above. As we already said, giving quantifiers intersentential scope not always gives the right results. In section (4) we argue that it is the conflict between the downward monotonicity of certain quantifiers and the upward monotonicity of information update (or, if you like, its incrementality) that is playing tricks on us here. On the basis of this diagnosis, then, we propose a structural solution to this problem within our flexible treatment of scope. We define a logical adjustment of the interpretation of the type changes, which preserves upward monotonicity of information update when the type changes apply to downward monotonic expressions. In section (5) we show that this adjusted interpretation does a good job in the troublesome examples. Section (6) is reserved for some conclusions and further discussion.

2 Dynamic interpretation and type flexibility

In this part we expose two basic techniques that are required in a compositional account of intersentential anaphoric relationships, when these are viewed as binding relations. We first introduce a theory of interpretation that provides for the mere possibility of dynamic binding. After that, we sketch Hendriks' compositional account of scope phenomena. We will speak of dynamic binding here, when a pronoun is bound, semantically, by a quantifying noun phrase that does not have the pronoun in its syntactic scope. We can achieve this if we enrich the traditional truth-conditional notion of meaning with a dynamic aspect. Janssen (1986) and Groenendijk and Stokhof (1989) have proposed an alternative for intensional logic which has just this expressive power: dynamic intensional logic (DIL). We will give only a sketch of (a simplification of) DIL here.

Dynamic intensional logic

DIL can be best understood in comparison with IL. First, think of typed extensional logic, where interpretation takes place with respect to two parameters: a model M and an assignment of values to variables g. Now, where IL allows for abstraction, so to speak, over the models of extensional logic, models that reappear as worlds in IL, DIL allows for abstraction over the other parameter of extensional logic, assignments, which reappear as states in DIL. Therefore, the intension of a formula in DIL is not the set of worlds in which the formula is true, but the set of assignments (states) with respect to which it is true. Intensions in DIL carry information about the values of variables, or better, information about the values of discourse markers since that is what the pronoun translations will be called.

For a first shot, like Groenendijk and Stokhof, we do not incorporate information about (possible states of) the world. This would amount to simultaneously abstracting over worlds and assignments with the intension operator. For the present purposes the restricted notion of information suffices. We also keep our system simple by admitting only discourse markers of type e. A generalization to a system with discourse markers for all types is possible, but would give unnecessary complications here.

We now turn to an explicit formulation of DIL's distinguishing properties. DIL is like IL, except for the following features:

• ***Syntax*** To the expressions of type e a set of discourse markers d, d'... is added. These discourse markers will function as pronoun translations in a dynamic Montague grammar. Quantified expressions are constructed with the help of these discourse markers: $\exists d\varphi$, $\forall d'\psi$. (The intension operator ^ is too important not to mention it yet.)
• ***Model*** Instead of worlds, a DIL-model contains a set of states. These states can be identified with the set of assignments of individuals to discourse markers. The interpretation function for the constants of the language is state-insensitive. (As we said, in the present set-up we only consider information about discourse marker assignments.)

• *Semantics* The differences in semantics only relate to the discourse markers. The interpretation of a discourse marker is the value it has in the state of evaluation (which is a discourse marker assignment); quantification over the values of discourse markers is mediated by quantification over these states (not over ordinary variable assignments); the intension operator abstracts over states (i.e. discourse marker assignments):

$$\| d \|_{M,s,g} = s(d)$$
$$\| \exists d\varphi \|_{M,s,g} = 1 \text{ iff } \exists d \in D_e\text{: } \| \varphi \|_{M,s[d/d],g} = 1$$
$$\| \forall d\psi \|_{M,s,g} = 1 \text{ iff } \forall d \in D_e\text{: } \| \psi \|_{M,s[d/d],g} = 1$$
$$\| {}^{\wedge}\varphi \|_{M,s,g} = \text{the function h such that for all states s': } h(s') = \| \varphi \|_{M,s',g}$$

Dynamics in DIL

The possibility to abstract over discourse marker assignments has the following pleasant consequence for λ-conversion in DIL. We have that $(\lambda x.\beta)({}^{\wedge}\gamma)$ equals $[{}^{\wedge}\gamma/x]\beta$ if all free variables in γ are free for x in β; **No** conditions on discourse markers in γ need obtain, because, although their interpretation is state dependent, ${}^{\wedge}\gamma$ is intensionally closed and hence not state dependent. A non-trivial example of such a reduction is the following:

$$(\lambda p_{(s,t)}.\exists d(\text{MAN}(d) \wedge {}^{\vee}p))({}^{\wedge}\text{WALK}(d)) \Leftrightarrow \exists d(\text{MAN}(d) \wedge \text{WALK}(d)).$$

We see that the λ-term $\lambda p.\exists d(\text{MAN}(d) \wedge {}^{\vee}p)$ in fact is dynamic in the sense indicated above: (indirectly) the embedded existential quantifier binds a discourse marker that occurs in argument-expressions of the λ-term, that is, a discourse marker *outside* of the syntactic scope of the quantifier. This is crucial. We can generalize this observation as follows. Dynamic expressions in DIL are λ-terms abstracting over a variable of an intensional type, an occurrence of which is preceded by the cup operator and in the scope of a quantifier: $\lambda x_{(s,a)}....\exists d(...{}^{\vee}x...)...$, or $\lambda x_{(s,a)}....\forall d(...{}^{\vee}x...)....$

Type change and scope

As we have said above, an account of intersentential binding is not to be based on Montague's quantification rules, because this would rule out the possibility of interpreting a discourse by processing its constituent sentences one by one. However, Hendriks (1989) has developed an alternative, and compositional, account of scope phenomena which suits our purposes when transposed to the framework of DIL. In Hendriks' flexible Montague grammar expressions of natural language are assigned, not translations of a categorially determined type, but sets of translations of types derivable from a categorially determined basic type by type change. At the heart of the proposal lies a type changing system that allows for correlated changes of the basic translation and type assigned to natural language expressions. These type changes, and their order of application, determine the relative scope of quantifiers in a sentence, in a way we will see presently.

In our exposition of the type changing system, we skip the rule of argument lowering, which is used only when scope bearing operators other than quantifiers get involved, intensional verbs, for instance. Our main concern here is the relation between quantifying noun phrases and pronouns, not intensionality. We add, on the other hand, a rule labeled generalized division. This rule largely overlaps with Hendriks' rule of argument raising.

In fact, the rule of division will eventually replace the rule of argument raising (for some arguments, see Dekker (1989)). Until we do so, however, it is convenient to have both rules at our disposal.)

In flexible Montague grammar, then, all atomic natural language expressions are assigned the basic type and translation they receive in standard Montague grammar, or a simplification of the traditional Montagovian one. Next, derivable types and translations are defined, and used in the construction of the translation of compound expressions. We will use the following three type shifting rules ($a: \alpha \Rightarrow b: \beta$ reads: *translation β of type b derives from translation α of type a*; $^\wedge a$ is short for the type (s,a); the vector $\vec{}$ indicates iteration):

Value Raising

[VR] $(\vec{a},b): \lambda\vec{x}.\varphi(\vec{x}) \Rightarrow (\vec{a},(^\wedge(^\wedge b,t),t)): \lambda\vec{x}\lambda P.\ {}^\vee P(^\wedge\varphi(\vec{x}))$

Value raising is the (generalized) raising of an object of type b into the corresponding generalized quantifier over objects of type b, of type $(^\wedge(^\wedge b,t),t)$. Here are some examples:

$$e: j \Rightarrow (^\wedge(^\wedge e,t),t): \lambda P.\ {}^\vee P(^\wedge j)$$

$$t: \varphi \Rightarrow (^\wedge(^\wedge t,t),t): \lambda R.\ {}^\vee R(^\wedge\varphi)$$

$$(^\wedge e,t): \text{WALK} \Rightarrow (^\wedge e,(^\wedge(^\wedge t,t),t)): \lambda x\lambda R.\ {}^\vee R(^\wedge\text{WALK}({}^\vee x))$$

The last two examples exemplify the raising of an expression into the type of quantifiers over propositions: the first of these, $\lambda R.{}^\vee R(^\wedge\varphi)$, denotes the set of properties of the proposition that φ; the last, $\lambda x\lambda R.{}^\vee R(^\wedge\text{WALK}({}^\vee x))$, denotes a function from individual concepts c to the set of properties of the proposition that the value of c walks. As we will see below, two properties of propositions that can be attributed to these quantifiers, are the property of being true in conjunction with the contents of further discourse, and the property of implying some consequent proposition.

Argument Raising

[AR] $(\vec{a},(^\wedge b,(\vec{c},t))): \lambda\vec{x}\lambda y\lambda\vec{z}.\ \varphi(\vec{x})(y)(\vec{z}) \Rightarrow$
$(\vec{a},(^\wedge(^\wedge(^\wedge b,t),t),(\vec{c},t))): \lambda\vec{x}\lambda Y\lambda\vec{z}.{}^\vee Y(^\wedge\lambda y.\ \varphi(\vec{x})(y)(\vec{z}))$

[AR] makes a function from objects of type b fit to apply to generalized quantifiers over objects of type b. This rule enables one, for instance, to assign extensional transitive verbs a basic translation of the simple type $(^\wedge e,(^\wedge e,t))$, and derive a translation that may be applied to (the intension of) quantifiers, a translation of type $(^\wedge(^\wedge(^\wedge e,t),t),(^\wedge(^\wedge(^\wedge e,t),t),t))$. Furthermore, there are two non-equivalent derivations into a translation of this higher type, and these underlie the two readings that a sentence may have when it is composed of such a transitive verb phrase flanked by two quantifying noun phrases. Of the following two examples, the first effectuates that the verb's second argument (its subject) will get wide scope, the next that its first argument (the object) gets wide scope:

$$\lambda y\lambda x.\ \text{LOVE}({}^\vee y)({}^\vee x) \Rightarrow_{[1AR][2AR]} \lambda T\lambda V.\ {}^\vee V(^\wedge\lambda x.\ {}^\vee T(^\wedge\lambda y.\ \text{LOVE}({}^\vee y)({}^\vee x)))$$
$$\lambda y\lambda x.\ \text{LOVE}({}^\vee y)({}^\vee x) \Rightarrow_{[2AR][1AR]} \lambda T\lambda V.\ {}^\vee T(^\wedge\lambda y.\ {}^\vee V(^\wedge\lambda x.\ \text{LOVE}({}^\vee y)({}^\vee x)))$$

(We write [nAR], if the n^{th} argument of a function is raised by an application of [AR]. The composition [X][Y] indicates that the derived translation results from applying the type changes [X] and [Y] successively.) Furthermore, if we posit a sequencing operator + that (functionally) conjoins the meanings (type ^t) of two sentences sequenced, it can be made to apply to a quantifier over propositions as well:

$$+ \quad \lambda p\lambda q.\ {}^{\vee}p \wedge {}^{\vee}q: ({}^{\wedge}t,({}^{\wedge}t,t)) \quad \Rightarrow_{[1AR]} \quad \lambda P\lambda q.\ {}^{\vee}P({}^{\wedge}\lambda p.\ {}^{\vee}p \wedge {}^{\vee}q): ({}^{\wedge}({}^{\wedge}({}^{\wedge}t,t),t),\ ({}^{\wedge}t,t))$$

Generalized Division

$$[GD] \quad (\vec{a},{}^{\wedge}(\vec{b},d),d)): \lambda\vec{x}_a\lambda P.\ \varphi(\vec{x}_a)(P) \Rightarrow$$
$$(\vec{a},{}^{\wedge}(\vec{b},({}^{\wedge}c,d)),({}^{\wedge}c,d))): \lambda\vec{x}_a\lambda R\lambda z_c.\ \varphi(\vec{x}_a)({}^{\wedge}\lambda\vec{y}_b.\ {}^{\vee}R(\vec{y}_b)(z_c))$$

This rule splits up an argument place of a function φ into two argument places, such that φ applies to the composition of the two argument expressions to which the division of φ applies. So the division makes a function φ applicable to argument expressions that themselves contain extra argument slots which the division of φ inherits. (We note that the semantics of this rule respects thematic structure.)

We give two examples. First, we divide, in a quantifier Q, the argument place for properties of individuals into an argument place for relations between individuals and the added argument place for individuals which is required then. The effect of this is that application to a relation R and to an individual x equals the application of Q to the property $^{\wedge}\lambda y.\ {}^{\vee}R(y)(x)$:

$$\lambda P_{{}^{\wedge}({}^{\wedge}e,t)}.\ \exists d(\text{MAN}(d) \wedge {}^{\vee}P({}^{\wedge}d)) \Rightarrow \lambda R_{{}^{\wedge}({}^{\wedge}e,({}^{\wedge}e,t))}\lambda x_{{}^{\wedge}e}.\ \exists d(\text{MAN}(d) \wedge {}^{\vee}R({}^{\wedge}d)(x))$$

Next, we divide the same argument place of a quantifier into an argument place for raised transitive verbs and the argument place for properties of propositions which is required in that case:

$$\lambda P_{{}^{\wedge}({}^{\wedge}e,t)}.\ \exists d(\text{MAN}(d) \wedge {}^{\vee}P({}^{\wedge}d)) \Rightarrow \lambda Q_{{}^{\wedge}({}^{\wedge}e,({}^{\wedge}({}^{\wedge}t,t),t))}\lambda R_{{}^{\wedge}({}^{\wedge}t,t)}.\ \exists d(\text{MAN}(d) \wedge {}^{\vee}Q({}^{\wedge}d)(R))$$

Now we can apply the quantifier to the raised translation of *walk* given above, such to the effect that a (really dynamic) quantifier over propositions results after application:

$$(\lambda Q\lambda R.\ \exists d(\text{MAN}(d) \wedge {}^{\vee}Q({}^{\wedge}d)(R)))({}^{\wedge}\lambda x\lambda R.\ {}^{\vee}R({}^{\wedge}\text{WALK}({}^{\vee}x))) \Leftrightarrow$$
$$\lambda R.\ \exists d(\text{MAN}(d) \wedge {}^{\vee}R({}^{\wedge}\text{WALK}(d)))$$

The resulting expression denotes the set of properties of the proposition that d walks, in a state that only differs from the state of evaluation in that it assigns a man to d.
We note in passing that this division rule generates all instantiations of the polymorphic quantifiers in Emms (1990). These polymorphic quantifiers were also designed for a treatment of scope phenomena.

To complete the type changing system, we add [Identity] of course, and [Transitivity], and [Application]. As usual, we let the rule of application be triggered by the category of natural language expressions. Therefore, we incorporate categorial information in the definition of this rule and, thus, link up the semantics with the syntax of a natural language fragment presented in the next section ('Y: y: ψ' abbreviates 'an expression of category Y with translation ψ of type y'):

$$\text{[Application/]} \quad A/B\text{: a: }\alpha\text{, B: b: }\beta \Rightarrow A\text{: d: }\delta({}^\frown\gamma)\text{ if}$$
$$\text{b: }\beta \Rightarrow \text{c: }\gamma\text{ and}$$
$$\text{a: }\alpha \Rightarrow ({}^\frown c,d)\text{: }\delta$$

The concatenation xy of two expressions x and y of category A/B and B respectively, has a translation $\delta({}^\frown\gamma)$ iff δ is a (derivable) translation of x and γ a (derivable) translation of y. The application of a left looking functor category B\A, is essentially similar.

3 A flexible dynamic Montague grammar

A fragment

Conceive of the following fragment of natural language. As basic categories we have: S (sentences), NP (noun phrases) and CN (common nouns). Complex categories are formed from the basic ones and the directed slashes '/' and '\'. We use the following abbreviations for frequently occurring complex categories: IV (= NP\S, intransitive verb phrases), TV (= IV/NP, transitive verb phrases), and DET (= NP/CN, determiners). With the categories the following basic types of interpretation are associated:

$$\text{bTYPE(S)} = t \qquad \text{bTYPE(NP)} = e \qquad \text{bTYPE(CN)} = ({}^\frown e,t)$$
$$\text{bTYPE(A\backslash B)} = \text{bTYPE(B/A)} = ({}^\frown\text{bTYPE(A)},\text{bTYPE(B)})$$

Expressions of natural language are translated into DIL-expressions. Basic translations of expressions of a certain category are of the basic type assigned to the category or of a type derivable from the basic type by type change. Here are some examples:

man	CN	$\lambda x.\ \text{MAN}({}^\smile x)$	
walks	IV	$\lambda x.\ \text{WALK}({}^\smile x)$	
loves	TV	$\lambda y \lambda x.\ \text{LOVE}({}^\smile y)({}^\smile x)$	
John	NP	j	
he$_i$	NP	d_i	
an$_i$	DET	$\lambda P \lambda Q.\ \exists d_i({}^\smile P({}^\frown d_i) \wedge {}^\smile Q({}^\frown d_i))$	
every$_i$	DET	$\lambda P \lambda Q.\ \forall d_i({}^\smile P({}^\frown d_i) \rightarrow {}^\smile Q({}^\frown d_i))$	
no$_i$	DET	$\lambda P \lambda Q. \neg\exists d_i({}^\smile P({}^\frown d_i) \wedge {}^\smile Q({}^\frown d_i)))$	
who	(CN\CN)/IV	$\lambda P \lambda Q \lambda x.\ {}^\smile Q(x) \wedge {}^\smile P(x))$	
not	S/S	$\lambda p.\ \neg{}^\smile p$	
+	S\(S/S)	$\lambda p \lambda q.\ {}^\smile p \wedge {}^\smile q$	
if	(S/S)/S	$\lambda p \lambda q.\ {}^\smile p \rightarrow {}^\smile q$	(also (S\S)/S)
or	S\(S/S)	$\lambda p \lambda q.\ {}^\smile p \vee {}^\smile q$	

The expressions of natural language are assigned translations here that will look familiar to anyone acquainted with Montague grammar. In Hendriks (1989) it is shown, that in a basic fragment like this one, with added type flexibility, a straightforward treatment of scope phenomena is possible. Furthermore, because we use DIL as the framework of interpretation, and because application is always to the intension of argument expressions, we have that a quantifier's scope is inextricably bound up with the binding of coindexed pronouns. Therefore, since the present system already allows us to account for quantifier scope and for the binding of anaphoric pronouns, rules of quantification can be dispensed with.

On top of that, we have added a functionally interpreted sequencing operation, and this suffices to extend the account of anaphoric dependencies to the (intersentential) level of discourse. The combination of DIL and type flexibility now allows a quantifier to take extrasentential scope over succeeding discourse *and* bind pronoun occurrences there. The following two examples exemplify this in a little detail:

Example (1)

(1) *A_i man walks. He_i whistles.*

A_i man	NP	$\lambda Q.\ \exists d_i(\mathrm{MAN}(d_i) \wedge {}^{\vee}Q({}^{\wedge}d_i))$
walks	IV	$\lambda x.\ \mathrm{WALK}({}^{\vee}x)$: $({}^{\wedge}e,t) \Rightarrow_{[VR][1AR]}$ $\lambda T\lambda R.{}^{\vee}T({}^{\wedge}\lambda x.{}^{\vee}R({}^{\wedge}\mathrm{WALK}({}^{\vee}x)))$: $({}^{\wedge}({}^{\wedge}({}^{\wedge}e,t),t), ({}^{\wedge}({}^{\wedge}t,t),t))$
A_i man walks.	S	$\lambda T\lambda R.{}^{\vee}T({}^{\wedge}\lambda x.{}^{\vee}R({}^{\wedge}\mathrm{WALK}({}^{\vee}x)))({}^{\wedge}\lambda Q.\ \exists d_i(\mathrm{MAN}(d_i) \wedge {}^{\vee}Q({}^{\wedge}d_i)))$ $\lambda R.\ \exists d_i(\mathrm{MAN}(d_i) \wedge {}^{\vee}R({}^{\wedge}\mathrm{WALK}(d_i)))$
+	S\(S/S)	$\lambda p\lambda q.\ {}^{\vee}p \wedge {}^{\vee}q$: $({}^{\wedge}t,({}^{\wedge}t,t)) \Rightarrow_{[1AR]}$ $\lambda P\lambda q.\ {}^{\vee}P({}^{\wedge}\lambda p.\ {}^{\vee}p \wedge {}^{\vee}q)$: $({}^{\wedge}({}^{\wedge}({}^{\wedge}t,t),t), ({}^{\wedge}t,t))$
A_i man walks. +	S/S	$\lambda P\lambda q.\ {}^{\vee}P({}^{\wedge}\lambda p.\ {}^{\vee}p \wedge {}^{\vee}q)({}^{\wedge}\lambda R.\ \exists d_i(\mathrm{MAN}(d_i) \wedge {}^{\vee}R({}^{\wedge}\mathrm{WALK}(d_i))))$ $\lambda q.\ \exists d_i(\mathrm{MAN}(d_i) \wedge \mathrm{WALK}(d_i) \wedge {}^{\vee}q)$
He_i whistles.	S	$\mathrm{WHISTLE}(d_i)$
A_i man walks. + He_i whistles.		$\lambda q.\ \exists d_i(\mathrm{MAN}(d_i) \wedge \mathrm{WALK}(d_i) \wedge {}^{\vee}q)({}^{\wedge}\mathrm{WHISTLE}(d_i))$ $\exists d_i(\mathrm{MAN}(d_i) \wedge \mathrm{WALK}(d_i) \wedge \mathrm{WHISTLE}(d_i))$

Comments There are three type changes in this derivation, of which the first is the crucial one. The lifting of the functional predicate WALK turns its value into the dynamic type of quantifiers over propositions. The other two type transitions, both instances of first argument raising, only make a functional expression fit to apply to a quantifier over objects of a type of its proper argument. (Notice that all λ-conversions are allowed.)

Example (2)

(2) *$Every_i$ chessbox comes with a_j spare pawn. It_j is taped on top of it_i.*

The derivation of the intended reading of example (2) is essentially similar to that of (1). Only one more instance of argument raising is required, because instead of an intransitive verb, we have a transitive verb here, accompanied by two quantifiers.

$Every_i$ chessbox	$\lambda Q.\ \forall d_i(\text{C-BOX}(d_i) \rightarrow {}^{\vee}Q({}^{\wedge}d_i))$
a_j spare pawn	$\lambda Q.\ \exists d_j(\text{S-PAWN}(d_j) \wedge {}^{\vee}Q({}^{\wedge}d_j))$

comes with $\lambda y\lambda x.\ \text{WITH}({}^{\vee}y)({}^{\vee}x) \Rightarrow_{[VR][1AR][2AR]}$
$\lambda T\lambda V\lambda R.\ {}^{\vee}V({}^{\wedge}\lambda x.{}^{\vee}T({}^{\wedge}\lambda y.\ {}^{\vee}R({}^{\wedge}\text{WITH}({}^{\vee}y)({}^{\vee}x))))$

Every$_i$ chessbox comes with a$_j$ spare pawn.
$$\lambda R.\ \forall d_i(\text{C-BOX}(d_i) \rightarrow \exists d_j(\text{S-PAWN}(d_j) \wedge {}^{\vee}R({}^{\wedge}\text{WITH}(d_j)(d_i))))$$

\+ $\lambda P\lambda q.\ {}^{\vee}P({}^{\wedge}\lambda p.\ {}^{\vee}p \wedge {}^{\vee}q)$

It$_j$ is taped on top of it$_i$. $\text{ON-TOP-OF}(d_i)(d_j)$

Every$_i$ chessbox comes with a$_j$ spare pawn. It$_j$ is taped on top of it$_i$.
$$\forall d_i(\text{C-BOX}(d_i) \rightarrow \exists d_j(\text{S-PAWN}(d_j) \wedge \text{WITH}(d_j)(d_i) \wedge \text{ON-TOP-OF}(d_i)(d_j)))$$

Compositional sequencing

So we see that intersentential anaphoric binding in fact can be treated compositionally with the help of Hendriks' type flexibility and the system of dynamic intensional logic. The combination of dynamics and flexibility allows us to interpret sentences as generalized quantifiers over propositions, which can be viewed as context change potentials. For instance, the quantifier over propositions $\lambda R.\ \exists d_i(\text{MAN}(d_i) \wedge {}^{\vee}R({}^{\wedge}\text{WALK}(d_i)))$ changes the context of interpretation into one in which d_i is a man, this because the variable R is the landing-site for extension of information by further discourse, and because it is in the scope of the existential quantifier.

Still, the notion of *sequencing* that we used in the examples above is not completely satisfactory yet. The result of sequencing two sentences is a translation of type t there. This means that the scope of the quantifiers in (1) and (2) is closed off after sequencing and that the quantifiers are unable to bind pronoun occurrences in further discourse. A more intuitive notion of sequencing, then, must be one that consists of the composition of the context change potentials of sentences. The result of sequencing two dynamic sentences should be dynamic as well, and noun phrases in any one of the sentences sequenced should be able to bind pronoun occurrences in subsequent discourse.

This more adequate notion of sequencing as composition is already present in a flexible dynamic Montague grammar. We derive it by raising the first argument of the sequencing operator, and dividing the second one:

$$\lambda p\lambda q.\ {}^{\vee}p \wedge {}^{\vee}q \Rightarrow_{[1AR][GD]} \lambda P\lambda Q\lambda R.\ {}^{\vee}P({}^{\wedge}\lambda p.\ {}^{\vee}p \wedge {}^{\vee}Q(R))$$

If we abbreviate the dynamic sentence type $({}^{\wedge}({}^{\wedge}t,t),t)$ to c, the type of this dynamic sequencing operation is $({}^{\wedge}c,({}^{\wedge}c,c))$. The derived operation takes two context change potentials as argument and returns a new one. If we apply this translation of sequencing to the intension of two dynamic formulas Φ and Ψ, the following translation results:

$$\lambda R.\ \Phi({}^{\wedge}\lambda p.\ {}^{\vee}p \wedge \Psi(R)) : ({}^{\wedge}({}^{\wedge}t,t),t)$$

Informally, this result of sequencing the quantifiers over propositions Φ and Ψ denotes the set of properties R of propositions such that Φ has the property of being true together with R's being a property of Ψ. We see here that if Φ and Ψ contain dynamic quantifiers, the variable R (type ${}^{\wedge}({}^{\wedge}t,t)$: the placeholder for subsequent discourse) is in the scope of quantifiers in Ψ, while Ψ itself, with R, is in the scope of quantifiers in Φ. Therefore, quantifiers in the first sentence Φ may bind coindexed pronouns in all of Φ's successors (the first one of which is Ψ) and quantifiers in Ψ may bind coindexed pronouns in all of Ψ's successors, provided that no coindexing of quantifiers occurs. The following trivial example may serve as an illustration:

A man whistles.

$\lambda R.\exists d_i(M(d_i)\wedge{}^{\vee}R({}^{\wedge}WH(d_i)))$

... *A woman sees him.*

$\lambda R.\exists d_i(M(d_i) \wedge WH(d_i) \wedge \exists d_j(WOM(d_j) \wedge {}^{\vee}R({}^{\wedge}SE(d_i)(d_j))))$

...*He loves her.*

$\lambda R.\exists d_i\exists d_j(M(d_i) \wedge WH(d_i) \wedge WOM(d_j) \wedge SE(d_i)(d_j) \wedge {}^{\vee}R({}^{\wedge}LO(d_j)(d_i)))$

...*He gives her a present.*

$\lambda R.\exists d_i\exists d_j\exists d_k(M(d_i)\wedge WH(d_i)\wedge WO(d_j)\wedge S(d_i)(d_j)\wedge LO(d_j)(d_i)\wedge P(d_k)\wedge{}^{\vee}R({}^{\wedge}G(d_k)(d_j)(d_i)))$

...*She doesn't like it.(THE END)*

$\exists d_i\exists d_j\exists d_k(M(d_i)\wedge WH(d_i)\wedge WO(d_j)\wedge S(d_i)(d_j)\wedge LO(d_j)(d_i)\wedge P(d_k)\wedge G(d_k)(d_j)(d_i)\wedge\neg L(d_k)(d_j))$

We conclude, that a flexible Montague grammar which uses the framework of DIL comprises a real dynamic interpretation of natural language. What is more, with the system of type change at hand we can also derive a correct interpretation of sentence sequences which exhibit rather complex scope dependencies, as the following example shows.

A more complicated example

(8) *Every customer is offered coffee, if he looks wealthy. He is sent up to me as quickly as possible. If he doesn't look wealthy, he can wait.*

We start with a sketch of the intended semantic structure of (8). First, notice that the first *if*-clause in (8) has syntactic scope over the sentential clause *Every customer is offered coffee*, whereas the pronoun *he* in the *if*-clause is bound by the quantifier *Every customer*. Next, notice that the second and third sentence of (8) must be interpreted as being in the scope of the quantifying noun phrase *Every customer*, but where the second sentence only relates to the cases in which the condition holds that the customer looks wealthy, the interpretation of the third sentence apparently must not be restricted to these cases, if it is not to be trivially true.

So, in the translation of the first sentence of example (8) we need two different landing sites, one for the continuation with the second sentence and one for the continuation with the third. Both landing sites must be in the scope of *Every customer*. Finally, in the translation of the (sub-)sentence *Every customer is offered coffee* we need one more landing site for the conditionalization *if he looks wealthy* to act upon. In order to get a plausible reading of (8), then, the interpretation of its sentential clauses should be put together as follows:

*Every' (customer')([..*R*..([is offered coffee']..*S*..)]..*T*..)*, with abbreviations:

R: *if he looks wealthy' (then he...)*

S: *[... and] he is sent up to me'*

T: *[... and] if he doesn't look wealthy, he can wait'*

Interpreting (8) in accordance with this scheme, crucial type transitions are two applications of [VR] on the intransitive verb phrase *is offered coffee*, and an application of both [GD] and [VR] on the conditional phrase *if he looks wealthy*:

[is offered coffee]$_{IV}$ $\lambda x.\ \text{C-OFF}(^{\vee}x) \Rightarrow_{[VR][VR][1AR]}$
$\lambda T\lambda R.^{\vee}T(^{\wedge}\lambda x.^{\vee}\mathbf{R}(^{\wedge}\lambda S.^{\vee}S(^{\wedge}\text{C-OFF}(^{\vee}x))))$

[Every customer is offered coffee]$_S$ $\lambda \mathbf{R}.\ \forall d(\text{CUST}(d) \rightarrow {}^{\vee}\mathbf{R}(^{\wedge}\lambda S.^{\vee}S(^{\wedge}\text{C-OFF}(d))))$

[if he looks wealthy]$_{S\backslash S}$ $\lambda q.(\text{WEALTHY}(d) \rightarrow {}^{\vee}q) \Rightarrow_{[GD][VR][1AR]}$
$\lambda P\lambda S\lambda T.\ {}^{\vee}\mathbf{P}(^{\wedge}\lambda Q.\ {}^{\vee}T(^{\wedge}(\text{WEALTHY}(d) \rightarrow {}^{\vee}Q(S))))$

[Every customer is offered coffee if he looks wealthy.]$_S$
$\lambda S\lambda T.\ \forall d(\text{CUST}(d) \rightarrow {}^{\vee}T(^{\wedge}(\text{WEALTHY}(d) \rightarrow {}^{\vee}S(^{\wedge}\text{C-OFF}(d)))))$

[+ He is sent up to me as quickly as possible.]$_{S\backslash S}$ $\lambda p.\ {}^{\vee}p \wedge \text{SENT-UP}(d)$
[+ If he doesn't look wealthy, he can wait.]$_{S\backslash S}$ $\lambda p.\ {}^{\vee}p \wedge (\neg\text{WEALTHY}(d) \rightarrow \text{WAIT}(d))$

Every customer is offered coffee, if he looks wealthy. He is sent up to me as quickly as possible. If he doesn't look wealthy, he can wait.

$$\forall d(\text{CUST}(d) \rightarrow ([\text{WEALTHY}(d) \rightarrow [\text{C-OFF}(d) \wedge \text{SENT-UP}(d)]] \wedge [\neg\text{WEALTHY}(d) \rightarrow \text{WAIT}(d)]))$$

Left over right sequencing

The discussion of the previous example may have convinced the reader of the power of combined dynamics and flexibility in a compositional theory of interpretation. As may be expected, however, there is a reverse to this. In Hendriks (1989), it is shown that a version of Montague grammar with added type flexibility, already derives all theoretically possible scopings of the quantifiers in a sentence. Now that we have extended the grammar to some (rudimentary) discourse grammar with a functional operation of sequencing, also all possible scopings of the quantifiers in a complete *text* are derivable. Clearly, we have a source of overgeneration here.

Consider the following schematic sequence plus derived scope configuration:

$\ldots + S + S' \ldots + \quad \mathbf{NP},\ NP\backslash S\ + S'' \ldots + S''' \ldots$	(sequence of expressions)
$\ldots + \varphi + \psi \ldots +^{*} \quad \mathbf{T},\ \pi\ + \chi \ldots + \xi \ldots$	(associated basic translations)
$\ldots \wedge \mathbf{T}(^{\wedge}\lambda x.\ \varphi \wedge \psi \ldots \wedge \pi(x) \wedge \chi \ldots \wedge \xi) \ldots$	(derivable translation)

The quantifying noun phrase **NP**, with translation **T**, occurs in a sentence in the middle of a stretch of text. However, the noun phrase takes wide scope over the piece of text if we use the following, derivable, translations of the IV (with basic translation π) and of the marked sequencing operator:

$$\pi \Rightarrow_{[VR][VR][1AR]} \quad \lambda T\lambda R.^{\vee}T(^{\wedge}\lambda x.^{\vee}R(^{\wedge}\lambda R.^{\vee}R(^{\wedge}\pi(x))))$$
$$+^{*} \Rightarrow_{[GD][2AR]} \quad \lambda p\lambda Q\lambda R.\ {}^{\vee}Q(^{\wedge}\lambda Q.\ {}^{\vee}p \wedge {}^{\vee}Q(R))$$

What we see here is that a quantifying noun phrase, like the one highlighted in the example, not only may bind pronouns occurring up to some point to the right of it (after the noun phrase's occurrence), but also up to some point to the left of it (before its occurrence). This last possibility is one we would like to be able to exclude. Of course, we acknowledge the possibility of backwards binding (cataphora), but we think this phenomenon is subject to severe restrictions, and not to be admitted to occur freely in the process of sequencing sentences. (But notice that, in cases where we do have backwards binding, our model is rich enough to account for it, as the scheme above shows.)

In order to complete this section, then, we want to show that we can exclude quantifying noun phrases from binding pronouns in preceding parts of discourse. The main purpose of this is to show that it is possible to have effective management of the number of derivable readings, something which, after all, is a slippery business in a flexible system. So, as an illustration, we propose to restrict our type changing system in the following way. First, we demand that type change and restrictions on type change be stored in the lexicon. This is a necessary condition, it seems, for restrictions on type change to be effective in general. (Without this preliminary demand, restrictions on type change might be easily circumvented in a flexible system.) Second, together with the sequencing operator we store, in the lexicon, the restriction that [VR] may not apply to it, and that only its first argument may be raised by [AR]. This means that only raising of the left argument of the sequencing operator is possible, and that the right one can only be divided. Taken together, these demands rule out the possibility that some quantifier may come to get scope over (any) sequencing operator to the left of it.

This restriction on the readings of the sequencing operator is a somewhat severe one. One might think of alternative, more liberal or even more severe, ways of forcing the sequencing operator to be directional. All of this need not bother us here, though. What is important, is that we can prune away, from a flexible dynamic Montague grammar, all readings of the sequencing operator that allow for backwards binding of pronouns by quantifying noun phrases. (But of course, the adoption of such a restriction on the interpretation of sequencing does not preclude *all* cases of cataphoric relationships. As we will see below, other sentence connectives need not be directional in this sense.)

4 Dynamic type change and monotonicity

In the introduction we already pointed out that we do not always get the right results if we treat intersentential anaphoric relationships simply by giving antecedent noun phrases extrasentential scope. On such an approach, the donkey sentences (3) and (4) and also the other examples (5) - (7) would be assigned a counterintuitive reading. Consider (6):

(6) *No$_i$ farmer beats a$_j$ donkey he$_i$ owns. He$_i$ doesn't kick it$_j$ either.*

If both *No$_i$ farmer* and *a$_j$ donkey he$_i$ owns* are given scope over this two-sentence discourse, we get a reading that there is no farmer that beats but does not kick a donkey he owns, or, equivalently, that every farmer who beats a donkey of his also kicks it. This is wrong. More in general, equally infelicitous readings result if we treat the anaphoric relationships in (3) - (7) just by extending the scope of the relevant quantifiers.

Now, one conclusion that one might draw from this observation, is that the treatment of anaphoric pronouns as bound variables just fails here, and that, apparently, other mechanisms must be taken to play a role in the interpretation of these examples. In what follows, however, we like to stick to the analysis of pronouns as bound variables, and try to make a diagnosis of the troublesome cases. We will not defend this analysis against possible alternatives, we just want to push it to the limit here. We will argue that the roots of the observed semantic misbehavior can be localized, and be cured. After that, in section (5), we will show how the resulting adjusted analysis applies to (3) - (7) as well as to one more puzzling example.

Monotonicity

In order to find the source of our troubles, we need a notion of monotonicity first. We define it as follows :

φ: $(\vec{a},((\vec{b},t),(\vec{c},t)))$ is upward monotonic in its n^{th} argument, $\varphi^{\uparrow n}$, iff
$$\forall M \forall x_{1_{a_1}} \ldots x_{(n-1)_{a_{(n-1)}}} \forall p,q\ (p \subseteq q \rightarrow (\|\varphi\|_M(\vec{x}_a)(p) \subseteq \|\varphi\|_M(\vec{x}_a)(q)))$$
φ: $(\vec{a},((\vec{b},t),(\vec{c},t)))$ is downward monotonic in its n^{th} argument, $\varphi^{\downarrow n}$, iff
$$\forall M \forall x_{1_{a_1}} \ldots x_{(n-1)_{a_{(n-1)}}} \forall p,q\ (p \subseteq q \rightarrow (\|\varphi\|_M(\vec{x}_a)(q) \subseteq \|\varphi\|_M(\vec{x}_a)(p)))$$

It is easy to spot the downward monotonic basic expressions in our fragment: they are *not* in its only argument, the determiner *no* in both its arguments, and the determiner *every* and the connective *if* both in their first argument. Armed with this notion of monotonicity, some examination of the puzzling examples (and other ones) reveals that dynamic non-readings result when either a downward monotonic quantifier is given extrasentential scope, or when a quantifier is raised out of the scope of a downward monotonic functional expression. In example (6), for instance, the downward monotonic quantifier No_i *farmer* must be given wide scope over the second sentence in order to bind the anaphoric pronoun He_i up there. And in example (4) a_i *farmer* figures in the downward entailing antecedent of *if*, and still it is to be given wide scope over the implication in order to bind the anaphoric pronoun he_i in the consequent. Now why should these type changes that interfere with downward monotonicity be so problematic? We take a closer look.

Downward monotonic quantifiers and update of information

Quite a plausible first principle of a discourse semantics, is that in a part of discourse every sentence sequenced contributes to information update. (The phenomena of revision or retraction are not under discussion here.) Also, the substitution of one sentence in a piece of discourse by a more informative one, should yield a more informative piece of discourse. In our flexible dynamic grammar, these principles will be observed if all dynamic sentence translations are upward monotonic. Now, in the examples considered in section (3), only upward monotonic sentence translations played a role. However, in order to cope with examples (3) - (7), downward monotonic expressions need to be raised or divided, and that would give rise to downward monotonic sentence translations. It is precisely this that causes troubles. Consider example (6) again:

(6) No_i *farmer beats* a_j *donkey* he_i *owns.* He_i *doesn't kick* it_j *either.*

A derivable dynamic translation of the first sentence of (6) reads:

$\lambda R.\neg\exists d_i\exists d_j[\text{FARMER}(d_i) \wedge \text{DONKEY\_OF}(d_i)(d_j) \wedge {}^{\vee}R({}^{\wedge}\text{BEAT}(d_j)(d_i))]$

This expression is downward monotonic. If the sentence, under this translation, is sequenced as in (6), no information update takes place, but information downdate: the proposition expressed by the first sentence (that there is no farmer who does this one thing of beating his donkey) will be weakened to the proposition that there is no farmer who does these two things of both beating his donkey, *and* not kicking it. Furthermore, if the first sentence of (6) were sequenced with a more informative continuation *He$_i$ doesn't kick it$_j$ nor yell at it$_j$*, the overall result would be an even weaker proposition, to wit the proposition that no farmer does each of the following three things: beating his donkey *and* not kicking it *and* not yelling at it. Clearly, the violation of the update principle suffices for rejecting this kind of dynamics.

At this point, one might think of two solutions to this problem within the approach to the dynamics of discourse that we have taken. The first solution simply consists of eliminating all downward monotonic sentence translations. However, apart from being ad hoc, this would rule out an account of the dynamic binding of pronouns by downward monotonic noun phrases, or by noun phrases in downward entailing contexts. Given the enterprise we are engaged in, and in view of examples (3) - (7), this isn't at all satisfactory. The second option is to generate upward monotonic sentence translations *also* on the basis of downward entailing constituent noun phrases. This, then, is the line that we pursue.

A dynamic translation of a downward monotonic quantifier

The variable R which is bound by the λ-operator in a dynamic sentence translation, is the place-holder for extension of information by continuations of current discourse. This being given, restoring proper monotonicity just amounts to enclosing this variable R by two negation signs. The two negations involved then constitute a local context of reversed monotonicity in between them, just for the extension of information to act within. Now, if no real update takes place, the extension of information is just the identity function, and the two negations cancel each other. Otherwise, if there really is some non-trivial discourse continuation, real update takes place. We propose, therefore, that the translation of the first sentence in (6) reads as follows (~ indicates generalized negation, ~R~ is the dual of R, ${}^{\wedge}\lambda p.{}^{\vee}({\sim}R)({\sim}p) = {}^{\wedge}\lambda p.\neg{}^{\vee}R({}^{\wedge}\neg{}^{\vee}p)$):

$$\lambda R.\ (\lambda R.\neg\exists d_i\exists d_j[\text{FARMER}(d_i) \wedge \text{DONKEY\_OF}(d_i)(d_j) \wedge {}^{\vee}R({}^{\wedge}\text{BEAT}(d_j)(d_i))])({\sim}R{\sim}) \Leftrightarrow$$
$$\lambda R.\neg\exists d_i\exists d_j[\text{FARMER}(d_i) \wedge \text{DONKEY\_OF}(d_i)(d_j) \wedge \neg{}^{\vee}R({}^{\wedge}\neg\text{BEAT}(d_j)(d_i))] \Leftrightarrow$$
$$\lambda R.\ \forall d_i\forall d_j[(\text{FARMER}(d_i) \wedge \text{DONKEY\_OF}(d_i)(d_j) \rightarrow {}^{\vee}R({}^{\wedge}\neg\text{BEAT}(d_j)(d_i))]$$

This expression turns out to be equivalent to the dynamic translation of *Every farmer does not beat a donkey he owns*, a sentence with the same truth-conditional content as the first sentence of (6).

Let us now consider the relevant monotonicity properties of the adjusted dynamic translation of a downward entailing determiner. According to our proposal, the dynamic translation of *No P is a Q*, should read:

$$\lambda R.\ \neg\exists d(P'(d) \wedge \neg{}^{\vee}R({}^{\wedge}\neg Q'(d)))$$

Evidently, this is an upward monotonic quantifier over propositions, as required. Also, if the sentence *No P is a Q*, under this translation, is sequenced with some proposition q, the result is more informative than if it is sequenced with a proposition weaker than q. In other words, the following expression is upward monotonic as well:

$$\lambda q.\ (\lambda R.\ \neg\exists d(P'(d) \wedge \neg{}^{\vee}R({}^{\wedge}\neg Q'(d))))({}^{\wedge}\lambda p.\ {}^{\vee}p \wedge {}^{\vee}q) \Leftrightarrow$$
$$\lambda q.\ \neg\exists d(P'(d) \wedge \neg(\neg Q'(d) \wedge {}^{\vee}q))$$

Finally, with regard to the truth-conditional content of the common noun and the intransitive verb, the schematic discourse under discussion is downward monotonic again, that is, the following λ-term is, properly, downward monotonic in both of its argument places again:

$$\lambda P\lambda Q.\ \neg\exists d(P(d) \wedge \neg(\neg Q(d) \wedge {}^{\vee}q))$$

An upward monotonic quantifier in a downward entailing context

Quite a similar story (the mirror image) can be told about a quantifier that comes to bind anaphoric pronouns occurring outside of the downward monotonic context in which the quantifier occurs. Consider the donkey sentence (4):

(4) *If a_i farmer owns a_j donkey (then) he_i beats it_j.*

The problem with (4) is the opposite of the problem with (6). If we just raise the antecedent clause here in order to let the noun phrases in it bind the pronouns in the consequent of (4), we do not reverse the upward monotonicity of the clause containing the pronouns, but we reverse the downward monotonicity of the clause containing the antecedent noun phrases. In other words, whereas the *if...then* operator should be downward entailing in its first argument (and upward entailing in the second one), just raising of this argument makes the operator upward monotonic there. Since the problem here is the opposite of the problem in (6), we can apply the opposite remedy (which is the same remedy technically) and surround such a raised argument by two negation signs in order to create a local context of restored downward monotonicity. So, instead of the expected raised translation of the implication $\lambda P\lambda q.\ {}^{\vee}P({}^{\wedge}\lambda p.\ \neg({}^{\vee}p \wedge \neg{}^{\vee}q))$, we opt for the following one ($\sim P\sim$, the dual of P, is ${}^{\wedge}\lambda R.\ {}^{\vee}(\sim P)(\sim R)$):

$$\lambda P.\ (\lambda P\lambda q.\ {}^{\vee}P({}^{\wedge}\lambda p.\ \neg({}^{\vee}p \wedge \neg{}^{\vee}q)))(\sim P\sim) \Leftrightarrow$$
$$\lambda P\lambda q.\ \neg{}^{\vee}P({}^{\wedge}\lambda p.\ {}^{\vee}p \wedge \neg{}^{\vee}q)$$

Clearly, this expression is downward monotonic in its first argument. Also, if this raised implication is applied to some upward monotonic dynamic sentence translation P, the result $\lambda q.\ \neg P({}^{\wedge}\lambda p.\ {}^{\vee}p \wedge \neg{}^{\vee}q)$ is upward monotonic as well, as is easily verified.

Dual type changes

Our findings thus far can be straightforwardly generalized. We propose alternative interpretations of the type changing rules when they relate to downward entailing argument expressions of a function φ onto which the type changing rules are applied. The difference in interpretation will consist of enclosing argument expressions in between two negation signs when these arguments invade, or escape out of, downward entailing contexts. Therefore, the raising and division of the nth argument place of a function $\varphi^{\downarrow n}$, is interpreted as follows ($(\vec{a},d)$ now abbreviates $(a_1,\ldots(a_{(n-1)},d)\ldots)$, b and c are required to be conjoinable types, X* is the dual of X, ${}^{\wedge}\lambda Y.\ {}^{\vee}(\sim X)(\sim Y)$, where ~ indicates generalized negation):

[DVR] $(\vec{a},(b,c))\colon \lambda\vec{x}\lambda y.\varphi(\vec{x})(y) \Rightarrow (\vec{a},({}^{\wedge}({}^{\wedge}(b,c),t),t))\colon \lambda\vec{x}_a\lambda R.\ {}^{\vee}R^*({}^{\wedge}\lambda y.\ \varphi(\vec{x}_a)(y))$

[DAR] $(\vec{a},({}^{\wedge}b,(\vec{c},t)))\colon \lambda\vec{x}\lambda y\lambda\vec{z}.\ \varphi(\vec{x})(y)(\vec{z}) \Rightarrow$
$(\vec{a},({}^{\wedge}({}^{\wedge}({}^{\wedge}b,t),t),(\vec{c},t)))\colon \lambda\vec{x}\lambda Y\lambda\vec{z}.{}^{\vee}Y^*({}^{\wedge}\lambda y.\ \varphi(\vec{x})(y)(\vec{z}))$

[DGD] $(\vec{a},({}^{\wedge}(\vec{b},c),c))\colon \lambda\vec{x}_a\lambda P.\ \varphi(\vec{x}_a)(P) \Rightarrow$
$(\vec{a},({}^{\wedge}(\vec{b},({}^{\wedge}({}^{\wedge}t,t),c)),({}^{\wedge}({}^{\wedge}t,t),c)))\colon \lambda\vec{x}_a\lambda Q\lambda R.\ \varphi(\vec{x}_a)({}^{\wedge}\lambda\vec{y}_b.\ {}^{\vee}Q(\vec{y}_b)(R^*))$

In [DAR] and [DVR], the dual of the raised argument is taken, and in [DAR] the dual is taken of the argument divided into. These duals preserve φ's monotonicity properties, when these are in danger of getting reversed by the invasion of argument expressions in a downward monotonic context, or their escape out of it. Below, we will show that the application of these rules gives us quite a few desirable results. Before that, two remarks need to be made.

First, we point at a remaining imperfection in the dual rules. The rule [DGD] only allows for division into type $({}^{\wedge}t,t)$, and, of course, we want a generalization of the rule such that we have division into all types. However, this presupposes a notion of a generalized dual which ensures monotonicity preservation also when higher dynamic types come into play. A related imperfection adheres to [DVR]. The value $\varphi(\vec{x}_a)(y)$ (type c) of the function φ that is raised, may be a quantifier over propositions. If this is the case, then the interpretation of the raising should also incorporate an internal dualization $\lambda R.\ \varphi(\vec{x}_a)(y)(R^*)$, and this, as well, should be generalized to dualization of higher dynamic types. However, we will not generalize the rules here, since that would takes us too far now, and because the imperfections are harmless for the present purposes.

The second remark concerns the aforementioned 'convenient' redundancy in our version of Hendriks' type system. Above, we remarked that the rule of division overlaps with the rule of argument raising and that the former should eventually replace the latter. The elimination of the rule of argument raising is opportune now for the following reason. If we use a system with [AR] and [DAR], a downward monotonic quantifier can be given extrasentential scope, *without* changing the type of the quantifier. In that case, information downdate is still possible after all. However, without [(D)AR], a quantifier can only be given wide scope by a division of it. In that case, the dynamization of the (downward monotonic) quantifier forces the application of a dual type shift, and, thus, guarantees proper monotonicity properties. We remark here that where applications of [AR] can be replaced by applications of [GD] and [VR], applications of [DAR] are equally replaceable by applications of [(D)GD] and [(D)VR], that is, provided that the dual type shifts are generalized in the way indicated above.

5 Applications

With the reservations made above we can now apply the modified system of type change to some examples. For ease of exposition we will keep using instances of [(D)AR], but we remind the reader that these are to be, and can be, eliminated.

Example (3)

(3) *Every$_i$ farmer who owns a$_j$ donkey beats it$_j$. He$_i$ is never nice to it$_j$.*

An interpretation of this example in which the pronouns are bound by the coindexed quantifiers can be arrived at in the following way. Starting from the functional translation of the relative pronoun *who* given above, a raised common noun translation is derivable:

who
$$\lambda P\lambda Q\lambda x.\ {}^{\vee}Q(x) \wedge {}^{\vee}P(x) \Rightarrow_{[3AR][1AR]}$$
$$\lambda P\lambda Q\lambda T.\ {}^{\vee}\mathbf{P}({}^{\wedge}\lambda P.\ {}^{\vee}T({}^{\wedge}\lambda x.\ {}^{\vee}Q(x) \wedge {}^{\vee}P(x))$$

owns a$_j$ donkey
$$\lambda T.\ \exists d_j(\text{DONKEY}(d_j) \wedge {}^{\vee}T({}^{\wedge}\text{OWN}(d_j)))$$

farmer who owns a$_j$ donkey
$$\lambda T.\ \exists d_j(\text{DONKEY}(d_j) \wedge {}^{\vee}T({}^{\wedge}\lambda x.\text{FARMER}({}^{\vee}x)\wedge\text{OWN}(d_j)({}^{\vee}x)))$$

Now, for the determiner *every* to be applicable to this translation, its first argument has to be raised. Since *every* is downward monotonic in this argument, the dual raising is used:

Every$_i$
$$\lambda P\lambda Q.\ \forall d_i({}^{\vee}P({}^{\wedge}d_i) \rightarrow {}^{\vee}Q({}^{\wedge}d_i)) \Rightarrow_{[1DAR]}$$
$$\lambda P\lambda Q.\ \neg{}^{\vee}\mathbf{P}({}^{\wedge}\lambda P.\ \neg\forall d_i({}^{\vee}P({}^{\wedge}d_i) \rightarrow {}^{\vee}Q({}^{\wedge}d_i)))$$

Every$_i$ farmer who owns a$_j$ donkey
$$\lambda Q.\ \neg\exists d_j(\text{DONKEY}(d_j) \wedge \neg\forall d_i((\text{FARMER}(d_i) \wedge \text{OWN}(d_j)(d_i)) \rightarrow {}^{\vee}Q({}^{\wedge}d_i)))$$
$$\Leftrightarrow \lambda Q.\ \forall d_j\forall d_i((\text{DONKEY}(d_j) \wedge \text{FARMER}(d_i) \wedge \text{OWN}(d_j)(d_i)) \rightarrow {}^{\vee}Q({}^{\wedge}d_i))$$

Further dynamic interpretation, similar to that in the examples (1) and (2), yields the following interpretation for the whole of (3):

$$\forall d_j\forall d_i((\text{DONKEY}(d_j) \wedge \text{FARMER}(d_i) \wedge \text{OWN}(d_j)(d_i)) \rightarrow (\text{BEAT}(d_j)(d_i) \wedge \text{NOT\_NICE\_TO}(d_j)(d_i)))$$

So, under these translations, example (3) in fact is assigned its proper truth conditions.

Example (4)

(4) *If a$_i$ farmer owns a$_j$ donkey he$_i$ beats it$_j$.*

This example has already been discussed to some extent. The sentence gets its familiar truth conditions, under a dynamic translation of the sentential clause *a$_i$ farmer owns a$_j$ donkey* and a dual raising of the first (downward monotonic) argument of *if*:

If
$$\lambda p\lambda q.\ {}^{\vee}p \rightarrow {}^{\vee}q \Rightarrow_{[1DAR]}$$
$$\lambda P\lambda q.\neg{}^{\vee}P({}^{\wedge}\lambda p.\neg({}^{\vee}p \rightarrow {}^{\vee}q))$$

a$_i$ farmer owns a$_j$ donkey
$$\lambda R.\ \exists d_i\exists d_j(\text{FARMER}(d_i) \wedge \text{DONKEY}(d_j) \wedge {}^{\vee}R({}^{\wedge}\text{OWN}(d_j)(d_i)))$$

If a$_i$ farmer owns a$_j$ donkey he$_i$ beats it$_j$.
$$\forall d_i\forall d_j((\text{FARMER}(d_i) \wedge \text{DONKEY}(d_j) \wedge \text{OWN}(d_j)(d_i)) \rightarrow \text{BEAT}(d_j)(d_i))$$

Example (5)

(5) *It is not true that Mary has no car. It is standing in front of the house.*

In the derivation of a dynamic translation of the first sentence of (5), two dual type changes are involved, a dual division of the downward monotonic determiner *no*, and one of the sentence negation:

has	$\lambda y \lambda x.\ \mathrm{OWN}({}^{\vee}y)({}^{\vee}x) \Rightarrow_{[VR][VR][GD][GD]}$ $\lambda T.\ {}^{\vee}T({}^{\wedge}\lambda y \lambda x \lambda R.\ {}^{\vee}R({}^{\wedge}\mathrm{OWN}({}^{\vee}y)({}^{\vee}x)))$
no car	$\lambda Q.\ \neg\exists d(\mathrm{CAR}(d) \wedge {}^{\vee}Q({}^{\wedge}d)) \Rightarrow_{[DGD][GD]}$ $\lambda Q \lambda x \lambda R.\ \neg\exists d(\mathrm{CAR}(d) \wedge {}^{\vee}Q({}^{\wedge}d)(x)(R^{*}))$
Mary has no car.	$\lambda R.\ \neg\exists d(\mathrm{CAR}(d) \wedge {}^{\vee}R^{*}({}^{\wedge}\mathrm{OWN}(d)(m)))$

A dual divided sentence negation is applied to (the intension) of the quantifier over propositions that is the interpretation of *Mary has no car*. The two negations cancel each other, and the same holds for the two duals ($R^{**} \Leftrightarrow {\sim\sim}R{\sim\sim} \Leftrightarrow R$):

It is not true that	$\lambda p.\ \neg{}^{\vee}p \Rightarrow_{[DGD]}$ $\lambda P \lambda R.\ \neg{}^{\vee}P(R^{*})$

It is not true that Mary has no car.

$$(\lambda P \lambda R.\ \neg{}^{\vee}P(R^{*}))({}^{\wedge}\lambda R.\ \neg\exists d(\mathrm{CAR}(d) \wedge {}^{\vee}R^{*}({}^{\wedge}\mathrm{OWN}(d)(m)))) \Leftrightarrow$$

$$\lambda R.\ \neg\neg\exists d(\mathrm{CAR}(d) \wedge {}^{\vee}R^{**}({}^{\wedge}\mathrm{OWN}(d)(m))) \Leftrightarrow$$

$$\lambda R.\exists d(\mathrm{CAR}(d) \wedge {}^{\vee}R({}^{\wedge}\mathrm{OWN}(d)(m)))$$

So, as a matter of fact, the derived translation of the first sentence of (5) equals a dynamic interpretation of the sentence *Mary owns a car*. The dynamic interpretation of the whole then is easily established to be:

$$\exists d(\mathrm{CAR}(d) \wedge \mathrm{OWN}(d)(m) \wedge \mathrm{IN\_FRONT\_OF\_THE\_HOUSE}(d))$$

Example (6)

(6) *No$_i$ farmer beats a$_j$ donkey he$_i$ owns. He$_i$ doesn't kick it$_j$ either.*

If we use the following translations, we get a reading of example (6) which accounts for the anaphoric relationships as indicated:

No$_i$ farmer	$\lambda Q \lambda R.\neg\exists d_i(\mathrm{FARM}(d_i) \wedge {}^{\vee}Q({}^{\wedge}d_i)(R^{*}))$
beats a$_j$ donkey he$_i$ owns	$\lambda T.\ {}^{\vee}T({}^{\wedge}\lambda x \lambda R.\exists d_j(\mathrm{DONK\_OF}(d_i)(d_j) \wedge {}^{\vee}R({}^{\wedge}\mathrm{BEAT}(d_j)({}^{\vee}x))))$

No$_i$ farmer beats a$_j$ donkey he$_i$ owns.

$$\lambda R.\neg\exists d_i(\mathrm{FARM}(d_i) \wedge \exists d_j(\mathrm{DONK\_OF}(d_i)(d_j) \wedge {}^{\vee}R^{*}({}^{\wedge}\mathrm{BEAT}(d_j)(d_i))))$$

$$\Leftrightarrow \lambda R.\forall d_i \forall d_j((\mathrm{FARM}(d_i) \wedge \mathrm{DONK\_OF}(d_i)(d_j) \rightarrow {}^{\vee}R({}^{\wedge}\neg\mathrm{BEAT}(d_j)(d_i)))$$

No$_i$ farmer beats a$_j$ donkey he$_i$ owns. He$_i$ doesn't kick it$_j$ either.

$$\forall d_i \forall d_j((\mathrm{FARM}(d_i) \wedge \mathrm{DONK\_OF}(d_i)(d_j)) \rightarrow (\neg\mathrm{BEAT}(d_j)(d_i) \wedge \neg\mathrm{KICK}(d_j)(d_i)))$$

Example (7)

(7) *There is no bathroom in the house or it is in a funny place. It is not on the first floor though.*

Here we have the famous bathroom example. *Or* is translated, basically, as a disjunction. Since a disjunction is upward monotonic in both of its arguments, we can have plain argument raising of its first argument, and division of its second:

$$Or \quad \lambda p\lambda q.\ \check{}p \vee \check{}q \Rightarrow_{[1AR][GD]} \quad \lambda P\lambda Q\lambda R.\ \check{}P(\hat{}\lambda p.\ \check{}p \vee \check{}Q(R))$$

The first disjunct is *No bathroom is in the house*. By a dual division of the negative determiner *no*, it is assigned the following dynamic translation:

$$\lambda R.\neg\exists d(\text{BATHROOM}(d) \wedge \check{}R^*(\hat{}\text{IN\_HOUSE}(d))) \Leftrightarrow$$
$$\lambda R.\forall d(\text{BATHROOM}(d) \rightarrow \check{}R(\hat{}\neg\text{IN\_HOUSE}(d)))$$

Using a raised translation of the second disjunct, the whole disjunction, after application, reduces to the following:

$$\lambda R.\ \forall d(\text{BATHROOM}(d) \rightarrow (\neg\text{IN\_HOUSE}(d) \vee \check{}R(\hat{}\text{FUNNY\_PLACE}(d))))$$

If the disjunction under this translation is sequenced with the second sentence *It is not on the first floor*, we arrive at the following interpretation of example (7):

$$\forall d(\text{BATHROOM}(d) \rightarrow (\neg\text{IN\_HOUSE}(d) \vee (\text{FUNNY\_PLACE}(d) \wedge \neg\text{FIRST\_FLOOR}(d)))) \Leftrightarrow$$
$$\forall d((\text{BATHROOM}(d) \wedge \text{IN\_HOUSE}(d)) \rightarrow (\text{FUNNY\_PLACE}(d) \wedge \neg\text{FIRST\_FLOOR}(d)))$$

Cataphoric pronouns

The preceding discussion shows that the analysis of pronouns as (indirectly) bound variables can be saved in spite of the seeming counterexamples (3) - (7). A logical adjustment, in well-defined cases, of the interpretation of the type changing rules suffices for deriving a correct interpretation of these examples, this, in a compositional way. We now briefly discuss two more examples, which again illustrate the generative force of the present approach to pronouns.

Above we described the sequencing operation as directional in the sense that it does not allow for right over left binding. However, we only opted for excluding cataphoric relationships between noun phrases that occur in two different sentences in a *sequence*. But, of course, not every sentence connective needs to be directional in the way the sequencing operation is, and a case in point is the conditional. Since we haven't imposed restrictions on type shifts of the conditional connective, we can raise its second argument:

$$If \quad \lambda p\lambda q.\ \check{}p \rightarrow \check{}q \Rightarrow_{[2AR]} \quad \lambda p\lambda Q.\ \check{}Q(\hat{}\lambda q.\ \check{}p \rightarrow \check{}q)$$

Using this derived translation, we can account for a cataphoric relationship that holds between noun phrases in the two sentential clauses of an implication:

(9) *If he is in danger, every man prays to God.*

he is in danger IN_DANGER(d)
every man prays to God $\lambda R.\ \forall d(\text{MAN}(d) \rightarrow {}^{\vee}R({}^{\wedge}\text{PRAY\_TO\_GOD}(d)))$
If he is in danger, every man prays to God.
$\forall d(\text{MAN}(d) \rightarrow (\text{IN\_DANGER}(d) \rightarrow \text{PRAY\_TO\_GOD}(d)))$

We see that a plausible reading of (9) results if the quantifying noun phrase *Every man* is given wide scope over the implication. Crucial again is that we use DIL as the framework of interpretation, this time in order to account for a *cataphoric* relationship. The next example is really a striking one. It exhibits backwards binding by a downward monotonic quantifier:

(10) *If he isn't insane, no man beats a donkey.*

If we were to give the quantifying noun phrase *no man* wide scope here, the following interpretation of (10) would be the result:

$\neg\exists d(\text{MAN}(d) \wedge (\text{SANE}(d) \rightarrow \text{BEAT\_A\_DONKEY}(d)))$

Now if one rephrases this formula for oneself, it may sound alright: *No man is such that if he is sane, he beats a donkey*. However, it is not correct. The formula amounts to the statement that every man is sane and does not beat a donkey, which is not the interpretation of (10). On our account a different interpretation results. The consequent in (10) can only take scope over the implication if it is translated as a quantifier over propositions. Because this consequent contains the downward monotonic noun phrase *no man*, a dynamic translation of the consequent requires a dual type shift, and the following translation is the result:

$\lambda R.\ \neg\exists d(\text{MAN}(d) \wedge {}^{\vee}R^*({}^{\wedge}\text{BEAT\_A\_DONKEY}(d))) \Leftrightarrow$
$\lambda R.\ \forall d(\text{MAN}(d) \rightarrow {}^{\vee}R({}^{\wedge}\neg\text{BEAT\_A\_DONKEY}(d)))$

Using this translation, then, (10) is assigned the following interpretation, which we take to be the correct one:

$\forall d(\text{MAN}(d) \rightarrow (\text{SANE}(d) \rightarrow \neg\text{BEAT\_A\_DONKEY}(d)))$

Logical equivalences at the dynamic level

We have proposed a system of type change with a dualized interpretation of type changes of downward monotonic expressions. We have seen that this system gives a correct interpretation of the examples which we set out to deal with. In order to conclude this section, we now very briefly discuss an interesting property of the present analysis. In our system, characteristic classical equivalences remain valid on the derived level of dynamic sentence translations of the type of quantifiers over propositions.

First, observe that the law of double negation still holds, also when two, divided, negations apply to a quantifier over propositions (although they are crucial, we omit the cups and caps for the sake of readability from now on):

Not not Φ $(\lambda P\lambda R.\neg P(R^*))((\lambda P\lambda R.\neg P(R^*))(\Phi')) \Leftrightarrow$
$\lambda R.\neg\neg\Phi'(R^{**}) \Leftrightarrow \lambda R.\Phi'(R) \Leftrightarrow \Phi'$

Example (5) may serve as a natural language illustration of this extended law of double negation. The double negation of the first sentence in (5) not only preserves truth conditional content, but also preserves the dynamic binding possibilities of the existential quantifier in the scope of both negations.

Next, observe that the classical interdefinability of sentential connectives remains valid at the derived level of quantifiers over propositions. Consider implication, disjunction and conjunction after raising the first argument and dividing the second:

If Φ then Ψ	$(\lambda P\lambda Q\lambda R.\ P^*(\lambda p.\ p \rightarrow Q(R)))(\Phi')(\Psi') \Leftrightarrow$
Not Φ, or Ψ	$(\lambda P\lambda Q\lambda R.\ P(\lambda p.\ p \vee Q(R)))(\lambda R.\neg\Phi'(R^*))(\Psi') \Leftrightarrow$
Not (Φ and not Ψ)	$\lambda R.\neg(\lambda P\lambda Q\lambda R.\ P(\lambda p.\ p \wedge Q(R)))(\Phi')(\lambda R.\neg\Psi'(R^*))(R^*) \Leftrightarrow$
	$\lambda R.\ \neg\Phi'(\lambda p.\ p \wedge \neg\Psi'(R))$

We can find natural language examples also of this dynamic interdefinability:

(7') *If there is a bathroom in the house, then it is in a funny place.*
(7) *Either there is no bathroom in the house or it is in a funny place.*
(7") *It is* not *true that there is a bathroom in the house and that it is* not *in a funny place.*

We take it that the continuation of these sentences with *It is not on the first floor*, in all three cases gives rise to the predicted reading, i.e. the reading of (7) that we derived above.

Finally, dividing the quantifiers in our fragment also preserves interdefinability:

Every Φ Ψ	$(\lambda Q\lambda R.\ \forall d(\Phi'(d) \rightarrow Q(d)(R)))(\lambda x\lambda R.\ \Psi'(x)(R)) \Leftrightarrow$
No Φ does not Ψ	$(\lambda Q\lambda R.\ \neg\exists d(\Phi'(d) \wedge Q(d)(R^*)))(\lambda x\lambda R.\ \neg\Psi'(x)(R^*)) \Leftrightarrow$
	$\lambda R.\ \forall d(\Phi'(d) \rightarrow \Psi'(d)(R))$

If we rephrase example (2), we have an example of this equivalence in natural language:

(2) *Every chessbox comes with a spare pawn. It is taped on top of it.*
(2') No *chessbox does* not *come with a spare pawn. It is (always) taped on top of it.*

We may admit here that the examples that we discussed are a bit marginal indeed, marginal in the sense that a similar interpretation of other examples with structures parallel to these examples, often would not give rise to plausible readings. However, we do not think that this makes up an argument against our analysis. In the first place, we just want to be able to derive all, and only, the possible readings that (sequences of) sentences may have. Our examples, then, illustrate the use of our rules. Second, if we discuss a reading of an example with dynamic binding by quantifiers, it is only one among many other possible readings it has. For one thing, traditional static readings remain derivable as well. So, what is lacking, is an extension of the theory that explains what readings of what sentences are less likely to be plausible. This topic, however, falls beyond the scope of the present paper.

We would have been very happy, then, to conclude that our analysis only generates possible readings of puzzling examples. Unfortunately, if we systematically survey all cases in which a quantifier escapes from a downward entailing context, it appears that a certain type of sentences still can be assigned counterintuitive readings. Difficulties arise when a dynamic implication, or a dynamic universal quantifier, occurs in the antecedent of an implication, or in the common noun phrase of a universal determiner.

As was intended, an existential quantifier in the antecedent of an implication, or in the restriction of a universal quantifier, gets universal force when given wide scope. This is what happens in the illustrious donkey sentence. However, in our theory the reverse of this is that a universal quantifier in the same context gets existential force when given wide scope. This runs counter to intuition. The impression urges itself upon us that, basically, it is the notion of dynamic implication that causes troubles here. More specifically, it seems that an implication itself may not be taken to imply something dynamically, that is, to take wide scope from within the antecedent of a superordinate implication.

We will not go into this matter now, though, but confine ourselves to referring to Chierchia (1990). There, a conservative notion of implication is defined that does not give rise to the present problems. However, the drawback of Chierchia's conservativity is that it rather drastically alters the flexible approach to the dynamics of discourse that we pursued in this paper. A discussion of dynamic implication, conservativity and flexibility, therefore, is postponed to some other occasion.

6 Conclusions

In this paper we discussed two adaptations of Montague grammar. We added (a version of) Hendriks' system of type change to it, and replaced the system of intensional logic by the system of dynamic intensional logic as the framework of interpretation. Together, these two modifications make up a compositionalization, so to speak, of Montague's rules of quantification. Whereas, on the one hand, Hendriks' type flexibility enables us to generate all possible scope configurations, on the other hand, the use of DIL ensures that quantifiers also bind coindexed pronouns in their scope.

Next, with an interpretation of the sequencing operator as functional conjunction, we brought in the possibility of (compositionally) assigning quantifiers extrasentential scope. Because quantifiers bind coindexed pronouns in their scope, this enabled us to give a compositional treatment of intersentential anaphoric relationships. Furthermore, the combined forces of dynamics and flexibility proved to be powerful enough to account for complex intersentential dependencies like the ones we found in example (8).

Finally, we discussed a problem concerning anaphoric binding by quantifying noun phrases that are in the scope of a negation or of other expressions of downward monotonicity. We proposed a solution that consists of using dualized interpretations of type shifts when these apply to downward entailing expressions. We showed that the resulting alternative type changing rules yield a correct interpretation of examples, some of which, on the face of it, are really perplexing.

On the other hand, our treatment of the dynamics of discourse raises further questions. For one thing, a counterintuitive interpretation results if in our system a universal quantifier in the antecedent of an implication is given wide scope. One might choose, therefore, to forbid such a wide scope reading, but this would strike us as too ad hoc. An alternative might be to adopt a conservative variant of the dynamic implication, as is proposed by Chierchia. However, it is not clear as yet how to derive this kind of (dynamic) conservativity by using type changing rules as general as the ones we used here.

Furthermore, an extension of our fragment of natural language with other kinds of quantifying noun phrases is not without problems. For instance, it just would not be adequate to assign extrasentential scope to noun phrases that are non-monotonic, like the noun phrase *exactly* n *men*. Clearly, further research is needed here, research into the precise data concerning (the dynamics of) other quantifying noun phrases, as well as into the possibility of accounting for these data by type flexibility.

A final question concerns the precise, and effective, formulation of restrictions on type change, restrictions excluding (certain cases of) backwards binding, as well as other plausible constraints on possible scope configurations. It is not to be excluded at forehand that such an effective management of the system of type change requires an adjusted formulation of the type changing rules themselves. Here as well, further comprehensive future research must help in settling the matter.

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Generalized Quantifiers join Naked Infinitives*

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INTRODUCTION

In English there are several ways to describe perception, as is witnessed by the following examples:

(1) James saw Samuel
(2) James saw Samuel's winking
(3) James saw Samuel eat a fish
(4) James saw Samuel eating a fish
(5) James saw that the sun was rising

Each of these sentences emphasizes a different facet of perception. Sentence (1) reports the perception of an object, sentence (2) that of an event. Sentence (5) states that James was perceptually aware of the sun's rising, but its truth does not require that James did directly perceive the sun as it rose. He could have seen the large orange spot on the wallpaper turn bright yellow and then blindening white. The sentences (1) to (4) on the other hand describe perceptions of a more direct kind, all of them independent of the mental state of the perceiver. Sentence (3) might be true, for instance, without James knowing (or believing) that it was Samuel he saw. A speaker might use (3) when he wants to remain neutral in regard to James' perceptual awareness.

Idealized versions of sentences of forms (3)—naked infinitive perception reports—and (4)—gerundive perception reports—are the object of study in this paper. The main difference between these kinds of sentence is aspectual. In English one uses (3) to report that James' perception includes that Samuel finished his fish. If Samuel was still eating the fish and James saw him do so, one uses (4) instead. The difference is indicated by (6) and (7):

(6)* We are seeing Mary cross the street
(7) We are seeing Mary crossing the street

There is a 'clash' between the progressive form of (6) and the aspectual character of NI-sentences. In (7) there is no such conflict.

Here as in most other papers on the subject, we shall abstract from matters of tense and aspect, and we shall not treat intensional phenomena other than those induced by the verb *see* either. This decision eliminates the differences between naked infinitive and gerundive perception reports to be captured by a semantic theory. Depending on one's point of view both report, either the perception of an event, or of an object as possessing a certain property. Having said this, we shall concentrate in the sequel on the naked infinitive variant, henceforth called 'NI sentences'.

The logical part of the work reported here is inspired by *A Scenic Tour through the Land of Naked Infinitives* under the guidance of Hans Kamp (1984, unpublished). Among other things, Kamp purports to show how Barwise's insights on NI sentences (Barwise, 1981) can be treated within a simple extension of first-order predicate logic. The main feature of Kamp's treatment consists of introducing partiality into a traditional semantic theory. He interprets an ordinary first-order model as a model of the world at a

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* This article is a shortened version of van der Does 1990. For the proofs of propositions and theorems—which are omitted here—I refer to this report.

I would like to thank Johan van Benthem, Robin Cooper, Martin Emms, Hans Kamp, Stanley Peters, Barry Richards and Frank Veltman for the stimulating discussions and the probing questions they provided. I am also grateful to Dick de Jongh and Hans Swart for detailed criticism on the final draft. A pleasing and stimulating place to work is the Centre for Cognitive Science (Edinburgh, U.K.), where previous drafts of this article were written. The work reported here is supported in part by ESPRIT Basic Research Action 3175, DYANA.

certain point in time, and then takes the partial submodels of this model to be situations in the world at that time. Given this set-up, the *see* relation between a visual perceiver and a situation seen can be defined as a relation among an element of the domain on the one hand and a partial submodel on the other. In fact all perception verbs that allow for naked infinitives can be treated along these lines.[1]

The simplicity of the semantic objects introduced by Kamp is appealing, especially if one compares them with the intricate structures defined in *Situations and Attitudes* (Barwise and Perry, 1983: 60-61). Unfortunately, a closer look reveals that the resulting semantics is wanting. This is mainly due to the syntactic structure of the first-order formulas used to model the complement of an NI sentence. This structure prevents one to classify quantifiers according to their veridical behaviour. In fact Kamp's semantics turns all first-order definable quantifiers into veridical ones. Since *nothing* is one of them, it is clear that something went amiss here: it does not follow from 'Danny hears nothing squeak' that nothing squeaks.

The remedy we propose is simple, but in our opinion fruitful: instead of a standard first-order language we take one with generalized quantifiers as our point of departure (Barwise and Cooper, 1981: 167–171). In this manner one can retain the elegance of Kamp's original idea and at the same time study the semantics of NI sentences in full generality.

Our semantics for NI sentences is similar in spirit to the one given by Asher and Bonevac (1985,1987), but there are differences. First and foremost we do not extend situation semantics with the apparatus necessary for a satisfactory semantic theory of NI sentences. We prefer to capture the phenomena at hand in a 'conservative' manner, thus countering the claim that this is "...difficult or even impossible..." (Barwise and Perry, *op.cit.*). In fact it is nice to see how easily one defines a set-theoretic semantics for the kind of sentences that gave rise to a new semantic paradigm.

The present article has the following structure. First there is a discussion on alternative syntactic structures for NI sentences and some of the semantical issues they might give rise to. This is followed by an overview of the logical properties NI sentences seem to have informally. Some of these properties are embodied at an atomic level in the set-theoretic semantics and deduction system defined next. That this is so, will be shown in the section where the inference patterns get a formal treatment. There we shall try to detect to what extent the principles assumed at the atomic level are preserved under the formation of complex kinds of sentence. Here one will also find a fairly detailed study of the interplay between different forms of negation.

1. SYNTACTIC STRUCTURE

The syntactic structure of NI sentences is heavily debated, but due to Barwise (1981) many logical semanticists have adopted the Small Clause structure:

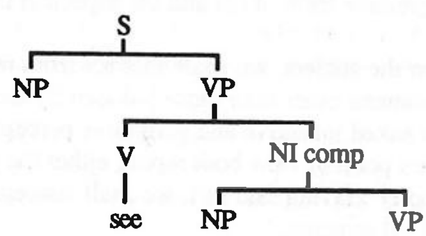

FIGURE 1

The choice is not obvious. Many linguists are sceptical in regard to the existence of Small Clauses, since it is difficult to find convincing evidence for this syntactic structure.

Barwise's preference for the Small Clause structure is probably due to the fact that he wants situations to be the starting point of his semantical enterprise. The intuitive motivation for this semantical theory derives mainly from the fact that at each point in time people perceive small parts of the world, *i.e.*, situations. Since NI sentences report on this kind of perception, it seems natural to take the NI

1 In this article we shall mainly use the verb 'see'. And when we talk about perception, perceivers and the like, we mean visual perception, etc.. In this connection, we shall often use 'perceiver' instead of the perhaps more accurate 'seer'.

complement to be a tenseless sentence—a Small Clause—which bears a particular relation to the situations perceived, also called *scenes*.

To support his position Barwise makes use of an interesting observation made by Gee (1976): perception verbs like *see* allow anaphors with respect to their complements.

(1) Whitehead saw Russell wink, and McTaggart saw it too.

Sentence (1) is used by Gee to support the claim that an NI complement is a syntactical unit, but this claim is countered by other observations. For instance, if NI complements are tenseless sentences, one would expect that passivization within the complement is possible, but it is not:

(2) Daniel saw Lucia phone Henry

(3)* Daniel saw Henry be phoned by Lucia

Sentence (2) also indicates that Barwise's semantics must allow the verification of the quasi-sentence 'Lucia phone Henry' by a scene even if one of the arguments of the perceived relation is missing. Barwise is aware of this slightly awkward phenomenon[2] and proposes the remedy to specify for each relation which arguments have to be present in the perceived scene (Barwise and Perry, 1983). One may wonder if this is possible in general.

Alternatives to the structure in Figure 1 are proposed by, among others, Akmajian, Higginbotham and Williams. Instead of taking the NI complement in Figure 1 to be a Small Clause, Higginbotham (1983) defends the view that it is a noun phrase. More 'radical' are Akmajian (1976) and Williams (1983) who postulate the following syntactic structure:

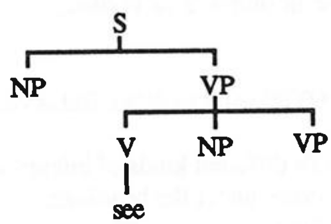

FIGURE 2

Adopting the syntactic structure in Figure 2 does not force one to reject Barwise's semantics, but it naturally gives rise to an interpretation which embodies a different view on perception; namely that one sees objects—referred to by the NP—as having certain properties—given by the VP. One way to defend this position would be to argue that the NP following *see*—named 'NP*' hereafter—has a different semantical status from the other NPs within the scope of this verb. In particular NP* might have a perceptual preference in that the objects it denotes—if any—are seen by the persons denoted by the main subject. The next few paragraphs shall be used to try to detect such a preference.

To begin with: does (4) imply (5)?:

(4) Daniel saw Lucia phone Henry

(5) Daniel saw Lucia

The following set-up purports to give a counterexample.[3] Imagine Lucia, Henry and Daniel each sitting in separate rooms. There are phones which enable Lucia and Henry to speak to each other, but only when Lucia phones Henry an oscilloscope in Daniel's room will show a pattern characteristic for Lucia's voice. Now suppose Daniel saw the pattern, can one report the fact by saying (4)?

Opinions on this matter differ. Some are opposed to using (4) under the circumstances, since there is no direct perception of the event or situation in which Lucia phoned Henry. For this reason they would rather have (6) instead of (4):

(6) Daniel saw that Lucia phoned Henry

Others find the use of (4) acceptable *simpliciter*, or argue that—at least in one sense—the requirement of direct perception is fulfilled. Perceiving the pattern on the oscilloscope is perceiving a representation of Lucia, much as perceiving a video-recording of her would have been. And clearly in the latter case (4) might be used.

Since the set-up described can easily be accommodated to all kinds of activity, there would be in general no perceptual preference for any of the NPs within the scope of *see*. And hence it would be

2 That is, awkward if NI complements are tenseless *sentences*.

3 We are indebted here to Robin Cooper.

impossible to specify for each relation which of its arguments have to be present in a perceived scene, as Barwise would have it (*Op. cit.*).

For those who reject the above counterexample, the syntactic structure of Figure 2 presents a wayout. This structure has an interpretation in which the truth conditions of (7) are those of (8) together with the requirement that as far as John Lee saw Scot he had the property of shooting no bears.

(7) John Lee saw Scot shoot no bears

(8) John Lee saw Scot

However, the solution is not unproblematic. As Gee (1976) has pointed out, with sufficient background knowledge (9) seems perfectly acceptable, even though the forest is behind a hill from John Lee's vantage point.

(9) John Lee saw the forest burn.

And what about (10)?:

(10) John Lee saw it rain

Taking *it rain* as a zero-place relation denoting a proposition or a truth value is not attractive, for does one see such abstract objects? And in this case *it* does not refer to material objects. If it refers at all, it rather takes its values among events.

At this point we conclude that there is no solid basis to accept the perceptual preference of NP*. We also remain undecided in regard to the syntactic structure of NI sentences. Since no semantical reason has been found to reject Small Clauses, we shall follow Barwise and opt for this structure. In this way a comparison of our semantics with those of others is facilitated.

2. PREFORMAL LOGICAL BEHAVIOUR

In the literature on NI sentences one finds different kinds of inference, some of which are judged valid. In this section we shall review them informally under the headings:

1. Import and Export of Quantifiers
2. Substitution
3. Veridicality
4. The Principle of Negation
5. The Principle of Logical Equivalence
6. Verb Phrase Intensionality
7. Export and Import of *and* and *or*

The inferences 1-5 and 7 are already to be found in Barwise's *Scenes and other Situations* (1981), while the VP phenomena named under 6 remain implicit in this paper. They make their first explicit appearance in Asher & Bonevac (1985).

According to Barwise 1-4 and 7 are at least valid if the sentences involved are 'simple'. In case arbitrary noun phrases are allowed they may fail to be valid. This was observed by Barwise with respect to 3, and we shall show that it is true for 1, 4 and 7 too. The inference 2, in contrast, is universally valid.

Barwise has defended that 5 should be invalid, but we think that he has overlooked some subtleties having to do with kinds of negation involved. Finally, there are some invalidities due to intensional phenomena in the VP of an NI complement. We shall try to detect a pattern underlying the valid and invalid inferences involving this VP.

IMPORT AND EXPORT OF QUANTIFIERS. Having assumed that NI complements do not contain intensional verbs, quantifiers can be moved freely in and outside the scope of *see*. The following example is from Barwise (1981):

(1) Ralph saw someone hide a letter under a rock

⇒ There was a particular person whom Ralph saw hide a letter under a rock

There is a proviso induced by the quantifiers themselves—and thus quite independent of the verb *see*—namely: one has to reckon with the scope of quantifiers, *e.g.* the equivalence:

(2) A man saw no philosopher scratch her head

⇔ No philosopher is such that a man saw her scratch her head

is invalid if one reads both sentences with the quantifier of their main subject as having scope over the remaining quantifier.

SUBSTITUTION. Inferences of the following kind are universally valid:

(3) James saw Samuel eat a fish
Samuel = Dr. Johnson
$\Rightarrow$ James saw Dr. Johnson eat a fish

In fact we need not consider this inference separately. Using type-lifted versions of individual denoting expressions, the validity of Substitution can be reduced to that of Import and Export of Quantifiers. For assume (a) and (b):

(a) James saw Q_{Samuel}x.x eat a fish
(b) Samuel = Dr. Johnson

Then by Export of Quantifiers:

(c) Q_{Samuel}x.James saw x eat a fish.

From (b) one infers that

(d) $Q_{Dr.Johnson}$x.James saw x eat a fish.

And hence by Import of Quantifiers:

(e) James saw $Q_{Dr.\ Johnson}$x.x eat a fish.

We have stated that Substitution holds universally, but we have also observed that Import and Export of Quantifiers is restricted by scope phenomena. Hence it might be so that in the above reduction the universal validity of Substitution is lost. Fortunately it is not, for quantifiers of the form $Q_t x$ are known to be scopeless.[4]

VERIDICALITY. In Barwise and Perry (*Op. cit.*, 184) one reads:

> *Seeing is clearly a causal relation, but also, and importantly, an information-preserving one. [...] The other attitudes we study are sensitive to how the information (or misinformation) actually influences the agent's thought and action. By contrast, epistemically neutral reports are concerned only with the nature of the situation about which information is detected by the agent.*

This preservation of information seems to explain why NI sentences can be used as evidence about the world:

> *Suppose, for example, that we sent a scout up to the top of a nearby hill to watch out for hostile pioneers. If the scout saw hostile pioneers come his way, then such pioneers came this way, and we would be well advised to get ready for them* (Op. cit., 181).

The inference used here:

(4) The scout saw hostile pioneers come his way
$\Rightarrow$ Hostile pioneers came his way

is an instance of Veridicality which can be stated as a schema:

(VER) NP see [NP' VP]
$\Rightarrow$ [NP' VP]*

In VER the sentence [NP' VP]* results from supplying the tenseless NI complement [NP' VP] with the tense of the premiss.

In this general form Veridicality cannot be upheld. It is not difficult to find sentences in accordance with VER for which the inference is invalid, for example:

(5) Donna saw few children play
$\Rightarrow$ Few children played

It is observed often that the non-validity of this argument is due to the kind of noun phrases occurring in the NI complement, an insight captured by the semantics defined below.

Although NI sentences are veridical their use as evidence about the external world is limited, for often we are in doubt as to the truth of the premiss. The kind of seeing NI sentences report on may well be information-preserving, but in describing it much (mis)information is invested which need not be true of the part of the world seen. As with individuals, one can refer to a perceived event by means of properties which the event in fact does not have. At a distance the scout from the example may not be able to discern hostile pioneers from angry friends. If the scout saw angry friends approach the camp, these friends are actually to be expected. But the scout's NI report: "I saw the enemy approach our camp" is then mistaken and should prompt the query: "Are you sure?". In short, one must discern sharply between *what* is seen and the *description* of what is seen.

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4 This is proved by Zimmerman, see Van Benthem 1989 and §5.

THE PRINCIPLE OF NEGATION. The interplay between the information used to describe a perceived event and the information given by the event itself, is particularly prominent when the NI complement contains negative elements of some sort. Before we reflect on this it must first be noted that a syntactical occurrence of negation within the scope of *see*, does not necessarily indicate a semantical such occurrence. Our mind hovers between a contradictory and a coherent reading of (6):

(6) Moore saw winking Russell not wink

It is plain that (6) can only be coherent if *not* is interpreted as sentence negation, *i.e.*, if (6) is equivalent to (7):

(7) Moore didn't see winking Russell wink

In that case both sentences could describe a situation in which Russell did wink while Moore saw him from behind, and thus did not see him wink.

It has been argued—or rather: taken for granted—that there is a form of negation which occurs within the scope of *see*, but which is not equivalent to sentence negation. An example of such a sentence could be:

(8) The policeman saw Andrew not stop for the traffic light[5]

In the Introduction it is stated that NI sentences describe the perception of events which are completed. Given the in general stative nature of negated sentences, one would expect that the negation used in (8) must somehow have a non-classical content. For if Andrew was ill and in bed while the policeman was on shift, (9) is true:

(9) The policemen didn't see Andrew stop for the traffic light

But Andrew's not stopping for the traffic light—when taken in this sense—is not completed during the policeman's shift, and it would be awkward to report this non-event by means of (8). A weaker reading of (8) is intended, one which implies (9) but is not equivalent to it. If this form of negation is to appear genuinely within the scope of *see*, it has to have a positive, active character which allows the property formed by means of it to be completed within an interval of perception. The negation in (8) can be seen to be of that type, if we read the sentence in the manner of Higginbotham (1983, 110-111):

(10) The policemen saw Andrew *refrain from* stopping for the traffic light

A similar use of negation is to be found in a story related to me by Pieter Seuren. Two English ladies have the habit of meeting each other daily, always at the same place roundabout five 'o clock. One day the clockwork above their heads doesn't function properly. The lady who had the event within earshot reported on it by saying: "I definitely heard the clock not strike five."

In contrast, statives invariably invoke marked appearances of negation:

(11)* The dwarfs saw Cinderella not own a penny

From the previous discussion it emerged that the behaviour of negation in NI sentences is closely connected with the aspectual character of these sentences, and we stated that we shall not treat aspectual matters here. However, in the semantics defined below we have the option of introducing two forms of negation, weak and strong negation, which seem to capture the main differences between, respectively, sentence negation and 'genuine' complement negation.[6] Using the logical semantics to be introduced shortly, we shall therefore study the interplay of strong and weak negation in some detail.

The observation that (8) with *not* interpreted as strong negation implies (9), was codified for the first time by Barwise (1981: 378) in his Principle of Negation:

(PN) Jens sees not φ

$\Rightarrow$ Jens doesn't see φ

At first sight the principle appears to be universally valid. For let Jens see Wendy not leave the room, then it is impossible that he sees her leave the room at the same time. However, one has to observe that different forms of negation may occur in premiss and conclusion, and that the validity of PN may depend on that. One also has to pay attention to scope phenomena involving negation and noun phrases if the latter are part of one's formal system. By way of example, if in the premiss the negation is weak negation outside the scope of *someone* the argument is valid, otherwise it is not, as in (12):

(12) Jens sees someone not wink

$\Rightarrow$ Jens doesn't see someone wink

Barwise notes in passing that the principle of negation "is not really new; it follows from two uses of the principle of veridicality, together with principles of logic that don't involve perception." (*op.cit.*

5 We owe this sentence to Elisabet Engdahl.

6 Below it will appear that the latter is closely related to Higginbotham's *antonyms* (*Op. Cit.*, 111).

379). The previous observations indicate that in general this reduction may break down (*Cf.* Higginbotham, *Op. Cit.*, 110).

K AND LOGICAL EQUIVALENCE. Suppose we know that

(13) Fred saw Mary enter Door 1.

Could we convey this information by using sentence (14) ?

(14) Fred saw [Mary enter Door 1 and (Brown enter Door 2 or Brown not enter Door 2)].

Plainly the answer is *no*. As each pragmatic theory would have it, information must be transferred so that the processing time required by the addressees is minimal.

In the literature on NI sentences one finds not a pragmatical, but a semantical reason for deeming (14) unacceptable as equivalent to (13). With respect to NI sentences the principle of logical equivalence is rejected without exception:

(LE) NP see S
S $\leftrightarrow$ S'
$\Rightarrow$ NP see S'

Barwise was the first who did so in order to solve his puzzle of logical equivalence (1981: 381). This puzzle contains a 'proof' of 'a famous modal logician *K*' which purports to demonstrate that the following situation is inconsistent:

(15) Mary saw Brown enter Door 2
(16) Fred didn't see Brown enter Door 2
(17) Fred saw Mary enter Door 1

An outline of *K*'s argument, as reported by Barwise (*op. cit.*: 387), begins by using veridicality to infer from (15) that Brown entered Door 2. Another instance of veridicality is:

(18) Fred saw Brown not enter Door 2
$\Rightarrow$ Brown did not enter Door 2

Given that Brown did enter Door 2, *K* concludes that Fred did not see Brown not enter Door 2. He goes on to argue that (17) together with logical equivalence gives (14). Distributing and exporting the disjunction in (14) will give a sentence of which the first and second disjunct are respectively:

(19) Fred saw [Mary enter Door 1 and Brown enter Door 2]
(20) Fred saw [Mary enter Door 1 and Brown not enter Door 2]

A consequence of (20) is that Fred did see Brown not enter Door 2, which contradicts the established fact that Fred did not. Hence (19) must be true. This, however, implies that Fred saw Brown enter Door 2, as opposed to what is given in (16). Contradiction!

The proof seems watertight, but is it? Our semantics will show to be a good instrument to analyze it.

VERB PHRASE INTENSIONALITY. The previous subsections—in particular the ones on Import and Exportation of Quantifiers and Substitution—showed that NI sentences are highly extensional in regard to noun phrases. This may evoke the conjecture that NI complements are completely transparent, a conjecture which can be easily refuted.

In our modern times it is still impossible to cycle without pedaling, but the next argument is invalid

(21)*[7] Paula saw Don cycle
Cycling involves pedaling
$\Rightarrow$ Paula saw Don pedal

Don may be passing by at the other end of a hedge, and Paula only happens to see his hands holding the handle bars together with the upper part of his body. Interestingly, not all inferences of this form are invalid:

(22) Paula saw Don eat greedily
Eating greedily involves eating
$\Rightarrow$ Paula saw Don eat

It is even possible to find valid arguments which do not use adverbial modification:

(23) Paula saw Don ramble
Rambling involves walking
$\Rightarrow$ Paula saw Don walk

7 Like grammatical markedness, we indicate an invalid inference with a '*'.

Can one discern a pattern behind these (in)validities? It is clear that the intensional features of NI sentences are not caused by the fact that they describe the observation of activities by a conscious being. In case of (21), for instance, Paula may be aware of the fact that cycling involves pedaling, but even then *Paula sees Don pedal* does not follow from *Paula sees Don cycle*. It seems rather that the intensionality is due to a subtle relation between the perceived event and the linguistic means used to described it. Here one must observe that the use of 'involves' in (21)-(23) is ambiguous. In (21) it means *is part of*: each cycle event contains a pedal event. And as with material objects, one can perceive an event without seeing all its parts. In (22) and (23), on the other hand, it means *is a specific form of*, *e.g.*: rambling is a leisurely form of walking. The validity of (22) and (23) is thus based on the fact that one may use more general descriptions of the event seen by Paula, if the specific nature of activity she saw is not relevant to the information one wants to convey.

IMPORT AND EXPORT OF *AND* AND *OR*. The inference patterns to be discussed now are those involving con- or disjunction in the NI complement, *e.g.*:

(24) Paul saw Ry sing or Flaco play the accordion
⇔ Paul saw Ry sing or Paul saw Flaco play the accordion

There are two such inference patterns—one for *and* and one for *or*—, but in English examples there is an ambiguity concerning the functioning of the junctions which possibly trivializes matters. For instance, the premiss of the above argument can be seen as a coordination of the conclusion, in which case the validity of the reasoning is obvious. In the sequel we shall therefore assume that the junctions genuinely occur within the scope of *see*.

Even if *or* is taken to occur within the scope of *see* (24) is valid, since in general *see* is scopeless with respect to disjunction. The situation is slightly different with the *and* variant of (24):

(25) Paul saw Ry sing and Flaco play the accordion
⇔ Paul saw Ry sing and Paul saw Flaco play the accordion

First of all note that due to the past tense of *see*, it has to be assumed that the perceptions described in the premiss of the ⇐-direction both occur at the same time. However, this assumption will not show up in the semantics given below, for we do not treat tenses. Barwise (1981) observed furthermore that the ⇐-direction of this argument is valid only if it is assumed that Paul has at most one field of vision, a reasonable assumption which shall be made in the sequel.[8]

Although the junctions can be moved from and into the scope of *see*, this is not so for the quantifiers occurring within an NI sentence. For instance, if the quantifier associated with the subject of an NI sentence is of an improper kind, invalidity will result, as (26) to (27) will make plain:

(26)* *Export of <u>and</u>*
Neither friend saw Ry sing and Flaco play accordion
⇒ Neither friend saw Ry sing and neither friend saw Flaco play accordion

(27)* *Import of <u>or</u>*
Neither friend saw Ry sing or neither friend saw Flaco play accordion
⇒ Neither friend saw Ry sing or Flaco play accordion

Doug saw Ry sing, but he did not see Flaco play the accordion, and his poor friend Ruth did not see anything at all. Taken together they provide counterexamples to (26) and (27).

Suppose there are three farmers. Two of them saw some pigs die and two saw the corn decay, but only one saw pigs die and corn decay. These farmers give a counterexample to (28):

(28)* *Import of <u>and</u>*
Most farmers saw some pigs die and most farmers saw the corn decay
⇒ Most farmers saw some pigs die and the corn decay

The next argument is invalid too:

(29)* *Export of <u>or</u>*
Most hunters saw a duck fly or a fox run
⇒ Most hunters saw a duck fly or most hunters saw a fox run

If there are two hunters and one of them saw a duck fly without seeing a fox while the other saw a fox run without seeing a duck, then the premiss of the argument is true and the conclusion false.

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8 It should be clear that the logical semantics defined in §3 can be easily changed in order to study the more general situation of allowing multiple fields of vision.

3. THE FORMAL SYSTEM

SYNTAX. The formal language to be presented shortly is intended to model the main logical features of NI sentences. First we specify the signs used in forming expressions. The list is fairly standard, except perhaps for the so-called 'quantifier variables' which come in different sorts.[9] In Section 2 we noticed that inference patterns are often valid for particular kinds of noun phrases only. Semantically noun phrases correspond with restricted quantifiers, and hence we need quantifiers of different sorts in order to be able to delimit for each inference the kinds of noun phrase which validate it.

ALPHABET. The expressions of $L(GQ)^{ni}$ are built from the following disjoint sets of signs:

Logical Signs

- Countably many individual variables x_n.
- The logical constants ~ (*weak* negation), $\wedge$, $\vee$, =.
- The two-place quantifier constant ***some***.
- The set constant ***thing***.

NI Signs

- A 2-place relation sign S (for the relation: *x* sees *y*).
- The symbol *SEE* (for the verb *see* as it occurs in NI sentences)

Nonlogical Signs

- Countably many individual constants c_n.
- For all $n \in \omega$ countably many pairs of n-ary relation signs R and $\neg$R, where $\neg$R will be interpreted as the antonym of R. We let ρ vary over these signs.
- The following sorts of quantifier will be used:[10]
  c = *MON*$\uparrow$ + consistent,
  sm = *MON*$\uparrow$ + symmetric,
  For each sort s there are countably many two-place quantifier variables Q^s. These sets of quantifier variables are pairwise disjoint.

Punctuation Signs

- Parenthesis:), (
- Comma: , .

The relation signs $\neg$R are thought of as being non-compound. Below we shall use the distinction among relation signs to introduce *strong* negation besides the already present weak negation.

Among the finite strings of signs the *expressions* model the relevant characteristics of NI sentences:

EXPRESSIONS OF $L(GQ)^{ni}$. The expressions of $L(GQ)^{ni}$, *i.e.*, formulas and set terms, are defined by a simultaneous induction as follows:

- If ρ is an n-place relation sign and t_i ($1 \leq i \leq n$) are individual constants or variables, then $\rho(t_1,\ldots,t_n)$ is an atomic formula;
- If t_i ($i \in \{1,2\}$) are individual constants or variables, then $t_1 = t_2$ is an atomic formula;
- If t is a variable or an individual constant and φ is a formula in which '*SEE*' and '=' does not occur, then *SEE*(t,φ) is an NI formula;
- If φ and ψ are formulas, so are $\varphi \wedge \psi$, $\varphi \vee \psi$ and $\sim\varphi$;
- All one-place relation signs *P* are set terms;
- If x is a variable and $\varphi(x)$ is a formula, then $\hat{x}.\varphi(x)$ is a set term;

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9 A point of terminology: in Van Benthem 1986 it is argued that quantifiers are determiners which satisfy additional constraints. Determiners are defined as relation among sets. Here we use ***quantifier*** as Van Benthem uses ***determiner***. A ***restricted quantifier*** is a quantifier in the sense of Barwise and Cooper (1981): a quantifier plus its first argument.

10 The following properties are defined relative to a particular domain *A*:

(i) Q is a *sieve* iff $\forall X: \emptyset \neq \{Y \mid QXY\} \neq \wp(A)$

(ii) Q is *monotone increasing* iff $\forall XYZ(QXY$ and $Y \subseteq Z \Rightarrow QXZ)$

(iii) Q is *symmetric* iff $\forall XY(QXY \Rightarrow QYX)$

For a definition of consistency, see Section 4, The Export and Import of Negation.

By definition, a restricted quantifier $Q(X,\ldots)$ has one of the above properties, if Q has. This is somewhat less general than the usual definitions. For example, it may be the case that $\forall YZ(QXY$ and $Y \subseteq Z \Rightarrow QXZ)$, *i.e.*, $Q(X,\ldots)$ is *MON*$\uparrow$ in the usual sense, while Q does not satisfy (i) above. However, this distinction is of no importance for the present inquiry.

- If υ is a set term and t is an individual constant or a variable, then $\upsilon(t)$ is a formula;
- If Q is ***some*** or a quantifier variable while υ and υ' are set terms, then $Q(\upsilon,\upsilon')$ is a formula.

Expressions are *closed*, if they do not contain free variables. Closed formulas are *sentences*.

An example of a closed set term and its intuitive meaning is: $\hat{y}.\boldsymbol{some}(\hat{x}.Px \wedge \neg Tx, \hat{z}.Dyz)$, the set of objects that dismiss a non-tough policeman. The closed NI sentence: $SEE(s, \boldsymbol{some}(\hat{x}.Px \wedge \neg Tx, \hat{z}.Dez)$, states that Silvester sees Eddie dismiss a non-tough policeman.

The logical connectives and quantifiers not mentioned in the alphabet are introduced in the familiar manner. We shall use the following abbreviations as well:

- $\exists x.\varphi \equiv_{def} \boldsymbol{some}(\boldsymbol{thing}, \hat{x}.\varphi)$
- $(Q\sim)(\hat{x}.\varphi, \hat{y}.\psi) \equiv_{def} Q(\hat{x}.\varphi, \hat{y}.\sim\psi)$ *Internal negation*
- $(\sim Q)(\hat{x}.\varphi, \hat{y}.\psi) \equiv_{def} \sim Q(\hat{x}.\varphi, \hat{y}.\psi)$ *External negation*
- $\boldsymbol{all}(\hat{x}.\varphi, \hat{y}.\psi) \equiv_{def} (\sim\boldsymbol{some}\sim)(\hat{x}.\varphi, \hat{y}.\psi)$

Although *MON*↑ quantifiers are taken as primitive, other kinds of quantifier are definable within this language, *e.g.*, *MON*↓ can be defined as the internal or external negation of a *MON*↑ quantifier. Since identity is part of the language, nonmonotone quantifiers can also be defined. For example:

$$\boldsymbol{n!}(\hat{x}.\varphi, \hat{y}.\psi) \equiv_{def} \exists z_1,\ldots,z_n \forall u[([^u/x]\varphi \wedge [^u/y]\psi) \leftrightarrow \bigvee_{1\leq i\leq n} u = z_i]$$

In the above definition the third clause disallows iterations of the operator '*SEE*'. This restriction is induced by the fact that a semantics for a language with iterations has to model a form of circularity. Such circularity is present, for instance, in a situation where two people mutually perceive each other. As is shown in *The Liar* of Barwise and Etchemendy (1987) and in Barwise's work on common knowledge (Barwise, 1988) the non-well-founded set-theory developed by Aczel can handle circularity in a natural way. In order to give a satisfactory semantics for 'nested' NI sentences along these lines, one has to develop a theory of generalized quantifiers using non-well-founded sets.

The third clause also disallows identities within the scope of '*SEE*'. There is no formal basis for this restriction, but from an intuitive point of view it seems clear that the highly stative identities do not make sense as part of an NI complement with its 'confinement to the active or transient' (Higginbotham, 1983, 117-118).

SEMANTICS. The models for the language defined are much like models for *L(GQ)* (Barwise & Cooper, 1981), which can be viewed as higher-order extensions of models for first-order predicate logic. To transform a first-order model into an $L(GQ)^{ni}$ model, one must expand its interpretation function to cover the quantifier signs and one must stipulate the meaning of the formalized NI sentences. The following definition shows how to do this for a language without function symbols.

$L(GQ)^{ni}$ MODELS. An $L(GQ)^{ni}$ model $\mathfrak{A}$ is a tuple $\langle A, *, S, \Sigma\rangle$ with:

- $A \neq \emptyset$;
- $*$ is an interpretation function for the logical and non-logical symbols with:
  $c^* \in A$, for all individual constants c,
  $\boldsymbol{thing}^* = A$,
  $\neg R^*, R^* \subseteq A^n$ and $\neg R^* \cap R^* = \emptyset$;
- $S^* = S \subseteq A^2$, i.e. the see relation S is a relation between objects;
- Σ is a function such that for each $d \in DOM(S)$ and each relation sign ρ:
  $\Sigma(d,\rho) \subseteq \rho^*$.
- $\boldsymbol{some}^* := \{\langle X,Y\rangle \in \wp(A)^2 \mid X \cap Y \neq \emptyset\}$
- For quantifier signs Q, $Q^* \subseteq \wp(A)^2$ and Q^* respects the sort of Q in the obvious way; *e.g.* Q^{c*} is *MON*↑ and consistent.

A typical characteristic of $L(GQ)^{ni}$-models is that they do not contain formal counterparts of the scenes or events perceived. In this respect the semantics differs from all other proposals in the literature. Using the relation S and the function Σ, only the influence of a field of vision on objects and relations is captured. In particular, the objects seen by d—$S_d = \{d' \mid Sdd'\}$—is but part of the objects present in the context. And the function Σ intends to give for each pair $\langle d,\rho\rangle$ the extension of the relation ρ in *d*'s field of vision, which is part of ρ's contextually given extension. In this connection, it is plain why one would want $\Sigma(d,\rho) \subseteq \rho^*$ rather than $\Sigma(d,\rho) = \rho^*$. For if we had used the latter, (1) would have been valid:

(1) Russell winked
⇒ Moore saw Russell wink

With the oscilloscope example in Section 1 in mind, we have refrained from imposing further conditions on the value of $\Sigma(d,\rho)$. For instance, (2) can be true, even though Daniel saw neither Lucia, nor Henry:

(2) Daniel saw Lucia phone Henry

Of course, there are other options here; *e.g.*, for suitable ρ one could define $\Sigma(d,\rho) = \{\langle d_1,\ldots,d_n\rangle \in \rho^* \mid \{d_1,\ldots,d_n\} \cap S_d \neq \emptyset\}$, stipulating that at least one of its arguments must be seen.

Now we are in the position to define the notion of satisfaction for formulas with respect to a $L(GQ)^{ni}$ model. Since set terms are defined by means of formulas, one has to define the value of a term simultaneously.

VALUES AND SATISFACTION IN MODELS. Let $\mathfrak{A} = \langle A, *, S, \Sigma\rangle$ be an $L(GQ)^{ni}$ model and a an assignment for $\mathfrak{A}$. The *value of a term t* in $\mathfrak{A}$ under a, notation $[\![t]\!]^{\mathfrak{A}}_a$, and the *satisfaction of a formula* χ in $\mathfrak{A}$ under a, notation $\mathfrak{A} \models \chi\,[a]$, are defined by:

- *Terms*
  - $[\![c]\!]^{\mathfrak{A}}_a = c^*$
  - $[\![x]\!]^{\mathfrak{A}}_a = a(x)$
  - $[\![\hat{x}.\varphi]\!]^{\mathfrak{A}}_a = \{d \in A \mid \mathfrak{A} \models \varphi\,[a^x_d]\}$
- *Formulas*
  - $\mathfrak{A} \models \rho t_1,\ldots,t_n\,[a] \quad\Leftrightarrow\quad \langle [\![t_1]\!]^{\mathfrak{A}}_a,\ldots,[\![t_n]\!]^{\mathfrak{A}}_a\rangle \in \rho^*$
  - $\mathfrak{A} \models Q(\hat{x}.\varphi,\hat{y}.\psi)\,[a] \quad\Leftrightarrow\quad Q^*([\![\hat{x}.\varphi]\!]^{\mathfrak{A}}_a,[\![\hat{y}.\psi]\!]^{\mathfrak{A}}_a)$
  - $\mathfrak{A} \models \sim\varphi\,[a] \quad\Leftrightarrow\quad$ not: $\mathfrak{A} \models \varphi\,[a]$
  - $\mathfrak{A} \models \varphi \wedge \psi\,[a] \quad\Leftrightarrow\quad \mathfrak{A} \models \varphi\,[a]$ and $\mathfrak{A} \models \psi\,[a]$
  - $\mathfrak{A} \models \varphi \vee \psi\,[a] \quad\Leftrightarrow\quad \mathfrak{A} \models \varphi\,[a]$ or $\mathfrak{A} \models \psi\,[a]$
- NI *formulas*
  - $\mathfrak{A} \models SEE(t,\rho t_1,\ldots,t_n))\,[a] \quad\Leftrightarrow\quad \langle [\![t_1]\!]^{\mathfrak{A}}_a,\ldots,[\![t_n]\!]^{\mathfrak{A}}_a\rangle \in \Sigma([\![t]\!]^{\mathfrak{A}}_a,\rho)$
  - $\mathfrak{A} \models SEE(t,Q(\hat{x}.\varphi,\hat{y}.\psi))\,[a] \quad\Leftrightarrow\quad Q^*([\![\hat{x}.\varphi]\!]^{\mathfrak{A}}_a,[\![\hat{y}.SEE(t,\psi)]\!]^{\mathfrak{A}}_a)$
  - $\mathfrak{A} \models SEE(t,\sim\varphi)\,[a] \quad\Leftrightarrow\quad \mathfrak{A} \models \sim SEE(t,\varphi)\,[a]$ [11]
  - $\mathfrak{A} \models SEE(t,\varphi \wedge \psi)\,[a] \quad\Leftrightarrow\quad \mathfrak{A} \models SEE(t,\varphi)\,[a]$ and $\mathfrak{A} \models SEE(t,\psi)\,[a]$
  - $\mathfrak{A} \models SEE(t,\varphi \vee \psi)\,[a] \quad\Leftrightarrow\quad \mathfrak{A} \models SEE(t,\varphi)\,[a]$ or $\mathfrak{A} \models SEE(t,\psi)\,[a]$

This covers all cases.

An alternative semantics—one which would stay close to the existing literature—is obtained by defining the notions 'scene in $\mathfrak{A}$' and 'verification and falsification of a formula by a scene' first, and then to use these notions to define 'the satisfaction of NI sentences in $\mathfrak{A}$' (See Kamp, 1984; Van der Does, 1987). Given our previous observations, it is quite clear that apart from the relation signs the reference of all other linguistic items in the scene coincide with their reference in $\mathfrak{A}$. So, technically speaking, parsimony dictates us to define the notions in one fell swoop.

It turns out that the set of sentences valid on $L(GQ)^{ni}$ models is recursively enumerable. The sentences can be viewed as theorems of the deduction system NI.

THE DEDUCTION SYSTEM NI. The deduction system NI is an extension of standard first-order logic. It consists of a first-order logic (with identity) which uses restricted quantifiers $some(\hat{x}.\varphi,\hat{y}.\psi)$ together with the following axiom schemas.

Abstraction Principles

α $\vdash \hat{x}.\varphi = \hat{y}.[^y/x]\varphi$ [12]

β $\vdash (\hat{x}.\varphi)(t) \leftrightarrow [^t/x]\varphi$

Quantifier Principles

q1 $\vdash (\hat{x}.\varphi = \hat{x}.\varphi' \wedge \hat{y}.\psi = \hat{y}.\psi') \rightarrow (Q(\hat{x}.\varphi,\hat{y}.\psi) \leftrightarrow Q(\hat{x}.\varphi',\hat{y}.\psi'))$ — *Extensionality*

q2 $\vdash \forall x(\varphi \rightarrow \psi) \rightarrow (Q^s(\hat{x}.\chi,\hat{y}.\varphi) \rightarrow Q^s(\hat{x}.\chi,\hat{y}.\psi))$ — *For all sorts s Q^s is a MON↑,*

$\vdash \sim Q^s(\hat{x}.\varphi,\hat{y}.\bot) \wedge Q^s(\hat{x}.\varphi,\hat{y}.\top)$ — *non-empty sieve*

q3 $\vdash Q^c(\hat{x}.\varphi,\hat{y}.\psi) \rightarrow \tilde{Q}^c(\hat{x}.\varphi,\hat{y}.\psi)$ — *Q^c is consistent*

q4 $\vdash Q^{sm}(\hat{x}.\varphi,\hat{y}.\psi) \rightarrow Q^{sm}(\hat{y}.\psi,\hat{x}.\varphi)$ — *Q^{sm} is symmetric*

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11 N.B. this principle only holds for *weak* negation.

12 In using '$[^t/x]\varphi$' it is presupposed that the term t is free for x in φ.

Consistency of antonyms

p $\vdash \forall x_1,\ldots,x_n(\neg R(x_1,\ldots,x_n) \rightarrow {\sim}R(x_1,\ldots,x_n))$

NI principles

NI1 *Veridicality for atomic sentences*

$\vdash \exists x.SEE(x,\rho(t_1,\ldots,t_n)) \rightarrow \rho(t_1,\ldots,t_n)$

NI2 *Export and Import of Quantifiers for all Q*

$\vdash SEE(t,Q(\hat{x}.\chi,\hat{y}.\psi)) \leftrightarrow Q(\hat{x}.\chi,\hat{y}.SEE(t,\psi))$ *provided* $x \not\models t$

NI3-5 *Export and Import of Logical Connectives*

NI3 $\vdash SEE(t,{\sim}\varphi) \leftrightarrow {\sim}SEE(t,\varphi)$ [13]

NI4 $\vdash SEE(t,\varphi \wedge \psi) \leftrightarrow SEE(t,\varphi) \wedge SEE(t,\psi)$

NI5 $\vdash SEE(t,\varphi \vee \psi) \leftrightarrow SEE(t,\varphi) \vee SEE(t,\psi)$

By definition one also has:

~1 $\vdash (Q{\sim})(\hat{x}.\varphi,\hat{y}.\psi) \leftrightarrow Q(\hat{x}.\varphi,\hat{y}.{\sim}\psi)$ *Internal negation*

~2 $\vdash ({\sim}Q)(\hat{x}.\varphi,\hat{y}.\psi) \leftrightarrow {\sim}Q(\hat{x}.\varphi,\hat{y}.\psi)$ *External negation*

Where $\Gamma \cup \{\varphi\}$ is a set of formulas, the notion $\Gamma \vdash \varphi$ is defined as usual. For the system NI one can prove:

SOUNDNESS AND COMPLETENESS THEOREM. $\Gamma \vdash \varphi$ if and only if $\Gamma \models \varphi$ with respect to all $L(GQ)^{ni}$ models. □

Above we remarked that scenes only do duty in determining the extensions of relations in the VP part of an NI complement. This is made precise by the following result:

SEE NORMAL FORM THEOREM. Each formula is provably equivalent to a formula in which *SEE* only has atomic sentences of the form $R(t_1,\ldots,t_n)$ and $\neg R(t_1,\ldots,t_n)$ within its scope. □

4. FORMAL TREATMENT OF INFERENCE PATTERNS

In Section 2 we discussed several inference patterns in an informal way. Often it appeared that the nature of the noun phrases occurring in the sentence—semantically: the quantifier associated with these noun phrases—played a decisive rôle in this respect. With the exception of Substitution—which we showed to be reducible to Export and Import of Quantifiers—we shall try to delimit in this section for each of the inference patterns the class of quantifiers for which it is valid. The deduction system NI embodies the strategy we pursue. At an atomic level we assume the inferences formulated in Barwise (1981) to be valid. Given this, we shall use properties of quantifiers to see how these principles can be extended to cover more complex sentences. The order of appearance is as in Section 2, *i.e.*:

1. Export and Import of Quantifiers
2. Veridicality
3. Export and Import of Negation
4. The Principle of Logical Equivalence
5. Verb Phrase Intensionality[14]

Keeping restriction sets implicit, we shall concentrate on sentences of the form:

$$SEE(t, Q_1(\hat{x}_1.Q_2\ldots Q_n(\hat{x}_n.\rho(x_1,\ldots,x_n))\ldots))$$

—with ρ a relation sign—which are abbreviated as: $SEE(t, Q_1\ldots Q_n(\rho))$. The notation summarizes all possible associations of quantifiers Q_i with argument places x_i of $\rho(x_1,\ldots,x_n)$. Thus we arrive at a suitable level of abstraction, since the results below are invariant under such associations. The kind of sentence we restrict ourselves to, cover all examples given in the literature. The price to pay is that the results are less general than one should like them to be.

EXPORT AND IMPORT OF QUANTIFIERS. Having assumed that complements contain nonintensional verbs only, quantifiers can be moved freely in and outside the scope of *SEE*. One only has to reckon with the scopes of the quantifiers involved. This basic fact concerning Export and Import of Quantifiers—EIQ for short—is expressed in the following proposition:

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13 Recall that ~ denotes *weak* negation.

14 The nonatomic cases for the ex- and import of *and* and *or* will not be studied here. (But see van der Does 1990).

PROPOSITION *(Export and Import of Quantifiers)*. $\vdash SEE(t, Q_1 \ldots Q_n(\varphi)) \leftrightarrow Q_1 \ldots Q_n(SEE(t, \varphi))$. □

The principle EIQ obtained here, is a substantial improvement over the exportation instances allowed by the semantics in *Situations and Attitudes* (*Op.cit.*: 181-192): *Distribution of indefinite description* and *Existential generalization* (*Op.cit.*: 182) are just special cases of a more general phenomenon.

Some have doubts whether every quantifier is scopeless in regard to *SEE*, and especially if the quantifier *all* is. Is (1) true?:

(1) Don saw every phonologist falter
⇔ Every phonologist is seen to falter by Don

Those who think not, will probably restrict only the first occurrence of *every phonologist* in (1) to the phonologist Don saw. In other words, they think (1) should be formalized by:

(2) $SEE(\text{Don}, \textit{every}(\hat{x}.S(\text{Don},x) \wedge \text{Phonologist}(x), \text{Falter}))$
⇔ $\textit{every}(\text{Phonologist}, \hat{y}.SEE(\text{Don}, \text{Falter}(y)))$

It is not difficult to see that, indeed, (2) does not hold. For take a model in which *Phonologist* is non-empty, and in which Don is blind. In such model the set terms $\hat{x}.(S(Don,x) \wedge Phonologist(x))$ and $\hat{y}.SEE(Don,Falter(y))$ denote the empty set. So, the upper part of (2) is true, whereas the lower part is not.

Asher and Bonevac explain the phenomenon by adopting a new form of consequence—monotonic implication—which embodies the assumption that the context of evaluation is possibly enlarged when interpreting the sentences in an inference. (1985: 212; 1987: 586). Monotonic implication invalidates the above reasoning too, for the class of phonologist may expand in the process of inference, independent of the fact whether or not this class is restricted by Don's field of vision. Those who take it to be valid, simply assume that there is no such growth.

Although monotonic implication does provide an explanation in terms of growth of extensions, it does not explain *why* there is such an expansion in the case considered. The simple explanation provided here, clarifies that the kind of expansion from which the invalidity results, is due to the elimination of the restriction to a particular field of vision.

An explanation of the phenomenon in terms of monotonic implication is also wanting in that it counters ones intuitions by rendering (3) false:

(3) Don hears no phonologist falter
⇔ No phonologist is heard to falter by Don [15]

Exportation and Import of Quantifiers is a powerful tool which will be used often to describe the logical behaviour of NI sentences. For instance, *The Principle of Increased Specificity* of Asher and Bonevac (1985, 204, 217) can be reduced to it. An instance of this principle is:

(4) John saw an airplane take off
That airplane was a Comanche
⇒ John saw a Comanche take off

In order to circumvent the *that* in the second premiss, we reformulate this as:

(5) An airplane which John saw take off was a Comanche
⇒ John saw a Comanche take off

Which, in turn, is a special case of the schema:

(6) DET N which *a* saw VP are N'
⇒ *a* saw DET N' VP.

Using import of quantifiers, one can prove that the principle of increased specificity is valid for symmetric, *MON*↑ determiners Q^{sm}:

$\mathfrak{A} \models Q^{sm}(\hat{x}.(Nx \wedge SEE(a, \varphi(x))), M)$
$\Rightarrow \mathfrak{A} \models Q^{sm}(M, \hat{x}.(Nx \wedge SEE(a, \varphi(x))))$ *Symmetry*
$\Rightarrow \mathfrak{A} \models Q^{sm}(M, \hat{x}.SEE(a, \varphi(x)))$ *MON*↑
$\Rightarrow \mathfrak{A} \models SEE(a, Q^{sm}(M, \hat{x}.\varphi(x)))$ *Import*

In a similar fashion other properties of quantifiers can be made to avail inside the scope of *SEE*.

VERIDICALITY. We have already seen that Veridicality stated as scheme (7) is not universally valid:

15 Asher and Bonevac, 1985: 212 - 215; 1987: 583 - 589, footnote 7

(7) NP see [NP' VP]
⇒ [NP' VP]* [16]

In this section we shall answer the question for which kind of noun phrase veridicality does hold. Checking some simple examples indicates that going from premiss to conclusion involves growth of extensions in the right argument of the determiners. So, there will at least be connection between the validity of Veridicality and monotone increasing (= *MON*↑) noun phrases.[17] There is, however, a subtlety which shows up in the next proposition:

PROPOSITION *(Veridicality)*. ⊢ *SEE*$(t, Q_1 \ldots Q_n(\rho)) \rightarrow Q_1 \ldots Q_n(\rho)$ holds iff all quantifiers Q_i $(1 \leq i \leq n)$ are *MON*↑ except for an even (possibly zero) number of *MON*↓ quantifiers.□[18, 19]

It is not obvious that one may allow an even number of *MON*↓ quantifiers. Is, for instance, (8) valid?:

(8) Bunk saw no saxophonist play no scales
⇒ No saxophonist played no scales

Only after having realized the (strong) equivalence of 'no saxophonist plays no scales' with 'all saxophonists play a scale' our judgment becomes positive.

That sequences containing non-monotone quantifiers or an odd number of *MON*↓ ones, lead to invalid instances of Veridicality is indicated by examples (9) and (10)

(9)* Bunk saw Artie play no scale
⇒ Artie played no scale

(10)* Bunk saw Artie play one scale
⇒ Artie played one scale

Here and in the sequel *one* is taken to mean: exactly one.

There has been some dispute, about the veridical behaviour of *all*. Is the following argument valid?

(11) Cathy saw all logicians laugh
⇒ All logicians laughed

As in the case of EIQ, the differences in opinion can be explained in terms of the (non-) restriction of the noun *logicians* by Cathy's field of vision. Again an explanation in terms of monotone implication does not explain why the extension of *logicians* in premiss and conclusion relate as they do.

EXPORT AND IMPORT OF NEGATION. In Barwise (1981: 378) one finds the principle:

(PN) Jens saw not φ
⇒ Jens didn't see φ

We have observed in section 2 that different forms of negation may occur in premiss and conclusion and that each may have its own influence on PN. It was also recalled that there are scope phenomena involving negation. Here these observations are made precise. We shall concentrate on the interaction of strong and weak negation inside the scope of *SEE*.

Above we saw that the occurrence of negation within an NI complement is sometimes marked. Other occurrences turn out to function semantically as sentence negation and yet others seem to be instances of 'genuine' complement negation. As an example of complement negation we used Engdahl's sentence:

(12) The policeman saw Andrew not stop for the traffic light.

There are three formalizations (12). Using shallow analysis, we get respectively:

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[16] Recall that the sentence [NP' VP]* results from supplying the complement [NP' VP] with the tense of the premiss.

[17] Compare: Higginbotham, 1983: 109 - 110; Asher and Bonevac, 1985: 207 and 1987: 577; Bouma and Ter Meulen, 1986: 75; Neale, 1988: 306.

[18] Compare Asher & Bonevac 1987. With respect to the loss of generality named in the introduction to this section, one could define:

VERIDICAL FORMULAS. The class of veridical formulas is defined to be the smallest set VF which contains all atomic formulas $R(t_1,\ldots,t_n)$ and $\neg R(t_1,\ldots,t_n)$, and which is closed under:

- if $\varphi(x)$ is in VF and t is a term, then $\hat{x}.\varphi(x)(t)$ is in VF;
- if $\varphi(x)$ is in VF and Q is *MON*↑, then $Q(\hat{x}.\varphi(x))$ is in VF;
- if φ and ψ are in VF, then so are $\varphi \wedge \psi$ and $\varphi \vee \psi$.

And then try to prove

PROPOSITION. ⊢ *SEE*$(t,\varphi) \rightarrow \varphi$ holds iff φ is strongly equivalent to a veridical formula.

[19] Here and in the sequel, it is presupposed that the quantifiers formed by means of ***some*** are sieves too, otherwise the proposition must be reformulated. For instance, (***some***~)(False,*S*) is always true, and hence this sentence is trivially veridical. But the non-sieve (***some***~)(False,...) is *MON*↓.

(13) The policeman saw $(\hat{x}.\neg$stop for the traffic light(x)(Andrew))
(14) The policeman saw $(\hat{x}.$stop for the traffic light(x)(Andrew~))
(15) The policeman saw $(\hat{x}.$stop for the traffic light(x)(~Andrew))

Sentence (13) is the most informative, it describes those situations in which the policeman saw Andrew *refrain from* stopping for the traffic light. *Not* is here: strong complement negation. Sentence (14) uses weak complement negation, *e.g.*, to state that the policeman saw something *incompatible* with Andrew's stopping for the traffic light.[20] Although this is not very informative—it does not indicate which action Andrew did perform—it is not thereby without meaning. The external negation of a quantifier as in (15) models sentence negation which cannot occur within an NI complement:

(16)* The policeman saw *not:* Andrew stop for the traffic light.
(17)* The policeman saw it not be so that Andrew stops for the traffic light.

This, however, is no sufficient reason to ban (15) to the realm of the meaningless. Below it will appear that (14) and (15) are equivalent to the useful:

(18) The policeman didn't see Andrew stop for the traffic light.

Thus far there has been an asymmetry between strong and weak negation in that only atomic sentences are strongly negated, whereas weak negation has no such limitations. The next definition brings both kinds of negation on a par.

DEFINITION *(Strong Negation)*. For an arbitrary formula φ its strong negation $\neg'\varphi$ is given by the following abbreviations:

$\neg' R(t_1,\ldots,t_n) \equiv \neg R(t_1,\ldots,t_n)$
$\neg'\neg R(t_1,\ldots,t_n) \equiv R(t_1,\ldots,t_n)$
$\neg'{\sim}\varphi \equiv {\sim}{\sim}\varphi$
$\neg'(\varphi \wedge \psi) \equiv \neg'\varphi \vee \neg'\psi$
$\neg'(\varphi \vee \psi) \equiv \neg'\varphi \wedge \neg'\psi$
$\neg' D(\hat{x}.\varphi, \hat{y}.\psi) \equiv \tilde{D}(\hat{x}.\varphi, \hat{y}.\neg'\psi)$ [21]
$\neg' SEE(t,\varphi) \equiv SEE(t,\neg'\varphi)$

In the sequel we conflate $\neg'$ and $\neg$, and use the latter only.[22]

A straightforward induction shows that the consistency property $\models \neg\alpha \rightarrow {\sim}\alpha$, which holds for atomic formulas, is preserved under the above definition:

PROPOSITION 1. For all formulas φ: $\models \neg\varphi \rightarrow {\sim}\varphi$. □

Proposition 1 together with the facts: $[SEE(t,\neg\varphi) \leftrightarrow \neg SEE(t,\varphi)]$, and: $[SEE(t,{\sim}\varphi) \leftrightarrow {\sim}SEE(t,\varphi)]$, give a version of Barwise's PN for arbitrary formulas:

(PN1) $\models SEE(t,\neg\varphi) \rightarrow {\sim}SEE(t,\varphi)$.

This version can be used to account for the logical relationships between the readings (13) and (14) of (12) above.

(13) The policeman saw $(\hat{x}.\neg$stop for the traffic light(x)(Andrew))
(14) The policeman saw $(\hat{x}.$stop for the traffic light(x)(Andrew~))

This can be seen as follows. By definition one has:

The policeman saw $(\hat{x}.\neg$stop for the traffic light(x)(Andrew))
$\Leftrightarrow$ The policeman saw $\neg(\hat{x}.$stop for the traffic light(x)(Andrew))

and similarly for weak negation, but now using the fact that proper names are self-dual. Given PN1 it is thus immediate that (14) follows from (13).

In Section 3 we recalled the definitions of the weak internal and external negation of a determiner. Having defined strong negation for all formulas, one could introduce the strong internal and external negation of a determiner in the obvious manner. As before this will induce the strong internal and external negation of a quantifier too. These kinds of quantifier negation give rise to two principles:

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20 The analogies between strong complement negation and to *refrain from* on the one hand and weak complement negation and *to do something incompatible with* on the other, are not perfect, but they are useful approximations.

21 See Feferman 1984.

22 One would like to have antonymic VP's of arbitrary complexity, but Higginbotham only defines this notion for 'simple' VP's. Here, this restriction is eliminated. In general strong negation can be taken to yield antonyms, and we have set terms $\hat{x}.\neg\varphi$ for all φ.

PROPOSITION 2. The principles PN2 and PN3 hold precisely if all quantifiers Q_i ($1 \leq i \leq n$) are *MON*↑ except for an even (possibly zero) number of *MON*↓ quantifiers:
(PN2) $\vdash SEE(t, Q_1 \ldots (Q_i\neg)Q_{i+1} \ldots Q_n(\rho)) \rightarrow SEE(t, Q_1 \ldots (Q_i\sim)Q_{i+1} \ldots Q_n(\rho))$
(PN3) $\vdash SEE(t, Q_1 \ldots (\neg Q_i)Q_{i+1} \ldots Q_n(\rho)) \rightarrow SEE(t, Q_1 \ldots (\sim Q_i)Q_{i+1} \ldots Q_n(\rho))$ □

It is somewhat hard to check PN2 against English examples, since in English there will be no syntactical difference between the strong and weak negation in premiss and conclusion. For instance, the premiss can be the strong and the conclusion the weak reading of *not* in:

(19) Tom saw not all philosophers wink

However, the difference can be brought to the fore by use of indices or paraphrases. For example, the strong and the weak reading can respectively be paraphrased as:

(20) Tom saw a philosopher refrain from winking
(21) Tom didn't see all philosophers wink

Using (20) and (21) it is quite clear that the weak reading of (19) is a consequence of its strong reading.
The quantifier *no*(girl) is *MON*↓, so according to PN2 (22) should be invalid:

(22)* Carol saw no girl not_s smoke
⇒ Carol saw no girl not_w smoke

Or formally:

(23)* $SEE(\text{Carol},(\textbf{\textit{no}}(\text{girl})\neg)(\text{smoke}))$
⇒ $SEE(\text{Carol},(\textbf{\textit{no}}(\text{girl})\sim)(\text{smoke}))$

One is inclined to read both premiss and conclusion as: Carol sees all girls smoke, in which case the inference is trivially valid. However, this reading would obliterate the subtle distinction between weak (not_w) and strong (not_s) negation. In fact the equivalence only holds for the conclusion, while the premiss should be read as: Carol sees no girl refrain from smoking, *i.e.*, all girls seen by Carol are involved in some action incompatible with refraining from smoking.

Under these readings (22) is, indeed, invalid. To falsify the conclusion we assume that Carol saw a girl which did not smoke. We assume further that all girls seen by Carol were younger than three years old. Since none of these girls was a habitual smoker, the intention required to refrain from smoking plainly cannot be ascribed to them. Hence, the premiss will be true.

There is a converse to Proposition 2:

PROPOSITION 3. The principles PN4 and PN5 hold precisely if all quantifiers Q_i ($1 \leq i \leq n$) are *MON*↑ except for an even (possibly zero) number of *MON*↓ quantifiers:
(PN4) $\vdash SEE(t, Q_1 \ldots (Q_i\sim)Q_{i+1} \ldots Q_n(\rho)) \rightarrow SEE(t, Q_1 \ldots (Q_i\neg)Q_{i+1} \ldots Q_n(\rho))$
(PN5) $\vdash SEE(t, Q_1 \ldots (\sim Q_i)Q_{i+1} \ldots Q_n(\rho)) \rightarrow SEE(t, Q_1 \ldots (\neg Q_i)Q_{i+1} \ldots Q_n(\rho))$ □

The conditions on PN5 state that the following should be valid:

(24) Don saw a dog eat (no biscuit~)
⇒ Don saw a dog eat (no biscuit¬)

Using equivalent formulations for premiss and conclusion the validity of (24) can be argued for in different ways depending on the relative scope of *a* with respect to *no*: In case *a* has wide scope the rationale is: if Don saw a dog eat all biscuits [= (no biscuits~)], he saw a dog refrain from eating no biscuits. Another possibility is that both instances of *no* have *a* within their scope. Now the validity of (24) is justified by: if all biscuits are such that Don saw a dog eat them, then no biscuit is such that Don saw all dogs refrain from eating it.

One might want to improve on Proposition 2 to obtain a result which stays closer to Barwise's original formulation of PN. In order to do so we shall introduce a necessary concept first (*Cf.* Zwarts, 1986, Chapter 6):

DEFINITION. A quantifier Q is *consistent* if and only if $Q \subseteq \tilde{Q}$ (i.e., $Q\sim \subseteq \sim Q$).

In other words, the consistent quantifiers are those which do not allow the simultaneous assignment of contradictory predicates. The improvement we have in mind is:

PROPOSITION 4. If all $Q_1 \ldots Q_n$ are *MON*↑ and all $Q_1 \ldots Q_{i-1}$ are consistent, then:
$\vdash SEE(t, Q_1 \ldots (Q_i\neg)Q_{i+1} \ldots Q_n(\rho)) \rightarrow \sim SEE(t, Q_1 \ldots Q_n(\rho))$. □ [23]

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[23] One would like to have a characterization of the inference as we have for the previous ones, but as yet we have not been able to find one.

An example of a *MON*↑ and consistent quantifier is *more than half of the*. So, Proposition 4 correctly 'predicts' the validity of (25):

(25) Magie heard (more than half of the clocks not_s) strike
⇒ Magie didn't$_w$ hear more than half of the clocks strike.

THE PRINCIPLE OF LOGICAL EQUIVALENCE. One of the reasons why Barwise wants to reject the Principle of Logical Equivalence:

(LE) NP see S
S ↔ S'
⇒ NP see S'

is that it seems to enable one to prove the absurdity that there is no situation in which (26), (27) and (28) are jointly true:

(26) Mary saw Brown enter Door 2
(27) Fred didn't see Brown enter Door 2
(28) Fred saw Mary enter Door 1

Barwise attributes the 'proof'—which is given in Section 2—to 'a famous modal logician *K*'. Probably *K* gained fame by displaying subtler modes of reasoning than the ones used below. To begin with, the proof involves the instance of veridicality (29) which can only be maintained for strong negation.[24]

(29) Fred saw Brown not_s enter Door 2
⇒ Brown did not_s enter Door 2

However, *K* is well-known for his work on partial semantics too, so he must have noticed that under this interpretation the VP's 'enter Door 2' and 'not_s enter Door 2' do not form a mutually exclusive pair. Consequently he would not have used the principle of logical equivalence, for he had seen that the sentences (28) and (30) are not equivalent in a partial environment.[25]

(30) Fred saw [Mary enter Door 1 and (Brown enter Door 2 or Brown not_s enter Door 2)]

However, this move towards partiality is not necessary. For if he had given the VP negation a weak interpretation, the principle of logical equivalence *is* valid. But that is of no avail to *K*'s proof, since then the above instance of veridicality does not hold! Under the weak interpretation (31)—which models the premiss of (29)—is equivalent to (32) which does not imply (33).

(31) $SEE(\text{Fred},\hat{x}.{\sim}\text{enter Door } 2(x)(\text{Brown}))$
(32) ${\sim}SEE(\text{Fred},\text{Brown enter Door } 2)$
(33) Brown didn't enter Door 2.

It seems more likely, therefore, that *K* had observed his impasse. The impression that the inconsistency of (26) to (28) can be proved in this manner, is solely due to a sloppy use of negation in a language lacking NP/VP structure.

It appeared that in the above argument the principle of logical equivalence is rather harmless: when strong negation is used it does not apply, and if weak negation is used it is of not much help. Be this as it may, some may find it worrying that the principle *is* valid for weak negation. We shall argue, however, that this is innocuous.

NI sentences are used to report on someone's fields of vision, and—as in all reports—there seems to be no *semantical* reason why they may not contain noninformative tautological parts. Under the weak interpretation (30) does not state that Fred saw Mary enter Door 1 and a tautology, it rather signals that the speaker who uses it—if any—is a little fastidious. When taken in this sense, (30) is just a roundabout way to report the truth of (28). In contrast, (34) is contingent if *not* is interpreted as strong negation.

(34) Fred saw Brown smile or Brown not_s smile

An observation which is in accordance with the given logical semantics.

VERB PHRASE INTENSIONALITY. Although NI sentences are highly extensional, they are not completely transparent. In Section 2 (35) was countered in an informal way:

(35)* Paula sees Don cycle
Cycling involves Pedaling
⇒ Paula sees Don pedal

24 The strong negation of a relation is treated as its antonym (*Cf.* Higginbotham, 1983, 111). So, this instance of Veridicality is as Neale (1988, 309) would have it.
25 This, of course, is Barwise's solution to the puzzle.

A formal countermodel is one with, *e.g.*:

$\mathfrak{A} \models SEE(\text{Paula,Cycler(Don)}) \wedge \textbf{all}(\text{Cycler,Pedaler}) \wedge \neg\exists x.SEE(\text{Paula, Pedaler}(x))$

Such a model is possible, since in Paula's scene the set of pedalers can be properly contained in the set of cyclers, even though in the model all cyclers are pedalers.

There are inferences of the same form as (35) which one wants to be valid, *e.g.*:

(36) Paula saw Don eat greedily
Eating greedily involves eating
⇒ Paula saw Don eat

(37) Paula saw Don ramble
Rambling involves walking
⇒ Paula saw Don walk

In (36) and (37) the conclusion describe the actions referred to in the premiss in a less specific manner. So, here we have two descriptions of the same perceived event. In (35), on the other hand, the conclusion describes the perception of only a part of the event described in the premiss. And one can perceive an event without seeing all its subevents.

The examples make plain that NI sentences come close to a kind of extensionality in regard to actions. The models used here contain nothing but individuals. As a consequence inferences of the above form are made valid by restricting oneself to models in which particular constraints (or, meaning postulates) are true. In $L(GQ)^{ni}$ constraints for n-place relation sign get the form:

- $R^n \sqsubseteq S^n :=_{def} \forall x,y_1,\ldots,y_n(SEE(x, Ry_1,\ldots,y_n) \rightarrow SEE(x, Sy_1,\ldots,y_n)) \wedge$
$\wedge \forall y_1,\ldots,y_n(R(y_1,\ldots,y_n) \rightarrow S(y_1,\ldots,y_n))$
- $R^n \equiv S^n :=_{def} R^n \sqsubseteq S^n$ and $S^n \sqsubseteq R^n$

Note that $\sqsubseteq$ is a partial order. In the definition of $R^n \sqsubseteq S^n$, the first conjunct ensures that in each field of vision the extension of R^n is contained in that of S^n. Since all fields of vision may be empty, this conjunct does not imply that the same holds in the model. That is why we introduced the second conjunct.

Perhaps a more natural way to go about, is to assume that a partial order $\sqsubseteq$ among relation signs of the same arity is given; mirroring, as it were, preformal semantical conventions or intuitions. This relation $\sqsubseteq$ will get its intended effect by stipulating that the interpretation function * and the function Σ in a model $\langle A, *, \Sigma, S\rangle$ are monotone with respect to it. *I.e.*, by stipulating: If $R^n \sqsubseteq S^n$, then $R^* \subseteq S^*$ and for all $d \in A$: $\Sigma(d,R) \subseteq \Sigma(d,S)$.

Now, it seems plain that we have:

- cycle $\not\sqsubseteq$ pedal.
- eat greedily $\sqsubseteq$ eat,
- ramble $\sqsubseteq$ walk,

Hence, given our linguistic conventions, (36) and (37) are valid with respect to the newly defined models, while (35) is not. This is as it should be.

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Polymorphic Quantifiers

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I want to propose that an elegant account of natural language quantifiers emerges if one assumes at the outset that they denote *polymorphic* functions. That is they denote functions whose domain and range are not confined to a particular type, as they are in the traditional Generalised Quantifier interpretation, but instead span a whole range of types. With this sophistication of the quantifiers in hand the way is opened to formulate a semantics whose syntactic algebra takes the form of a categorial grammar. Despite the extreme simplicity of such an algebra, the semantics achieves the full measure of descriptive adequacy, not only succeeding in interpreting the full range of quantified sentences but also assigning to each its full range of scope possibilities. The system that achieves this is outlined in Sections 2, 3 and 4. In Section 1 as a preliminary I look at how little is achieved by categorially based models if the conventional Generalised Quantifier interpretation is adopted. Section 5 contains a miscellany of further virtues of the model advocated in Section 4.

1 The Lambek Calculus and Generalised Quantifiers

Categorial grammar could be said to consist in the attempt to render grammatical truths as logical laws. Thus just as propositional logic involves the infinite language of propositions, defined by their recursive construction out of atomic propositions and propositional connectives, so categorial grammar involves the infinite language of *categories*, defined by their recursive construction out of basic categories and categorial connectives. For example 'p → q' is a composite proposition, built with the connective '→' out of the atomic propositions 'p' and 'q'. Relatedly, 's\np' is a composite category built with connective '\' out of the basic categories 's' and 'np'.

Also common to both propositional logic and categorial grammar is a notion of entailment $x_1, x_2, \ldots x_n \Rightarrow y$, the obtaining of which is expected to be contingent upon the pattern of connectives found in the x_i and y. Thus there are names for characteristic schemas of such entailments, valid for any substitution for the variables. For propositional logic, we have Modus Ponens:

(1) $\quad x \rightarrow y,\ x \ \Rightarrow y$

For categorial grammar we have Function Application

(2) $\quad y/x,\ x \ \Rightarrow y$

In the categorial grammars to be discussed here, the extension of the derivability relation is determined by adaptations of Gentzen's sequent calculus for predicate logic. In case the reader is in the dark as to how such logic-like calculi come to be related to the matter of grammatical analysis, the case is roughly that the valid entailments play the role of grammatical rules, and are to be read 'if I have an x_1, followed by an x_2, ...followed by an x_n then I have a y.

Attempts to formulate grammars of this kind date back to Ajdukiewicz, who proposed to construct the categories using the single connective '/' and to have as the single rule of the calculus, the Function Application schema given above. Since then arguments have been made for additions to the categorial vocabulary, and to the calculus. Bar-Hillel argued for a second connective '\'. Montague grammar is the inspiration for the rule of Type Raising, Geach's "Program for Syntax" paper argues for the Geach rule, and in papers by Ades and Steedman, Dowty and others a composition rule is argued for:

(3) **Type Raising** $X \Rightarrow Y / (Y \backslash X)$

Geach $X/Y \Rightarrow (X/Z) / (Y/Z)$

Composition $X/Y, Y/Z \Rightarrow X/Z$

In 1958 Lambek published a calculus which, for a vocabulary based on three connectives: / , \and ·(the product), can be seen as bringing to an end this process of finding categorial laws. [1]

The rule schema just given are all derivable within Lambek's calculus, but more than this, there is a natural *interpretation* of the connectives with respect to which the calculus is a sound and complete logic. Without going too far into the details, this natural interpretation of the categorial language is simply that every category is to be interpreted as a subset of the strings of a language. The recursive structure of the language is reflected in the fact that the *base* categories may be freely assigned to a string subset, but the compound categories are subject to a clause governing the interpretation of its connective. For example:

(4) $[\![X \backslash Y]\!] = \{a : \forall b \in [\![Y]\!], ba \in [\![X]\!]\}$

This is a natural interpretation of the connective \ because it was always the case that one would assign the category X \Y to a string only if preceding it with any string of category Y would give a string of category X. Otherwise overgenerations would be given rise to by that categorisation.

One can assess whether any given categorial entailment is logically valid one in terms of this interpretation, and the Lambek calculus allows one to derive precisely the valid entailments.

It is with respect to the product (·) free part of Lambek's calculus that we shall begin our discussion of categorial grammar and quantifiers. This is useful because a demonstration that something cannot be done within the Lambek calculus, (such as for example accounting for quantifier scopings), suffices to demonstrate the inadequacy of a number of weaker systems. Here then is the calculus, (in a notation such that in both x/y and x\y, x is the result and y the argument.)

Definition 1 (The Lambek Calculus)

(Ax) $x\colon \alpha \rightarrow x\colon \alpha$

(Cut) $$\frac{U, x\colon \alpha, V \Rightarrow w \qquad T \Rightarrow x\colon \alpha}{U, T, V \Rightarrow w}\text{Cut}$$

(/ E) $$\frac{U, y\colon \alpha\beta, V \Rightarrow w \qquad T \Rightarrow x\colon \beta}{U, y/x\colon \alpha, T, V \Rightarrow w}/\,E$$

(/ I) $$\frac{T, x\colon \alpha \Rightarrow y\colon \beta}{T \Rightarrow y/x\colon \lambda\delta\beta}/\,I$$

(\E) $$\frac{U, y\colon \alpha\beta, V \Rightarrow w \qquad T \Rightarrow x\colon \beta}{U, T, y\backslash x\colon \alpha, V \Rightarrow w}\backslash E$$

(\I) $$\frac{T, x\colon \alpha \Rightarrow y\colon \beta}{T \Rightarrow y\backslash x\colon \lambda\delta\beta}\backslash I$$

The reader for the moment should try to ignore the italicised parts occurring after colons. In their absence we would have a straightforward statement of the Lambek calculus. The notation conventions would then be that U,T,V stand for sequences of categories, with U and V possibly empty. w,x,y stand for single categories. It is a calculus of sequents, where a sequent is T $\Rightarrow$ x, whose rules mimic exactly Gentzen's sequent calculus for the propositional connective '$\rightarrow$'. A proof of a sequent is constructed in an upwards direction from conclusion sequent to one or more premise sequents. The construction can terminate only in arrival at an axiom sequent. Leaving Cut to one side for the moment, it is always the case that a connective of the conclusion sequent is absent from the premise sequents. This leads to the fact that there is a simple decision procedure for the Cut-free calculus. The rule names (/E) and (/I) are read / elimination and /introduction, and this is owed to the natural deduction rules to which the propositional versions correspond.

[1] In fact the arguments for adding additional rules postdate the Lambek paper, so it concluded a process that was yet to begin.

We shall now explain the significance of the italicised parts occurring after colons in the above calculus. It is a familiar requirement of Universal Grammar that every string combining syntactic rule should be associated with a meaning combining rule, taking the meanings associated with the syntactic parts and defining in terms of these a meaning associated with the syntactic whole. In this way a homomorphism is specified between an algebra of derivation trees and an algebra of meanings. Basically the colon in the sequent serves to indicate a pairing under such a homomorphism. As sequents may be derived from simpler sequents, then the image of the derived sequent in the semantic algebra must be specified in terms of the images of the premise sequent. We shall seek that the goal category of a derived sequent shall have associated with it a typed λ calculus term, closed but for the variables that are associated with the antecedent categories. In using this sequent in the analysis of a particular sentence we will have particular translations to substitute for the variables paired with the categories of the antecedent.

The notation conventions then are that U,T,V abbreviate sequences of category:translation pairs, U and V possibly empty. w is a single category:translation pair. x and y are single categories. $\alpha, \beta and \delta$ are single λcalculus terms, with δ a variable. If a term is associated with a category, the type of that term must be that given by the following category to type correspondence: ν(s)= t, ν(np) = e, $\nu(A|B) = (\nu(B) \rightarrow \nu(A)$. It should also be borne in mind that the pairing of λ calculus terms with a syntactic category is the adaptation to the categorial setting of the Curry-Howard isomorphism between proofs of implicational logic and typed λcalculus expressions. The adaptation to the unidirectional Lambek calculus with a rule of permutation was carried out by van Benthem (86), whilst the association for the bidirectional calculus without permutation is due to Moortgat.

Lets use the notation $\vdash$ *sequent* to mean the *sequent* is derivable. Then one uses the calculus to determine the meaning and syntactic type of a given string in the following way:

Definition 2 (Procedure for Assigning Type and Meaning) *An assignment of syntactic types and translations to lexical strings is assumed: then Φ' is the translation of a compound string and t is its syntactic type if $\vdash \tau \Rightarrow t : \Phi$ where τ is a sequence $[t_i; \delta_i]$ such that the t_i correspond to the categories of the* lexical *items in the string, and Φ' is Φ with the translations of the lexical items substituted for the variables δ_i*

For example assuming a lexicon { *john* $\rightarrow$np:*j'* , *mary* $\rightarrow$np:*m'*, *loves* $\rightarrow$(s\np)/np: *love'* }, one can prove that *john loves mary* has the category s and translation *love'*(*m'*)(*j'*) by first obtaining the proof:

(5)

$$\cfrac{\cfrac{s{:}\ u2(u3)(u1) \Rightarrow s{:}\ u2(u3)(u1) \qquad np{:}\ u1 \Rightarrow np{:}\ u1}{np{:}\ u1\ ,\ s\backslash np{:}\ u2u3 \Rightarrow s{:}\ u2(u3)(u1)}\backslash E \qquad np{:}\ u3 \Rightarrow np{:}\ u3}{np{:}\ u1\ ,\ (s\backslash np)/np{:}\ u2\ ,\ np{:}\ u3 \Rightarrow s{:}u2(u3)(u1)}/\,E$$

and then calculating the substitution:

(6) u2(u3)(u1) $[j'/u1, love'/u2, m'/u3]$

As this gives us no notion of decomposition other than into lexical parts, this is best taken as defining a way of *discovering* the type and translation of a string rather than as the definition of what it is to be the type and translation of a string. The actual definition of the type and translation of a string is:

Definition 3 (Type and Meaning of a string)
If the strings $\alpha_1, \ldots \alpha_n$ are assigned the types and translations $t_1 : \alpha'_1 \ldots t_n : \alpha'_n$ respectively and $\vdash t_1 : x_1 \ldots t_n : x_n \Rightarrow t : \Phi$, then then the string β got by concatenating the strings $\alpha_1, \ldots \alpha_n$ has the type and translation $t : \Phi'$ where Φ' is Φ with α_i substituted for x_i throughout.

That the discovery procedure assigns strings the very same types and translations as Defintion 3 depends on Cut elimination: Wherever an assignment of a type and translation is effected means of the decomposition of a string into non-lexical parts, this can be reflected by a use of Cut in the discovery procedure. In his 58 paper, Lambek adapted Gentzen's Cut Elimination proof to the categorial calculus, to show that exactly the same sequents may be derived without the use of Cut as with. From the vantage point of the Curry-Howard isomorphism, Cut eliminations correspond to β reductions, and β reductions are known to be meaning preserving. Thus one might expect that Cut elimination preserves the set of meanings that may be assigned by the calculus. Moortgat and Hendriks have recently both confirmed this.

These considerations affect how we shall choose to present the derivation of a sentence. To present the derivation is to present the evidence that the string is a sentence. We shall present this evidence according to definition 3 and not according to Definition 2. Thus we will not give sequent calculus proofs but tree-like derivations, where in place of the usual phrase structure rules we will have derivable sequents. For example, instead of 5:

(7)

$$\dfrac{\dfrac{\text{John}}{\text{np}} \qquad \dfrac{\dfrac{\text{loves}}{(\text{s}\backslash\text{np})/\text{np}} \quad \dfrac{\text{Mary}}{\text{np}}}{\text{s}\backslash\text{np}}\,1}{\text{s}}\,2$$

Each horizontal line is numbered and with that mth step is associated a certain sequent calculus proof and term Φ_m. In the above $\Phi_1 = \text{u1(u2)}$, and $\Phi_2 = \text{u2(u1)}$. More often than not we will leave this sequent proof unillustrated. Here is how we shall present the translation of the syntactic analysis into a semantic one:

(8)

1	**loves** $\Longrightarrow$ *love'*	*From the lexicon*
2	**mary** $\Longrightarrow$ *m'*	*From the lexicon*
3	**loves mary** $\Longrightarrow \Delta_1(\Delta_2)$	*By step*1 $\Phi_1 = u1(u2)$
4	$\Longrightarrow$ *loves'(m')*	*Unabbreviating*Δ_1*and*Δ_2
5	**john** $\Longrightarrow$ *j'*	*From the lexicon*
6	**John loves mary** $\Longrightarrow \Delta_4(\Delta_5)$	*By step*2 $\Phi_2 = u2(u1)$
7	$\Longrightarrow$ *love'(m)(j)*	*Unabbreviating* Δ_4*and*Δ_5

We wish to show that it is not possible to account for natural language quantification with a system of this design and a commitment to the orthodox Generalised Quantifier interpretation quantifying expressions. (Barwise and Cooper 1981)

We will choose a particular lexicon and demonstrate the uninterpretability of basic quantified sentences. Then there will be the question of other choices of lexicon to be considered.

First of all the lexicon adheres to a strict category to type correspondence ν, such that $\nu(\text{s}) = \text{t}$, $\nu(\text{np}) = \text{e}$, $\nu(A|B) = (\nu(B) \rightarrow \nu(A))$. Leaving types to default according to this recursion, I will consider the following categorisations:Proper names will be categorised as np. Verbs intransitive, transitive and ditransitive will be categorised s\np ,(s\np)/np and ((s\np)/np)/np, respectively. In order to be able to consider some Dutch examples, I include a category appropriate for Dutch transitive verbs in subordinate clauses:(s\np)\np Given that the quantified noun phrases are going to be interpreted by Generalised Quantifiers what categorisations are possible ? They cannot be assigned to np as this would deviate from the category to type correspondence above and as a reflex of this would force items of logical type $((e \rightarrow t > \rightarrow t)$ to be given as arguments to functions of type $(e \rightarrow t)$, which is impossible . The other natural categorisation is s/(s\np)and s\(s/np). On this basis one arrives at the quantified lexicon indicated below

every boy	$s/(s\backslash np)$, $s\backslash(s/np)$	$:\lambda P[\forall x(boy'x \rightarrow Px)] = EB_{GQ}$
a chocolate	$s/(s\backslash np)$, $s\backslash(s/np)$	$:\lambda P[\exists x(choc'x \wedge Px)] = AC_{GQ}$
a nun	$s/(s\backslash np)$, $s\backslash(s/np)$	$:\lambda P[\exists x(nun'x \wedge Px)] = AN_{GQ}$
elke schurk	$s/(s\backslash np)$, $s\backslash(s/np)$	$:\lambda P[\forall x(crook'x \rightarrow Px)]$
een detective	$s/(s\backslash np)$, $s\backslash(s/np)$	$:\lambda P[\exists x(detective'x \wedge Px)]$

With this lexicon we shall try to interpret the following mixture of Dutch and English sentences.

IV Every boy cried.

TV A nun liked every boy.

TTV Margaret gave every boy a chocolate.

Dutch TV (...dat) elke schurk een detective vreest.(every crook fears a detective)

We will distinguish 2 levels of success. At the first we will expect that all quantified sentence are at least *interpretable*. That is, they all get assigned at least one possible meaning. This minimum requirement will be referred to as Criterion A. The second is that each quantified sentence be assigned its full range of scope readings. This shall be Criterion B.

When we see how a semantics based on either the B-H calculus or the Lambek tackles the set of sentences above we meet with failure. Neither meets Criterion A and so a fortiori neither meets Criterion B. The particular cases of failure/success for each of the calculi are listed in the table below.

	IV	TV	TTV	Dutch TV
Bar-Hillel	yes	no	no	no
Lambek	yes	yes	no	no

In the rest of this section we explain some of the entries in the table. Working out the entries for the Bar-Hillel calculus is a simple matter of trial and error given its very simple nature . For the more flexible Lambek calculus it is easier to exploit its property of string semantic soundness and completeness. We use string semantic arguments to explain both the success of the calculus on the TV and its failure on the Dutch TV.

Consider then the TV sentence. The question is whether $\vdash s/(s\backslash np), (s\backslash np)/np, s\backslash(s/np) \Rightarrow s$. This will be true if $\vdash (s\backslash np)/np, s\backslash(s/np) \Rightarrow s\backslash np$. It can be shown that this must be derivable as it is valid.

Proof:suppose $\not\models (s\backslash np)/np, s\backslash(s/np) \Rightarrow s\backslash np$. Then $\exists$M,a model,in which $\exists a,b(a\in[(s\backslash np)/np], b\in[s\backslash(s/np)], a\cdot b\notin[s\backslash np])$. Then $\exists c(c\in[np], c\cdot a\cdot b\notin[s])$. Now one can quickly show that $c\cdot a\in[s/np]$. Hence, $\exists d(d\in[s/np], d\cdot b\notin[s]$. Hence $b\notin[s\backslash(s/np)]$, which is a contradiction.

Consider now the dutch TV sentence, The question is whether $\vdash s/(s\backslash np), s/(s\backslash np), (s\backslash np)\backslash np \Rightarrow s$. It can be shown that this must not be derivable as it is not valid.

Proof:suppose $\models s/(s\backslash np), s/(s\backslash np), (s\backslash np)\backslash np \Rightarrow s$. Then there is no model M in which $\exists a,b,c(a\in[s/(s\backslash np)], b\in[s/(s\backslash np)], c\in[(s\backslash np)\backslash np], a\cdot b\cdot c\notin[s])$. We shall construct such an M. Let M be that model such that $[np]=\{\beta\}$, $[s]=\{\beta\beta\alpha, \gamma\beta\alpha\}$. Therefore derived categories are $[s\backslash np]=\{\beta\alpha\}$, $[(s\backslash np)\backslash np]=\{\alpha\}$, $[s/(s\backslash np)]=\{\beta, \gamma\}$. Notice $\gamma\gamma\alpha \notin[s]$. So $\exists a,b,c(a\in[s/(s\backslash np)], b\in[s/(s\backslash np)], c\in[(s\backslash np)\backslash np], a\cdot b\cdot c\notin[s])$.

One could construct a similar proof for the case of the TTV, as it shares with the Dutch example the feature that the verbal element seeks 2 successive arguments in the same direction.

The choice of lexicon we have just made has come to be known as the strategy of "least possible type assignment ". There is more to be recommend this strategy than simply an appeal to simplicity. Consider one of the most striking deviations from this strategy: Montague's type assignment to transitive verbs, having them take as object arguments not entity typed things but quantifier type things (in fact intensional versions of these). Adopting such typings guarantees success in terms of Criterion A. But the cost of meeting Criterion A in this way is failure to meet that other ubiquitous criterion of semantics, *the need to account for logical implications.* So for example, *without the invocation of meaning postulates*, the following implication would be lost:

(9) 1 Margaret liked every boy

2 John is a boy

3 so Margaret liked John

The principal advantage of the minimal type assignment strategy is therefore that *if* one succeeds in interpreting a quantified sentence at all *then* one will at the same time account for its logical force, and one will do this without the invocation of meaning postulates.

From this point of view the lexical choices we just made are the only possible ones, and they just do not work. If one is not of this point of view, then many other choices will appear equally motivated. Herman Hendriks has carried through this survey of other choices and so we will not repeat it here. One finds one can meet Criterion A, but not Criterion B.

Therefore from either point of view there are reasons for looking beyond the combination of the Lambek calculus with Generalised Quantifiers. Either to secure plain interpretability, maintain inference and eschew meaning postulates, or to find an explanation for scope ambiguity. In the following sections we will advocate a shift to *polymorphism* in the interpretation of generalised quantifiers. We will find that this allows us to secure all of these goals.

2 Polymorphic Semantics

Where there was a failure to obtain an analysis on the categorisation of quantifiers adopted above, there is always an alternative categorisation that would have allowed a successful analysis. The table below lists these categorisations for the sentence types considered above.

np-type	categorisation
IV-subject	s/(s\np)
TV-object	(s\np)\((s\np)/np)
TTV-object	((s\np)/np) \(((s\np)/np)/np)
Dutch TV-object	(s\np)/((s\np)\np)

From this it may appear the only way to succeed would be to assume that the quantifier is assigned to a multitude of categories. We would like to say, however, that there is a uniformity to all of these different categorisations. Informally speaking it is just that the quantifier is able to "take as an argument" any item of type X/np, and will "return" X. There is no way to write down a type of the conventional categorial language that has this logical behaviour. What we propose to do is to *enlarge* the categorial language in order that it become able to express the distributional property that a quantifier has and this expansion will be the inclusion of what we shall call *polymorphic types.* The use to which such a categorial language can be put in meeting Criteria A and B are discussed in section 4, but in this section and section 3 we will be concerned with the formal details of a polymorphic categorial system.

In 2.1 we define the system of types and terms that we will use to capture polymorphism semantically. With the system defined, we proceed to address the question of the type and translation and denotation of quantifiers in 2.2 and 2.3.

In section 3 we define a syntactic type system suitable to be matched with the polymorphic semantic type system.

2.1 Polymorphic Types and Terms

To introduce the polymorphic quantifiers it is instructive to consider what would be the spectrum of quantifier interpretations that would naturally accompany the spectrum of categories in table above. These appear in the table below.

np-type	interpretation
IV-subject	$\lambda P^{e\rightarrow t)}[GQ\lambda x[Px]]$
TV-object	$\lambda P^{(e\rightarrow(e\rightarrow t))}\ \lambda y[GQ\ \lambda x[Pxy]]$
TTV-object	$\lambda P^{(e\rightarrow(e\rightarrow(e\rightarrow t)))}\ \lambda y\ \lambda z\ [GQ\ \lambda x[Pxyz]]$
Dutch TV-object	$\lambda P^{(e\rightarrow(e\rightarrow t))}\ \lambda y[GQ\ \lambda x[Pxy]]$

There is uniformity to this group of quantifier translations. Let us try to describe this uniformity in terms of what they denote. Each expression is an abstraction over predicates of type, $(e\rightarrow(\ldots\rightarrow t)\ldots)$. Such a predicates P, is the curried characterisation of a set of n+1 tuples, $n \geq 0$, drawn from the domain $D_e \times \vec{D_X}$, where $\vec{D_X}$ indicates a sequence of n domains. The result of combining with such a predicate is a new function F which is the curried characterisation of a set of n-tuples drawn from the domain $\vec{D_X}$. A given n-tuple $\vec{X}$ is in the set so characterized according to the relationship between the generalised quantifier GQ and the set 1 tuples got by restricting P by the n tuple $\vec{X}$. In the TV case, F is $\lambda y GQ(\lambda x Pxy)$, a set of 1 tuples, and $\lambda x Pxy$ is the set of 1 tuples got from P by restricting its first argument place by y. In the TTV case, F is $\lambda y \lambda z GQ(\lambda x Pxyz)$, the curried characterisation of a set of 2 tuples $< y, z >$, and $\lambda x Pxyz$ is the set of 1 tuples got from P by restricting its 2nd and 3rd argument positions with the 2 tuple $< y, z >$.

What we wish to develop is a λ calculus in which to express the denotational uniformity just noted above. Minimally this should allow us to write a single formula which, under different typings of the variables involved, denotes what the spectrum of quantifier translations denotes. This kind of approach to polymorphism by ambiguity of typing is an approach that has beed taken to capturing the type-neutrality of the identity function. If we express the identity by:

(10) λx.x

then there are infinitely many possible typings of x that make this a well typed formula, and this has been taken to be a sense in which the typed lambda calculus can express the polymorphic identity. However, there is no way that the typed λ calculus familiar from IL will allow us to give expression to the kind of polymorphism we are seeking for quantifiers. The best that we can do is to express what one might call the trivial polymorphic quantifiers - those corresponding to the generalised quantifiers of type raised individuals. For a of type e, a polymorphic type raised individual is

(11) λP Pa

As P varies in type across all possible instantiations of the schema $(e\rightarrow(\vec{X}\rightarrow t))$, the single formula denotes a function of type $(\vec{X}\rightarrow t)$.

In describing the uniformity across the quantifier translations, we referred to n-tuples and the fact that a typical predicate denotation is a curried characterisation of a set of n-tuples. We propose to introduce terms, typings and conversions appropriate to describe such tuples and the process of currying.

We shall develop these types and terms on the model of the Curry-Howard isomorphism between intuitionistic propositional proofs and typed λ calculus expressions. Thus we will introduce sequent calculi for the type language itself and then proofs in this calculus with typed terms. Conversion between the terms will be constrained to correspond to reductions between proofs. We will then be able to express the polymorphism of quantifiers with a single expression that may be typed many ways.

We proceed in the introduction of this system in stages. In 2.1.1 a simple type language TG will be defined that includes List types. A sequent calculus will be defined that characterises the derivability relationship for TG, and further, an association of λterms with proofs in the calculus will be defined. These terms will include syntax that concerns *lists* and several kinds of term reduction appropriate to lists will be defined, as well as β reduction. In 2.1.2 we start to expand the type language, introducing variables. This is TV. Noting that certain expressions that can be assigned to indefinitely many different types can be assigned types containing variables, in 2.1.3 we take the natural next step to TP which allows universal quantification of type variables. In the wake of this expansion of the language come increases in the calculus, increases in the term language to be associated with proofs and additional notions of reduction. In 2.2 we return to the polymorphic quantifiers.

2.1.1 Types with Lists

To begin with we define a semantic type language which is just like the language of implicational propositional logic, except for the fact that it additionally contains List types. These will be the types of n tuples.

Definition 4 (Types of TG)
a. e,t are TYPES
b. if *a and b are TYPES* then *(a →b) is a TYPE*
c. ⊥ is a LIST
d. if *a is a TYPE and L is a LIST* then *[a,L] is a LIST*
e. if *L is a LIST* then *L is a TYPE*

We call this TG with the intention that this stand for *Types Grounded.* Here are some example types. (e →t) is a TYPE, [e,⊥] is a LIST and a TYPE, [e,[e,⊥]] is a LIST and a TYPE, ([e,[e,⊥]] →t) is a TYPE .

As the List notation is cumbersome we may abbreviate thus $[e,e] \stackrel{df}{=} [e,[e,\perp]]$

In 5 a sequent calculus is defined where we allow that $X \Rightarrow y$ is a sequent if X is a sequence of types, not possibly empty, and y is a single type. [2] The following notation conventions are observed in the rules: U,V,T stand for sequences of types, U,V possibly empty. w,x,y stand for single types. L stands for a List type. π (U) stands for any permutation of U. There are no further structural rules.

Definition 5 (Calculus for TG)

(Ax) $x \rightarrow x$

(Ax) $(\perp \rightarrow x) \Rightarrow x$

(→ E) $$\frac{U,y,V \Rightarrow w \quad T \Rightarrow x}{U,(x \rightarrow y),T,V \Rightarrow w}\rightarrow E$$

(→ I) $$\frac{T,x \Rightarrow y}{T \Rightarrow (x \rightarrow y)}\rightarrow I$$

([]E) $$\frac{U,x,L,V \Rightarrow w}{U,[x,L],V \Rightarrow w}[]E$$

([]I) $$\frac{T \Rightarrow L}{x,T \Rightarrow [x,L]}[]I$$

(⊥E) $$\frac{U,V \Rightarrow w}{U,\perp,V \Rightarrow w}\perp E$$

(⊥ I) $x \Rightarrow [x, \perp]$

(Perm) $$\frac{\pi(U) \Rightarrow w}{U \Rightarrow w}Perm$$

As far as the rules (Ax), (→ E), (→ I), ([]E), ([]I) and (Perm) these are exactly the same as those found in van Benthem's unidirectional Lambek calculus. (van Benthem 86) The difference is the list rules. In (12a) we have a proof of (a →(b →c)) ⇒ ([a,[b, ⊥]] →c). This is an example of an inference moving from a more to a less curried type. In (12b) we have a proof of ([a,L] →c) ⇒ (a →(L →c)), which indicates motion in the opposite direction

[2] actually what should decide us whether X may be empty is unclear. The prohibition has the effect that *closed* terms of the λ calculus language cannot be paired with proofs. The map from proofs to terms is not onto, as it is in the case of the Curry-Howard isomorphism, but into.

(12)

a.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{b, a, (a \rightarrow (b \rightarrow c)) \Rightarrow c}{a, b, (a \rightarrow (b \rightarrow c)) \Rightarrow c}\text{Perm}}{a, b, \perp (a \rightarrow (b \rightarrow c)) \Rightarrow c}\perp E}{a, [b, \perp], (a \rightarrow (b \rightarrow c)) \Rightarrow c}[]\,E}{[a,[b, \perp]], (a \rightarrow (b \rightarrow c)) \Rightarrow c}[]\,E}{(a \rightarrow (b \rightarrow c)) \Rightarrow ([a,[b, \perp]] \rightarrow c)}\rightarrow I$$

b.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{L \Rightarrow L}{a, L \Rightarrow [a,L]\ c \Rightarrow c}[]\,I}{a, L, ([a,L] \rightarrow c) \Rightarrow c}\rightarrow E}{L, a, ([a,L] \rightarrow c) \Rightarrow c}\text{Perm}}{a, ([a,L] \rightarrow c) \Rightarrow (L \rightarrow c)}\rightarrow I}{([a,L] \rightarrow c) \Rightarrow (a \rightarrow (L \rightarrow c))}\rightarrow I$$

In ??, for future reference, generalisations of the inferences of 12 are noted. $\vec{X}$ here in *not* part of the type language, but is a schema over it. L ,the list type, is a part of the type language.

(13)

Useful facts about List Types

$(\vec{X} \rightarrow ([a,L] \rightarrow Y)) \Rightarrow (\vec{X} \rightarrow (a \rightarrow (L \rightarrow Y)))$

$(\vec{X} \rightarrow (a \rightarrow (b \rightarrow Y))) \Rightarrow (\vec{X} \rightarrow ([a,b] \rightarrow Y))$

With proofs in the calculus we want to associate typed expressions of λ calculus. The present case will differ from "conventional " typed λ calculus by the provision of syntactical devices that are intended to allow the representation of Lists

We define what it is to be a TYPED WFF. The association of terms to proofs will be such as to guarantee that a term of type t is associated with a proof of t. We assume 2 sets of variables VAR and LISTVAR. x,y,z are in VAR. X,Y, Z are in LISTVAR

Definition 6 (Typed Wff)

1. if $\alpha \in$ *VAR(LISTVAR), a is a NONLIST(LIST) TYPE,* then α^a *is a TYPED VAR(LISTVAR)*
2. *TYPED VAR and TYPED LISTVAR are TYPED WFFS*
3. if $\alpha^{(a \rightarrow b)}$ *and* β^a *are TYPED WFFS,* then $(\alpha^{(a \rightarrow b)}\beta^a)^b$ *is a TYPED WFF*
4. if α^a *is a TYPED VAR or TYPED LISTVAR and* β^b *is a TYPED WFF* then $(\lambda\alpha^a\beta^b)^{(a \rightarrow b)}$ *is a TYPED WFF*
5. $\perp$ *is a TYPED LIST*
6. if α^a *is a TYPED LISTVAR* then α^a *is a TYPED LIST*
7. if α^a *is a TYPED WFF and* β^b *is a TYPED LIST,* then $[\alpha^a, \beta^b]^{[a,b]}$ *is a TYPED LIST*
8. if $\alpha^{[a,b]}$ *is a TYPED LIST,* $(\alpha_H^{[a,b]})^a$ *is a TYPED WFF, and* $(\alpha_T^{[a,b]})^b$ *is a TYPED LIST*
9. if α^a *is a TYPED LIST,* then α^a *is TYPED WFF*

We associate terms with the calculus in the fashion specified in 7

Definition 7 (Terms and the TG calculus)

(Ax) $\text{x}: \alpha^{x} \rightarrow \text{x}: \alpha^{x}$

(Ax) $(\bot \rightarrow \text{x}): \alpha^{(\bot \rightarrow x)} \Rightarrow \text{x}: \alpha^{x}$

(→ E) $$\frac{\text{U},\text{y}: (\alpha^{(x \rightarrow y)}\beta^{a})^{x},\text{V} \Rightarrow \text{w} \quad \text{T} \Rightarrow \text{x}: \beta^{x}}{\text{U},(\text{x} \rightarrow \text{y}): \alpha^{(x \rightarrow y)},\text{T},\text{V} \Rightarrow \text{w}} \rightarrow \text{E}$$

(→ I) $$\frac{\text{T},\text{x}: \alpha^{x} \Rightarrow \text{y}: \beta^{y}}{\text{T} \Rightarrow (\text{x} \rightarrow \text{y}): (\lambda\delta^{x}\beta^{y})^{(x \rightarrow y)}} \rightarrow \text{I}$$

([]E) $$\frac{\text{U},\text{x}: (\alpha^{[x,L]}{}_{H})^{x},\text{L}: (\alpha^{[x,L]}{}_{T})^{L},\text{V} \Rightarrow \text{w}}{\text{U},[\text{x},\text{L}]: \alpha^{[x,L]},\text{V} \Rightarrow \text{w}} []\text{E}$$

([]I) $$\frac{\text{T} \Rightarrow \text{L}: \beta^{L}}{\text{x}: \alpha^{x},\text{T} \Rightarrow [\text{x},\text{L}]:[\alpha^{x},\beta^{L}]^{[x,L]}} []\text{I}$$

(⊥E) $$\frac{\text{U},\text{V} \Rightarrow \text{w}: \alpha^{w}}{\text{U},\bot,\text{V} \Rightarrow \text{w}: \alpha^{w}} \bot\text{E}$$

(⊥ I) $\text{x}: \alpha^{x} \Rightarrow [\text{x}, \bot]:[\alpha^{x}, \bot]^{[x,\bot]}$

(Perm) $$\frac{\pi(\text{U}) \Rightarrow \text{w}: \alpha^{w}}{\text{U} \Rightarrow \text{w}: \alpha^{w}} \text{Perm}$$

The notation conventions are now that U,V,T represent sequences of pairs of types and terms, U,V possibly empty. w is a single type:term pair. We shall have occasion several more times to define first a calculus in isolation and then one with terms associated. The change in notation conventions accompanying this will be assumed.

For illustration the two list converting proofs are repeated below this time together with their term annotations. (14a) is the proof of $y^{(a \rightarrow (b \rightarrow c))} \Rightarrow \lambda x^{[a,b]}[y(x_{h})^{a}((x_{T})^{b})_{H}]^{([a,[b,\bot]] \rightarrow c)}$. As intended the term associated with this type transition is uncurrying operation. (14b) is the proof of $x^{([a,L] \rightarrow c)} \Rightarrow \lambda y^{a}\lambda Z^{L}[x([y,Z])]^{(a \rightarrow (L \rightarrow c))}$, this time currying.

(14)

a.

$$\text{b}: (\text{x}_{T})_{H},\text{a}: \text{x}_{H}, (\text{a} \rightarrow (\text{b} \rightarrow \text{c})): \text{y} \Rightarrow \text{c}: y(x_{H})(x_{T})_{H}$$

——— Perm

$$\text{a}: \text{x}_{H},\text{b}: (\text{x}_{T})_{H},(\text{a} \rightarrow (\text{b} \rightarrow \text{c})):\text{y} \Rightarrow \text{c}: y(x_{H})(x_{T})_{H}$$

——— ⊥ E

$$\text{a}: \text{x}_{H},\text{b}: (\text{x}_{T})_{H}, \bot\ (\text{a} \rightarrow (\text{b} \rightarrow \text{c})):\text{y} \Rightarrow \text{c}: y(x_{H})(x_{T})_{H}$$

——— [] E

$$\text{a}: \text{x}_{H}, [\text{b}, \bot]:\text{x}_{T}, (\text{a} \rightarrow (\text{b} \rightarrow \text{c})):\text{y} \Rightarrow \text{c}: y(x_{H})(x_{T})_{H}$$

——— [] E

$$[\text{a},[\text{b}, \bot]]:\text{x}, (\text{a} \rightarrow (\text{b} \rightarrow \text{c})):\text{y} \Rightarrow \text{c}: y(x_{H})(x_{T})_{H}$$

——— → I

$$(\text{a} \rightarrow (\text{b} \rightarrow \text{c})): \text{y} \Rightarrow ([\text{a},[\text{b}, \bot]] \rightarrow \text{c}): \lambda x[y(x_{H})(x_{T})_{H}]$$

b.

$$\text{L} :Z \Rightarrow \text{L} :Z$$

——— [] I

$$\text{a} :y,\text{L} :Z \Rightarrow [\text{a},\text{L}] :[y, e]:x([y, Z]) \Rightarrow \text{c} :x([y, Z])$$

——— → E

$$\text{a} :y,\text{L} :Z, ([\text{a},\text{L}] \rightarrow \text{c}) :x \Rightarrow \text{c} :x([y, Z])$$

——— Perm

$$\text{L} :Z,\text{a} :y,([\text{a},\text{L}] \rightarrow \text{c}) :x \Rightarrow \text{c} :x([y, Z])$$

——— → I

$$\text{a} :y, ([\text{a},\text{L}] \rightarrow \text{c}) :x \Rightarrow (\text{L} \rightarrow \text{c}) :\lambda Z[x([y, Z])]$$

——— → I

$$([\text{a},\text{L}] \rightarrow \text{c}) :x \Rightarrow (\text{a} \rightarrow (\text{L} \rightarrow \text{c})) :\lambda y\lambda Z[x([y, Z])]$$

The terms have associated with them a notion of reduction, of which there are 5 kinds: β, H, T, $\bot ab$, and $\bot ap$.

Definition 8 (Reduction)

Beta $\lambda\alpha[\beta](a) \rhd \beta[\alpha/a]$

H $[\alpha,\beta]_{H} \rhd \alpha$

T $[\alpha,\beta]_{T} \rhd \beta$

$\bot ab$ $\alpha(x^{\bot}) \rhd \alpha.$

$\bot ap$ $\lambda x^{\bot}.\alpha \rhd \alpha$

These reductions reflect the proof reductions possible between proofs with corresponding terms. That is, if a Cut rule were to be added, the term reductions would reflect Cut eliminating proof reductions. Space prohibits the illustration of this. One should note that if 2 terms are interconvertible using the L and T reductions, this indicates an equivalence not an identity. The curried and the uncurried versions of a given function are isomorphic to each other but are nonetheless are not the same function. This is unlike β equivalence in IL, which is interpreted as identity of denotations.

2.1.2 Variable Types

We will now expand the Type language TG to a larger one TV:"Types with variables".

Basically we will simply add type variables to our language of types. These variables will be of 2 sorts. First there will be additional *base types*. We will assume a set of these, VAR′, and represent them with lower case greek: π, θ, ϕ. Second there will be additional Lists. We assume set of these, LISTVAR′, and represent them with upper case Greek: Π, Θ, Φ. Then added to the definition of TG is:

Definition 9 (TG Types added to TV)
a. if $\theta \in VAR'$, then θ *is a TYPE b.* if $\Theta \in LISTVAR'$. then Θ *is a LIST*

Given the definition of TYPED WFF, we can generate wffs whose types contain variables, as in 15a . When this is so one could also have generated the similar wff differing only by a substitution for the type variable,as in 15b and c.

(15)

a $\lambda y^{(e\rightarrow\pi)}[y(x^e)]^{((e\rightarrow\pi)\rightarrow\pi)}$

b $\lambda y^{(e\rightarrow e)}[y(x^e)]^{((e\rightarrow e)\rightarrow e)}$

c $\lambda y^{(e\rightarrow([e,e]\rightarrow t))}[y(x^e)]^{((e\rightarrow([e,e]\rightarrow t))\rightarrow([e,e]\rightarrow t))}$

Correspondingly, in the calculus if we can prove $(e \Rightarrow ((e \rightarrow\pi) \rightarrow\pi))$, then we can prove a similar sequent that replaces π with any type at all.

This is reminiscent of the rule of universal introduction of natural deduction, and it seems that we have a calculus that allows us to find the grounds of a universal quantification without being able to express what follows from these grounds. Girard, in the early 70's, originally proposed a type theory with variables and made the same observation, moving on from there to include quantification in the syntax of types. Reynolds independently came up with essentially the same system in 74. We shall follow their lead, and proceed next to a language allowing for the universal quantification of type variables.

2.1.3 Polymorphic Types

We now expand the language TV to a larger one still TP: "Types with Polymorphism". TP has the same definition as TV, with the addition of clauses governing a new type constructor $\forall$

Definition 1 (TP types added to TV)

1. if $\theta \in VAR'$ *and a is a TYPE.* then $\forall\theta.a$ *is a TYPE*

2. if $\Theta \in LISTVAR'$ *and a is a TYPE, then* $\forall\Theta.a$ *is a TYPE*

For example,

(16) $\forall\pi((e\rightarrow\pi)\rightarrow\pi)$ is a TYPE.

We will first specify the sequent rules governing this type constructor and then describe the terms that will have these types.

Definition 10 (Calculus for TP types)

(∀E) $$\frac{U,x[p/a],V \Rightarrow w}{U,\forall p.x,\ V \Rightarrow w}\ \forall E$$ *[a is some chosen type]*

(∀I) $$\frac{T \Rightarrow x}{T \Rightarrow \forall p.x}\ \forall I$$ *[p is not free in T]*

In 15a below, a polymorphic version of type raising is illustrated: $e \Rightarrow \forall\pi((e\to\pi)\to\pi)$, whilst 15b shows a polymorphic type may combine with other types: $(e \to t),\ \forall\pi((e\to\pi)\to\pi) \Rightarrow t$

(17)

a. $$\cfrac{\cfrac{\cfrac{e \Rightarrow e \quad \pi \Rightarrow \pi}{(e\to\pi),e \Rightarrow \pi}\to E}{e \Rightarrow ((e\to\pi)\to\pi)}\to I}{e \to \forall\pi((e\to\pi)\to\pi)}\forall I$$

b. $$\cfrac{\cfrac{(e\to t) \Rightarrow (e\to t) \quad t \Rightarrow t}{(e\to t),\ ((e\to t)\to t) \Rightarrow t}\to E}{(e\to t),\ \forall\pi((e\to\pi)\to\pi) \Rightarrow t}\forall E$$

To associate terms with these proofs we need to expand the language used to notate proofs and we do so, following Girard, by adding another kind of term formation "abstraction over types".

Definition 2 (Typed Wffs with Extraction)

1. if $p \in VAR' or LISTVAR'$ *and* α *is a TYPED WFF of type* x, then $\Delta p.\alpha^{\forall p.x}$ *is a TYPED WFF. (p must not be free in a the type of free variable of* α.
2. if $\alpha^{\forall p.x}$ *is a TYPED WFF and a is a TYPE, then* $\alpha(a)^{x[p/a]}$ *is a TYPED WFF.*

It should be noted that vitally p, the variable being abstracted over is a *type* variable and is not a variable of the λ calculus used hitherto. That one should have terms whose types are universally quantified will seem bizarre to anyone accustomed to equate types with the function spaces of IL. In due course the denotation of these terms will be considered, but in advance of this they can be explained by reference to the Curry-Howard isomorphism. According to this, one can look upon typed lambda calculus expressions as an alternative notation for proofs of intuitionistic propositional logic - that is why their types are implicational formulae. Naturally then a notation for proofs in a language containing quantification will give terms the types of quantified formulae. We now define the association of type-abstraction, and type-application formulae with proofs involving type quantification.

Definition 11 (Terms associated with TP types)

(∀E) $$\frac{U,x[p/a]{:}\alpha(a)^{x[p/a]},V \Rightarrow w}{U,\forall p.x{:}\alpha^{\forall p.x},\ V \Rightarrow w}\ \forall E$$ *[a is some chosen type]*

(∀I) $$\frac{U \Rightarrow x{:}\ \alpha^{x}}{U \Rightarrow \forall p.x{:}\ {:}(\Delta p.\alpha^{x})^{\forall p.x}}\ \forall I$$ *[p is not free in U]*

With the new terms comes an associated reduction. It is exactly analogous save that the substitution is now into the type part of a term. We shall call this β' reduction. It is again the case that this term reduction can be related to Cut eliminating term reductions.

Definition 3 (β' reduction)

$\beta'\ \Delta p.\alpha^{x}(a) \triangleright \alpha^{x[p/a]}$

2.2 Polymorphic Quantifiers

We are now in a position to propose a *polymorphic quantifier* corresponding to any given *generalised quantifier*. It shall use quantification of List types.

Definition 12 (Polymorphic Quantifier)
To any given Generalised Quantifier Q_G, of type ((e →t) →t) there corresponds a Polymorphic Quantifier Q_P, of type $\forall\Pi((e\rightarrow(\Pi\rightarrow t))\rightarrow(\Pi\rightarrow t))$, given by

$$\Delta\Pi\lambda y^{(e\rightarrow(\Pi\rightarrow t))}\lambda X^{\Pi}[Q_G(\lambda x[yxX^{\Pi}])]$$

With this we achieve what we set out to do by way of notation: a single formula whose denotation as we vary the types of certain of its variables, gives the spectrum of quantifier meanings. We also have abstracted upon the type that we wish to vary. Doing this and identifying this with universal quantification rationalises the λ terms and reductions, by making them conform to the Curry-Howard isomorphism. Notice that whatever list type the quantifier is applied to, the result will be a function taking arguments of conjoinable type, (where Type is Conj if Type = t or if Type = (a →b) where b is Conj).

One can actually *derive* this polymorphic quantifier from the generalised one within the calculus by attempting to prove the sequent: ((e →t) →t) ⇒ $\forall\Pi((e\rightarrow(\Pi\rightarrow t))\rightarrow(\Pi\rightarrow t))$:

(18)

$$\begin{array}{cl}
\Pi{:}X,\ e{:}x,\ (e\rightarrow(\Pi\rightarrow t)){:}y\ \Rightarrow\ t{:}\ yxX^{\Pi} & \\ \hline
e{:}x,\ \Pi{:}X,\ (e\rightarrow(\Pi\rightarrow t)){:}y\ \Rightarrow\ t{:}\ yxX^{\Pi} & \text{Perm} \\ \hline
\Pi{:}X,\ (e\rightarrow(\Pi\rightarrow t)){:}y\ \Rightarrow\ (e\rightarrow t){:}\ \lambda x[yxX^{\Pi}]\ \ t{:}Q(\lambda x[yxX^{\Pi}])\ \Rightarrow\ t{:}Q(\lambda x[yxX^{\Pi}]) & \rightarrow I \\ \hline
\Pi{:}X,\ (e\rightarrow(\Pi\rightarrow t)){:}y,\ ((e\rightarrow t)\rightarrow t){:}\ Q\ \Rightarrow\ t{:}Q(\lambda x[yxX^{\Pi}]) & \rightarrow E \\ \hline
(e\rightarrow(\Pi\rightarrow t)){:}y,\ ((e\rightarrow t)\rightarrow t){:}\ Q\ \Rightarrow\ (\Pi\rightarrow t){:}\lambda X^{\Pi}Q(\lambda x[yxX^{\Pi}]) & \rightarrow I \\ \hline
((e\rightarrow t)\rightarrow t){:}\ Q\ \Rightarrow\ ((e\rightarrow(\Pi\rightarrow t))\rightarrow(\Pi\rightarrow t)){:}\lambda y\ \lambda X^{\Pi}Q(\lambda x[yxX^{\Pi}]) & \rightarrow I \\ \hline
((e\rightarrow t)\rightarrow t){:}Q\ \Rightarrow\ \forall\Pi((e\rightarrow(\Pi\rightarrow t))\rightarrow(\Pi\rightarrow t)){:}\ \Delta\Pi\lambda y^{(e\rightarrow(\Pi\rightarrow t))}\lambda X^{\Pi}[Q_G(\lambda x[yxX^{\Pi}])] & \forall I
\end{array}$$

One cannot prove the reverse. e of course can be mapped into this type also. [3]

We will set down here a useful property of the translations quantifiers. It will often be the case that an instantiated polymorphic quantifier $Q_P(L)$ is applied to an n-ary function, translated as

$$\lambda x^e\lambda X^L[\Phi^t].$$

We can then perform the following conversion:

$$PQ(L)(\lambda x^e\lambda X^L[\Phi^t])\triangleright\lambda X^L[Q_G(\lambda x^e\Phi^t]$$

This shall be referred to as *Variable Exportation*

2.3 Interpretation of types and terms

Familiar from IL will be the idea of a domain of type τ. As we have types in our applicative language, we will prefer to see these domains as the *interpretations* of types. To begin with there are those TG types that are also IL types:

(19) Some set D is assumed

$$[\![e]\!] = D; [\![t]\!] = \{0,1\}; [\![(a\rightarrow b)]\!] = [\![b]\!]^{[\![a]\!]}$$

Next we interpret the List types:

(20) $[\![\perp]\!] = \emptyset.\ [\![\,[a,L]\,]\!] = [\![\,a\,]\!]\times[\![\,L\,]\!]$

[3] **Incidentally the Conjoinable might be expressed $\exists\Pi(\Pi\rightarrow t)$. Then if we express the quantifier type as something "looking for" functions into Conjoinable types, we would get a kind of Donkey sentence in types:**

$$(e\rightarrow(\exists\Pi(\Pi\rightarrow t)\rightarrow(\Pi\rightarrow t)))\approx\forall\Pi((e\rightarrow(\Pi\rightarrow t))\rightarrow(\Pi\rightarrow t))$$

Some examples:

(21) $[\![\bot \to t]\!] = \{0,1\}^{\emptyset} = \{0,1\}$

This is the idea familiar from algebra of identifying a constant with a nullary function. One can just take it as convention, (cf. $2^1 = 2$)

(22) $[\![[[a,b],\bot]]\!] = [\![e]\!] \times ([\![e]\!] \times \emptyset) = [\![e]\!] \times [\![e]\!]$

This relies on $[\![e]\!] \times \emptyset = [\![e]\!]$. One should note $[\![[[a,b],c]]\!] \neq [\![[a, [b,c]]]\!]$. For example if $<< x,y >,z >$ is in the first it will not be in the second. The interpretations are isomorphic though.

The interpretations of these types form the basis for the interpretation of the other types we have been discussing. Let $\mathcal{D}$ be the set of all interpretations of TG types, and let $\vec{\mathcal{D}}$ be the subset of this interpreting list types. Then type variables will be will receive values in these domains via an assignment. Choosing to interpret the variables in this way determines how the polymorphic types will be interpreted. Before giving the interpretation of this type, lets consider the interpretation of terms that have this type.

Suppose for the moment that we have a specification of the space of all possible type interpretations, including the polymorphic types. Call this T. Then the interpretation of a term $\Delta\Pi\beta^b$ of type $\forall\Pi b$, would be:

(23) $[\![\Delta\Pi\beta^b]\!]^g$ is a function h: T $\to\cup$ T, mapping interpretations of types into objects of those types, such that for any x $\in$ T, h(x) = $[\![\beta^b]\!]^{g'}$, where $g \stackrel{x}{=} g'$, $g'(\Pi)$ being x.

There is a circularity in this definition. If T includes all types, it includes $\forall\Pi b$. Since a type is identified with the set of possible objects of that type, $[\![\Delta\Pi\beta^b]\!]^g$ is being defined as a function of a set containing $[\![\Delta\Pi\beta^b]\!]^g$, something not set theoretically possible.

We have chosen to avoid this problem by specifying that type variables be assigned to interpretations within $\mathcal{D}$. Evidently then any type abstraction will be a function on this same space and not upon the space of all possible types. This will make the interpretation of application to a polymorphic type uninterpretable. We should therefore revise our definition of what is a typed wff. Replace clause 2 of the definition Typed Wffs with Abstraction to : "*if* $\alpha^{\forall p.x}$ is a TYPED WFF and a is a Ground Type, then $\alpha(a)^{x[p/a]}$ is a TYPED WFF."

We shall also make the simplifying assumption that we shall need to interpret no term with abstraction over more than one type variable, and then we can state the interpretation of the polymorphic types as

(24) $[\![\forall\pi b]\!]^g = \{ <T, [\![b]\!]^{g'} >: T \in \mathcal{D}, g \stackrel{\pi}{=} g', g'(\pi) = T \}$

$[\![\forall\Pi b]\!]^g = \{ <T, [\![b]\!]^{g'} >: T \in \vec{\mathcal{D}}, g \stackrel{\pi}{=} g'\ g'(\pi) = T \}$

Faced with the circularity of the naive set theoretic interpretation of polymorphism, we have chosen a particular tactic to avoid the problem. There are other alternatives in the literature. In particular, Girard has recently presented a semantics for this kind of polymorphism. It requires a far more complex notion of what the interpretation of a type might be than we have been discussing. The rest of the paper should be taken as exploring the hypothesis that we can account for natural language phenomena using polymorphism in the restricted way that we have just indicated. But if this turns out not the be the case, then we will have look to the more sophisticated semantics of Girard.

This completes our discussion of the semantic machinery that we intend to deploy in using polymorphism. Just before we turn to the question of how this is to be wed with a categorial grammar, we shall just check that the translations we have given to quantifiers, gives them kind of interpretation that we informally attributed to them at the very outset.

We must evaluate $[\![\Delta\Pi\lambda y^{(e\rightarrow(\Pi\rightarrow t))}\lambda X^{\Pi}[GQ_{GQ}\lambda z[yzX^{\Pi}]]\!]$. It is that function h such that $h\Pi yX = 1$ iff

$\Pi = D_{X_1} \times D_{X_2} \ldots D_{X_n}$ *some* $X_1 \ldots X_n \in TG$
$y \subset D_e \times (D_{X_1} \times D_{X_2} \ldots D_{X_n})$
$X \in D_{X_1} \times D_{X_2} \ldots D_{X_n}$
and $\{x :< x, X > \in y\} \in GQ$

This indeed seems to capture the uniformity across different types noted at the outset.

3 Polymorphic Syntax

We propose to extend the Lambek calculus of section 1 adding syntactic type variables and Universal Quantification. What will not be carried over from the semantic calculus are the List types and rules. This seems to be an important difference between semantic types and syntactic types- you cannot in general uncurry a syntactic type. As in 1, we will pair the syntactic proofs with λ calculus terms, using however the larger language we have just described. In 3.2 we give the polymorphic categorisations of the quantifiers. In 3.3 the string semantics appropriate to the polymorphic categories is discussed.

3.1 Syntactic Types and Terms

First we will revise the definition of what can be a Syntactic Type, adding Variables and Universal Quantification. The resulting language shall be known as PC: Polymorphic Categories. We assume a set of type variables CATVAR. X,Y and Z are particular variables in CATVAR.

Definition 13 (Syntactic Types of CP)
a. s,np and cn are TYPES
b. if $\theta \in CATCAR$, then *θ is a TYPE*
c. if *a and b are TYPES,* then *a/b and a \b are TYPES*
d. if *$\theta \in CATVAR$ and a is a TYPE, then $\forall\theta.a$ is a TYPE.*

The calculus, without its associated λ terms is given in below.

Definition 14 (Polymorphic Lambek Calculus)

(Ax) $\quad x \rightarrow x$

(/ E) $\quad \dfrac{U,y,V \Rightarrow w \quad T \Rightarrow x}{U,(y/x),T,V \Rightarrow w}/E \qquad$ (/ I) $\quad \dfrac{T,x: \Rightarrow y}{T \Rightarrow y/x}/I$

(\E) $\quad \dfrac{U,y,V \Rightarrow w \quad T \Rightarrow x}{U,T,y\backslash x,V \Rightarrow w}\backslash E \qquad$ (\I) $\quad \dfrac{T,x \Rightarrow y}{T \Rightarrow y\backslash x}\backslash I$

($\forall$ E) $\quad \dfrac{U,x[p/a],V \Rightarrow w}{U,\forall p.x, V \Rightarrow w}\forall E$

($\forall$ I) $\quad \dfrac{T \Rightarrow x}{T \Rightarrow \forall p.x}\forall I \qquad$ *[p is not free in U]*

The translation of a quantifier abstracts over List types, and as indicated, the domain of a particular List type [a1, ... an] will be a set of n-tuples: $D_{a1} \times \ldots D_{an}$. The syntactic categorisation of a quantifier will be a quantification over syntactic variables, but no particular instantiation of such a variable will have as its *direct* semantic counterpart, a List type. This would seem to create a difficulty if we are ever to establish an association of typed terms with this syntactic calculus.

In fact the difficulty can be overcome, because for all the instantiations of the syntactic variables that we envisage we expect that they shall correspond to *conjoinable types.* The interpretation of such a types is related in a canonical way to the interpretation of some list type. We shall set up the term association such that when a $\forall$ is eliminated and one would expect the translation of the polymorphic item to be applied to a List type, it shall not be applied to the semantic type directly corresponding to the syntactic type used in the elimination, but to a particular isomorphic List type: in place of $(a1 \rightarrow (\ldots (an \rightarrow t) \ldots)$ we will have the isomorphic list type $([a1, \ldots an] \rightarrow t)$

However there are other difficulties. In the semantic calculus, there is a proof combining the types $(a \rightarrow b)$ and a' when a and a' are isomorphic to each other. Thus a quantifier which has as an uncurried argument type may be combined with an item having the corresponding curried version of the type. This will remain the background situation when we are using the syntactic types, only there is nothing in the syntactic foreground witnessing the meaning transformation that occurs in traversing the isomorphism from curried to uncurried. This is the classic style of situation in which a notion of type-driven translation is usually argued for. We shall also make such an appeal.

This might be done by weakening the condition of type correspondence stipulated for the term association for the Lambek calculus. There it was required that a term be paired with a category only if it is of the type specified by the map ν, from categories to types. Herman Hendriks, Benthem, Groenendijk and Stokhof have advocated that generally in type-theoretical semantics ν should rather be a relation, with general principles generating type-alternatives of the single type one would have under a functional ν. In the present case, we will use 2 principles that we were already noted in Section 2.1.1 as 'Useful facts about list types'. We repeat them here as '+Curry' and '-Curry':

(25) -Curry $((\vec{X}) \rightarrow ([a,L] \rightarrow Y)) : \Phi \Rightarrow (\vec{X} \rightarrow (a \rightarrow (L \rightarrow Y))) : \lambda X^{\vec{X}} \lambda L^{[a,b]} \lambda Y^{\vec{Y}} [\Phi \vec{X} L_H L_T \vec{Y}]$

+Curry $((\vec{X}) \rightarrow (a \rightarrow (b \rightarrow Y))) : \Phi \Rightarrow ((\vec{X}) \rightarrow ([a,b] \rightarrow Y)) :$
$\lambda X^{\vec{X}} \lambda x^a \lambda y^b \lambda Y^{\vec{Y}} [\Phi \vec{X} [x,y] \vec{Y}]$

If one wished to have a functional map from category to type then it will a map into an equivalence class of types.

We can now give the calculus and its associated terms.

Definition 15 (Polymorphic Lambek Calculus)

(Ax) $\quad$ x: $\alpha \rightarrow$ x:α

(/ E) $\quad \dfrac{U, y{:}\alpha\beta, V \Rightarrow w \qquad T \Rightarrow x{:}\ \beta}{U, (y/x){:}\alpha, T, V \Rightarrow w}\ /E$

(\E) $\quad \dfrac{U, y{:}\alpha\beta, V \Rightarrow w \qquad T \Rightarrow x{:}\ \beta}{U, T, y \backslash x{:}\ \alpha, V \Rightarrow w}\ \backslash E$

($\forall$ E) $\quad \dfrac{U, x[p/a]{:}\alpha(L), V \Rightarrow w}{U, \forall p.x{:}\alpha, V \Rightarrow w}\ \forall E$ $\qquad$ *[where $\nu(a) \Rightarrow (L \rightarrow t)$]*

($\forall$ I) $\quad \dfrac{T \Rightarrow x{:}\ \alpha}{T \Rightarrow \forall p.x{:}\Delta p.\alpha}\ \forall I$ $\qquad$ *[p is not free in T]*

(/ I) $\quad \dfrac{T, x{:}\ \alpha \Rightarrow y{:}\ \beta}{T \Rightarrow y/x{:}\ \lambda\delta\beta)}\ /I$

(\I) $\quad \dfrac{T, x{:}\ \alpha \Rightarrow y{:}\ \beta}{T \Rightarrow y \backslash x{:}\ \lambda\delta\beta}\ \backslash I$

The side condition for $\forall E$ repeats what we said above: that application shall be to an uncurried version of the canonical type of the instantiating category. The category to type mapping ν now give the canonical semantic type. It is given by the old ν plus the following clauses

Definition 16 (Mapping from Syntactic Types to Semantic Types)

1. *ν for types containing variables is defined in terms of another map τ, from CATVAR to VAR.*

   (a) $\tau(X) = \Pi$, $\tau(Y) = \Theta$

   (b) if *p is a variable,* $\nu(p) = (\tau(p) \rightarrow t)$

(c) $\tau(\forall p.a) = \forall\tau(p)\nu(a)$

As an example the category to type mapping is giving in 26

(26)

$$\begin{aligned}\tau(\forall X(X/(X\backslash np))) &= \forall\tau(X)\nu(X/(X\backslash np))\\ &= \forall\Pi(\nu(X\backslash np)\rightarrow\nu(X))\\ &= \forall\Pi(\nu(np)\rightarrow\nu(X))\rightarrow\nu(X))\\ &= \forall\Pi(e\rightarrow(\tau(X)\rightarrow t))\rightarrow(\tau(X)\rightarrow t)\\ &= \forall\Pi((e\rightarrow(\Pi\rightarrow t))\rightarrow(\Pi\rightarrow t))\end{aligned}$$

3.2 Polymorphic Quantifiers

We can now define the syntactic type of a Quantifier.

Definition 17 (Polymorphic Quantifier)

Syntactic Type *$\forall X.X/(X\backslash np)$ and $\forall X.X\backslash(X/np)$*

Translation $\Delta\Pi\lambda y^{(e\rightarrow(\Pi\rightarrow t))}\lambda X^{\Pi}[G_{GQ}\lambda x[yxX^{\Pi}]$ *where G is a generalised quantifier*

3.3 String Semantics

In this section we will discuss the string semantic interpretation polymorphism. For the conventional syntactic types, interpretation is as follows:

Definition 18 (String Interpretations)

- *Assume a freely generated semigroup $< A, \cdot >$*
- $[\![x/y]\!] = \{\alpha : \forall\beta \in [\![y]\!], \alpha\cdot\beta \in [\![x]\!] \}$
- $[\![x\backslash y]\!] = \{\alpha : \forall\beta \in [\![y]\!], \beta\cdot\alpha \in [\![x]\!] \}$

For the interpretation of Polymorphic Types it will be necessary to assume that types are interpreted relative to an *assignment*; the assignment will fix the interpretation of *variable*, and will interpret them as subsets of A, just as other types are interpreted. (I shall say "string set" for subset of A sometimes)

The polymorphic types will also be interpreted as string sets, and not as any thing functional as was the case with the semantic types.

A first guess is that we should allow the quantification to be over arbitrary subsets of A. But this will not then allow quantifier word to have the polymorphic quantifier type. To see why, imagine the interpretation of polymorphic types is given as in:

(27) $[\![\forall X.\alpha]\!]^g = \{a : \forall g' \stackrel{X}{=} g, a \in [\![\alpha]\!]^{g'} \}$

Now consider the interpretation of the polymorphic type of quantifiers: ∀X.X/(X\np)

(28)

$$\begin{aligned}a \in [\![\forall X.X/(X\backslash np)]\!]^g &\leftrightarrow \forall g' \stackrel{X}{=} g(a \in [\![\forall X.X/(X\backslash np)]\!]^{g'})\\ &\leftrightarrow \forall g' \stackrel{X}{=} g(a\cdot[\![X\backslash np]\!]^{g'} \subset [\![X]\!]^{g'}) \quad (a\cdot S = \{as : s \in S\}\\ &\leftrightarrow \forall g' \stackrel{X}{=} g(a\cdot\{b : [\![np]\!]\cdot b \subset [\![X]\!]^{g'}\} \subset [\![X]\!]^{g'})\\ &\leftrightarrow \forall S(a\cdot\{b : [\![np]\!]\cdot b \subset S\} \subset S)\end{aligned}$$

Now we can check whether a typical quantifier satisfies this string semantic condition on what it is to be a quantifier. So putting a = "every man" and assuming that ⟦ np ⟧ = { john, mary }, lets consider the case when S = { john walks, mary walks } Then,

(29)
$$\{b : [\![np]\!] \cdot b \subset S\} = \{walks\}$$

and so with a = *every man*

(30)
$$\begin{aligned} a \cdot \{b : [\![np]\!] \cdot b \subset S\} &= everyman \cdot \{walks\} \\ &= \{everyman\ walks\} \end{aligned}$$

But then { every man walks } $\not\subset$ S and so it ought to be the case that *every man* $\notin$ ⟦ ∀X.X/(X\np)⟧^{g}

So if we want non-np's to have quantifier types, as we do, we then cannot allow X in ∀X.X/(X\np)) to range over arbitrary string sets. Instead we allow X to range only over string sets that are the interpretations of other types. But again here we can run into the problem of circularity: since ∀X α is a type ⟦ ∀X α ⟧ will depend on ⟦ ∀X α ⟧.

Again we will choose a solution to this that allows X to vary only over the sets that interpret *ground types*, i.e. those constructed from s,np, and cn using / and \. Thus we have the following for the string semantic interpretation of polymorphic types

Definition 19 (Polymorphic Types)

- $[\![\forall X\alpha]\!]^g = \{a : \forall g' \overset{X}{=} g, a \in [\![\alpha]\!]^{g'}\}$
  where $g' = [\![\beta]\!]$ *for some ground type* β

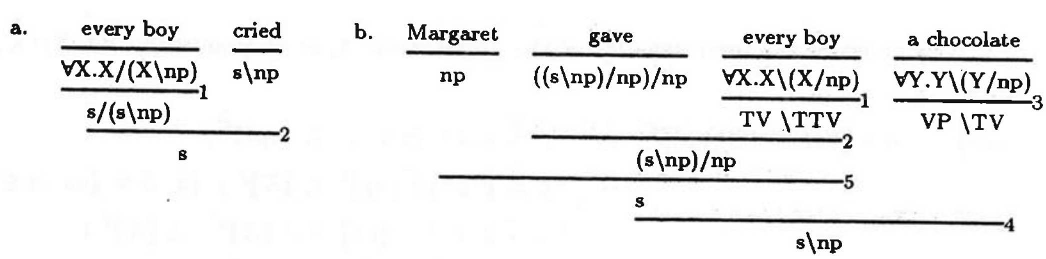
This allows us to legitimately include *every man* in the type ∀X.X/(X\np). Recall before the counterexample arose because we could chose as a value for X, S= {john walks, mary walks }. But this is not the interpretation of a type. The nearest set which is, is S = ⟦ s ⟧ = { john walks, mary walks, every man walks }. Now the required inclusion { *every man* } ⊂ S, holds.

4 Applications of Polymorphism

4.1 Criterion A

Let us call the combination of the polymorphic quantifiers with the Polymorphic Lambek calculus, the Polymorphic proposal. We look now at whether the combination of the Polymorphic proposal secures success by criterion A. The answer is that it does and furthermore to do so only the Polymorphic B-H calculus need be used. (that is the B-H calculus combined with the rules for polymorphism. As illustrations of this, there follow analyses of two of the sentences considered in Section 1, the IV and the TTV. Recall that the TTV was one of the sentences that was underivable in either B-H or L.(Notation VP abbreviates s\np, TV abbreviates (s\np)/np, and TTV abbreviates ((s\np)/np)/np.)

(31)



(32) translation of (31a)

1	**every boy** $\Longrightarrow \Delta\Pi\lambda P^{(e\rightarrow(\Pi\rightarrow t))}\lambda X^{\Pi}[EB_{GQ}\lambda x[PxX^{\Pi}]]$	*from the lexicon*
2	$\Longrightarrow \lambda P^{(e\rightarrow(\perp\rightarrow t))}\lambda X^{\perp}[EB_{GQ}\lambda x[PxX^{\perp}]]$	$since \nu(s) = t, t \Rightarrow (\perp\rightarrow t)$
3	**cried** $\Longrightarrow cry'^{(e\rightarrow t)}$	*from the lexicon*
4	**every boy cried** $\Longrightarrow \lambda P^{(e\rightarrow(\perp\rightarrow t))}\lambda X^{\perp}[EB_{GQ}\lambda x[PxX^{\perp}]](cry')$	$by\ step2, where\ \Phi_2 = u1u2$
5	$\Longrightarrow \lambda P^{(e\rightarrow(\perp\rightarrow t))}\lambda X^{\perp}[EB_{GQ}\lambda x[PxX^{\perp}]](\lambda x[cry'x])^{e\rightarrow\perp\rightarrow t}$	*−Curry on cry'*
6	$\Longrightarrow EM_{GQ}(cry')$	*β and ⊥reductions*

(33) translation of (31b)

1	**every boy** $\Longrightarrow \Delta\Pi\lambda P^{(e\rightarrow(\Pi\rightarrow t))}\lambda X^{\Pi}[EB_{GQ}\lambda x[PxX^{\Pi}]]$	*from the lexicon*
2	$\Longrightarrow \lambda P^{(e\rightarrow([e,e]\rightarrow t))}\lambda X^{[e,e]}[EB_{GQ}\lambda x[PxX^{[e,e]}]]$	$since\ \nu(TV) \Rightarrow ([e,e]\rightarrow t)$
3	**gave** $\Longrightarrow \lambda x\lambda y\lambda z[gave'(z,x,y)]$	*from the lexicon*
4	**gave every boy** $\Longrightarrow \Delta_2(\Delta_3)$	$by\ step2,\ where\ \Phi_1 = u2u1$
5	$\Longrightarrow \Delta_2(\lambda x\lambda Y\Delta_3 xY_H Y_T)$	$-Curry\ on \Delta_3$
6	$\Longrightarrow \lambda Y^{[e,e]}[EB(\lambda x\Delta_3 xY_H Y_T)]$	*Variable export*
7	**a chocolate** $\Longrightarrow \Delta\Pi\lambda Q^{(e\rightarrow(\Pi\rightarrow t))}\lambda X^{\Pi}[AC_{GQ}\lambda y[QyX^{\Pi}]$	*from the lexicon*
8	$\Longrightarrow \lambda Q^{(e\rightarrow([e]\rightarrow t))}\lambda X^{[e]}[AC_{GQ}\lambda y[QyX^{[e]}]$	$since\ \nu(s\backslash np) \Rightarrow ([e]\rightarrow t)$
9	**gave every boy a chocolate** $\Longrightarrow \Delta_8(\Delta_6)$	$by\ step4,\ where\ \Phi_2 = u2u1$
10	$\Longrightarrow \Delta_8(\lambda y^e\lambda z^e[EB(\lambda x\Delta_3 xyz])$	*+Curry*
11	$\Longrightarrow \lambda z^e AG(\lambda y EB(\lambda x\Delta_3 yz^e)$	*exportation*
12	**margaret** $\Longrightarrow m'$	*from the lexicon*
13	**margaret gave every boy a chocolate** $\Longrightarrow \Delta_{11}(m')$	$by\ step3, where\ \Phi_3 = u1u2$
14	$\Longrightarrow AG(\lambda y EB(\lambda x\Delta_3 ym')$	*β*
15	$\Longrightarrow AC_{GQ}\lambda y[EB_{GQ}\lambda x[gave'(m',x,y)]]$	*unabbreviateD3*

This illustrates the proposal in just 2 cases, namely the English IV and TTV. The other cases will follow according to a similar pattern.

What this shows is that the move to polymorphism brings a solution to the basic combinatorial equations required by quantification. Criterion A is therefore satisfied. In Section 4.2 the possibility that Criterion B is also satisfied is investigated.

4.2 Criterion B

4.2.1 Bar-Hillel

The Polymorphic Bar-Hillel calculus is able to analyse each of the sentence types only one way. Basically the calculus will assign a reading in which the scope relations reverse exactly the order of the np arguments of the expression being quantified. So for example, an English transitive of category (s\np)/np subcategorises for the np's in the order $DO > SU$,(Direct Object, Subject). In combinations with quantifiers the scope relations will then be precisely the reverse, $SU > DO$. With this as a rule one arrives at the following table of scopings assigned by the B-H calculus, where the quantified np's are numbered according to the width of their scope.

IV	*(every boy)*$_1$ *cried*
TV	*(a nun)*$_1$ *liked (every boy)*$_2$
TTV	*(most nuns)*$_1$ *gave (every boy)*$_3$ *(a chocolate)*$_2$
DutchTV	*...dat (elke schurk)*$_1$ *(een detective)*$_2$ *vreest*

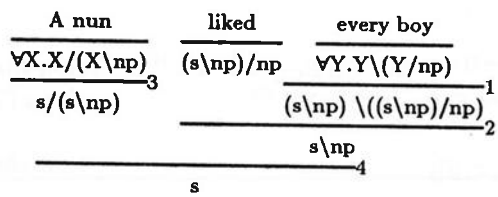
This scoping of the sentence will be subsequently referred to as the ***pure applicative scoping.***

4.2.2 Lambek

The Polymorphic Lambek calculus naturally is able to find all the readings found in the B-H calculus. The question then is whether the Lambek calculus provides derivations of any of the ***non-applicative*** readings.

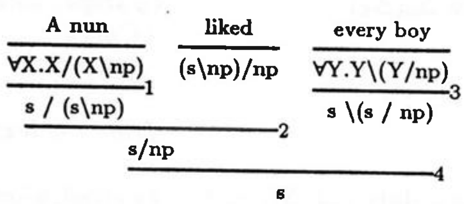
We consider first the TV, ***a nun liked every boy.*** Of the two scope possibilities for this, the scoping ***a nun > every boy*** is the pure applicative one, and can therefore be arrived by an analysis entirely within the Bar-Hillel system. For the sake of comparison with the derivation of the other reading, this applicative analysis is illustrated below:

(34)



To obtain the reading in which this scope relationship is reversed, intuitively what is needed is for the semantic material provided by *a nun* and *liked* to be combined as some single item which is then given as an argument to *every boy.* This is exactly what the calculus allows licensing it with the sequent: s/(s\np), (s\np)/np→ s/np, which the reader may confirm may be proved in such a way that the term $\lambda u3[u1(u2(u3)$ is associated with the goal category. Below we illustrate the analysis and set down its translation.

(35)



(36) Translation of (35)

1	**a nun** $\Longrightarrow \Delta\Pi\lambda Q^{(e\rightarrow(\Pi\rightarrow t))}\lambda X^{\Pi}[AN_{GQ}\lambda y[QyX^{\Pi}]$	*from the lexicon*
2	$\Longrightarrow \lambda Q^{(e\rightarrow(\perp\rightarrow t))}\lambda X^{\perp}[AN_{GQ}\lambda y[QyX^{\perp}]$	*since* $\nu(s) \Rightarrow \perp\rightarrow t$
3	**liked** $\Longrightarrow \lambda x\lambda y[like'(y,x)]$	*from the lexicon*
4	**a nun liked** $\Longrightarrow \lambda u3[\Delta_2(\Delta_3 u3)]$	*by step2, using* $\Phi 2$
5	$\Longrightarrow \lambda u3[\Delta_2(\lambda x like'(u3)(x)))]$	β
6	$\Longrightarrow \lambda u3[\lambda X^{\perp}[AN(\lambda y like'(u3)(y)X^{\perp})]]$	$\beta on \Delta_2$
7	$\Longrightarrow \lambda u3[AN(\lambda y like'(u3)(y))]]$	$\perp$
8	**every boy** $\Longrightarrow \Delta\Pi\lambda P^{(e\rightarrow(\Pi\rightarrow t))}\lambda X^{\Pi}[EB_{GQ}\lambda x[PxX^{\Pi}]$	*from the lexicon*
9	$\Longrightarrow \lambda P^{(e\rightarrow(\perp\rightarrow t))}\lambda X^{\perp}[EB_{GQ}\lambda x[PxX^{\perp}]$	*since* $\nu(s) \Rightarrow \perp\rightarrow t$
9	**a nun liked every boy** $\Longrightarrow \Delta_9(\Delta_7)$	*by step4*
10	$\Longrightarrow \lambda P^{(e\rightarrow(\perp\rightarrow t))}\lambda X^{\perp}[EB_{GQ}\lambda x[PxX^{\perp}](\Delta_7)$	*unabbreviating*
11	$\Longrightarrow \lambda X^{\perp}[EB_{GQ}\lambda x[\Delta_7 xX^{\perp}]]$	β
12	$\Longrightarrow EB_{GQ}(\lambda x[\lambda u3[AN_{GQ}(\lambda y like(u3)(y))]x])$	*unabbreviate*
13	$\Longrightarrow EB_{GQ}(\lambda x[AN_{GQ}(\lambda y like(x)(y))])$	β

We consider now the TTV, *Margaret gave every boy a chocolate.* There are 2 possible scope relations of the quantified np's, and the non-applicative one is in this case has *every boy* < *a chocolate.* The first thing to note is that the intuition that we used in the previous sentence cannot be followed here. For in the case at hand, combining the semantic material provided by *a chocolate* and *gave* into some single item to which *every boy* might then apply is impossible. It is impossible because the items needing combination are not adjacent.

This observation may seem a fatal one, and lead one to consider calculi in which such non-adjacent combinations are possible. But in fact we have yet not exhausted the range of readings that may be assigned by the Lambek calculus.

One avenue to be explored takes its lead from Herman Hendriks type flexibility proposal. In such a case as this, Hendriks would use the semantic type shift he calls Argument Raising, given in (37a). We can mimic this in the syntactic type system with the sequent given in (37b).

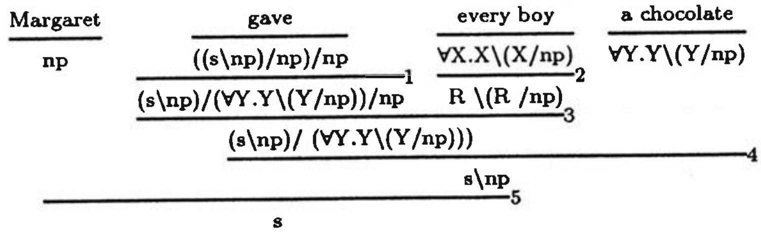
(37)

a (e →e →e →t) ⇒ (e →(et,t) →e →t)

b ((s\np)/np)/np ⇒ ((s\np)/ (∀Y.Y\(Y/np)))/np

If incorporates this into an analysis in the fashion shown in 43 and calculates through the associated terms, the desired reading is obtained. R abbreviates (s\np)/ (∀Y.Y\(Y/np)).

(38)



However, we have foregone this analysis with our restrictions on the instantiations of type variables. It will be recalled that we made the stipulation that a variable could only be instantiated to a ground type. But to carry out the above derivation the variable Y must, in the ∀Y elimination, be instantiated to (s\np)/ (∀Y.Y\(Y/np)), which is not a ground type.

Nonetheless, even within the restrictions we have laid down, a derivation can be found which assigns the same reading.

We are led to this be considering the meaning transition accompanying the syntactic type transition of polymorphic argument raising indicated in 37b. Where f is the translation of some string of category ((s\np)/np)/np, the meaning shift is:

(39) $f^{(eeet)} \Rightarrow \lambda x^{e} \lambda Q^{(e\Pi t \rightarrow \Pi t)}[Q([e])(fx)]^{([e] \rightarrow t)}$

Let us call this derived function Φ. About Φ we should note that though this a function taking as arguments polymorphic functions, it is sensitive to only to the $[e]$ part of the polymorphic function. Thus if one has Q1 and Q2 as 2 functions of quantifier type (i.e. $\forall\Pi(e\Pi t \rightarrow \Pi t)$) such that Q1([e]) = Q2([e]), then,

$$\Phi a Q1 = \Phi a Q2$$

As Φ characterises a set of triples, $< a, Q, b >$ what this shows is that these triples fall into equivalence classes according to the value of Q at [e]. Since the value at [e] will be some function of type $(e[e]t \rightarrow [e]t)$, Φ will be isomorphic to some function characterising a set of triples $< a, R, b >$, with R of type $(e[e]t \rightarrow [e]t)$. This suggests that we should be able derive the same reading by effecting a transition on $f^{(eeet)}$ into something of type $(e \rightarrow (e[e]t \rightarrow [e]t) \rightarrow [e] \rightarrow t)$. As it happens this is the type transition characteristic of another the Type shifts that Hendriks has proposed in the context of semantic type flexibility : Value Raising.

Value raising is a generalised form of Type Raising, one of the categorial laws we mentioned in Section 1 as being derivable in the Lambek calculus. The categorial form could be defined as follows:

(40) if C has Value X, then a Value-Raise of C has Y/(Y \X) in place of X.

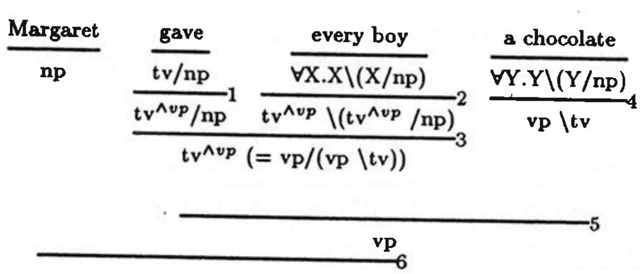
The instance of this we need here is:

(41) ((s\np)/np)/np:
$u \Rightarrow ((s\backslash np)/((s\backslash np)\backslash(s\backslash np)/np))/np : \lambda v^{e} \lambda w^{(eet,et)}[w(uv)]^{et}$

For a more compact notation we will use

(42) $X^{\wedge Y}$ = Y/(Y \X), and $X^{\vee Y}$ = Y \(Y / X)

(43)



The translation of this derivation follows:

(44) Translation of (43)

1	**gave** $\Longrightarrow gave'$	*From the lexicon*
2	$\Longrightarrow \lambda v^{e} \lambda w^{(eet \rightarrow et)}[w(\Delta_1 v)]$	*by step 1*
3	**every boy** $\Longrightarrow \Delta\Pi\lambda P^{(e \rightarrow (\Pi \rightarrow t))}\lambda X^{\Pi}[EB_{GQ}\lambda x[PxX^{\Pi}]$	*from the lexicon*
4	$\Longrightarrow \Delta_3([(eet \rightarrow et), e])$	*since* $\nu(tv^{\wedge vp}) \Rightarrow ([eet \rightarrow et, e] \rightarrow t)$
5	**gave every boy** $\Longrightarrow \Delta_4(D2)$	*by step 3*
6	$\Longrightarrow \Delta_4(\lambda v^{e} \lambda Z^{[(eet \rightarrow et), e]}[\Delta_2 v Z_H Z_T]$	$-Curryon\Delta_2$
7	$\Longrightarrow \lambda Z^{[(eet \rightarrow et), e]}[EB(\lambda v \Delta_2 v Z_H Z_T)]$	*exportation*
8	$\Longrightarrow \lambda z_1^{(eet \rightarrow et)} \lambda z_2^{e}[EB(\lambda v \Delta_2 v z_1 z_2)]$	*+Curry*
9	**a chocolate** $\Longrightarrow \Delta\Pi\lambda P^{(e \rightarrow (\Pi \rightarrow t))}\lambda X^{\Pi}[AC_{GQ}\lambda y[PyX^{\Pi}]$	*from the lexicon*
10	$\Longrightarrow \lambda P^{(e \rightarrow ([e] \rightarrow t))}\lambda X^{[e]}[AC_{GQ}\lambda y[PyX^{[e]}]$	*since* $\nu(vp) \Rightarrow ([e] \rightarrow t)$
11	$\Longrightarrow \lambda P^{eet} \lambda z_3^{e}[AC(\lambda y P y z_3^{e})]$	*+Curry*
12	**gave every boy a chocolate** $\Longrightarrow \Delta_8(\Delta_{11})$	*by step 5*
13	$\Longrightarrow \lambda z_2^{e}[EB(\lambda v \Delta_2 v \Delta_{11} z_2)]$	β
14	**margaret** $\Longrightarrow m'$	*from the lexicon*
15	**margaret gave every boy a chocolate** $\Longrightarrow EB(\lambda v \Delta_2 v \Delta_{11} m')$	*Step6*
16	$\Longrightarrow EB(\lambda v[\Delta_{11}(\Delta_1 v)m'])$	$Unabbreviating\Delta_2$
17	$\Longrightarrow EB(\lambda v[AC(\lambda y(\Delta_1 v)ym'])$	$Unabbreviating\Delta_{11}$
19	$\Longrightarrow EB(\lambda v[AC(\lambda y gave' v y m')])$	*unabbreviating* Δ_1

The analysis of the Dutch TV sentence, *(... dat) elke schurk een detective vreest* is structurally very similar to the English TTV just considered.

5 Further Applications

It may seem that we have laboured long and hard for only the moderate reward of being able to assign all readings to 4 sentences. Of course we can do more than this with the system developed. We haven't the space to illustrate how the proposal assigns interpretations to a wider class of sentences - particularly sentence embedding constructions. However we may persuade anyone already familiar with Hendriks flexiblity proposal by a conversion argument.

Of the 3 type shifts Hendriks uses 2 correspond to valid categorial laws: Argument Lowering and Value Raising. Their role in a Hendriks derivation may thus be taken over by equivalent syntactic type shifts. The conversion for the other shift, Argument Raising splits into 2 cases. AR can occur in a a succesful Hendriks derivation if this AR anticipates combination with a quantifier, or with some item that has beed Value Raised. We have just looked at how the first case is represented in the categorial system,basically replacing the instance of AR with an instance of VR. In the second case, when AR anticipates latter combination with a VR-ed item, the AR may again be replaced by an instance of VR. We do the following replacement:

$$(a \rightarrow ((b \rightarrow t) \rightarrow t) \rightarrow c \rightarrow t) \textit{ is changed to } (a \rightarrow (bct \rightarrow bt) \rightarrow c \rightarrow t)$$

The translations to be compared are:

$$\lambda T^{(bt,t)} \lambda y T(\lambda z \Phi x z y) \textit{ compared to } \lambda x \lambda U^{(bct \rightarrow ct)} \lambda y U(\Phi x) y$$

These 2 are are isomorphic if the only possible values for T and for U are type raised versions of some element b, as the reader may confirm. This very strongly suggests that we should be able represent every derivation in Hendriks system with a valid categorial derivation within the Polymorphic Lambek calculus.

Moving away now from quantifiers, one can also see applications of the system to other logical constants, such as *not* and *and*. For these it has been maintained for some time that they be seen as polymorphic - having some part of their interpretation occurring in every possible type of domain. Keenan and Faltz (85) is a notable case of this. We can express these polymorphic functions very easily using the language of abstraction over types:

(45) **And** $\Delta\Pi\lambda P^{(\Pi\rightarrow t)}\lambda Q^{(\Pi\rightarrow t)}\lambda X^{\Pi}[PX \wedge QX]$

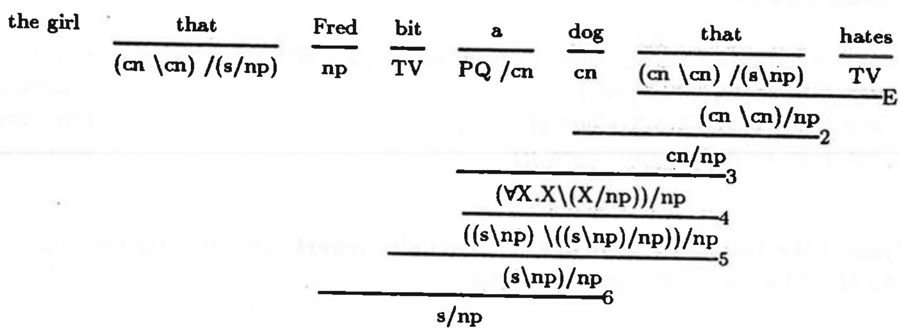
Not $\Delta\Pi\lambda P^{(\Pi\rightarrow t)}\lambda X^{\Pi}[\neg(PX)]$

Furthermore just as the Lambek calculus proved successful in accounting for quantifièr scope ambiguities, one can look also to it to account for conjunction scope ambiguities. Moortgat (88) has also advocated this approach to conjunction ambiguity.

Finally, with scope ambiguity anchored in the syntax it is possible to explain the so-called Island Constraint on scoping, (see Rodman 76). The constraint claims that np's that are 'islanded' for extraction purposes also have necessarily narrow scope. Just such a co-occurrence of constraints on extraction and scoping is fully to be expected on the current model. An illustration of this follows for the case of the complex np island constraint.

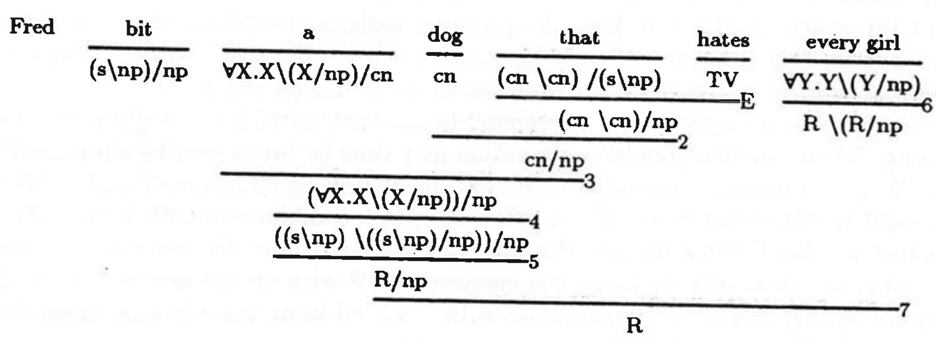
In 46 we have illustrated a derivation that violates the complex np island constraint. PQ abbreviates a polymorphic quantifier.

(46)



This should be compared with a derivation that gives an island-occupying np wide scope, shown in 47. (R abbreviates (s\np) \((s\np)/np).

(47)



Common to both of these is the step marked E, a step licensed by the valid sequent: (cn \cn)/(s \np), (s\np)/np ⇒ (cn \cn)/np. This sequent is derivable by use of /I. If we could build into the grammar a sensitivity to the fact that such /I derived sequents cannot be used on strings containing the Relativiser, we would prevent at the same time these extractions and wide-scopings. Building in such sensitivity shall not be attempted here. All we shall add is that such a modification could be seen as quite principled from the string semantics point of view simply by assuming that the relativiser is an exception to the associativity of the string combining operation.

6 Conclusions

Researchers who have looked at the possibility of building an account of quantifier phenomena within a categorial grammar have all hitherto come to the conclusion that it cannot be done, and have argued then for the adjunction of mechanisms apt to handle the problems of quantification. I have in mind particularly here Hendriks' use of a semantic calculus of type flexibility. What we have emphasized here is that the problems for categorial grammar and quantification start earlier then the problem of scope ambiguity: Categorial grammar faces a problem at the level of straightforward interpretability. Then, without an eye to scope ambiguity, we have built into categorial grammar the ability to at least meet the interpretability criterion. With this, an account of scope ambiguity simply comes for free, upon adopting the Lambek calculus. Therefore the principal attraction of the system is that no part is included solely for the purposes of scope ambiguity; scope ambiguity is an emergent property.

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EXPLORING EXCEPTION PHRASES[1]

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1. Overture.

"The exception proves the rule", according to a familiar saying. Certainly there are few rules without exceptions, special cases or implicit restrictions to certain domains. Natural languages have developed a great many ways of making allowance for such exceptions, some of which have crystallized in the form of what I will refer to here as exception-phrase constructions. To give an indication of just what I have in mind here, I have listed a number of examples of such phrases, drawn from a variety of languages, under (1).

(1) I. English.

allesbehalve

a. The man was anything but polite.
b. Except for me, nobody came.
c. The police caught all save one.
d. Give her anything short of the car.

II. German.

e. Außer ihm war keiner da.
   except him was nobody there
   "Except for him, nobody was there"
f. Er hat nichts als Unsinn geredet[2]
   he has nothing than nonsense talked
   "He talked nothing but nonsense"

III. French.

g. Je sorte tout le temps sauf s'il pleut
   "I go out all the time except if it rains
h. Que lui ai-je dit que d'indifférent?[3]
   what him have-I said but of indifferent
   "What did I tell him but indifferent things?"

IV. Dutch.

i. Behalve Frits was iedereen tevreden.
   "Except for Frits everybody was content"
j. Op Jan na zijn allen veilig teruggekeerd.
   "Except for Jan, all have returned safely"
k. Ik heb, afgezien van Zeeland, alles gezien.
   "I have, except for Zeeland, seen everything."

V. Latin.

l. De grammaticis nihil nisi bene.
   of grammarians nothing but good

I will make a terminological distinction here between exception *phrases* on the one hand and exception *markers* on the other. An exception phrase is any phrase such as *except for Mussolini* or *behalve Frits* which serves to indicate an exception to a generalization. An exception marker is a lexical item which heads an exception phrase, such as *except (for)*, *but* and *behalve*. Exception markers have various historical origins and are often highly polysemous. For instance, German *außer* does not just mean "except", it can also mean "outside of" or "without", meanings which also show up with English *but*, a descendant of Old English *butan* "outside" (cf. also Modern Dutch *buiten*), as in (2).

(2) But for your help, I would not have made it.
  = Without your help/were it not for your help, ...

Precisely how these shades of meaning hang together and how they have developed historically, is not discussed in this paper (I refer the reader to Moignet (1973) for a discussion of French exception phrases and to Mourin (1980) and König and Kortmann (1987) for a comparative perspective).

The goal of this paper is to describe in some detail the distributional properties of exception phrases and to present evidence that these properties are partly to be described in semantic terms; reference to surface structure or logical form does not suffice to handle the full range of data. I present a new semantic account of what I call free exception phrases and suggest that this account can be extended fruitfully to constructions other than those which serve to express exceptions, such as English *besides*-phrases. Some attention is also paid to the role of focus in the interpretation of exception phrases.

2. Previous analyses.

Before presenting my own thoughts on the matter, I briefly review some of the small literature on exception phrases. I focus on work directly related to the concerns of this paper, in particular proposals by Keenan and Stavi and by Reinhart. A third proposal by myself will be discussed later on (see section 3.2.) and rejected on the basis of some new evidence.

2.1. Keenan and Stavi (1986).

The analysis of exception markers given in Keenan and Stavi (1986) is no more than a thumbnail sketch, but it is a good point of departure for a more thorough investigation. Keenan and Stavi develop a theory of determiner denotations within an algebraic version of generalized quantifier theory and show that strings such as *every .. but Jim* can be viewed as complex discontinuous determiners semantically. In (3), definitions for two such determiners are given. Instead of Keenan and Stavi's algebraic notation, I am using the simpler but equivalent set-theoretic notation of Zwarts (1983) and van Benthem (1986).

(3) *every .. but Jim* (A, B) iff $\{j\} = A - B$
  no .. but Jim (A, B) iff $\{j\} = A \cap B$

  where E is the domain of quantification,
  A, B subsets of E and j an element of E.

According to these definitions, every A but Jim is a B just in case Jim is the only A who is not a B. Furthermore, no A but Jim is a B just in case Jim is the only A who is also a B. If we assume these definitions, then the sentences (4b-d) are entailments of (4a).

(4) a. No student but Jim is a stamp collector.
  b. No student other than Jim is a stamp collector.
  c. Jim is a student.
  d. Jim is a stamp collector.

We can check this by inspecting the corresponding metalinguistic clauses in (5).

(5) a. $\{j\} = A \cap B$
b. $A - \{j\} \cap B = 0$
c. $j \in A$
d. $j \in B$

In an earlier paper (Hoeksema 1987), I raised the question whether c and d are entailments, rather than Gricean implicatures. If we take (5b), rather than (5a), as the proper representation of the truth-conditional meaning of (9a), then (5c) and (5d) no longer follow. Instead, they re-emerge as conversational implicatures, if we assume that exception phrases are not used vacuously. If either (4c) or (4d) were false, then the exception phrase could be omitted without affecting the truth of the statement. Contexts such as (5) suggest to me that the clauses in (4c) and (4d) may indeed by mere implicatures, which can be lifted.

(5) Well, except for Dr. Samuels everybody has an alibi, inspector. Let's go see Dr. Samuels to find out if he's got one too.

Assuming that this line of reasoning is correct, we arrive at definitions as in (6). If it is not correct, not much harm is done, as the definitions which I employ here and later on can easily be adjusted.

(6) *every .. but Jim* (A,B) iff $A-\{j\} = A \cap B$
no .. but Jim (A,B) iff $A-\{j\} \cap B = 0$

These definitions, by the way, are compatible with the conservatity condition for determiner meanings, defined in (7).

(7) *Conservativity.*

Q is conservative iff Q(A,B) implies Q(A, $A \cap B$)

(8) Conservativity of *every .. but Jim.*

every .. but Jim (A,B) <=> $A-\{j\} = A \cap B$
<=> $A-\{j\} = A \cap (A \cap B)$
<=> every .. but Jim (A, $A \cap B$)

(9) Conservativity of *no .. but Jim*

no .. but Jim (A,B) <=> $A-\{j\} \cap B = 0$
<=> $A-\{j\} \cap (A \cap B) = 0$
<=> no .. but Jim (A, $A \cap B$)

I am assuming here, for the sake of simplicity, that the NP following the exception marker is a referring term and can be represented by a variable over individuals, as in (15). As a matter of fact, the situation is not so simple, as we will see later on.

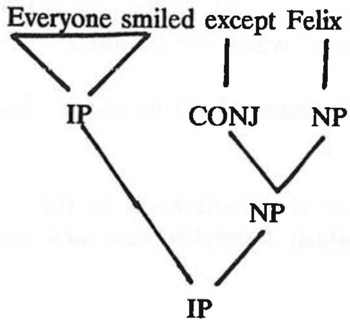
There is more to be said about the complex determiner analysis, but before I do so, I want to point out a major limitation. There is no account of 'free' exception phrases such as the English and German ones in (10).

(10) a. Except for John, I did not see anybody.
b. Außer dem Franz kenne ich keinen Linguist.
except for Franz know I no linguist
"Except for Franz, I don't know any linguist"

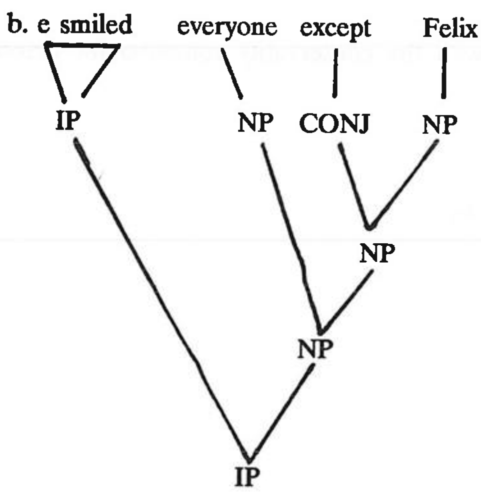
2.2. Reinhart (1989).

To bring sentences such as the ones in (10) within the scope of the Keenan and Stavi theory, Reinhart (1989) proposes a rule of restructuring at the level of Logical Form, which brings together the exception phrase and the quantified term which it operates on. The trees in (11) are taken from Reinharts paper to illustrate the transformation in question. It is perhaps helpful to point out that IP (the maximal projection of INFL, the verbal inflection node) is what used to be called S. Reinharts syntactic framework is that of Chomsky's monograph *Barriers* (Chomsky 1986), but the details of that theory, in particular the notion of a bounding node, will not concern us here.

(11) a.



b.



Reinhart treats the word *except* as a conjunction sign, not as a preposition or determiner modifier. In this respect her analysis differs from the one proposed in Keenan and Stavi, and indeed it is necessary to change the Keenan and Stavi semantics somewhat to accomodate the different syntactic structure. It is more similar to the proposal made in Harris (1982). Harris also views exception markers as conjunctions, but unlike Reinhart he takes them to be essentially sentential conjunctions. So (11a), for instance, is derived from sentence (12) by means of a reduction transformation (or "zeroing" as Harris calls it).

(12) Everyone smiled, except Felix did not smile.

However, this works best for languages such as English where exception markers can also be used as conjunctions. For Dutch *behalve*, on the other hand, there is no such use (cf. fn. 5), and an analysis along the lines sketched by Harris becomes problematic.

Reinhart's account crucially assumes that exception phrases are licenced by quantified noun phrases. To support her movement account, she notes the existence of island phenomena with exception phrases, such as Complex NP Condition effects, e.g.:

(13) a. *The people who loved every composer arrived except Mozart.
b. *?He recognized the books on every shelf yesterday except the second.

The universal quantifiers in these sentences are locked up, so to speak, in syntactic islands out of which they cannot move. Since the exception phrases occur outside of the islands, they cannot be combined and the result is unacceptable. It might be noted here that an earlier proposal involving a transformational relation between a quantified NP and an exception phrase was given in Landman and Moerdijk (1980). That paper also discussed a number of island phenomena in connected with the movement analysis. Landman and Moerdijk's proposal differs from the Reinhart proposal however in that the movement rule does not promote the quantifier to the level of the exception phrase, but rather lowers the exception phrase onto the quantifier site. I will not compare the two theories in detail here.

As it stands, however, the relation between movement and licencing of exception phrases is somewhat problematic. Notice for example the possibility of linking exception phrases with quantifiers in Wh-islands in (14):

(14) a. Except for the FBI, I don't know who to call.
b. Except for Joan, I wonder if anyone was interested.

LF-movement along the lines sketched by Reinhart would give a representation like (14') for (14a).

(14') For which x, x $\neq$ the FBI, I don't know (whether) to call x

This representation clearly does not represent the correct interpretation of (14a). Note also that the position of *anyone* in (14b) is not one from which extraction is permitted. It appears that some violations of the wh-island constraint are grammatical with exception phrases, but not all. The examples in (15), for example, are not acceptable.

(15) a.*Except for the FBI, I wonder why you invited every service.
b.*Except for Joan, I wonder whether I saw every student.
c.*Except for Joan, I wonder whether everyone was invited.

I conclude that some of the island evidence for a movement analysis is problematic, although more study is needed. Much more directly problematic for the movement account are cases where there is no quantifier to move around. Such cases abound in the Brown corpus of contemporary American prose. Consider in this connection the sentences in (16), all taken from that corpus.

(16) a. Du Pont would be denied the right to acquire any additional General Motors stock except through General Motors' distributions of stock or subscription rights to its stockholders.
b. The vast, dungeon kitchens seem hardly worth using except on occasions when one is faced with a thousand unexpected guests for lunch.
c. The Indians who came aboard the ship to collect the mail also interested her greatly, even if she was suitably shocked, according to the customs of the society in which she had been reared, to find them "naked, except a piece of cotton cloth wrapped around their middle".
d. As for states' rights, they have never counted in the thinking of my liberal friends except as irritations of a minor and immoral nature which exist now only as anachronisms.

Take for example sentence (16c). Here, the exception phrase clearly attaches to the predicate *naked*. Yet that adjective is not a quantifier of the usual sort and not an element for which it is plausible to assume movement at Logical Form. It does not engage in scope ambiguities, for example. It is also not possible to view *naked except a cotton cloth* as a conjunction of any kind. Conjunctions have the special property that their parts are alike. But *naked* is used as a predicate, while *a cotton cloth* is used as an argument term. In (16d), likewise, there is no candidate for the prepositional phrase introduced by *except as* to conjoin with.

A related problem is createdby sentences such as (17).

(17) Except for you, I don't know a living soul in New York.

Here the only candidate for adjoining to the exception phrase is the indefinite polarity item *a living soul*. Normally, however, indefinite noun phrases do not licence the use of exception phrases. Only in negative contexts is this possible, cf. the ungrammaticality of (18).

(18) *Except for you, I know somebody in New York.

These examples provide us with a paradox for Reinhart's theory. The exception phrase in (17) requires raising of the object NP, but the polarity status of that NP forbids moving it out of the scope of negation, as (19) illustrates.

(19) a. That guy, I don't know.
b.*A living soul, I don't know.

And even if raising were possible, it would adjoin an indefinite NP to the exception phrase, which should not yield an admissible structure. Obviously, what one would need here is a richer representation, as in predicate logic, where an existential quantifier under the scope of negation corresponds to a wide scope universal quantifier. The theory of LF movement does not permit such substitutions.

A final comment that I want to make about Reinhart's proposal is that it is too general in that it does not distinguish between various kinds of exception phrases. It is not the case that exception phrases can appear at any adjunction site c-commanding a universal quantifier. There is a major asymmetry between *but*-phrases and *except*-phrases. *But*-phrases may be adjoined to an NP or occur in extraposition, while *except*-phrases may also appear in topic or sentence-initial position. Compare (21) with (20).

(20) a. Everybody but Jamie was invited.
b. Everybody was invited but Jamie.
c.*But Jamie, everybody was invited.

(21) a. Everybody except for Jamie was invited.
b. Everybody was invited except for Jamie.
c. Except for Jamie, everybody was invited.

Sentence-initial uses of *but*-phrases are restricted to special cases such as (2). The Brown corpus contains only two cases of fronted *but*-phrases, listed in (22). In both cases, *but for* is used to express the reason why some hypothetical state of affairs is not an actual state of affairs.

(22) a. Perhaps the moralities of world law are not advanced by stealing American diplomatic papers and planes, but the Kennedy administration can always file a demurrer to the effect that, but for its own incompetence in protecting American interests, these things would not happen.
b. But for my presence, they would have been at each others throat.

I consider this use to be separate from the use of *but* as a marker of exceptions.[4] I conclude that *but*-phrases are primarily used to modify quantified noun phrases and that like so many postmodifiers of noun phrases they can occur in sentence-final position. *Except*-phrases, on the other hand, also have an important use as sentential modifiers, and thus may occur in all positions typical of sentential adverbs, viz. sentence-initial, sentence-medial and sentence-final position. Reinhart's account, which treats all exception markers as phrasal conjunctions, and

Harris' account, which treats all of them as sentential conjunctions, are both too global to deal with the differences between *except* and *but*. Nevertheless, it turns out that there is some interesting evidence, especially in Dutch and German, that some aspects of Reinhart's conjunction theory are on the right track. I review this evidence in section 4 below. This concludes my discussion of earlier studies of exception phrases.

3. Types of Exception Phrases.

I assume two distinct but related types of exception phrases, which I have termed free exception phrases and connected exception phrases in Hoeksema (1987). Connected phrases are linked to a phrase, usually a noun phrase, while free phrases are sentential operators and occur whereever sentential operators may occur. The positional possibilities of connected exception phrases are usually more limited than those of free exception phrases. I ignore here a third important use of at least the English exception markers *except* and *but*, namely their use as adversative conjunctions.

(23) a. I would like to come but I can't.
b. I would like to come except I can't.

It is remarkable that both markers have developed very similar uses as sentential connectives. Dutch *behalve* entirely lacks this function[5], whereas German *außer* has a different meaning when used as a connective, one that roughly corresponds to English *unless* (see Abraham 1979 for further discussion).

(24) Du mußt deine Suppe nicht essen, außer du magst sie.
you must your soup not eat, except you like it
"you don't have to eat your soup, unless you like it"

3.1. Connected Exception Phrases.

I will first consider connected exception phrases, describe some of their syntactic properties and then give a non-extensional semantics for them. This semantics has the advantage that it can be extended in a straightforward way to sentential exception phrases.

As Keenan and Stavi noted, exception phrases can be used to form complex determiners. However, this use is by no means their most frequent or most central one and can be viewed as parasitic on the primary use as noun phrase modifiers. Some relevant examples are given in (25).

(25) a. All but two of the students were ready.
b. All but at most 20% of the fish was spoiled.
c. None but the very best of us can compete.

The examples in (25) all involve partitive constructions. Note that not all such constructions are grammatical. In particular, (26) is ill-formed.

(26) *None but Jim of us can compete.

This may seem remarkable in light of the fact that (27 a,b) are fine.

(27) a. None but Jim could compete.
b. None of us but Jim could compete.

The contrast between (26) and (25c) suggests that *but* behaves like a conjunction in these partitive constructions. For a sequence *A but B of NP* to be acceptable, both A and B must be able to head a partitive. Since both *none* and *the best* can head a partitive (cf. *none of us* and *the best of us*), the combination *none but the best* may also head a partitive. Proper names, on the other hand, do not head partitives (cf. **Jim of us*) and so they resist conjunction with *none* in partitive structures. In (27a), the syntactic context imposes fewer constraints and because both *none* and *Jim* are fine as subjects of this sentence, their conjunction is also fine. Example (27b) is acceptable for the same reason.

Some cases of conjunction-like combinations from the Brown corpus are given in (28).

(28) a. sponsors rarely use any but white models in commercials.
b. any but a limited use of economic pressure
c. By 1960 there were such schools in all but 4 states.

I take it that the structure of these examples is as indicated in (28').

(28') a. [[any but white] models]
b. [[any but a limited] use]

rather than

(28") a. [any [but white models]]
b. [any [but a limited use]]

Both structures are possible, but they correspond to different interpretations. The following two contexts may help to bring out these differences:

(29) A: Would you like to meet some women tonight?
B: Nah, I don't want to meet any but white models.

(30) A: We should advertize this product on TV.
B: If you do, don't use any but white models.

In (29) *any* is used as a pronoun and stands for *any women*. The implicit predicate 'women' is provided by the preceding question. In (30), *any* is used as a determiner and the predicate is not taken from the discourse context but is the syntactic argument of *any*.

The interpretation of *but* in these cases is, as I mentioned earlier, parasitic on the interpretation of *but* as an operator on noun phrases. More precisely, we can use a point-wise definition as in (31):

(31) (Det_1 *but* Det_2)(Noun) = Det_1(Noun) *but* Det_2(Noun)

Of course, this begs the question of how *but* is defined as a noun phrase operator, a point to which I return shortly.

There is no reason to assume that NP-final exception phrases form a complex but discontinuous constituent with the determiner, in spite of obvious cooccurrence restrictions. The same problems that beset the so-called Det-S or complex determiner analysis of relative clauses (see especially Vergnaud 1974 for discussion) also apply to the complex determiner analysis of exception phrases. In particular the possibility of conjunctions as in (32) is problematic for any simple version of the complex determiner analysis.

(32) Every man and every woman but Adam and Eve were born in sin.[6]

Note in particular that it does not help to treat these cases as right-node raising of the exception phrase from each conjunct because the exception phrase crucially modifies the conjunction as a whole. In we treat exception phrases as operators on noun phrases, cases like (32) do not pose special problems. The same is true for phrases like *nothing but the* *truth*, where a complex determiner analysis would force one to break up *nothing* into its components *no* and *thing* and to treat it as the result of applying *no .. but the truth* to *thing*, disregarding the fact that *nothing* is a lexical unit.

I now turn to the semantics of connected exception phrases. I am assuming the generalized quantifier notation of Barwise and Cooper for the following definitions. There are a number of options to be made in the semantics, depending on what one takes to be the meaning and what pragmatic implicatures associated with that meaning. In my 1987 paper I simplified matters a bit by assuming that the NPs in exception phrases are always referring terms. However, this is not actually true, to judge from examples such as (33).

(33) a. Jones hates all foreigners except for at most a third of the criminally insane among them.
b. Life teaches no lessons except all the expensive ones.

As van Benthem noted (p.c.), negative quantifiers are excluded:

(34) a. *I love everybody except for no students.
b. *We welcome everyone except for not Jack.

These example differ minimally from the ones in (35) which are acceptable but not exception phrases.

(35) a. I love everybody except no students.
b. We welcome everyone except not Jack.

The latter cases exemplify the so-called Stripping construction, which arises through the possibility of using *except* as a sentential connective. As examples (35a,b) show, *except for* does not have this use. Moreover, as noted before, Dutch *behalve* is never used as a sentential connective, and hence we predict that the Dutch counterparts to (35a,b) are ungrammatical. This prediction is correct, witness (36).

(36) a. *Ik houd van iedereen behalve geen studenten.
b. *We heten iedereen welkom behalve niet Jack.

The difference in acceptability between (34) and (36) on the one hand and (35) on the other creates a further head-ache for Reinhart's account, which would predict all of these cases to be equally acceptable since she tries to analyze exception phrases as cases of Stripping.

Assuming that we can somehow rule out negative noun phrases as arguments of exception markers, I suggest the following definition as a first approximation of the meaning of connected exception phrases:

(37) $||NP_1 \text{ but } NP_2|| = \{X < E\text{: for some } Y,\ X\text{-}Y \in ||NP_1||_{E\text{-}Y} \text{ and } Y \in ||NP_2||\}$

The main idea in this definition is that the exception phrase serves to introduce a set Y of cases that are left out of consideration. The domain of quantification is retricted by subtracting Y. To make the disregarded cases true exceptions, one might add clause (38).

(38) not $(X \in ||NP_1||_E)$

or treat (38) as a Gricean implicature, along the lines sketched earlier.

The choice of set Y is rather unconstrained. This is necessary to deal with cases such as (33a), where every set in the quantifier may provide the set of exceptions. It is much less appropriate for referring terms. Using the standard characterization of referring terms as ultrafilters generated by an individual, we get the undesirable result that 'all except John are happy' is true if all but John and Bill are happy, since any set containing John is a member of the ultrafilter and so the set consisting of John and Bill would also be a possible set of exceptions. For example, the entailments in (39) become valid under this interpretation.

(39) All but A, B and C are D ==> All but A and B are D ==> All but A are D

The fact that we usually interpret 'All but John were happy' as 'Only John was unhappy' could be interpreted as a Gricean scalar implicature, but that seems to weak an account. Compare (40a) with (40b).

(40) a. John has three kids. In fact, he has five.
b. All but John are dead. #In fact, Jim is not dead either.

Unlike (40a), a typical example of a scalar implicature lifted by further information, (40b) strikes one as contradictory in nature. A way out of the problem might be found if we take seriously the suggestion of much recent work to treat referring terms as denoting entities rather than generalized quantifiers. In this way they would not fall under definition (37) but require a separate definition. This definition is given in (41).

(41) $||\text{NP but a}|| = \{X < E: X-\{a\} \in ||\text{NP}||_{E-\{a\}}\}$

In addition to this definition, it is necessary to characterize the class of NPs that can be modified in this way. It turns out that the two closure properties defined in (42) below select the proper class of noun phrases. These properties could be restated in terms of properties of the determiners, in particular the properties of left downward monotonicity and anti-additivity, which are perhaps familiar from the literature on generalized quantifiers, were it not for the fact that we have choses to treat exception phrases as NP-operators.

(42) a. Closure under Submodels.

If $E' < E$ and $X \in Q_E$, then $X \cap E' \in Q_{E'}$

b. Closure under Model Unions.

If $X \cap E \in Q_E$ and $X \cap E' \in Q_{E'}$, then $X \cap (E \cup E') \in Q_{E \cup E'}$

Examples of noun phrases with the first property is listed in (43), examples of noun phrases with the second property are listed in (44). The property of closure under submodels entails that the quantified sentence is true at the empty model. This rules out all kinds of existential quantifiers and referring expressions.

(43)		(44)	
	all men		all men
	every bat		every bat
	at most three boys		John
	no pets		no pets
	few books		the students

The property of closure under model unions is needed to rule out quantifiers such as *at most three boys*. It can be checked that noun phrases which occur in one list but not in the other do not combine with exception phrases.

(45) all men but Harry
every bat but Dracula
*at most three boys but Fred
no pets but snakes
*few books but this one
*John but Sam
*the students but us

Somewhat problematic in this picture is the behavior of *only*. It can be checked that the noun phrase *only girls* has both required properties, yet (46) is ungrammatical.

(46) *Only girls but Rex were invited.

On the other hand, free exception phrases are fine with *only*, as (47) shows.

(47) Except for Rex, only girls were invited.

I interpret this anomaly as follows. *Only*, being an adverb and not a determiner, takes widest scope in the noun phrase, so that the proper parsing of the subject of (52) is as given in (48).

(48) [only [girls but Rex]]

Since *girls but Rex* is ungrammatical, *only girls but Rex* is also ruled out. The fact that *only* must be the outermost operator is further illustrated by the data in (49).

(49) only Japanese from Tokio
Japanese from Tokio only
*Japanese only from Tokio

Only differs in this respect from *not*.[7] I have argued elsewhere (Hoeksema 1986a) on independent grounds that *not* attaches to determiners, rather than noun phrases. Consequently, (50) is predicted to be bad, given that *not every* as a complex determiner lacks the required closure properties.

(50) *[[not every] student but Jim]

Under a different bracketing, as in (51), this phrase ought to be acceptable.

(51) [not [every student but Jim]]

Since (50) is never acceptable, I conclude that only the parsing in (50) is correct.

The behaviour is *neither* is easier to account for. If we follow Barwise and Cooper's (1981) suggestion that *neither* is only defined for sets with exactly two members, it follows that *neither diplomat* lacks closure under subsets. In spite of the close semantic similarity between *neither* and *no* we therefore predict that *neither* differs from *no* in not licencing connected exception phrases. This prediction is correct, as (52) shows.

(52) *Neither boy but Sam was pleased.

Truly problematic is the behaviour of *little* and *few*, which licence exception phrases, even though the noun phrases they introduce lack the closure under unions property. Thus, (53) is grammatical, but the inference in (54) is not valid.

(53) We had little choice but to comply.

(54) Little coffee was left in the can.
Little coffee was spilled on the table.

Little coffee was left in the can or spilled on the table.

If *little coffee* had the property of closure under unions, the inference in (54) ought to be valid. However, regardless of whether one interprets *little* as 'relatively little' or in a more absolute sense as 'less than some contextually specified measure', the inference is invalid. The status of sentences such as (53) is also somewhat peculiar in another respect, since exception phrases are normally used whenever the corresponding sentence without the exception phrase would be false. However, in (53) it seems questionable that the set of options would not be small if complying is included. Here we run into the problem of vagueness, in the guise of the Paradox of the Heap. Just as adding a grain to something which is not a heap does not make it a heap suddenly, adding one option to a small set does not make the set large. Presumably this is the reason why *but* can be read as equivalent to *besides* in (53).

3.2. Free exception phrases.

Free exception phrases are by far the most interesting and complex class. In my (1987) paper, I used a straightforward extension of the treatment of connected phrases to deal with the distribution of free exception phrases. The idea was that a free exception phrase serves to restrict the models of the sentence, as in (55).

(55) $||\text{Except for A, S}||_E$ is True iff $||S||_{E-\{||A||\}}$ = True

There is a subtle distinction in acceptability between the sentences in (56), where an existential quantifier separates the exception phrase from the universal quantifier and the sentences in (57), where there is a referring term instead of an existential NP. To account for this observation, I added the requirement that the sentence modified by the exception phrase has the property of closure under model unions (but not under abitrary extensions of the model).

(56) a. *Except for this Cadillac, somebody damaged every car.
b. *Except for Mark, a professor left messages for every student.
c. *Except for Lily, I sometime detest all my siblings.
(57) a. Except for this Cadillac, he damaged every car.
b. Except for Mark, I left messages for every student.
c. Except for Lily, I detest all my siblings.

Sentences in which an existential quantifier has wide scope over a universal quantifier lack the property of closure under model unions. To see this, consider a simple example. If 'Someone hates every professor' is true at your university, and the same statement is true at mine, it does not follow that the statement also holds if we extend the domain of quantification to the union of the two university populations. Sentences in which a universal has wide scope over an existential quantifier do have the required property, and they can easily be modified by free exception phrases, cp. (58). Examples d and e show that the surface order of the quantifiers is not relevant, but rather their scope behavior, since they contain universal quantifiers which for some reason have scope over the existential quantifiers which precede (and in the case of (58d) also c-command) them.

(58) a. Except for Jones, every lawyer has a drinking problem.
b. Except for Henry, all senior partners owned a Cadillac.
c. Except for February, every month has at least 30 days.
d. Except for Padua, there was a delegate from every Italian city.
e. Except for August, I have a conference every month.

Interestingly, the examples in (56) are much better when the exception phrase occurs in sentence-final position.

(59) a. Someone damaged every car, except for this Caddilac.
b. Someone left messages for every student, except Mark.
c. I sometimes detest all my siblings, except for Lily.

This can be explained if we assume that the exception phrase occurs in or adjoined to the VP in these examples, outside the scope of the subject. Without introducing yet a different type for exception phrases, we could treat them as sentence operators by translating them as exemplified in (60).

(60) damage every car, except for this Cadillac ==>
lambda x: damage every car (x) except for this Cadillac

Here the formula that the exception phrase combines with contains a variable, rather than a quantifier. It can be checked that this formula has the required property of closure under model unions. For exception phrases in sentence-initial position, such an analysis is not available, and so we have a principled explanation for the difference between (56) and (59).

4. A New Theory of Free Exception Phrases.

4.1. Problems for the domain restrictor theory.

There are a few nagging problems with the theory that exception phrases are operators which change the domain of quantification. First, consider (61), taken from the Brown corpus.

(61) On Thursday nobody but Charlie Coe was thinking of Charlie Coe.

This sentence is also acceptable if we put the exception phrase in sentence initial position:

(62) Except for Charlie Coe, nobody was thinking of Charlie Coe.

To interpret the second occurrence of the proper name Charlie Coe, it seems we cannot restrict the domain of discussion to everybody who is not Charlie Coe. This problem (noted in Hoeksema 1987 and von Fintel 1989) seems solvable, if we make a distinction between the domain of discussion and the domain of quantification, or, in other words, between the way in which quantifiers are assigned an interpretation and the way in which proper names are interpreted. This seems reasonable, as quantifiers are often interpreted as implicitly restricted to some contextually understood set, which may or may not include denotations for proper names, and pronouns. Some relevant examples are given in (63):

(63) a. I can see everybody quite well from here.
b. Nobody is as tall as Henrietta.

Obviously, the normal interpretation of (63a) is one in which the speaker is excluded from the set over which the quantifier *everybody* ranges. Likewise, *nobody* ranges over all individuals but Henrietta in (63b). Exception phrases, then, might be said to manipulate the sets relevant to quantification, not the larger sets used to interpret proper names and pronouns. I will refrain from spelling out the details of such a theory, however, because of additional problems that we run into. As already noted in Hoeksema (1987), requiring closure under model unions incorrectly rules out sentences in the universal-existential-universal kind, sentences, that is, in which a universal quantifier has scope over an existential one which in turn has scope over an existential one. Such sentences lack the property of closure under model union, yet allow modification by exception phrases, as (64) shows.

(64) a. Except for Jim, every pimp has a reason to hate every cop.
b. Except for Van Pelt, every tycoon donated a book to every library.

Interestingly, these sentences have only one interpretation. The exception phrase is understood as a restriction of the first, and not of the second universal quantifier. So in (64b), Van Pelt is understood as an exceptional tycoon, rather than an exceptional library. Both the fact that the sentences in (64) are acceptable and the fact that they have this reading are not predicted.

A third problem is perhaps the most interesting one, and one which has gone largely unnoticed in the literature. Exception phrases allow, to varying degrees, pied piping. By 'pied piping' I refer to the phenomenon that an exception phrase may contain more than just the noun phrase which denotes the exception to some universal quantifier. Just as fronted wh-phrases may come along with their prepositions, the arguments of exception markers may come adorned with prepositions. Consider first the Dutch examples in (65).

(65) a. Behalve met Jan heb ik met niemand gesproken.
except with Jan have I with nobody spoken
b. We spraken over alles, behalve over geld
we spoke about everything except about money
c. Behalve hem ken ik hier niemand.
except him know I here nobody
d. Behalve hij kent niemand mij hier.
except he knows noone me here

The prepositions in the exception phrases are copies of the prepositions introducing the quantifiers. Likewise the case marking on the pronoun following the exception marker is the same as the case that would be appropriate for the quantifier *niemand.* If exception phrases are operators on sentences, they ought to be insensitive to the internal structure of these sentences. Patterns such as in (65) are striking evidence for a conjunction analysis à la Reinhart. The prepositions and cases involved are normally assigned only once, except in conjunction structures (cf. e.g. *looking neither for money nor for power* or *dance with colleagues and with lovers*). However, there is considerable variation in the pied piping behavior of the various exception markers. For instance, the Dutch discontinuous marker *op .. na* does not exhibit it, nor does English *except for*.

(66) a. *Op met Jan na sprak ik met iedereen.
on with Jan after spoke I with everyone
"*I spoke with everyone except with Jan"
b. *Op hij na kent niemand mij hier.
on he after knows noone me here

Most interesting in this connection are examples such as (67).

(67) Except for the parents of John, we talked to the parents of every pupil.

According to my (1987) proposal, this sentence is true if it is the case that we talked to the parents of every student in a universe from which the parents of John have been removed. However, that still leaves us with John, a pupil whose parents we did not talk to, by assumption. Clearly, to get the right truth conditions, we should require that John be removed from consideration, rather than his parents. Note that the structural position of the name 'John' corresponds to the structural position of the quantifier 'every pupil'.

4.2. A substitutional theory of exception phrases.

To deal with pied piping and cases such as (67), I will introduce a new type of account, based on the idea of substitution. We can view (67) as some compound of the sentences in (68).

(68) a. We talked to the parents of every pupil.
b. We talked to the parents of John.

(68b) is derived from (68a) by substituting the argument of the exception marker for the quantifier. Sentence (67) is true in a given model if that model falsifies (68a,b) and if a minimal change in the model to change the truth-value of (68b) also changes the truth-value of (68a). To make this idea a little bit more precise, it is useful to consider partial models and the notion of a minimal model.

(69) A model M is a pair <E,F> where E is some set and F a function such that

(i) F(A) ϵ E (for all names A)
(ii) F(B, x) = 1,0 or undefined (for all monadic predicates B and individuals x)
(iii) F(C,x,y) = 1,0 or undefined (for all binary predicates C and individuals x,y)
etc.

We assume in the following definitions that the domain E is fixed for all models and that F may vary.

(70) The intersection of two models M = <E,F> and M' = <E,F'>
is defined as M" = <E, F">, where for all A,B, C etc.:

F"(A) = F'(A) = F(A)
F"(B,x) = F'(B,x) if F'(B,x) = F(B,x), undefined otherwise
F"(C,x,y) = F'(C,x,y) if F'(C,x,y) = F(C,x,y) and undefined otherwise
etc.

(71) M is a submodel of M' iff M $\cap$ M' = M.

(72) M is a miminal model of S iff M $\Vdash$ S and
all submodels M' of M such that M' $\Vdash$ S equal M.

(73) MinMod(S) is the set of minimal models of S.

(74) M~M' is the model M as modified by M'. Formally:
M~M' = M", where E=E'=E" and F(A) = F'(A) = F"(A) for any A and
F"(B,x) = F'(B,x) if F'(B,x) is defined and = F(B,x) otherwise
F"(C,x,y) = F'(C,x,y) is F'(C,x,y) is defined and = F(C,x,y) otherwise

(75) M~MinMod(S) = {M': M' = M~M" for some M" in MinMod(S)}

These definitions sketch the following picture. Models are basically lists of atomic propositions, as in Situation Semantics, which can be true, false or undefined. There is a natural subset relation on these models and the notion of minimal model. A miminal modification of a model can be thought of now as a model modified by a minimal model. This gives us the semantics for exception phrases in (76). This definition, by the way, is a simplification and will be revised shortly.

(76) M $\Vdash$ Except A, S[NP] iff any M' in M~MinMod(S[A]) is such that M' $\Vdash$ S[NP]
in case $\neg$ M $\Vdash$ S[A] and any M' in MinMod(-S[A]) is such that M' $\Vdash$ S[NP] otherwise.

In English prose, this boils down to the following. To evaluate a sentence with an exception phrase, we consider two other sentences, viz. that sentence without the exception phrase and the sentence which comes from substituting the argument of the exception phrase, here indicated by A, for the targeted NP. The first sentence is indicated here by S[NP], the second by S[A]. In sentences with more than one possible target NP, the intonation will help to disambiguate. I will come back to this point in a moment. The sentence indicated by S[NP] is not true in the model, but a minimal change in M which makes S[A] true in case it was false and which makes S[A] false in case it were true produces only models M' which make true S[NP]. This semantic interpretation was inspired by the famous Ramsey test for conditionals. Sentences with exception phrases are like counterfactual conditionals of a special kind. Cp.:

(77) Except for Chris, everybody wept.

This sentence can be paraphrased in a fairly faithful way by 'Had Chris (also) wept, everybody would have wept'. The main difference between conditional and exception sentences is the greater intesionality of the former. The conditional paraphrase I gave of (77) is only partly correct. If everybody but Chris wept, it does not follow necessarily that if Chris also weeps everybody will be shedding tears. There might be some causal connection which prevents the conditional from being true, for instance, because the sight of Chris weeping is enough to cheer up everybody else. Such causal connections make it tough to express the semantics of conditional sentences in terms of simple revisions of sets of atomic propositions in as explicit a fashion as proposed here for sentences with exception phrases. To state it bluntly, if I add the antecedent of a conditional to my stock of beliefs and then make appropriate adjustments to maintain consistency, the outcome may vary greatly, depending upon the various causal connections that I am willing to allow for. It might be said that exception phrase sentences are a better application for a Ramsey-type semantics than conditional sentences because of their extensional character. Their truth can be established by considering a single state of affairs; modal connections between states of affairs (as formalized by accessibility relations or selection fuctions) need not be considered. There is also an obvious connection between the semantic interpretation proposed above and the notion of minimal entailment which figures in theories of nonmonotonic reasoning, such as McCarthy's theory of circumscription (cf. McCarthy 1980). The connection is perhaps not too surprising, given that nonmonotonic reasoning is about generalizations with implicit or explicit exceptions. Returning now to example (77), note that this sentence is true in the model described in (78).

(78) M = <E,F>

E= {a,b,c}
F(Chris) = c
F(wept, a) = 1
F(wept, b) = 1
F(wept, c) = 0

Since 'Chris wept' is false in this model, we have to consider minimal models in which this sentence is true. There is one such model, given in (79).

(79) M' = <E,F'>

E={a,b,c}
F(Chris) = c
F(wept, a) = undefined
F(wept, b) = undefined
F(wept, c) = 1

M as modified by M' is the model M" in (80). This model makes true the sentence "Everybody wept", as required.

(80) M" = <E,F">

E={a,b,c}
F(Chris) = c
F(wept, a) = 1
F(wept, b) = 1
F(wept, c) = 1

4.3. Consequences of the substitutional theory.

Consider now how this account works for a number of ungrammatical cases. First, take a look at (81).

(81) *Except for Joan, Jim came.

There is a simple reason why this is bad. The truth of the sentence 'Jim came' is not sensitive to what we do to the truth value of 'Joan came'. Whether or not we change the model to accomodate that sentence, 'Jim came' will keep its truth value. Hence the exception phrase serves no purpose and sentences like (81) have no use.

I might add that counterparts to (81) are grammatical in German and Dutch:

(82) a. Ausser der Karl war auch der Franz da. (German)
except Karl was also Franz there
"Besides Karl, Franz was also present"
b. Behalve met Karel heb ik ook met Hans gesproken
besides with Karel have I also with Hans spoken
"Besides Karel, I also talked with Hans"

However, as the English translations indicate, the meaning here is crucially different: the exception markers are used here not as exception markers but with the meaning 'besides'.[8] Example (82b) shows the pied piping effect mentioned earlier. This construction can also be given a natural interpretation using substitution. Paardekooper (1966) as well as Landman and Moerdijk (1980) pointed out a number of differences between the use of *behalve* as an exception marker and its use as an operator meaning 'besides'. One of the most striking differences is that *behalve*-phrases used as exception phrases can occur either in sentence-final or in sentence-initial position, whereas they are restricted to a position preceding the target NP in their use as 'besides' phrases:

(83) a. Behalve met Karel heb ik ook met Hans gesproken (=82b)
b. Ik heb behalve met Karel ook met Hans gesproken
c.*Ik heb ook met Hans gesproken behalve met Karel

In examples like (84), two interpretations are possible:

(84) Behalve Jan heeft Piet Klaas gezien
besides Jan has Piet Klaas seen
"Besides Jan, Piet has seen Klaas"

Here the interpretation is either as in (85a) or (85b).

(85) a. Piet heeft Klaas gezien en Jan heeft Klaas gezien
b. Piet heeft Klaas gezien en Piet heeft Jan gezien

This ambiguity does not arise in the spoken language though, because the NP for which the object of *behalve* substitutes is marked clearly by focus intonation. Focus intonation is also what tells us to substitute for which NP in exception phrase constructions.

Another type of sentence that we need to rule out is given in (86a). To see why it does not have the reading readily associated with (86b), consider what our semantics says about it.

(86) a. *Except for this Cadillac, someone damaged every car.
b. Someone damaged every car, except for this Cadillac.

A typical model for (86b) is given in (87). For the sake of brevity, I won't write out all atomic propositions, but use set-theoretic notation.

(87) M = <E,F>
E = {a,b,c,x,y}
F(this Cadillac) = c
F(car) = {a,b,c}
F(person) = {x,y}
F(damage) = {<x,a>, <x,b>}

It is clear that changing this model minimally such that 'Someone damaged this Cadillac' becomes true does not guarantee that the sentence 'Someone damaged every car' becomes true. In particular, if we add the pair <y,c> to the extension of *damage*, we still haven't made (86a) true on the reading where the existential quantifier has wide scope. Only the other reading will be validated automatically. Notice that I have required in (81) that any minimal change of the appropriate kind should validate the sentence without the exception phrase. It can be checked that the models for (86a) which have this property are precisely the ones in which there is just one person. But in such cases, existential quantifiers are not used because definite descriptions are more informative and this is, perhaps, what causes the oddness of (86a).

As I just noted in passing, universal-existential sentences do not have the problem that not every minimal change of the requested kind satisfies the sentence without the exception phrase. I also note that in sentences of the universal-existential-universal kind the account given here predicts that the exception phrase may operate on the first or outermost universal quantifier but not on the last or innermost universal quantifier (unless, of course, the exception marker itself is within the scope of the existential quantifier). This is a major advance over my earlier account. In sentences of the universal-universal kind, the exception phrase is predicted to operate on either quantifier, and the examples in (88) show that this is correct.

(88) a. Except for Ned, every prof has been to every conference.
b. Except for GLOW, every prof has been to every conference.

The intended interpretations for these sentences are given by the paraphrases in (88'a,b).

(88') a. Every prof but Ned has been to every conference.
b. Every prof has been to every conference but GLOW.

For negative universal quantifiers, the situation is somewhat different. Consider what happens when one quantified NP is headed by *no* or when both are.

(89) a. Except for Harry, every prof saw no student.
b. Except for Harry, no prof saw every student.
c. Except for Harry, no prof saw no student.

Sentence (89a) is interpreted as (89'a), (89b) as (89'b) and (89c) as (89'c).

(89') a. Every prof but Harry saw no student.
b. No prof but Harry saw every student.
c. No prof but Harry saw no student.

To be excluded are the readings in (89").

(89") a. Every prof saw no student but Harry.
b. No prof saw every student but Harry.
c. No prof saw no student but Harry.

How can we explain these observations? First consider (89a). I maintain that the reading (89"a) is unavailable, given the semantic account of free exception phrases presented above. Let us assume a model in which the sentence 'Every prof saw no student' is false and where the student Harry is the only exception which prevents this sentence from being true. This could be because some professors saw Harry or even because all professors saw Harry. Intuitively, it should not matter whether all or just some professors saw Harry. All models in which at least one prof saw Harry ought to make the reading we are considering here true. But this is not how the formal account works. The formal account requires models such that every minimal change which changes the truth-value of "every prof saw Harry" will also change the truth-value of "every prof saw no student". Consider first the case where some but not all professors saw Harry. Changing the truth value of "Every prof saw Harry" by a minimal change in the model now means that we make this sentence true by giving the value 1 to every atomic proposition of the form (*saw, x, Harry*), where x is some professor. But that would not change the truth-value of 'Every prof saw no student': This sentence would still be false. Next consider the case where "Every prof saw Harry" is true in the initial model. Then changing it minimally so as to make this sentence false involves making one of the atomic propositions (*saw, x, Harry*) false, where x is a professor. However, this would not suffice to make the sentence "every prof saw no student" true, unless there is, in fact, only one professor in the model. But if it is obvious to both the hearer and the speaker that this is the only situation within which sentence (89a) can be used on the reading intended, then the universal quantifier *every prof* is inappropriate and a definite description would have to be preferred. And indeed, when a definite description is used, the intended reading becomes readily available, cf.:

(90) Except for Harry, the prof saw no student.

What about sentence (89b)? To see what is the matter with this sentence, it is useful to first consider a simpler case:

(91) *Except for Harry, somebody left.

Given our semantics, this sentence is true just in case changing the truth-value of 'Harry left' creates a model in which 'somebody left' is true. It is easy to see that the only models which qualify are the ones in which 'nobody left' is true. Clearly, it would have been simpler to express the content of (91) by that sentence. Moreover, it appears to me that (91) fails to satisfy a reasonable pragmatic requirement which one might impose on sentences with exception phrases, to wit, that the exception phrase be "especially appropriate". By this requirement I mean that reference to another exception is not possible. This requirement is met by sentences such as (88a), in which the exception phrase applies to a universal quantification. There, the choice of another exception would not have been correct. However, when the quantifier is an existential one, as is the case in (91), any individual could have been chosen for the exception phrase. That is to say, if the persons in the domain of discourse are Tom, Dick and Harry, then instead of (91), it would have been equally appropriate to use (92a) or (92b).

(92) a. *Except for Tom, somebody left.
b. *Except for Dick, somebody left.

In light of this requirement, consider again (89b). By assumption, the sentence "no prof saw every student" is false. Hence also "no prof saw Harry" is false, if Harry is a student. Changing the model minimally so that "no prof saw Harry" is validated will clearly also validate "No prof saw every student". However, the requirement that the exception phrase is uniquely appropriate is not met in the general case. If we proceed from the assumption that there is more than one student in the model (an assumption warranted by the choice of the quantifier), then any student would have served as a proper exception. In other words, if some prof saw every student, then any minimal change which validates "no prof saw x", where x is some student, will validate "no prof saw every student". Hence there was no need to single out Harry, as in (89b). More generally, we can say that occurrences of universal quantifiers under the scope of an odd number of negative operators do not licence exception phrases, because of their equivalence to wide scope existential quantifiers. This immediately explains why (89c) does not have the reading given in (89"c).

The domain-restrictor theory makes a different, but quite interesting prediction for sentences with two universal quantifiers. It causes BOTH quantifiers to be simultaneously restricted to a smaller domain. For sentences such as "Except for Jeff, nobody trusted everybody", the predicted reading is "Nobody but Jeff trusted everybody but Jeff". Such a reading, I take it, is not actually available for this sentence. The actual reading is 'Nobody but Jeff trusted everybody'. This, then, provides us with a further argument against the domain-restrictor theory.

Finally consider example (67) again:

(67) Except for the parents of John, we talked to the parents of every pupil.

Consider a model where not every pupil's parents have been talked to by us. This model will make (67) true just in case changing it minimally so that the sentence 'We talked to the parents of John' becomes true suffices to make 'We talked to the parents of every pupil' true.

4.4. Superlatives and their ilk.

Superlatives have much in common with quantificational expressions (cf. e.g. Szabolcsi 1985 and Hoeksema 1986b for some discussion of the similarities). One such common feature is the possibility of modification by exception phrases. However, before looking at superlatives, let us first take a look at a closely related expression, *the only*[9]. An intriguing prediction that falls out from the present substitutional analysis is the difference in acceptability between the sentences in (93), noted (but not explained) in Hoeksema (1987).

(93) a. Except for Richard, I am the only realtor.
b.*Except for Richard, I hate the only realtor.

In the first example, *the only* occurs in a predicate nominal, and in the second example it occurs in a direct object. Why should this make a difference? First consider (93a). Assume that Richard and I are realtors. So I am not the only realtor. Now if we change this situation minimally so as to make (94a), the result of substituting *Richard* for *the only realtor*, true, then we see how this might also make (94b) true.

(94) a. I am Richard.
b. I am the only realtor.

Now consider example (93b). Assume that it is not true that I hate the only realtor, perhaps because there are several realtors, or because the one existing realtor is not someone I hate. Changing the situation minimally so that (95a) becomes true does not influence the truth of (95b), unless Richard is the only realtor. But that goes against the presupposition or entailment generally associated with exception phrases, namely that leaving them out would make the sentence false.

(95) a. I hate Richard.
b. I hate the only realtor.

I note in passing that example (93a) also shows that the interpretation of *the* as *the only*, due to Russell (1905) and found in one form or another in more recent work such as Montague (1973) or Keenan and Stavi (1986), is incorrect. If we replace *the only* by its putative equivalent *the*, this sentence becomes ungrammatical.

Exception phrases with superlatives are rather rare in the Brown corpus. I found only two cases. Presumably, this is due to the relative infrequent use of superlatives vis à vis quantified NPs. The two cases are given below in (96).

(96) a. It was the largest house he had ever been in, almost the largest building, except for a hotel.
b. We saw Giuseppe Berto at a party once in a while, tall, lean, nervous and handsome, and, in our opinion, the best novelist of them all except Pavese, and Pavese is dead.

Note that the superlatives in these examples are used predicatively, just as the NP *the only realtor* in the earlier example. In other uses, an exception phrase may not be permitted. Some pertinent examples of such incompatibility are given in (97).

(97) a. *Except for Johanna, she looked at the prettiest girl.
b. *Except for these jerks, I am tired of the worst scum.

However, it would be misleading to suggest that only superlatives in predicate nominals licence exception phrases. Szabolcsi (1985) has drawn attention to a class of superlatives which she calls comparative superlatives (see also Hoeksema 1983 for additional discussion). Comparative superlatives such as (98a) have characteristic paraphrases such as (98b).

(98) a. Fred made the fewest mistakes.
b. Fred made fewer mistakes than anyone else.

Often, sentences with superlatives are ambiguous due to the option of giving the superlative a comparative reading or not. Consider for example the case of a man whose wives committed suicide by jumping from the Empire State Building and from the Chrysler building respectively. If asked the question "Who jumped from the tallest building", the man could answer "Nobody", since none of them jumped from Sears Tower in Chicago, the tallest building. Or else, he could say "My first wife", if she was the one who jumped from the Empire State Building, since that is the taller of the two. The first answer is appropriate on an absolute interpretation of the superlative, the second on a comparative interpretation.

Interestingly, Szabolcsi (1985) relates the comparative reading of superlatives to the presence of a focussed constituent, which serves as the object of comparison. Compare for example the two sentences in (99).

(99) a. JOHN caused Mary the fewest problems.
b. John caused MARY the fewest problems.

These examples can be paraphrased as in (99').

(99') a. The number of problems that John caused Mary is smaller than the number of problems that anyone else caused her.
b. The number of problems that John caused Mary is smaller than the number of problems that he caused anyone else.

The relevance of Szabolcsi's observations for the semantics of exception phrases is obvious. First of all, the paraphrase she suggests contains a universal quantifier, which explains why exception phrases are possible with comparative superlatives. Second, the semantic account proposed here requires one to find a minimal model for the result of substituting the NP following the exception marker for a focussed NP. Thus we predict two different readings for the sentences in (100).

(100) a. Except for Adam, JOHN caused Mary the fewest problems.
b. Except for Adam, John caused MARY the fewest problems.

According to the present semantic account, (100a) is true is model with a partial order '<' representing the relation 'less troublesome to Mary than' of which Adam is the least member and John is the least member of the restriction of '<' to E-{Adam}. Only in such a model would a minimal change which falsifies "ADAM caused Mary the fewest problems" verify (99a).[11] Likewise, it follows that (100b) is true in a model with a partial order '<<' representing the relation 'bothered less by John than', of which Adam is the least element and Mary the least element of its restriction to E-{Adam}. Again, this is the only scenario which would allow any minimal modification falsifying "John caused ADAM the fewest problems" to verify (99b).

A point left implicit in the discussion so far is that I am assuming that substitution for a focussed NP leaves the focus structure intact. In other words, when I substitute *Adam* for *Mary* in (100b), I suppose that *Adam* bears focus just as *Mary* did before.

Finally I point out that although the exception phrases in the above examples are licenced by superlatives, they do not substitute for these superlatives. In this respect, then, comparative superlatives are essentially different from universal quantifier constructions, where the target of substitution is always the quantifier and not some associated focus expression. The role of focus is also evident in a final observation, also due to Szabolcsi. Besides focussed NPs, WH-elements, including relative pronouns, can also create structures in which comparative readings are possible for superlatives. Since relative pronouns are never focussed, we expect to find that in such circumstances exception phrases are not permitted. The examples in (101) show that this expectation is fulfilled.

(101) a. There is a prize for the student who states this principle in the fewest words.
b. *Except for Mary, there is a prize for the student who states this principle in the fewest words.
c. The student who writes the best essay will receive $ 500.
d. *Except for Jim, the student who writes the best essay will receive $ 500.

5. Conclusion. Remaining problems.

In conclusion, I would like to mention a few further avenues of research and some of the remaining problems that need to be handled. First of all, I note the need for study of expressions such as 'instead of', which also might profitably be studied in terms of a substitution analysis. It seems that sentence (102a) is true just in case (102b) is true.

(102) a. Instead of a pay raise, we need a vacation.
b. [We need a vacation] and not [we need a pay raise]

Note that the NP *a pay raise* in (102a) is read De Dicto, just as a substitution analysis would predict. (The substitution rule is quite different from Montague's rule of quantifying-in because the latter only produces wide-scope readings.)

Problematic for the substitution analysis are first of all cases where exception phrases modify verbs or adjectives (with regard to these cases, then, the present analysis does not fare better than the one in Reinhart (1989)). The Brown corpus has several examples of *but*-phrases with the adjective *naked*. In Dutch, the discontinuous marker *op..na* would have to be used in such cases, rather than *behalve*, cp. (103a). English permits both *but* and *except*; the latter is attested in (103b), from the letter of a dean:

(103) a. Op een lendendoek na was hij naakt.
but a loin cloth was he naked
"He was naked but for a loin cloth"

(cf. *Behalve een lendendoek was hij naakt.)

b. This form is intended to indicate that, except for minor alterations, the dissertation is ready for final defense.

The class of adjectives that can occur with exception phrases is rather interesting. It appears to be the same class that permits modification by adverbs such as 'virtually', which select the same class of quantificational noun phrases as exception phrases. Some data are given in (105).

(104) a. naked but for a loin cloth.
b. virtually naked
c. all but the best
d. virtually all

e. *sick but for a fever

f. *virtually sick
g. *several but the best
h. *virtually several

In Dutch, we note a close affinity between *op .. na* and *vrijwel* "almost, virtually". The latter expression was studied in Zwarts (1985). Since it was shown before that *op .. na* does not show some of the crucial features that suggested the substitution account, in particular pied piping properties, a different analysis is needed for this expression anyway. The main conclusion to be drawn from this is that exception markers exhibit properties which range from typical connective properties to properties more typical of prepositions. As Paardekooper (1979) already saw, exception markers may have properties of both word classes, and attempts to treat either as pure-bred prepositions (as in Landman and Moerdijk 1980) or as connectives (as in Harris 1982, Reinhart 1989) are bound to fail.

Due to the space limits of this publication, it was not possible to treat exception phrases in questions. The following quote from the apostle Paul has two exception phrases, one with a quantification expression, the other with a wh-operator.

(105) For who else can know a man's thoughts, except the man's own spirit that is within him? So no one else can know God's thoughts, but the spirit of God. (1. Cor. 2:11-12)

Horn and Bayer (1984) pointed out that sentences such as (106a) below are only acceptable as rhetorical questions, when the presupposed answer is "Nobody!".[10] When such an interpretation is not available, and the question is used as a genuine request for information, the use of *but* is impossible (cf. 106b), although *except for* may still be used (cf. 106c).

(106) a. Who but a total idiot would say a thing like that?
b.*Who but John do you think is coming to the party?
c. Who is coming to the party, except for John?

The above examples are interesting among other things because they show that the category of rhetorical questions is grammaticized in English.

Finally a word must be said about cases where there is no overt quantifier for the exception phrase to substitute for. Some such cases were mentioned before as a problem for Reinhart's theory. Obviously, they also pose a problem for my own account. In the Brown corpus such cases are quite common, in particular in negative sentences and generic or habitual sentences. More than 30% of the 151 cases of phrases with the marker *except* that I selected from the corpus were licenced solely by negation and not by some overt quantifier. It is attractive to see this as evidence for the notion 'implicit argument'. An implicit argument is usually interpreted as an existential quantifier. In the context of negation, this quantifier is equivalent to a wide scope universal one, and can be shown to licence exception phrases. A typical example from the Brown corpus is given in sentence (107).

(107) But I once again assure all peoples and all nations that the United States, except in defense, will never turn loose this destructive power.

The adverb *never*, being a temporal quantifier, cannot be modified by the exception phrase because the exception phrase does not contain a temporal expression. Rather, the exception phrase seems to be possible through the existence of a implicit argument or, if you will, modifier, which can be made overt as "in any circumstances". By introducing such entities into the analysis, it becomes rather more abstract than the earlier domain restriction account, but this may be the price one has to pay for greater empirical coverage.

Notes.

1. I thank Johan van Benthem for sending me his own thoughts on exception phrases and Megan Moser for sending me the material she extracted from the Brown corpus.

2. The possibility of using a comparative as exception phrase also exists in Dutch and is restricted to the negative pronouns *nichts* and *niemand*. Presumably these combinations derive from the longer form *nichts anderes als* and *niemand anders als* by omission of *anders* "other, else". That we are dealing with a real exception-phrase construction here, and not just a case of *anders* deletion between a pronominal quantifier and an *als*-phrase, is shown by the fact that *iemand als Piet* "somebody than Piet" does not have the interpretation "somebody else/other than Pete". Only when the *als*-phrase can be used to indicate an exception, viz. when the preceding quantifier is universal in character, is this reading possible. See also note 10.

3. Example from Chateaubriand, Génie du Christianisme, I,1,2, cited in Moignet (1973: 162).

4. Dutch *behalve* does not have this particular use at all. However, English *except for* also exhibits it, witness the following example from the Brown corpus:

(i) (..) many historians maintain that except for Northern meddling it would have ended in states like Virginia years before it.

This suggests that this use is distinct from, but related to the use of *but* and *except* as exception markers.

5. To be sure, *behalve* can be used to introduce a subordinate clause, but it cannot be used to conjoin to clauses. Cf.:

(i) Ik weet niets, behalve dat het regent
I know nothing except that it rains

(ii) *Iedereen is ziek, behalve Jan is niet ziek
Everyone is sick, except Jan is not sick

6. This kind of sentence also poses a problem for a third analysis, proposed in Von Fintel (1989), according to which exception phrases are common-noun modifiers.

7. As a matter of fact, *only* can also be used as an operator on determiners, for instance when it is used with a numeral (cf. Jacobs 1983 for motivation of this claim). I am assuming that this use is not involved in (46).

8. English *except for* may also exhibit the meaning 'besides', although this interpretation is quite rare. The Oxford English Dictionary mentions a few cases, such as the example in (i) from 1578, but notes that it is obsolete and rare.

(i) Excepte fleshe, fishe and eldinge ... this Ile has a pasture .. that may feid sum wethiris.

However, the OED fails to observe that in questions this interpretation seems to be alive and well:

(ii) Except for Dick and Sue, who do you know in Tucson?

9. For a discussion of the relationship between superlatives and (the Dutch counterpart of) *the only*, see Hoeksema (1986b). It is noted there, among other things, that in substandard Dutch *the only* is rendered as *de enigste*, with superlative morphology.

10. More precisely, we need an antisymmetric, transitive and reflexive relation, better paraphrased perhaps as "at most as bothersome to Mary as".

11. In Dutch, the comparative can be used in such rhetorical questions (e.g. *wie dan God alleen* "who but God only"). Again, a close affinity with 'nobody' can be noted here: Not only is this the expected answer, but *niemand* "nobody" (and *niets* "nothing") are also the only quantifiers which take comparative PPs as exception phrases (cf. note 2).

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Applications of Intensional Logic to Program Semantics [1]

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Abstract

We apply intensional logic to the semantics of programming languages which contain arrays, blocks and procedures with parameters. Intensional logic is a logic with which one can reason about the intension or sense of an expression. We take the intensional value of an expression as a function from states (possible worlds) to (extensional) values. One of the motivations of applying intensional logic to program semantics is that there are intensional contexts within assignments and parameter passings. Furthermore location references can easily be modeled.

Programming language features such as blocks and procedures with parameters are difficult to deal with in both denotational and axiomatic semantics. The difficulty lies in (i) the formalism of the notion of location, (ii) the local storage structures, and (iii) the occurrences of aliasing among variables inside the procedure bodies. Recently there have been several new approaches to modeling storage structure such as using set-theoretic equivalence relations, possible worlds and category theory.

Our semantics is given by translating program expressions to intensional logic expressions. We also give a translation of the Hoare logic formulas to intensional logic formulas. The semantics is compositional, in which the meaning of a compound expression is a function of the meanings of its parts. We have a nice treatment of pass-by-value, pass-by-address, pass-by-name and pass-by-value-result parameters. In addition there are semantic correspondences between blocks and procedures.

1 Introduction

1.1 Blocks and Procedures

A block is a definition mechanism in which identifiers are declared and the declared identifiers are then visible inside the statement which follows the declarations. Nested blocks are possible, so that the same identifier name can be redeclared inside the nested declarations. Procedure declarations allow the programmer to package computations and parameterize their behavior. A procedure can be invoked anywhere in the program text. To facilitate our discussion, the syntax of blocks and procedure declarations are given.

Blocks :

begin new x := e ; S end

begin alias y = v ; S end

begin macro z = v ; S end

In the new-block above, the new identifier x is declared with the initial value e and S is the body (statement) of the block. The statment S is also referred to as the *scope* of the

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identifier x. It is the region of program text over which x is in effect. This resembles the quantification in the first order logic, and the **let** constructs in Lisp and Scheme. In the alias-block, v stands for an integer variable which can be a simple identifier x, or an array element a[e]. Within the alias-block the identifier y has the same location reference as the integer variable v. The macro-block behaves the same as the statement obtained by replacing all occurrence of z in S by v.

Procedure Declarations :

$$P \Leftarrow \; < \textbf{value } w \, ; \, \textbf{address } x \, ; \, \textbf{value-result } y \, ; \, \textbf{name } z \; > \quad S \textbf{ end}$$

where P is the procedure name, S is the procedure body (statement), w is a pass-by-value parameter, x is a pass-by-address parameter, y is a pass-by-value-result parameter, and z is a pass-by-name parameter. All the parameters in the declaration are called the *formal parameters.* They have scopes over the procedure body. Procedure P can be invoked by a procedure call say P(1+2,a,b,c). We call 1+2,a,b,c the *actual parameters.*

We consider four different parameter passing mechanisms given as follows. (1) *Pass by Value* : The actual parameter is evaluated and the value is copied into the location of the formal parameter. (2) *Pass by Address (Reference)* : The location reference of the actual parameter is passed as the location reference of the formal parameter. Both the formal and the actual parameters share the same location. (3) *Pass by Name* : The formal paramater is not bound to a location at the point of call, it is bound to a (possibly different) location each time it is used in the procedure body. Pass by name mechanism can be thought of as a textual substitution of all occurrences of the formal parameter in the procedure body by the actual parameter. (4) *Pass by Value-Result* : The actual parameter must be an integer variable. The formal parameter is considered as a new local variable inside the procedure body. Its value is initialized to the value of the actual parameter. On completion of execution of the procedure, the content of the formal parameter is copied to the actual parameter.

We adopt a design principle for programming languages called the *principle of correspondence* which states that for any parameter passing mechanism, there should be a corresponding definition (block) mechanism, and *vice versa* [Tennent 81]. Here pass-by-value parameter passing corresponds to new-block, pass-by-address parameter passing corresponds to alias-block, pass-by-name parameter passing corresponds to macro-block and pass-by-value-result is a special case of new-block. This principle will be justified by the semantic correspondences between blocks and procedures in our semantics.

1.2 Compositionality Principle

A *meaning* for expressions of a language is a function $[\![\cdot]\!]$ mapping syntactic entities into semantic entities. Let Δ be a syntactic domain, and $\mathcal{D}$ its semantic domain. For any $\delta \in \Delta$, we write $[\![\delta]\!]$ to denote the meaning of δ.

We assume that all compound expressions of a language are constructed from primitive expressions by finitely many applications of certain *syntactic operators.* The meaning function of the language is said to satisfy the *compositionality principle* if for every such n-ary syntactic operator $\Phi : \Delta_1 \times \Delta_2 \times \ldots \times \Delta_n \rightarrow \Delta_{n+1}$, there exists a function $\Psi : \mathcal{D}_1 \times \mathcal{D}_2 \times \ldots \times \mathcal{D}_n \rightarrow \mathcal{D}_{n+1}$ such that for every compound expression $\Phi(\delta_1, \delta_2, \ldots, \delta_n) \in \Delta_{n+1}$ formed from the components $\delta_1 \in \Delta_1, \delta_2 \in \Delta_2, ..\delta_n \in \Delta_n$ with the syntactic operator Φ, $[\![\Phi(\delta_1, \delta_2, \ldots, \delta_n)]\!] = \Psi([\![\delta_1]\!], [\![\delta_2]\!], \ldots [\![\delta_n]\!])$

This principle says that the meaning of a compound expression is obtained from the meanings of its parts. It is carefully formulated and discussed in Janssen's thesis [Janssen 86].

1.3 Difficulties and Objective

Blocks and procedures with parameters are problematic features which are difficult to deal with in both axiomatic and denotational semantics [Manna & Waldinger 81]. The difficulty lies in (i) the formalism of a notion of location, (ii) the structure of local storages, and (iii) the occurrence of aliasing among integer variables inside the procedure bodies.

To handle memory locations, addresses were introduced into the semantic domain. *Syntactic application* was used to model parameter passing [De Bakker 80], which resulted in semantics which were not compositional. Recently, several attempts have been made to model local storage structures [Meyer & Sieber 88]; Oles uses category-theoretic methods [Oles 85] and Brookes uses set-theoretic aliasing relations [Brookes 85]. Most of the researchers are looking for semantic models which are *fully abstract*, which means that in such models, two terms will be semantically equivalent if they induce the same *behavior* in all program contexts.

Most proof systems for procedures with parameters suffer from a number of restrictions [Hoare 71, Olderog 84]. Some proof systems for aliasing [Cartwright & Oppen 81] are complicated. Reynolds invents *specification logic* for reasoning about the correctness of procedures by handling *interference* among parameters and variables [Reynolds 81].

Our objective is to use intensional logic to develop a compositional semantics which correctly models location references, and a larger set of programming features including blocks, procedures, and different parameter passing mechanisms. Furthermore, intensional logic can be used naturally for reasoning about the correctness of the programs.

2 Intensional Logic

There are two important notions in the meaning of an expression in languages: *extension*, or *denotation*, and its *intension* or *sense*. Intensional logic is a logic with which one can reason about the intension or sense of an expression.

Frege was one of the first who investigated the notion of sense. A well-known example is as follows. Consider the sentence

Necessarily the morning star is the morning star.

The sentence is obviously true. We know that extensionally 'the morning star' and 'the evening star' refer to the planet Venus. Replacing the second 'the morning star' by 'the evening star' in the above sentence, we get

Necessarily the morning star is the evening star.

The new sentence fails to be true because one can easily imagine a world in which these two names refer to different objects. Hence the substitution of a co-designating name in the sentence does not preserve the truth value. The text after 'Necessarily' is said to be *referentially opaque* (opposite to *referentially transparent*, in which the truth value under such a substitution is preserved). The word 'Necessarily' is said to introduce an *intensional context*.

Church made his first attempt at axiomatizing intensional logic in [Church 51]. Based on Kriple's *possible world semantics* for modal logic [Kripe 63], Montague gave possible world semantics to intensional logic and successfully translated English fragments to formulas of intensional logic [Montague 74]. The main characteristic of Montague's approach is that it is compositional.

Our approach to intensional logic is along the lines of [Montague 74], [Gallin 75] and [Janssen, van Emde Boas 77]. We denote the logic by **IL**. The intensional value of an expression is taken as a function from possible worlds to (extensional) values. It is a higher order typed logic with two special operators, namely the intension operator ˆ and the extension operator ˇ. Intuitively, for any expression α, $\hat{}\alpha$ stands for the values of α in all possible worlds and $\check{}\alpha$ stands for the value of α in the current world.

3 Motivations

3.1 Intensional Contexts

Computer scientists have long realized the presence of intensional contexts in programming languages [Stoy 77]. The right hand side of an assignment statment is referentially transparent, and the left hand side is referentially opaque.

Consider the procedure declaration

$$P \Leftarrow \ < \textbf{value}\ x\ ;\ \textbf{address}\ y\ >\ y := x\ \textbf{end}$$

where x is a pass-by-value parameter and y is a pass-by-address parameter. Suppose the identifiers u, v have the same value 7. Obviously, the procedure calls P(u,z), P(v,z), P(7,z) have the same effect that z becomes 7. However P(0,u) has the effect that u becomes 0 and v remains unchanged, while P(0,v) has the effect that v becomes 0 and u remains unchanged. From this example, we can observe that pass-by-value parameters are referentially transparent and pass-by-address parameters are referentially opaque. In addition pass-by-name and pass-by-value-result parameters are referentially opaque. This shows how intensional contexts are introduced in the procedure calls.

3.2 Parameter Passing Mechanisms

Now let us distinguish between pass-by-address, pass-by-name and pass-by-value-result mechanisms. Consider the *correctness formula* $\{\ \phi_1\ \}\ S\ \{\ \phi_2\ \}$, where S is a program (statement), ϕ_1 is a first order formula which describes the computer states *before* the execution of S, ϕ_2 is the formula which describes the computer states *after* the execution of S. When the correctness formula is valid, we write $\models \{\ \phi_1\ \}\ S\ \{\ \phi_2\ \}$. Here we consider only *total* correctness in which the execution of S will always halt with a final state.

Consider the following three procedure declarations:

$$P^a \Leftarrow \ < \textbf{address}\ x\ >\ y := 1;\ x := x + y\ \textbf{end}$$

$$P^n \Leftarrow \ < \textbf{name}\ z\ >\ y := 1;\ \$z := \$z + y\ \textbf{end}$$

$$P^r \Leftarrow \ < \textbf{value-result}\ x\ >\ y := 1;\ x := x + y\ \textbf{end}$$

The following hold :

$$\models \{\ y = 0\ \}\ P^a(y)\ \{\ y = 2\ \}$$

$$\models \{\, y = 0 \,\}\, P^n(y)\, \{\, y = 2 \,\}$$
$$\models \{\, y = 0 \,\}\, P^r(y)\, \{\, y = 1 \,\}.$$

It is interesting that without arrays, there is no difference between pass-by-address and pass-by-name parameter passing mechanisms [De Bakker 80].

Consider the three procedures $\mathtt{swap}^a, \mathtt{swap}^n, \mathtt{swap}^r$ with three different parameter passing mechanisms. The procedure $\mathtt{swap}^a$ is declared as follow.

```
swapᵃ ⇐ < address u; address v >
        begin new w := u;
              u := v;
              v := w;
        end
     end
```

The following hold:

$$\models \{\, a[0] = 1 \wedge i = 0 \,\}\, \mathtt{swap}^a(i, a[i])\, \{\, a[0] = 0 \wedge i = 1 \,\}$$
$$\models \{\, a[0] = 1 \wedge i = 0 \,\}\, \mathtt{swap}^n(i, a[i])\, \{\, a[0] = 1 \wedge i = 1 \,\}$$
$$\models \{\, a[0] = 1 \wedge i = 0 \,\}\, \mathtt{swap}^r(i, a[i])\, \{\, a[0] = 0 \wedge i = 1 \,\}.$$

It has been discovered that one cannot swap the contents of two memory locations if pass-by-name is used [Fleck 76]. We believe that there is a good conceptual reason for this, namely that pass-by-address is needed to manipulate location parameters.

4 Possible Worlds

IL is a typed higher order language which has functions and functionals. There are two basic types n and t, which are corresponding to the natural numbers $\mathcal{N}$ and Boolean values. For any type τ, let $\mathcal{D}_\tau$ be the set of τ objects, VAR_τ be the set of τ variable symbols, and CON_τ be the set of τ constant symbols. The higher order types of IL are compound types of the form $(\tau_1 \rightarrow \tau_2)$, which are the types of functions from $\mathcal{D}_{\tau_1}$ to $\mathcal{D}_{\tau_2}$. Since senses of expressions are in our object language, there are also compound types $(\mathsf{s} \rightarrow \tau)$ which are the types of functions from possible worlds to τ objects. We call $(\mathsf{s} \rightarrow \tau)$ the type sense or concept of τ.

We identify possible worlds in the context of intensional logic as the internal states (memory configurations) of computers. We denote this set of states by *State* and its elements by σ. Let REF be a subset of $CON_{(\mathsf{s} \rightarrow \mathsf{n})}$, and $ARRAY$ be a subset of $CON_{(\mathsf{n} \rightarrow (\mathsf{s} \rightarrow \mathsf{n}))}$. Their definitions are given in detail in the next section. Intuitively, REF is the set of reference symbols and $ARRAY$ is the set of array symbols. To characterize the set of states (possible worlds), we need three postulates given as follows [Janssen 86].

Distinctness Postulate

For all $\sigma_1, \sigma_2 \in State$,

$$(\text{ for all } X \in REF,\ [\![X]\!](\sigma_1) = [\![X]\!](\sigma_2)\ ) \Longleftrightarrow \sigma_1 = \sigma_2$$

This postulate states that two states which agree in the values of all reference symbols are not distinguishable. This postulate puts a restriction on the possible domains of states and we can take states as functions.

Update Postulate

This postulate states that any change of the value of a reference symbol will create a new state. That is for all $\sigma \in State, X \in REF, n \in \mathcal{N}$, there exists a $\sigma' \in State$ such that

$$\begin{aligned} [\![X]\!](\sigma') &= n \\ [\![Y]\!](\sigma') &= [\![Y]\!](\sigma) \text{ for all } Y \in REF \text{ where } Y \not\equiv X \end{aligned}$$

This postulate ensures the richness of *State*. We denote σ' by $\sigma\{X/n\}$.

Properness Postulate

For any $A \in ARRAY, k \in \mathcal{N}$, there exits an $A^k \in REF$ such that $[\![A]\!](k) = [\![A^k]\!]$. This postulate ensures that every array element refers to a location.

The set *State* is not vacuous, we can construct a mathematical model for *State* which satisfies the above three postulates. The values of senses or concepts (type $(s \rightarrow \tau)$) can then be derived accordingly.

The set of states *State* can be taken as a non-empty subset of functions $\sigma : REF \longrightarrow \mathcal{N}$ which are closed under the update and satisfies the properness postulates. The meaning of each constant symbol $X \in REF$ is $[\![X]\!] = \lambda\sigma \cdot \sigma(X)$, and the meaning of each constant symbol $A \in ARRAY$ is $[\![A]\!] = \lambda k \cdot [\![A^k]\!]$. We call $[\![X]\!]$ a *reference*, which is a projection function. The meaning of an array is then a row of references.

5 Syntax and Semantics of IL

5.1 Basic Symbols

We denote our logical language by **IL**. The set of types $TYPE$ is defined inductively as follows:

1. $n, t \in TYPE$,
2. $\tau \in TYPE \Rightarrow (s \rightarrow \tau) \in TYPE$,
3. $\tau_1, \tau_2 \in TYPE \Rightarrow (\tau_1 \rightarrow \tau_2) \in TYPE$.

The set of basic symbols

$$S = S_1 \cup S_m \cup S_\lambda \cup ARRAY \cup REF_g \cup REF_a$$

where

$S_1 = \{0,\ 1,\ +,\ -,\ \times,\ =,\ >,\ true,\ \neg,\ \wedge,\ if\_then\_else\_fi\}$
$\quad \cup \{\exists n \mid n \in VAR_{\mathsf{n}}\}$
$S_m = \{\hat{}\,,\ \check{}\,,\ <>\}$
$S_\lambda = \{\lambda x \mid x \in VAR_{\mathsf{n}} \cup VAR_{(\mathsf{s} \to \mathsf{n})} \cup VAR_{(\mathsf{s} \to (\mathsf{s} \to \mathsf{n}))} \cup VAR_{(\mathsf{s} \to \mathsf{t})}\}$
$\quad \cup \{()\}$
$ARRAY = \{L,\ A,\ ...\}\quad L,\ A,... \in CON_{(\mathsf{n} \to (\mathsf{s} \to \mathsf{n}))}$
$REF_g = \{X,\ Y,...\}\quad X,... \in CON_{(\mathsf{s} \to \mathsf{n})}$
$REF_a = \{A^1, A^2, ..\}\quad A^1, A^2, .. \in CON_{(\mathsf{s} \to \mathsf{n})}$

S_1 is the set of symbols for the first order arithmetic. In our definition S_1 is not finite. S_m is called the set of *modal operators*, where $\hat{}$ is called the *intension* operator, $\check{}$ is called the *extension* operator, and <> is called the *state switcher*. S_λ contains the *functional abstraction* symbol, and the *functional application* symbol. $ARRAY$ is the set of *array constants* . REF_g contains the set of *global reference* symbols. REF_a contains the set of reference symbols for array elements. REF_g and REF_a are disjoint. Let $REF = REF_g \cup REF_a$.

Note: Usually we use X to range over all symbols in REF and A to range over all symbols in $ARRAY$. The set of variables of all types is denoted by VAR, and the set of constants of all types is denoted by CON.

5.2 Meaningful Expressions

The set of meaningful (well-formed) expressions ME_τ of any type τ is defined inductively as follows. In general, α, β... range over expressions of arbitrary type, φ ranges over ME_{t}, ε ranges over ME_{n}, δ (array index) ranges over ME_{n}, and ξ ranges over $ME_{(\mathsf{s} \to \mathsf{n})}$.

1. $0,\ 1,\ \in ME_{\mathsf{n}}, true \in ME_{\mathsf{t}}, X, A^k \in ME_{(\mathsf{s} \to \mathsf{n})}, A \in ME_{(\mathsf{n} \to (\mathsf{s} \to \mathsf{n}))}$,
2. $x \in VAR_\tau \Rightarrow x \in ME_\tau$,
3. $\varepsilon_1,\ \varepsilon_2 \in ME_{\mathsf{n}} \Rightarrow (\varepsilon_1 + \varepsilon_2),\ (\varepsilon_1 - \varepsilon_2),\ (\varepsilon_1 \times \varepsilon_2) \in ME_{\mathsf{n}}$,
4. $\varepsilon_1,\ \varepsilon_2 \in ME_{\mathsf{n}} \Rightarrow (\varepsilon_1 = \varepsilon_2),\ (\varepsilon_1 < \varepsilon_2) \in ME_{\mathsf{t}}$,
5. $\varphi \in ME_{\mathsf{t}} \Rightarrow \neg\varphi \in ME_{\mathsf{t}}$,
6. $\varphi_1,\ \varphi_2 \in ME_{\mathsf{t}} \Rightarrow (\varphi_1 \wedge \varphi_2) \in ME_{\mathsf{t}}$,
7. $\varphi \in ME_{\mathsf{t}},\ \varepsilon_1, \varepsilon_2 \in ME_{\mathsf{n}} \Rightarrow if\ \varphi\ then\ \varepsilon_1\ else\ \varepsilon_2 fi \in ME_{\mathsf{n}}$,
8. $\varphi \in ME_{\mathsf{t}}, n \in ME_{\mathsf{n}} \Rightarrow (\exists n \varphi) \in ME_{\mathsf{t}}$,
9. $x \in VAR_{\tau_1}, \alpha \in ME_{\tau_2} \Rightarrow (\lambda x \cdot \alpha) \in ME_{(\tau_1 \to \tau_2)}$,
10. $\alpha \in ME_{(\tau_1 \to \tau_2)}, \beta \in ME_{\tau_1} \Rightarrow \alpha(\beta) \in ME_{\tau_2}$,
11. $\alpha \in ME_\tau \Rightarrow (\hat{}\alpha) \in ME_{(\mathsf{s} \to \tau)}$,
12. $\alpha \in ME_{(\mathsf{s} \to \tau)} \Rightarrow (\check{}\alpha) \in ME_\tau$,
13. $\alpha \in ME_\tau, \varepsilon \in ME_{\mathsf{n}}, \xi \in ME_{(\mathsf{s} \to \mathsf{n})} \Rightarrow (\alpha<\xi/\varepsilon>) \in ME_\tau$,

Note

A state switcher gives a syntactic version of substitution (see Theorem 6). Typically, ξ is a reference X. We allow ξ to be array element $A(\delta)$. For the usage of state switchers see section 7.1, definition of (v:=e)$'$. Whenever parentheses are omitted, $\neg$, $\hat{}$, $\check{}$ have the highest binding power. For example $\check{}q{<}\xi/\varepsilon{>}$ should be read as $(\check{}q){<}\xi/\varepsilon{>}$. The numeral $\overline{m}$ stands for $1+1+..+1$ with m 1's, and $(\varphi_1 \longrightarrow \varphi_2)$ is a short form of $\neg(\varphi_1 \wedge (\neg\varphi_2))$.

5.3 Semantics

Let $State = \mathcal{N}^{REF}$, the set of functions from constant symbols $X, Y, ., A^1, A^2 ...$ to natural numbers. A *generalized frame* is a family of domains $\{\mathcal{D}_\tau\}_{\tau \in TYPE}$ satisfying

1. $\mathcal{D}_{\mathsf{n}} = \mathcal{N}, \mathcal{D}_{\mathsf{t}} = \mathcal{B}$,
2. $\mathcal{D}_{(\mathsf{s} \rightarrow \tau)}$ is a non-empty subset of $\mathcal{D}_\tau{}^{State}$, it includes all the functions inductively defined in the following semantic equations.
3. $\mathcal{D}_{(\tau_1 \rightarrow \tau_2)}$ is a non-empty subset of $\mathcal{D}_{\tau_2}{}^{\mathcal{D}_{\tau_1}}$, it includes all the functions inductively defined in the following semantic equations.

A *generalized model* $\mathcal{M}$ is a system $((\mathcal{D}_\tau)_{\tau \in TYPE}, \kappa)$, where $\{\mathcal{D}_\tau\}_{\tau \in TYPE}$ is a generalized frame based on $State$ and κ is a mapping assigning values to constants :

1. $\kappa(0) = \mathbf{0},\ \kappa(1) = \mathbf{1},\ \kappa(true) = \boldsymbol{true}$
2. $\kappa(X) = \lambda\sigma \cdot \sigma(X)$
3. $\kappa(A^k) = \lambda\sigma \cdot \sigma(A^k)$
4. $\kappa(A) = \lambda k \in \mathcal{N} \cdot \lambda\sigma \cdot \sigma(A^k)$

Let Val be the set of valuations over $\mathcal{M}$. Each element ρ in Val is a variable assignment function which assigns to each variable x of type τ a value in the domain $\mathcal{D}_\tau$. Assume the types of the expressions are the same as the types indicated in the syntactic definition in the previous section, σ ranges over $State$ and ρ ranges over Val. The values of the expressions in **IL** are defined inductively relative to σ and ρ as follows.

1. $[\![C]\!]\ \sigma\rho = \kappa(C)$ where $C \in CON$
2. $[\![x]\!]\ \sigma\rho = \rho(x)$ where $x \in VAR$
3. $[\![\varepsilon_1 + \varepsilon_2]\!]\ \sigma\rho = [\![\varepsilon_1]\!]\sigma\rho + [\![\varepsilon_2]\!]\sigma\rho$ and similar for $-, \times$
4. $[\![\varepsilon_1 = \varepsilon_2]\!]\ \sigma\rho = \begin{cases} \boldsymbol{true} & \text{if } [\![\varepsilon_1]\!]\sigma\rho = [\![\varepsilon_2]\!]\sigma\rho \\ \boldsymbol{false} & \text{otherwise} \end{cases}$
5. $[\![\varepsilon_1 < \varepsilon_2]\!]\ \sigma\rho = \begin{cases} \boldsymbol{true} & \text{if } [\![\varepsilon_1]\!]\sigma\rho < [\![\varepsilon_2]\!]\sigma\rho \\ \boldsymbol{false} & \text{otherwise} \end{cases}$
6. $[\![\neg\varphi]\!]\ \sigma\rho = \begin{cases} \boldsymbol{false} & \text{if } [\![\varphi]\!]\sigma\rho = \boldsymbol{true} \\ \boldsymbol{true} & \text{otherwise} \end{cases}$

7. $[\![\varphi_1 \wedge \varphi_2]\!]\ \sigma\rho \;=\; \begin{cases} \textbf{\textit{true}} & \text{if } [\![\varphi_1]\!]\sigma\rho \;=\; \textbf{\textit{true}}, \text{ and } [\![\varphi_2]\!]\sigma\rho \;=\; \textbf{\textit{true}} \\ \textbf{\textit{false}} & \text{otherwise} \end{cases}$

8. $[\![\mathit{if}\ \varphi\ \mathit{then}\ \varepsilon_1\ \mathit{else}\ \varepsilon_2\ \mathit{fi}]\!]\ \sigma\rho \;=\; \begin{cases} [\![\varepsilon_1]\!]\sigma\rho & \text{if } [\![\varphi]\!]\sigma\rho \;=\; \textbf{\textit{true}} \\ [\![\varepsilon_2]\!]\sigma\rho & \text{otherwise} \end{cases}$

9. $[\![\exists n\varphi]\!]\ \sigma\rho \;=\; \begin{cases} \textbf{\textit{true}} & \text{if } [\![\varphi]\!]\sigma\rho\{\,n/\boldsymbol{n}\} \;=\; \textbf{\textit{true}} \text{ for some } \boldsymbol{n} \in \mathcal{N} \\ \textbf{\textit{false}} & \text{otherwise} \end{cases}$

10. $[\![\lambda x \cdot \alpha]\!]\ \sigma\rho \;=\; \lambda \boldsymbol{d} \cdot [\![\alpha]\!]\sigma\rho\{\,x/\boldsymbol{d}\}$ where $\boldsymbol{d} \in \mathcal{D}_{\tau_1}$

11. $[\![\alpha(\beta)]\!]\ \sigma\rho \;=\; ([\![\alpha]\!]\sigma\rho)([\![\beta]\!]\sigma\rho)$ function application

12. $[\![\,\hat{}\,\alpha]\!]\ \sigma\rho \;=\; \lambda\sigma' \cdot [\![\alpha]\!]\sigma'\rho$

13. $[\![\,\check{}\,\alpha]\!]\ \sigma\rho \;=\; ([\![\alpha]\!]\sigma\rho)(\sigma)$

14. $[\![\alpha{<}\xi/\varepsilon{>}]\!]\ \sigma\rho \;=\; \begin{cases} [\![\alpha]\!]\sigma\{\,X/[\![\varepsilon]\!]\sigma\rho\}\ \rho & \text{if } [\![\xi]\!]\sigma\rho = [\![X]\!] \\ & \text{for some } X \in \mathit{REF} \\ [\![\alpha]\!]\sigma\rho & \text{otherwise} \end{cases}$

Remarks :

1. By update postulate, for all $X,Y \in \mathit{REF}, X \not\equiv Y \Rightarrow$ there exists σ, such that $[\![X]\!](\sigma) \neq [\![Y]\!](\sigma)$, therefore (for all $\sigma, [\![X]\!](\sigma) = [\![Y]\!](\sigma)) \Rightarrow X \not\equiv Y$. This guarantees that the X in equation 14 is unqiue.

2. The interpretation of terms is 'two dimensional'. All quantifier variables are interpreted with valuations ρ and state modifiers are interpreted with states σ. Equations 12,13,14 are interesting cases.

3. For our translated expressions of program expressions, the expression $\xi \in ME_{(\mathsf{s} \to \mathsf{n})}$ usually ranges over reference constants, hence in 14, it is always the first case which holds. When $\xi \equiv X$ we have the typical case

$$[\![\alpha{<}X/\varepsilon{>}]\!]\ \sigma\rho \;=\; [\![\alpha]\!]\sigma\{\,X/[\![\varepsilon]\!]\sigma\rho\}\ \rho$$

When $\xi \equiv A(\delta)$ we have

$$[\![\alpha{<}(A(\delta))/\varepsilon{>}]\!]\sigma\rho \;=\; [\![\alpha]\!]\sigma\{A^k/[\![\varepsilon]\!]\sigma\rho\}\ \rho$$

where $[\![\delta]\!]\sigma\rho \;=\; k$.

Two expressions $\alpha, \beta \in ME$ are *semantically equivalent*, denoted by $\alpha \cong \beta$ if and only if for all $\sigma \in \mathit{State}$ and for all $\rho \in \mathcal{V}al$,

$$[\![\alpha]\!]\sigma\rho \;=\; [\![\beta]\!]\sigma\rho$$

An expression α is *rigid* if for all $\sigma_1, \sigma_2 \in \mathit{State}$, and $\rho \in \mathcal{V}al$,

$$[\![\alpha]\!]\sigma_1\rho \;=\; [\![\alpha]\!]\sigma_2\rho.$$

5.4 State-Switcher Reductions

Theorem 1 For any $\xi \in ME_{(\mathrm{s} \to \mathrm{n})}$, $\varepsilon \in ME_{\mathrm{n}}$,

1. $C<\xi/\varepsilon> \;\cong\; C$ for all $C \in CON$
2. $v<\xi/\varepsilon> \;\cong\; v$ for all $v \in VAR$
3. $(\varepsilon_1 + \varepsilon_2)<\xi/\varepsilon> \;\cong\; \varepsilon_1<\xi/\varepsilon> + \varepsilon_2<\xi/\varepsilon>$
   similar for $-, \times, =, <$
4. $(\neg\varphi)<\xi/\varepsilon> \;\cong\; \neg(\varphi<\xi/\varepsilon>)$
5. $(\varphi_1 \wedge \varphi_2)<\xi/\varepsilon> \;\cong\; \varphi_1<\xi/\varepsilon> \wedge \varphi_2<\xi/\varepsilon>$
6. $(\mathit{if}\ \varphi\ \mathit{then}\ \varepsilon_1\ \mathit{else}\ \varepsilon_2\ \mathit{fi})<\xi/\varepsilon>$
   $\cong\ \mathit{if}\ \varphi<\xi/\varepsilon>\ \mathit{then}\ \varepsilon_1<\xi/\varepsilon>\ \mathit{else}\ \varepsilon_2<\xi/\varepsilon>\ \mathit{fi}$,
7. $(\exists n\varphi)<\xi/\varepsilon> \;\cong\; \exists n[\varphi<\xi/\varepsilon>]$ where $n \notin \mathbf{free}(\xi) \cup \mathbf{free}(\varepsilon)$
8. $(\lambda x \cdot \alpha)<\xi/\varepsilon> \;\cong\; \lambda x \cdot (\alpha<\xi/\varepsilon>)$ where $x \notin \mathbf{free}(\xi) \cup \mathbf{free}(\varepsilon)$
9. $\alpha(\beta)<\xi/\varepsilon> \;\cong\; \alpha<\xi/\varepsilon>(\beta<\xi/\varepsilon>)$
10. $({}^\wedge\alpha)<\xi/\varepsilon> \;\cong\; {}^\wedge\alpha$

Proof : By the semantic definition of the state switchers.

Theorem 2 For all $X, Y \in REF, A, B \in ARRAY,\ \delta \in ME_{\mathrm{n}}$,

1. $({}^\vee X)<X/\varepsilon> \;\cong\; \varepsilon$
2. $({}^\vee Y)<X/\varepsilon> \;\cong\; {}^\vee Y$ where X, Y are distinct.
3. $({}^\vee X)<A(\delta)/\varepsilon> \;\cong\; {}^\vee X$
4. $({}^\vee(A(\delta)))<X/\varepsilon> \;\cong\; {}^\vee(A(\delta<X/\varepsilon>))$
5. $({}^\vee(A(\delta')))<A(\delta)/\varepsilon> \;\cong$
   $\mathit{if}\ (\delta'<A(\delta)/\varepsilon> = \delta)\ \mathit{then}\ \varepsilon\ \mathit{else}\ {}^\vee(A(\delta'<A(\delta)/\varepsilon>))\ \mathit{fi}$
6. $({}^\vee(B(\delta')))<A(\delta)/\varepsilon> \;\cong\; {}^\vee(B(\delta<A(\delta)/\varepsilon>))$

Proof : By the semantic definition of the state switchers.

Note: The state switcher reduction is not total. The terms like ${}^\vee x<X/\varepsilon>$, ${}^\vee X<x/\varepsilon>$ can not be simplified using the above theorems.

Let $\alpha[x/\beta]$ denote the term obtained by replacing all free occurrences of x in α by β, with possibly renaming of bound variable in α.

Theorem 3 (β-reduction) Let $(\lambda x \cdot \alpha)(\beta) \in ME_\tau$ and either one of the following two conditions holds : (1) β is rigid, (2) no occurrence of x in α lies within the scope of ${}^\wedge$ and $<>$, then

$$(\lambda x \cdot \alpha)(\beta) \;\cong\; \alpha[x/\beta]$$

Proof: This is an extension to [Janssen 86] and [Dowty 85, p.178]. To show for any $\sigma \in State, \rho \in Val$

$$[\![(\lambda x \cdot \alpha)(\beta)]\!]\sigma\rho = [\![\alpha[x/\beta]]\!]\sigma\rho$$

Let $d = [\![\beta]\!]\sigma\rho$.

$[\![(\lambda x \cdot \alpha)(\beta)]\!]\sigma\rho$
$=[\![\lambda x \cdot \alpha]\!]\sigma\rho([\![\beta]\!]\sigma\rho)$ by definition
$=(\lambda x \cdot [\![\alpha]\!]\sigma\rho\{\, x/x\}\,)(d)$ by definition
$=[\![\alpha]\!]\sigma\rho\{\, x/d\}$

That is to prove

$$[\![\alpha]\!]\sigma\rho\{\, x/d\} = [\![\alpha[x/\beta]]\!]\sigma\rho$$

We will prove this by induction on α. The basic part can be shown easily. We show only the interesting induction parts. Suppose the induction hypothesis.

1. $\alpha \equiv\ \hat{}\gamma$

1.1 Suppose β is rigid

$LHS \equiv [\![\hat{}\gamma]\!]\sigma\rho\{\, x/d\}$
$= \lambda\sigma' \cdot [\![\gamma]\!]\sigma'\rho\{\, x/d\}$ by definition
$= \lambda\sigma' \cdot [\![\gamma]\!]\sigma'\rho<x/[\![\beta]\!]\sigma'\rho$ since β is rigid, $d = [\![\beta]\!]\sigma\rho = [\![\beta]\!]\sigma'\rho$
$= \lambda\sigma' \cdot [\![\gamma[x/\beta]]\!]\sigma'\rho$ by induction hypothesis
$= [\![\hat{}(\gamma[x/\beta])]\!]\sigma\rho$ by definition
$= [\![(\hat{}\gamma)[x/\beta]]\!]\sigma\rho$ by definition
$\equiv RHS$

1.2 Suppose x does not occur inside the scope of $\hat{}$, then $x \notin$ **free**(γ)

$LHS \equiv [\![\hat{}\gamma]\!]\sigma\rho\{\, x/d\}$
$= \lambda\sigma' \cdot [\![\gamma]\!]\sigma'\rho\{\, x/d\}$ by definition
$= \lambda\sigma' \cdot [\![\gamma]\!]\sigma'\rho$ since $x \notin$ **free**(γ)
$= [\![\hat{}\gamma]\!]\sigma\rho$ by definition
$= [\![(\hat{}\gamma)[x/\beta]]\!]\sigma\rho$ since $x \notin$ **free**(γ)
$\equiv RHS$

2. $\alpha \equiv \gamma<\xi/\varepsilon>$ and let $n = [\![\varepsilon]\!]\sigma\rho\{\, x/d\}$

$LHS \equiv [\![\gamma<\xi/\varepsilon>]\!]\sigma\rho\{\, x/d\}$

$$= \begin{cases} [\![\gamma]\!]\sigma\{\, X/n\}\ \rho\{\, x/d\} & \text{if } [\![\xi]\!]\sigma\rho\{\, x/d\} = [\![X]\!] \\ & \text{for some } X \in REF \\ [\![\gamma]\!]\sigma\rho\{\, x/d\} & \text{otherwise} \end{cases}$$

$RHS \equiv [\![(\gamma<\xi/\varepsilon>)[x/\beta]]\!]\sigma\rho$
$= [\![\gamma[x/\beta]<\xi[x/\beta]/\varepsilon[x/\beta]>]\!]\sigma\rho$
by induction hypothesis
we have $[\![\varepsilon[x/\beta]]\!]\sigma\rho = [\![\varepsilon]\!]\sigma\rho\{\, x/d\} = n$

$$= \begin{cases} [\![\gamma[x/\beta]]\!]\sigma\{\, X/n\}\ \rho & \text{if } [\![\xi[x/\beta]]\!]\sigma\rho = [\![X]\!] \\ & \text{for some } X \in REF \\ [\![\gamma[x/\beta]]\!]\sigma\rho & \text{otherwise} \end{cases}$$

by definition

$$= \begin{cases} [\![\gamma]\!]\sigma\{\, X/n\}\ \rho\{\, x/d'\} & \text{if } [\![\xi]\!]\sigma\rho\{\, x/d\} = [\![X]\!] \\ & \text{for some } X \in REF \\ [\![\gamma]\!]\sigma\rho\{\, x/d\} & \text{otherwise} \end{cases}$$

where $d' = [\![\beta]\!]\sigma\{X/n\}\,\rho$
this is by induction hypothesis

2.1 Suppose β is rigid
then $d' = [\![\beta]\!]\sigma\{X/n\}\,\rho = [\![\beta]\!]\sigma\rho = d$
Therefore $LHS = RHS$

2.2 Suppose x does not occur inside the scope of <>, then $x \notin \mathbf{free}(\gamma)$
then $[\![\gamma]\!]\sigma\{X/n\}\,\rho\{x/d\} = [\![\gamma]\!]\sigma\{X/n\}\,\rho\{x/d'\}$
Therefore $LHS = RHS$.

Theorem ($\check{}\,\hat{}$**-cancellation**) For any $\alpha \in ME$, $\check{}\,\hat{}\alpha \cong \alpha$.
Proof : By semantic definition of $\hat{}$ and $\check{}$.

6 Program Expressions and Hoare Formulas

6.1 Syntax of the Programming Language

Integer Constants	m, ...
Integer Identifiers	x, y, ...
Procedure Identifiers	P ::= P^v, P^a, P^n, P^r
Integer Variables	v ::= x | \$z | a[e]
Integer Terms	e ::= m | v | $(e_1 + e_2)$ | $(e_1 - e_2)$ | $(e_1 \times e_2)$ | if b then e_1 else e_2 fi ...
Boolean Terms	b ::= true | $(e_1 = e_2)$ | $(e_1 > e_2)$...
Blocks	K ::= begin new x := e ; S end | begin alias y = v ; S end | begin macro z = v ; S end
Statements	S ::= skip | v := e | $(S_1 ; S_2)$ | if b then S_1 else S_2 fi | K | $P^v(e)$ | $P^a(v)$ | $P^r(v)$ | $P^n(v)$
Procedure Bodies	B ::= B^v | B^a | B^n | B^r

where
$B^v \equiv$ < value x > S end
$B^a \equiv$ < address x > S end
$B^r \equiv$ <value-result x > S end
$B^n \equiv$ < name z > S end

Procedure Declarations D ::= $< P^i \Leftarrow B^i >$
where $i \equiv v, a, r, n$

Programs R ::= $< D_1; D_2; ...D_n ; S >$

A program is *well-formed* if all its procedure names are declared before their calls appear. This means that there exists tree-like dependency structures among the procedures. A procedure P_1 is a parent of another procedure P_2 if the body of P_1 contains a call to P_2. Hence, the procedures which are the leaves of the forest are declared first, then are the procedures at higer levels, and the procedures at the roots are declared last. A well-formed program does not contain recursive procedures. The reason we consider only well-formed programs is because we do not want to deal with fixed-point semantics here. Furthermore, the reference \$z must occur inside the scope of macro-blocks or pass-by-name procedure bodies. The integer identifier j and array identifier l are the reserved identifiers. They are used in the translation in section 7.1, and they are not visible to the programmers.

6.2 Syntax of the Assertion Language

Assertion Integer Variables	$n, \ldots$
Assertion Integer Terms	$e ::= n \mid m \mid x \mid a[e] \mid (e_1 + e_2) \mid (e_1 - e_2) \mid (e_1 \times e_2) \mid$ *if* b *then* e_1 *else* e_2 *fi* $\mid \ldots$
Assertion Booleans	$b ::= true \mid (e_1 = e_2) \mid (e_1 > e_2) \mid \ldots$
Program Assertions	$\phi ::= b \mid \neg\phi \mid (\phi_1 \wedge \phi_2) \mid \exists n \phi$
Hoare Formulas	$F ::= \{\phi_1\}\, S \{\phi_2\} \mid \{\phi_1\}\, R \{\phi_2\}$

7 Translations

Since integer identifiers are location references and we want to treat formal parameters as lambda variables, we translate program identifiers to variables in $VAR_{(s \to n)}$. We take an array as a row of references. An array identifier is a function in $CON_{(n \to (s \to n))}$. Realizing that a subset of states can be characterized by a state predicate of type $(s \to t)$, we regard a program as a backward state predicate transformer [Dijkstra 76] which is expressible in IL. Intuitively, it transforms the predicate which describes the states after the execution of the program to the *weakest* predicate which describes the *largest* set of possible states before the execution. We follow Janssen's approach that the translations of statements and programs are of type $((s \to t) \to t)$.

Integer variable names such as x, a[x] are location references at given states. The name a[x] may denote different references at different states. Hence, we regard integer names as *senses* of references, and we translate them into variables in $VAR_{(s \to (s \to n))}$. The name expressions are translated into IL expression of type $(s \to (s \to n))$ accordingly. In [Horowitz 84, p205], pass-by-name arguments are treated as self-contained parameterless procedures with an environment, and it is evaluated using the environment inside the called procedure. We think that $\hat{}v$ (meaning $\lambda \cdot \sigma[\![v]\!]\sigma\rho$) captures some of the intuition.

Procedure identifiers are translated into functional variables whose values are functions from argument types to predicate transformers. The argument type for pass-by-value is number n, the argument type for pass-by-address and pass-by-value-result is reference $(s \to n)$, and the argument type for pass-by-name is sense of reference $(s \to (s \to n))$.

7.1 Translations of Program Expressions

Let E be an expression in our programming language, and let E′ be the translation of E into intensional logic.

Integer Constants and Identifiers

For each integer identifier x we assign a unique variable x of IL of type $(s \to n)$.

$$\mathbf{m}' \equiv \overline{m} \in CON_n$$
$$\mathbf{x}' \equiv x \in VAR_{(s \to n)}$$
$$\mathbf{z}' \equiv z \in VAR_{(s \to (s \to n))}$$

$$
\begin{aligned}
\mathbf{a}' &\equiv A \in ARRAY \\
\mathsf{p}^{v\prime} &\equiv p^{v} \in VAR_{\mathsf{s} \to (\mathsf{n} \to ((\mathsf{s} \to \mathsf{t}) \to \mathsf{t}))} \\
\mathsf{p}^{a\prime} &\equiv p^{a} \in VAR_{\mathsf{s} \to ((\mathsf{s} \to \mathsf{n}) \to ((\mathsf{s} \to \mathsf{t}) \to \mathsf{t}))} \\
\mathsf{p}^{n\prime} &\equiv p^{n} \in VAR_{\mathsf{s} \to ((\mathsf{s} \to \mathsf{n}) \to ((\mathsf{s} \to \mathsf{t}) \to \mathsf{t}))} \\
\mathsf{p}^{r\prime} &\equiv p^{r} \in VAR_{\mathsf{s} \to ((\mathsf{s} \to \mathsf{n}) \to ((\mathsf{s} \to \mathsf{t}) \to \mathsf{t})}
\end{aligned}
$$

Program Integer Variables ($ME_{(\mathsf{s} \to \mathsf{n})}$)

$$
\begin{aligned}
\mathbf{x}' & \quad \text{see translation for integer identifiers} \\
(\mathsf{a}[\mathsf{e}])' &\equiv \mathsf{a}'(\mathsf{e}') \\
(\$\mathsf{z})' &\equiv {}^{\vee}\mathsf{z}'
\end{aligned}
$$

Integer Terms (ME_{n})

$$
\begin{aligned}
\mathsf{m}' &\equiv \overline{m} \\
\mathsf{v}' &\equiv {}^{\vee}\mathsf{v}' \quad (\mathsf{v}' \text{ see translation of integer variables}) \\
(\mathsf{e}_1 + \mathsf{e}_2)' &\equiv (\mathsf{e}_1' + \mathsf{e}_2') \quad (\text{similar for - and } \times) \\
(\textbf{if } \mathsf{b} \textbf{ then } \mathsf{e}_1 \textbf{ else } \mathsf{e}_2 \textbf{ fi})' &\equiv \mathit{if}\ \mathsf{b}'\ \mathit{then}\ \mathsf{e}_1'\ \mathit{else}\ \mathsf{e}_2\ \mathit{fi}
\end{aligned}
$$

......

Note that the translation of v′ of an integer variable is considered as an *integer term*, it is the *extensional value* of its translation considered as a program variable.

Program Booleans (ME_{t})

$$
\begin{aligned}
\mathsf{true}' &\equiv \mathit{true} \\
(\mathsf{e}_1 = \mathsf{e}_2)' &\equiv (\mathsf{e}_1' = \mathsf{e}_2')
\end{aligned}
$$

......

Statements ($ME_{((\mathsf{s} \to \mathsf{t}) \to \mathsf{t})}$)

From now on, $q \in VAR_{(\mathsf{s} \to \mathsf{t})}$.

$$
\begin{aligned}
\mathbf{skip}' &\equiv \lambda q \cdot {}^{\vee}q \\
(\mathsf{v} := \mathsf{e})' &\equiv \lambda q \cdot {}^{\vee}q\langle \mathsf{v}'/\mathsf{e}' \rangle \\
(\mathsf{S}_1; \mathsf{S}_2)' &\equiv \lambda q \cdot \mathsf{S}_1'({}^{\wedge}(\ \mathsf{S}_2'(q))\ ) \\
\textbf{if } \mathsf{b} \textbf{ then } \mathsf{S}_1 \textbf{ else } \mathsf{S}_2 \textbf{ fi}' &\equiv \lambda q \cdot (\mathsf{b}' \wedge \mathsf{S}_1'(q)) \vee (\neg\mathsf{b}' \wedge \mathsf{S}_2'(q)) \\
\mathsf{K}' & \quad \text{see blocks below} \\
\mathsf{P}^{v}(\mathsf{e})' &\equiv {}^{\vee}\mathsf{P}^{v\prime}(\mathsf{e}') \\
\mathsf{P}^{a}(\mathsf{v})' &\equiv {}^{\vee}\mathsf{P}^{a\prime}(\mathsf{v}') \\
\mathsf{P}^{n}(\mathsf{v})' &\equiv {}^{\vee}\mathsf{P}^{n\prime}({}^{\wedge}\mathsf{v}') \\
\mathsf{P}^{r}(\mathsf{v})' &\equiv {}^{\vee}\mathsf{P}^{r\prime}(\mathsf{v}')
\end{aligned}
$$

Note that the state switchers are used in the assignment statements.

Blocks $(ME_{((s \to t) \to t)})$

$$(\textbf{begin new } \mathbf{x} := \mathbf{e} \,;\, \mathsf{S} \textbf{ end})' \equiv$$
$$\lambda q \cdot \exists n[(((\lambda \mathbf{x}' \cdot \mathsf{S}')(L(n)))({}^\wedge[n = {}^\vee j \wedge {}^\vee q\text{<}j/{}^\vee j+1\text{>}]))\text{<}L(n)/\mathbf{e}'\text{>}]$$

where $n \in VAR_{n}$, $\mathbf{j}' \equiv j \in VAR_{(s \to n)}$ and $\mathbf{1}' \equiv L \in ARRAY$.

$$(\textbf{begin alias } \mathbf{y} = \mathsf{v} \,;\, \mathsf{S} \textbf{ end})' \equiv (\lambda \mathbf{x}' \cdot \mathsf{S}')(\mathsf{v}')$$
$$(\textbf{begin macro } \mathbf{z} = \mathsf{v} \,;\, \mathsf{S} \textbf{ end})' \equiv (\lambda \mathbf{z}' \cdot \mathsf{S}')({}^\wedge \mathsf{v}')$$

Procedure Bodies

$$(<\textbf{ value } \mathbf{x} > \mathsf{S} \textbf{ end})' \equiv$$
$$\lambda m \cdot \lambda q \cdot \exists n[(((\lambda \mathbf{x}' \cdot \mathsf{S}')(L(n)))({}^\wedge[n = {}^\vee j \wedge {}^\vee q\text{<}j/{}^\vee j+1\text{>}]))\text{<}L(n)/m\text{>}]$$

$$(<\textbf{ address } \mathbf{x} > \mathsf{S} \textbf{ end})' \equiv \lambda \mathbf{x}' \cdot \mathsf{S}'$$
$$(<\textbf{ name } \mathbf{z} > \mathsf{S} \textbf{ end})' \equiv \lambda \mathbf{z}' \cdot \mathsf{S}'$$

$$(<\!\textbf{value-result } \mathbf{x} > \mathsf{S} \textbf{ end})' \equiv$$
$$\lambda y \cdot \lambda q \cdot \exists n[(((\lambda \mathbf{x}' \cdot \mathsf{S}')(L(n)))({}^\wedge[n = {}^\vee j \wedge {}^\vee q\text{<}j/{}^\vee j+1\text{><}y/{}^\vee L(n)\text{>}]))\text{<}L(n)/{}^\vee y\text{>}]$$

where $y \in VAR_{(s \to n)}$.

Programs

$$(\, << \mathsf{P}_1 \Leftarrow \mathsf{B}_1 >; < \mathsf{P}_2 \Leftarrow \mathsf{B}_2 >; \ldots < \mathsf{P}_k \Leftarrow \mathsf{B}_k >; \mathsf{S} >\,)' \equiv (\lambda \vec{\mathsf{P}}' \cdot \mathsf{S}')({}^\wedge \vec{\mathsf{B}}')$$

7.2 Translations of Assertions

Assertion Integer Variables (VAR_{n})

For each assertion variable n we associate with it an **IL** variable n.

$$\mathbf{n}' \equiv n$$

Integer Terms (ME_{n})

$$\begin{aligned}
\mathbf{n}' & \quad \text{see variables} \\
\mathbf{m}' &\equiv \overline{m} \\
\mathbf{x}' &\equiv {}^\vee \mathbf{x}' \\
(\mathbf{a}[\mathbf{e}])' &\equiv {}^\vee \mathbf{a}'(\mathbf{e}') \\
(\mathbf{e}_1 + \mathbf{e}_2)' &\equiv (\mathbf{e}_1' + \mathbf{e}_2') \quad \text{similar for } -, \times \\
(\textit{if } \mathbf{b} \textit{ then } \mathbf{e}_1 \textit{ else } \mathbf{e}_2)' &\equiv \textit{if } b' \textit{ then } e_1' \textit{ else } e_2' \textit{ fi}
\end{aligned}$$

......

Atomic Booleans (ME_t)

$$true' \equiv true$$
$$(e_1 = e_2)' \equiv (e_1' = e_2')$$
$$......$$

Assertions (ME_t)

$$b' \quad \text{see booleans}$$
$$(\neg\phi)' \equiv \neg\phi'$$
$$(\phi_1 \wedge \phi_2)' \equiv \phi_1' \wedge \phi_2'$$
$$(\exists n\phi)' \equiv \exists n'\phi'$$

Correctness Formulas (ME_t)

$$(\{\phi_1\}\, S\{\phi_2\}\,)' \equiv \phi_1' \rightarrow S'(\hat{}\phi_2')$$
$$(\{\phi_1\}\, R\{\phi_2\}\,)' \equiv \phi_1' \rightarrow R'(\hat{}\phi_2')$$

8 Results

Suppose $\{ x_1, x_2, .., x_k \}$ is the set of free $(s \rightarrow n)$ variables in the terms α and β. Then the two terms α, β are *equivalent relative to reference instantiation* denoted by $\alpha \simeq \beta$, if for any set of distinct symbols $X_1, X_2, ..X_k \in REF_g$

$$\alpha[\vec{x}/\vec{X}] \cong \beta[\vec{x}/\vec{X}].$$

Lemma 5 For any assertion integer term e, Boolean term b, program integer term e

$$e'<x'/e'> \simeq (e[x/e])'$$

$$b'<x'/e'> \simeq (b[x/e])'.$$

Proof : By simultaneous induction on e and b.

Theorem 6 (Substitution Property of State Switchers)

For any assertion formula ϕ, program term e

$$\phi'<x'/e'> \simeq (\phi[x/e])'.$$

Proof : By induction on ϕ, and use Lemma 5.

Lemma 7 For any assertion integer term e_0, Boolean term b_0, program integer term e, assertion integer term d there exists an assertion integer term e_1, and Boolean term b_1 such that

$$e_0'<a[d]'/e'> \simeq e_1'$$
$$b_1'<a[d]'/e'> \simeq b_1'.$$

Proof : By simultaneous induction on e_0 and b_0, and use the theorems relating arrays.
Theorem 8 For any assertion formula ϕ, program term e, there exists an assertion formula ψ such that

$$\phi'\langle v'/e'\rangle \simeq \psi'.$$

Proof : By induction on ϕ, and use the lemma and theorem above. Also, the integer variable v can allow assertion integer variables as array indices.

Lemma 9 Suppose y does not occur in S. Then

$$(\textbf{begin new } x := e;\ S\ \textbf{end})' \cong (\textbf{begin new } y := e;\ S[x/y]\ \textbf{end})'$$

where S[x/ y] is the statement obtained by replacing all free occurrences of x in S by y.
Proof : Since

$$\lambda x' \cdot S' \cong \lambda y' \cdot S'[x'/y']$$

Similar lemmas can be obtained for alias-blocks and macro-blocks.

Lemma 10 Suppose S does not contain any procedure calls. Then

$$(S[x/1[n]])'(\hat{\ }[n = j \wedge \phi]') \simeq \psi'$$

where ψ does not contain any term $1[m]$, for any $m \neq n$, and any occurrence of j will be in the form of $m = j' + \bar{k}$.
Proof : (outline) By induction on S. Suppose we encounter the next introduction of a new bound integer variable n due to a nested new-block inside S. We can rename the bound variable to m. Hence we will have in the post-condition $m = j'$ and n(the original n) = $j' + 1$. The state switcher $\langle 1[m]'/e'\rangle$ in the translation of the nested blocks will have the effect of replacing $1[m]'$ by e', and $1[n]'\langle 1[m]'/e'\rangle \cong 1[n]'$ because $n \neq m$. Therefore the resultant pre-condition is state-switcher free.

Theorem 11 (Expressibility of Weakest Preconditions)

For any program statement S, S does not contain any procedure call. Then for any program assertion ϕ, there exists a program assertion ψ such that

$$S'(\hat{\ }\phi') \simeq \psi'.$$

Proof : We firstly extend the programming language to allow assertion integer variables as array indices. Then prove by induction on S.

1. S $\equiv$ **skip**

skip$'(\hat{\ }\phi')$	
$\equiv (\lambda q \cdot \check{\ }q)(\hat{\ }\phi')$	by def. of translation
$\cong \check{\ }\hat{\ }\phi'$	by β-reduction theorem
$\cong \phi$	by $\check{\ }\hat{\ }$-cancellation theorem

2. S $\equiv$ v := e

$(v := e)'(\hat{\ }\phi')$	
$\equiv (\lambda q \cdot \check{\ }q\langle v'/e'\rangle)(\hat{\ }\phi')$	by def. of translation

$\cong$ $\check{}\hat{}\phi'$<v'/e'> by β-reduction
$\cong$ ϕ'<v'/e'> by $\check{}\hat{}$-cancellation
$\simeq$ ψ' by Theorem 8

Suppose the induction hypothesis.

3. $S \equiv S_1;S_2$
$(S_1;S_2)'(\hat{}\phi')$
$\equiv (\lambda q \cdot S'(\hat{}(S_2'(q))))(\hat{}\phi')$ by def. of translation.
$\cong S_1'(\hat{}(S_2'(\hat{}\phi')))$ by β-reduction
$\simeq S_1'(\hat{}\psi_1')$ for some ψ_1 by induction hypothesis
$\simeq \psi_2$ for some ψ_2 by induction hypothesis

4. S $\equiv$ **if** b **then** S_1 **else** S_2 **fi**
(**if** b **then** S_1 **else** S_2 **fi**)$'(\hat{}\phi')$
$\equiv (\lambda q \cdot (b' \wedge S_1'(q)) \vee (\neg b' \wedge S_2'(q)))(\hat{}\phi')$ by def. of tran.
$\cong (b' \wedge S_1'(\hat{}\phi')) \vee (\neg b' \wedge S_2'(\hat{}\phi'))$ by β-reduction
$\simeq (b' \wedge \psi_1') \vee (\neg b' \wedge \psi_2')$ for some ψ_1, ψ_2 by induction hypothesis
$\equiv ((b \wedge \psi_1) \vee (\neg b \wedge \psi_2))'$ by def. of translation.

5. S $\equiv$ **begin new x := e ; S end**
(**begin new x := e ; S end**)$'(\hat{}\phi')$
$\equiv (\lambda q \cdot \exists n[(((\lambda x' \cdot S')(L(n)))(\hat{}[n = \check{}j \wedge \check{}q<j/\check{}j+1>]))<L(n)/e'>])(\hat{}\phi')$
$\cong \exists n[(((\lambda x' \cdot S')(L(n)))(\hat{}[n = \check{}j \wedge \phi'<j/\check{}j+1>]))<L(n)/e'>$
by β reduction and $\hat{}\check{}$cancellation
$\cong \exists n[(((\lambda x' \cdot S')(1[n]')(\hat{}[n = j' \wedge \phi'<j'/\check{}j'+1>]))<1[n]'/e'>$
$\cong \exists n[((S[x/1[n]])'(\hat{}[n = j' \wedge (\phi[j/j+1])')) <1[n]'/e'>$
$\simeq \exists n[\psi_1'<1[n]'/e'>]$
by induction hypothesis and Lemma 10, ψ_1 contains no $1[m]$ where $m \not\equiv n$ and ψ contains j only in the form of $m' = j' + \overline{k}$
$\cong \exists n[\psi_2']$ where ψ_2 contains no 1 and j
$m' = j' + \overline{k}$ are removed from ψ_1 without harming the truth value.

6. S $\equiv$ **begin alias x = v; S end**
(**begin alias x = v; S end**)$'(\hat{}\phi')$
$\equiv ((\lambda x' \cdot S')(v'))(\hat{}\phi')$ by def. of translation
6.1 when $v \equiv y$
$\cong S'[x'/y'](\hat{}\phi')$ since x' is rigid
$\cong (S[x/y])'(\hat{}\phi')$
$\simeq \psi'$ by induction hypothesis
6.2 when $v \equiv a[e]$
$\cong \exists n[n = e' \wedge S'[x'/a[n]'](\hat{}\phi')]$
$\cong \exists n[n = e' \wedge (S[x/a[n]])'(\hat{}\phi')]$
$\simeq \exists n[n = e' \wedge \psi']$ by induction hypothesis
$\equiv (\exists n[n = e \wedge \psi])'$

7. S $\equiv$ **begin macro z = v; S end**
(**begin macro z = v; S end**)$'(\hat{}\phi')$

$\equiv ((\lambda \cdot z' \cdot S')({}^\wedge v'))({}^\wedge\phi')$	by def. of translation
$\cong S'[z'/v']({}^\wedge\phi')$	by β reduction since ${}^\wedge v'$ is rigid
$\equiv (S[z/v])'({}^\wedge\phi')$	by definition of translation.
$\simeq \psi'$	by induction hypothesis.

Example

To illustrate Theorem 11, suppose we have the following statement and post-condition.

```
S ≡ begin  new x :=1
           begin  alias y = x
                  begin  new x :=2
                         w := w +x +y
                  end
           end
    end
```

$\phi \equiv [w = 6]$

We put index to each bound variable to improve the readability.

$$S' \equiv \lambda q_1 \cdot \exists n_1[(((\lambda\, x' \cdot (\lambda y' \cdot \lambda q_2 \cdot \exists n_2[(((\lambda x' \cdot \lambda q_3 \cdot {}^\vee q_3<w'/{}^\vee w' + {}^\vee x' + {}^\vee y'>)(L(n_2))) ({}^\wedge[n_2 = {}^\vee j \wedge {}^\vee q_2<j/{}^\vee j+1>])<L(n_2)/2>])(x')) (L(n_1)))({}^\wedge[n_1 = {}^\vee j \wedge q_1<j/{}^\vee j+1>]))<L(n_1)/1>]$$

$$\cong \lambda q_1 \cdot \exists n_1[(\exists n_2\, [([n_2 = {}^\vee j \wedge [n_1 = {}^\vee j \wedge {}^\vee q_1<j/{}^\vee j+1>]<j/{}^\vee j+1>]<w/{}^\vee w + {}^\vee(L(n_2)) + {}^\vee(L(n_1))>) <L(n_2)/2>])<L(n_1)/1>]$$

$\phi' \equiv [{}^\vee w = 6]$

$$S'({}^\wedge\phi')$$

$$\cong \exists n_1[(\exists n_2[(\,[n_2 = {}^\vee j \wedge [n_1 = {}^\vee j \wedge [{}^\vee w = 6]<j/{}^\vee j+1>]<j/{}^\vee j+1>]<w/{}^\vee w + {}^\vee(L(n_2)) + {}^\vee(L(n_1))>) <L(n_2)/2>])<L(n_1)/1>]$$

$$\simeq \exists n_1[(\exists n_2[([n_2 = {}^\vee j \wedge [n_1 = {}^\vee j+1 \wedge {}^\vee w = 6]<w])/{}^\vee w + {}^\vee(L(n_2)) + {}^\vee(L(n_1))> <L(n_2)/2>])<L(n_1)/1>]$$

$$\simeq \exists n_1[(\exists n_2[([n_2 = {}^\vee j \wedge [n_1 = {}^\vee j+1 \wedge {}^\vee w + {}^\vee(L(n_2)) + {}^\vee(L(n_1)) = 6]])<L(n_2)/2>])<L(n_1)/1>]$$

$$\simeq \exists n_1[(\exists n_2[[n_2 = {}^\vee j \wedge [n_1 = {}^\vee j+1 \wedge {}^\vee w + 2 + {}^\vee(L(n_1)) = 6]]])<L(n_1)/1>] \quad \text{since } n_1 \neq n_2$$

$$\simeq \exists n_1[\exists n_2[n_2 = {}^\vee j \wedge [n_1 = {}^\vee j+1 \wedge {}^\vee w + 2 + 1 = 6]]]$$

$$\cong (w + 2 + 1 = 6)'$$

Hence $S'({}^\wedge(w = 6)') \simeq (w = 3)'$.

Theorem 12 For any program R, and program assertion ϕ, there exists a program assertion ψ such that

$$R'({}^\wedge\phi') \simeq \psi'.$$

Proof

$$
\begin{aligned}
& \mathsf{R}'({}^{\wedge}\phi') \\
\equiv\ & (<<<\mathsf{P}_1 \Leftarrow \mathsf{B}_1>;..;<\mathsf{P}_k \Leftarrow \mathsf{B}_k>>\ ;\ \mathsf{S}>)'({}^{\wedge}\phi') \\
\equiv\ & ((\lambda \vec{\mathsf{P}}' \cdot \mathsf{S}')({}^{\wedge}\vec{\mathsf{B}}'))({}^{\wedge}\phi') \\
\cong\ & (\mathsf{S}'[\mathsf{P}_1'/\mathsf{B}_1']..[\mathsf{P}_k'/\mathsf{B}_k'])({}^{\wedge}\phi') \text{ by } \beta\text{-reduction}
\end{aligned}
$$

Since R is well-formed, every procedure must be declared before it is used, B_k does not contain any P_i for $i \geq k$. Hence $\mathsf{S}'[\mathsf{P}_1'/\mathsf{B}_1']..[\mathsf{P}_k'/\mathsf{B}_k']$ contains no procedure variable. Using Lemma 9, we can rename the bound integer identifiers inside S to get a statement S_0 such that

$$\mathsf{S}'[\mathsf{P}_1'/\mathsf{B}_1']..[\mathsf{P}_k'/\mathsf{B}_k'] \cong (\mathsf{S}_0[\mathsf{P}_/\mathsf{B}_1]..[\mathsf{P}_k/\mathsf{B}_k])'.$$

Then by the same techniques as in Theorem 9 we can prove there is a ψ such that

$$(\mathsf{S}_0[\mathsf{P}_1/\mathsf{B}_1]..[\mathsf{P}_k/\mathsf{B}_k])'({}^{\wedge}\phi') \simeq \psi'.$$

We have to further consider four different kinds of procedure calls in S.

1. *pass-by-value parameter* For any procedure call $\mathsf{P}^{v}{}_{i}(\mathsf{e})$ inside S or $\vec{\mathsf{B}}$, its translation is ${}^{\vee}\mathsf{P}^{v}{}_{i}'(\mathsf{e}')$ which after substitution becomes

$$
\begin{aligned}
& ({}^{\vee\wedge}\mathsf{B}^{v}{}_{i}')(\mathsf{e}') \\
\cong\ & \mathsf{B}^{v}{}_{i}'(\mathsf{e}') \text{ by } {}^{\vee\wedge}\text{-cancellation} \\
\equiv\ & (<\ \textbf{value}\ \mathbf{x}\ >\ \mathsf{S}_i)'(\mathsf{e}') \\
\equiv\ & (\lambda m \cdot \lambda q \cdot \exists n[\ (((\lambda \mathbf{x}' \cdot \mathsf{S}_i')(L(n)))({}^{\wedge}[n = {}^{\vee}j \wedge {}^{\vee}q<j/{}^{\vee}j+1>]))<L(n)/m>])(\mathsf{e}')
\end{aligned}
$$

Since m does not lie within ${}^{\wedge}$ and <>, by β reduction, we have

$$
\begin{aligned}
\cong\ & \lambda q \cdot \exists n[\ (((\lambda \mathbf{x}' \cdot \mathsf{S}_i')(L(n)))({}^{\wedge}[n = {}^{\vee}j \wedge {}^{\vee}q<j/{}^{\vee}j+1>]))<L(n)/\mathsf{e}'>] \\
\equiv\ & (\textbf{begin new } \mathbf{x} := \mathbf{e}\ ;\ \mathsf{S}_i\ \textbf{end})'
\end{aligned}
$$

2. *pass-by-address parameter* For any procedure call $\mathsf{P}^{a}{}_{i}(\mathsf{v})$ inside S or $\vec{\mathsf{B}}$ its translation is ${}^{\vee}\mathsf{P}^{a}{}_{i}'(\mathsf{v}')$ which after substitution becomes

$$
\begin{aligned}
& ({}^{\vee\wedge}\mathsf{B}^{a}{}_{i}')(\mathsf{v}') \\
\cong\ & \mathsf{B}^{a}{}_{i}'(\mathsf{v}') \\
\equiv\ & (<\ \textbf{address}\ \mathbf{x}\ >\ \mathsf{S}_i)'(\mathsf{v}') \\
\equiv\ & (\lambda \mathbf{x}' \cdot \mathsf{S}_i')(\mathsf{v}') \\
\equiv\ & (\textbf{begin alias } \mathbf{x} = \mathbf{v}\ ;\ \mathsf{S}_i\ \textbf{end})'
\end{aligned}
$$

3. *pass-by-name parameter* Similar.

4. *pass-by-value-result parameter* For any procedure call $\mathsf{P}^{r}{}_{i}(\mathsf{v})$ inside S or $\vec{\mathsf{B}}$ its translation is ${}^{\vee}\mathsf{P}^{r}{}_{i}'(\mathsf{v}')$ which after substitution becomes

$$
\begin{aligned}
& ({}^{\vee\wedge}\mathsf{B}^{r}{}_{i}')(\mathsf{v}') \\
\cong\ & \mathsf{B}^{r}{}_{i}'(\mathsf{v}') \\
\equiv\ & (<\ \textbf{value-result}\ \mathbf{x}\ >\ \mathsf{S}_i)'(\mathsf{v}') \\
\equiv\ & (\lambda y \cdot \lambda q \cdot \exists n[(((\lambda \mathbf{x}' \cdot \mathsf{S}_i')(L(n)))({}^{\wedge}[n = {}^{\vee}j \wedge {}^{\vee}q<j/{}^{\vee}j+1><y/{}^{\vee}L(n)>]))<L(n)/{}^{\vee}y>])(\mathsf{v}')
\end{aligned}
$$

4.1 when $\mathsf{v} \equiv \mathbf{w}$

$$
\begin{aligned}
\cong\ & \lambda q \cdot \exists n[(((\lambda \mathbf{x}' \cdot \mathsf{S}_i')(L(n)))({}^{\wedge}[n = {}^{\vee}j \wedge {}^{\vee}q<j/{}^{\vee}j+1><\mathbf{w}'/{}^{\vee}L(n)>)))<L(x)/{}^{\vee}\mathbf{w}'>] \\
\cong\ & (\textbf{begin new } \mathbf{x} := \mathbf{w}\ ;\ \mathsf{S}_i\ ;\ \mathbf{w} := \mathbf{x}\ \textbf{end})'
\end{aligned}
$$

4.2 when $\mathsf{v} \equiv \mathsf{a}[\mathsf{e}]$

$$\cong \lambda q \cdot \exists n[n = \mathbf{e}' \wedge (\textbf{begin new x := a}[n] \; ; \; \mathsf{S}_i \; ; \; \mathbf{a}[n] := \mathbf{x} \; \textbf{end})'(q)] \quad \Box$$

9 Comparisons with Janssen's Approach

There are some differences in our approach from Janssen's. Firstly, we interpret the constants rigidly. This is suggested by Goodman as a kind of simplification so that we do not need the rigidness postulate as in [Janssen 86]. In Janssen's approach, he allows the integer identifier **xory** to denote either the reference of **x** or **y** depending on the current state. Secondly, our array type is $n \rightarrow (s \rightarrow n)$ while Janssen's array type is $s \rightarrow (s \rightarrow n)$. This is a technical reason so as to pass array elements as parameters like double(a[x]). In Janssen's approach, array assignments $a_1:=a_2$ are possible. Thirdly, Janssen translates integer identifiers to *constants* of type $(s \rightarrow n)$, we translate integer identifiers to *variables* of type $(s \rightarrow n)$. Our reason is to treat procedure parameters as lambda variables.

10 Conclusion

We have given a translation of statements and Hoare logic formulas for a programming language which allows blocks, procedures and the four parameter passing mechanisms into IL expressions. Since the semantics of IL is compositional, we have obtained a compositional semantics for the programming language. Currently we are applying intensional logic to pointers and record structures.

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Models for discourse markers

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1. Introduction

Several theories have been proposed for the treatment of pronouns in discourse, the most well known are discourse representation theory (DRT, Kamp 1981) and dynamic Montague grammar (DMG, Groenendijk and Stokhof 1989, 1990). To a certain degree these approaches are the same. Some formal logical representation is constructed in a systematic way from a discourse. In this process each term introduces a discourse marker in the logical representation, and during the process of building the representation somehow the links between the different discourse markers are laid. The main difference is that Groenendijk and Stokhof work in the compositional Montagovian tradition, whereas Kamp presents a unorthodox approach in which representations are essential.

In this paper we will formalize certain properties of discourse markers by means of a formal restriction on models: 'the 'update postulate'. The main issue of this paper is the question whether a set-theoretical model can be constructed which satisfies this postulate. It seems reasonable to assume that the question arise in both in discourse representation theory and in dynamic Montague grammar. The difference between the two approaches becomes evident when the answers to the question are considered. The first model that we will construct turns out to be acceptable for only one of the theories. Then a second, more complex model will be build. Finally some general characteristics of models satisfying the postulate are presented.

2. The update postulate

Below we will make three observations concerning discourse markers. These observations will below be formalized as 'the update postulate'.

i) In the discourse

A man enters Mary's room. He smiles.

the *he* in the second sentence introduces a discourse marker, say dm_3. This discourse marker gets as interpretation the man who entered Mary's room. For every individual in the model it is possible think of a situation (of a possible world) in which he enters Mary's room. Hence, under suitable circumstances, any individual can be the value of dm_3. Generally formulated, a discourse marker of type τ, can have any object of type τ as value.

ii) Consider the discourse

A man enters Mary's room . He smiles. A woman enters Mary's room. She smiles. She sees him.

In the first sentence a discourse marker is introduced for *a man*, and the discourse marker of the second sentence gets as interpretation this man. Third and fourth sentence sentence introduce new discourse markers. But the *him* in the last sentence is again the man who entered. The interpretation of the discourse markers referring to the man from are not changed by the process of interpreting the discourse markers in the third and fourth sentence. Generally formulated, the interpretation of a discourse marker, leaves the interpretation of all other discourse markers untouched.

iii) There are discourse markers of any type. For instance, the *she* in the second sentence of the discourse

Miss Universe is well known. She is elected every year.

does not say the one and the same individual is elected every year. It rather says that the property of being elected every year is a property of the individual concept *Miss Universe*. So the *she* introduces a discourse marker of type <s,e>. And the *it* in

John loves Mary. He likes it.

denotes the property of loving Mary . Hence the *it* introduces a discourse marker of type <s,<e,t>>.

The above observations give a motivation for

the update postulate

For each discourse marker dm_τ each object d of type τ, and each state s, there is a state s' such that

$[dm_\tau]_{s'} = d,$

whereas for all other discourse markers *dm'* holds

$[dm']_{s'} = [dm']_s.$

Although Kamp (1981) only deals with discourse markers of type e, the approach can in principle be extended with discourse markers of other types. Then the update postulate can be incorporated in that theory.

The update postulate originates from Janssen & Van Emde Boas 1977, where it is introduced for describing a phenomenon in the semantics of programming languages. The assignment statement is an instruction in programming languages that gives a value to an identifier. This value can be any value of the appropriate type, and other identifiers remain unchanged by such an assignment statement. So the update postulate formalizes the semantics effects of such assignments on identifiers. For more details, see Janssen & Van Emde Boas 1981 or Janssen 1986.

3. The problem

The combination of the update postulate with discourse markers of type <s,τ> gives rise to a fundamental problem. Let us consider the simplest case: type <s,e>. The update postulate would require that for discourse marker *dm*, each function $d \in D_{<s,e>}$ and each state $s \in S$ there is a state $s' \in S$ such that $[dm]_{s'} = d$. As a matter of fact, the number of functions in $D_e{}^S$ is $|D_e|^{|S|}$, so this is the number of possible values for the right hand side of the equality $[dm]_{s'} = d$. For the left hand side this equals $|S|$. Elementary set theory learns us that $|S| < |D_e|^{|S|}$. For instance, if $|S| = 10$ and $|D_e| = 2$ then $|D_e|^{|S|} = 2^{10} = 1024$. So there are much more values than states. This also holds for the infinite case. If S is countable and $|D_e|$ is finite, then is $|D_e|^{|S|}$

uncountable. This means that neither in the finite case, nor in the infinite case there are enough states to have a state for each value in $D_e{}^S$.

This raises the question whether it is possible at all to satisfy the update postulate, and whether dynamic Montague grammar and discourse representation theory have models at all.

There is, however, hope for a positive answer. The update postulate requires that for each value in $D_{<s,e>}$ there is a state in which a given discourse marker has that value. Consequently, there have to be more states than elements in $D_{<s,e>}$, hence $|S| \leq |D_{<s,e>}|$. Since set theory learns us that $|S| < |D_e|^{|S|}$, we might conclude that $D_{<s,e>} \subset D_e{}^S$. This means that we have to work with a generalized model in the sense of Henkin 1950, see also Gallin 1975.

The idea to use a generalized model does not solve the fundamental problem, since we do not know yet whether a generalized model exists that satisfies the update postulate. It gives us a direction in which a solution might be found. And that is what will be done in the next sections.

The above discussion gives us the following heuristics . If we would take the set S as primitive, then the set $D_e{}^S$ has hardly any structure. In such a situation it is difficult to indicate some subset as $D_{<s,e>}$. Therefore we will not take S as primitive, but build it from values for discourse markers.

The same problem as described above arises in the semantics of programming languages. Pointers are identifiers which have identifiers as values. Such pointers can be considered as functions of type <s, <s,e>> and the above above problem arises for assignments to pointers. The solution of Janssen & van Emde Boas 1977 can, however not be used in the situation considered here.

4. Presentation of a simple model

In this section we will consider a simple, but not ideal, model satisfying the update postulate, and in the next section a richer model. It is for two reasons useful to present it the simple model. Firstly, it is satisfactory as a model for discourse representation theory. Secondly, it is useful as a preparation for the richer model that will be presented in the next section.

The model formalizes the following two observations concerning discourse markers:

i.) The interpretation of constants (like *John* or *walk*) is independent of the interpretation of discourse markers. For instance, in the discourse

A man enters. He smiles.

the meaning of *smiles* is independent of the person referred to by the discourse marker introduced by *he*.

ii) The interpretation of a discourse marker is fully determined by the previous discourse. In particular, if a discourse marker is of the type of type e, then it is associated with some entity that is introduced in the previous discourse. Hence, given the previous discourse, the value associated with a discourse marker is a value from the ordinary model (i.e. the model for sentences with discourse markers). Below, two examples will illustrate this point. Consider the discourse

A man enters. He cries.

The discourse marker that corresponds with the *he* in the second sentence refers to an entity that is introduced by the term *A man* in the first sentence, and that entity would also arise in the standard Montague model for the sentence *A man enters* (i.e in the model without discourse markers).

The second example is

John loves Mary. It is a pleasant feeling

The *it* in the second sentence refers to a property (loving Mary) that arises in the standard model.

These observations lead us to the following construction. We start with the Montagovian model for sentences without discourse markers. Then we enrich the reference points (possible world with time index) with information concerning the values of discourse markers in that world. Finally we build a model for discourses in which we use these enriched possible worlds as states in our model for the discourse markers.

Following Montague 1973, we start with the following three primitive sets

E the set of basic entities

I the set of possible worlds

J the set of moments of time

From these sets we firstly build a standard Montague frame with Montague-Domains:

$MD_e = E$

$MD_t = \{1,0\}$

$MD_{<s,a>} = MD_a^{I \times J}$

$MD_{<a,b>} = MD_b^{MD_a}$

A model for Intensional Logic with Discourse Markers is defined as follows:

$S = \Pi_\tau \Pi_N (MD\tau) \times I \times J$

Hence an $s \in S$ is a triple (d,i,j), where

$d = d_{\tau_1,1}, d_{\tau_1,2}, \ldots. d_{\tau_2,1}, d_{\tau_2,2}, \ldots d_{\tau_3,1}, d_{\tau_3,2} \ldots$

So d is an infinite series of values: for each discourse marker there is a corresponding value of the appropriate type.

We are now prepared to define the domains D_τ.

$D_e = E$

$D_t = \{0,1\}$

$D_{<a,b>} = D_b^{D_a}$

$D_{<s,a>} \subset D_a^S$, more in particular it are those functions that are independent of the information aspect of the state. So it are functions that correspond with functions in the Montague Domains . Formally

$D_{<s,a>} =$

$\{f \mid \text{there is a } g \in MD_{<s,a>} \text{ such that for all } (w,d) \in S \text{ holds } f(w,d) = g(w)\}.$

The interpretation of discourse markers proceeds by means of the information component of the state, whereas the ordinary constants get Montague-like interpretations depending only on I and J (e.g. the interpretation of *walk* is not depending on the information aspect of the state).

We firstly introduce the (Montague - like) interpretation of ordinary constants, viz. a function MF: $CON_\tau \times I \times J \rightarrow D_\tau$. The function F interpreting the constantsand discourse markers in our model is then defined as follows:

F: $CON_\tau \times S \rightarrow D_\tau$ and F has the following two properties

1. for alle normal constants c_τ: $F(c_\tau, (d,i,j)) = MF(c_\tau, i,j)$
2. for all discourse markers dm_{τ_n,i_n}: $F(dm_{\tau_n,i_n}, (d,i,j))$ = the n-projection of d.

This model satisfies the update postulate: by changing the n-th component of the state to the given value, we obtain a new state in which discourse marker dm_{τ_n,i_n} gets that value, whereas all other constants have the same value as in the original state.

5 Discussion of the simple model

If the logical representation of a discourse is evaluated with respect to the simple model, then the desired results are obtained. The references for discourse markers are found, and the correct truth-values are assigned to the sentences of the discourse. The simple model yields correct outcomes for completed discourses and even for (abruptly) discontinued discourses.

However, this model is not sufficient for dynamic Montague grammar because of the principle of compositionality of meaning. That principle (generalized from sentences to discourses) states the *meaning* of a discourse is build from the *meanings* of the sentences of the discourse. This will be illustrated by an example. Consider the following discourse.

John enters. He smiles.

In dynamic Montague grammar it is required that the sentence *He smiles* has a meaning of its own, i.e. independently of the discourse in which it occurs. It must be possible to interpret the logical representation (with its discourse marker) even if the previous discourse is unknown. The meaning of *he smiles* will be some function from states to truth values. In particular it will be a function that varies from state to state, depending on the referent in such states of the discourse marker corresponding with *he*. Unfortunately such functions are not in the simple model. Therefore the simple model is unsuitable for dynamic Montague grammar.

The situation is different for discourse Representation theory . The interpretation of the presented discourse proceeds in that theory roughly as follows. For the discourse marker in the representation of the second sentence, a suitable discourse marker is to be found in the representation of the previous discourse. Only when these connections are laid, the model-theoretic interpretation can take place. The details of this process depend on the representation of the previous discourse. Therefore, the representation itself of the first sentence has to be available when interpreting the second sentence, and not its meaning (some abstract function). If the representation of the previous discourse is not available, then the interpretation of the last sentence cannot take place.

The above discussion explains that there is in discourse representation theory no need for assigning a meaning to the representation of a discourse sentence like *He smiles*. Hence no functions are needed which vary from state to state, and which depend on the referent in such states of the discourse marker corresponding with *he*. Therefore the simple model might be suitable for DRT whereas it is, due to compositionality, not suitable for DMG.

In discourse representation theory the logical *representation* of a discourse is build step by step from the *representations* for the sentences of the discourse. So one might consider this as a form of compositionality, viz. as compositionality of *representations*. But discourse representation theory does not aim at semantic compositionality. A model theoretic evaluation can only be performed on a final discourse representation. One might try to change DRT in such a way that it would obey the principle of compositionality of meaning. As a consequence the meaning of a discourse would then not be obtained from interaction of representations but from interaction of meanings. In such a situation the discourse representations would play the same role as the translations in Montague grammar, viz. figuring as representations of meanings, but playing no essential role. Then the representations could, in principle, be omitted (just as is the case with the IL-translations in Montague grammar). But in such a situation DRT would loose one of its essential features since it would not longer be a theory of discourse *representations*. As a matter of fact, dynamic Montague grammar started out as a reformulation of discourse representation theory that obeys the principle of semantic compositionality.

We may summarize this discussion as follows. Since DRT is a theory of representations, the simple model is a suitable model satisfying the update postulate. This model is not suitable for DMG because that theory, being a theory of meaning, aims at semantic compositionality.

6 A richer model

In this section a model will be presented that is richer than the model from the previous sections. Before doing so, we will make some observations concerning discourse markers that are of heuristic value.

Consider the following discourse

Mary enters. John loves her. He likes it.

This discourse expresses (in its most likely reading) that John likes loving Mary. The pronoun *her* in the second sentence introduces a discourse marker of entity type, say d_3 (which gets as interpretation Mary). The pronouns *he* and *it* in the third sentence introduce the discourse markers, say d_4 and d_5 of entity type and of property type respectively. Discourse marker d_4 is of course associated with *John*, and d_5 with the property expressed by *love her* in the previous sentence. So to d_5 is assigned, due to the interpretation of d_3 the property 'loving Mary'.

In this example we can make the following two observations.

i) In each stage of the discourse only a finite number of discourse markers is relevant. After the second sentence this number is one, after the last sentence three.

ii) Each of the relevant discourse markers is in finitely many steps connected with basic interpretations (i.e. interpretations which do not involve discourse markers). The d_5 is after two steps associated basic information (or after some larger number, depending on the precise definition of the notion 'step').

We can summarize the above by 'the amount of discourse information is finite, and the complexity of this information is finite'.

A model for dynamic montague grammar will be build in which the above observations are formalized. We will firstly introduce a series $DD_{\tau,m}$ of Domains Depending on the first m discourse markers. From these series we will make a generalized model in which only functions arise which depend on a finite number of discourse markers. The model will obey the update postulate, and allow for a compositional interpretation of the discourses we have considered, but in another respects it seems to be not completely satisfactory, see discussion at the end.

Step 1 The series $DD_{\tau,m}$

The first step in the construction of the model is the introduction of a a sequence of all discourse markers:

$dm_{\tau_1,i_1},\ dm_{\tau_2,i_2},\ \ldots,\ dm_{\tau_n,i_n},\ \ldots$

Now we define for all m

$DD_{e,m} = E$ and $DD_{t,m} = \{0,1\}$

And by induction we define

$DD_{<a,b>,m+1} = \{f \in DD_{b,m}{}^{DD_{a,m}} : f \text{ depends on } dm_{\tau_m,i_m}\}$

$DD_{<s,a>,m+1} = \{f \in DD_a{}^{\Pi_{1\le n\le m} DD_{\tau_n} \times I \times J} : f \text{ depends on } dm_{\tau_m,i_m}\}.$

By the condition 'f depends on dm_{τ_m,i_m}' we require that the m-th argument is relevant for determining the value of f. In other words, that f varies with its m-th argument. This condition is not essential for the construction of the model, but it eliminates a lot of ambiguities. The formal version of this requirement is:

there are $d_1 \in DD_{\tau_1}, \ldots, d_{m-1} \in DD_{\tau_{m-1}}$ and $a_1, a_2 \in DD_{\tau_m}$ such that $f(d_1, d_2, \ldots, d_{m-1}, a_1) \neq f(d_1, d_2, \ldots, d_{m-1}, a_2)$.

Finally we define

$DD_\tau = \cup_m DD_{\tau_m}$

Step 2 the model

We define

$S = \Pi_{1\le n} DD_{\tau_n} \times I \times J$, $D_e = E$, and $DD_t = \{0,1\}$.

$D_{<s,a>} = \{f \in D_a{}^S : f \text{ depends only on the first m discourse markers}\}$

or, more formally

$D_{<s,a>} = \{f \in D_a{}^S : \exists m\ \exists g \in DD_{<s,a>,m} : f \text{ is an extension of g to infinity tuples}\}$

We call f an extension of g to infinity tuples as arguments if for all

$d_{m+i} \in D_{\tau_{m+i}}$ holds that $f(<w,d_1,..d_m,d_{m+1},..,d_{m+i},...>) = g(<w,d_1,..d_m>)$.

$D_{<a,b>} =$

$\{f \in D_b{}^{D_a}$: the argument of f depends only on the first m discourse markers, and so does the value of f$\}$

or, more formally,

$\{f \in D_b{}^{D_a} : \exists m\ \exists g \in DD_{<a,b>,m}$: f is an extension of g to infinity tuples$\}$.

This extension has to take place (in argument or value of f) when there are subtypes of the form $<s,\tau>$.

Step 3 The discourse markers

As last step we will define the interpretation of discourse markers, and show that he model we satisfies the update postulate. The idea is that the interpretation of discourse marker dm_{τ_n,i_n} in state s is the n-th coordinate of the state. However, in case these coordinates are of an intensional or a functional type, then they are defined for a finite number of elements only. Therefore, the interpretation of a discourse marker is defined as the extension of such a function to an infinite tuple of arguments.

Definition $[dm_{\tau_n,i_n}]_s$ = n-th coordinate of s, extended to infinite tuples

Now the update postulate holds as will be explained below.

Let be given discourse marker dm_i, state s, and value $d \in D_{\tau_n}$. Then we find the state s' required by the update postulate as follows.

In case that τ_n=e or τ_n=t we change the n-th coordinate of s into d, and obtain in this way state s'. Consider next the case that $\tau_n = <s,a>$ for some type a. Then we know $d \in D_{<s,a>}$, so $\exists m\ \exists g \in DD_{<s,a>,m}$ such that d is an extension of g to infinity tuples. We obtain s' from s by changing the n-th coordinate into g. Finally we consider the case that $\tau_n = <a,b>$ for some types a and b. Then we know that $\exists m\ \exists g \in DD_{<a,b>,m}$: f is an extension of g to infinity tuples. Also in this case we change n-th coordinate into g.

Thus we have obtained a state s' in which the given discourse marker has the given value, and in which all other discourse markers have kept their original value. So the update postulate is satisfied.

This model is, however, in one respect not satisfactory. There is not a single identity function of type $<<s,\tau>,<s,\tau>>$. According to the definition of $D_{<<s,\tau>,<s,\tau,>>}$ we have an identity function that is defined for objects which depend on the first 5 discourse markers, one which depends on the first 6 discourse markers and so on. So there are an infinite number of identity functions of this type (and for certain other types as well), but not the one we would expect. I thank M. van den Berg for bringing this point to my attention.

7 General results.

It is striking to see that both models have the same structure; viz. cartesian product of values for discourse markers with the other parameters. In this section we will show that all models satisfying the update postulate are somehow of this nature.

Definition

Let DM be a set of discourse markers. Let M with S as set of states, be a model which satisfies the update postulate for all discourse markers in DM. By $\equiv_{DM}$ we understand the equivalence relation of having the same value for all discourse markers in DM.

Theorem 1

$S /_{\equiv\{dm_n\}}$ is isomorphic with D_{τ_n}.

Proof

Let f be a mapping from $S /_{\equiv dm_n}$ to D_{τ_n}. Then f is injective because states with the same value for dm_n are identified under the equivalence relation. The mapping is surjective due to the update postulate which requires that dm_n can take all values.

End of Proof

If we define $\equiv dm_{n1,n2}$ as having the same values for discourse markers dm_{n1} and dm_{n2}. then it is easy to see that

$S/_{\equiv dm_{n1,n2}}$ is isomorphic with $D_{\tau_{n1}} \times D_{\tau_{n2}}$

Theorem 2

Let DM be finite. Then is $S/_{\equiv DM}$ is isomorphic with $\Pi_{dm_n \in DM} D_{\tau_n}$.

Proof

Analogous to theorem 1.

Several special cases of this theorem are presented in Priatelj 1987.

Remark

The result of theorem 2 does not generalize to the case that DM is countably finite. Of course, the full product $\Pi_{dm_n \in DM} D_{\tau_n}$ is a correct model satisfying the update postulate. But also models are possible in which not the full product is used. This observation and the following model are due to P. van Emde Boas.

Let $s = (d_1, d_2, \ldots d_i \ldots)$ be an element of $\Pi_{dm_n \in DM} D_{\tau_n}$.

Define $S^{\sim}$ =

$$\bigcup_{D \text{ is a finite subset of DM}} \Pi_{dm_n \in D} (\underline{\text{if}}\ \tau_n \in D\ \underline{\text{then}}\ D_n\ \underline{\text{else}}\ \{d_n\})$$

All elements in $S^{\sim}$ have the property that they differ in finitely many coordinates from s. The update postulate is satisfied because it requires the change of one component, thus yielding another state that differs in only one coordinate from s.

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POLARITY SENSITIVE ANY AND FREE CHOICE ANY

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1. INTRODUCTION

As is well known, any can function in two different ways. On the one hand, it can be a negative polarity item - we will call any on this use **polarity sensitive** any (PS); on the other hand it has, what is called, a 'free choice' interpretation - we will call any on this use **free choice** any (FC). In this paper, we will propose a unified analysis of the semantic and pragmatic effects of any, that is, an analysis which applies to any on both its PS and FC uses.

The use of any as a negative polarity item is illustrated in (1) and (2):

(1) I don't have any potatoes.
(2) *I have any potatoes.

Ladusaw 1979's well known analysis says that negative polarity items (NPIs) are only licensed if they are in the scope of a downward entailing operator. A downward entailing (DE) operator is an operator that reverses the entailment, roughly (using ==> for entailment):

(3) O is a DE operator iff if A ==> B then O(B) ==> O(A).

Example (1) is OK on Ladusaw's account, because any is in the scope of negation which, as illustrated in (4), is a DE operator:

(4) swim ==> move
I don't move ==> I don't swim

In example (2), any is not licensed, because there is no DE operator that any is in the scope of.

Ladusaw's analysis elegantly accounts for a wide range of examples, and improves upon previous accounts - which relied on the presence of a negation operator - in that the notion of DE also deals with cases without a negation, like (5)-(8).

(5) At most three girls saw anything.
(6) *At least three girls saw anything.
(7) Every girl who saw anything was happy.
(8) *Some girl who saw anything was happy.

Assuming, with Generalized Quantifier Theory, that determiners are two place relations between a nominal property and a verbal property, Ladusaw predicts that (5) and (7) are OK because the determiner <u>at most three</u> is DE on both arguments, and the determiner <u>every</u> is DE on its first argument. (6) and (8) are out because <u>at least three</u> and <u>some</u> are not DE on either argument (in fact, both are upward entailing on both arguments).

While Ladusaw's analysis is generally quite successful, there are also some problems and questions. In the first place, there are various empirical problems. Linebarger 1987 discusses many such problems; we will only be able to comment upon a small selection of those in this paper. These empirical problems are generally of the following sorts:

(i) we find that sometimes NPIs fail to be licensed in a DE context (we will discuss the example of comparatives);

(ii) we find NPIs in contexts that seem not to be DE (we will discuss the case of adversative predicates);

(iii) We find NPIs in contexts where it is not clear how the notion of DE could be applied (we will discuss questions).

Secondly, there are two more theoretical issues that Ladusaw's theory does not address:

I. What is it about the meaning of NPIs that forces them to occur in certain contexts and not others?

What Ladusaw gives us is a description of the distribution of NPIs: the DE feature is a feature of the contexts in which the NPIs occur. As such it is quite successful. However, given that NPIs occur only in DE contexts, one may still want to know **why** that is so, i.e. **why** it is that NPIs are sensitive to this feature of the context, and how this sensitivity is related to the **meaning** of the NPI itself. We will suggest an answer to this question for the NPI <u>any</u>.

II. What is the connection between polarity sensitive <u>any</u> and free choice <u>any</u>?

(9) and (10) are examples of free choice <u>any</u>:

(9) You may take any apple.
(10) Any fool could tell you that.

The problem is this. Ladusaw 1979 offers a whole battery of arguments that show beyond doubt that PS <u>any</u> is an indefinite with an existential meaning. But FC <u>any</u> in (9) and (10) seems to have a universal meaning. And this goes beyond mere appearance. Carlson 1981 gives several arguments that FC <u>any</u> in fact is a universal quantifier. A strong argument is the behavior of <u>almost</u>. <u>Almost</u> is an operator that can modify only universal determiners, as shown in (11) to (13):

(11) Almost every fool can tell you that.
(12) Almost no fool can tell you that.
(13) *Almost some fool can tell you that.

As (14) and (15) show, almost can modify FC any but not PS any, arguing strongly that FC any, but not PS any, is universal.

(14) Almost any fool can tell you that.
(15) *I don't have almost any potatoes.

It seems then, that we should draw the conclusion (towards which Carlson 1981 leans) that any is lexically ambiguous: PS any is an existential quantifier, and FC any is a universal quantifier.

But there are also arguments that any is lexically unambiguous (some of which are discussed in Carlson's paper, see Carlson 1981 and references there). One very suggestive argument is that the existential/universal flip-flop that we observe in any has a parallel in a disjunction/conjunction flip-flop in or: in the same contexts where FC any is allowed, it is possible to interpret or as free choice disjunction: a disjunction with a conjunction meaning and the same feeling of 'choiciness':

(16) You may take an apple or a pear.
(17) Mary or Sue could tell you that.

and it seems unattractive to assume that or is lexically ambiguous as well, especially given the fact that, though not every language has something corresponding to any, free choice interpretations of disjunction do appear cross-linguistically.

In fact there is some further striking cross-linguistic evidence against the ambiguity hypothesis (and in fact in favor of the analysis that we will propose). In Hindi (Veneeta Srivastav, personal communication) there is a type of expression, kuch-bhi, koi-bhi, with the same meaning as any in English: in PS contexts these expressions have an existential meaning and behave like negative polarity items; in the contexts where in English we typically find FC any, they occur with a universal meaning. The interesting thing is that these expressions consist of a normal **indefinite** kuch (something), koi (someone) with the particle -bhi that brings in the any-effect.

We believe that any is unambiguous. But then we have to explain why it has two uses, and how it gets interpreted as an existential on one use and as a universal on another. Ladusaw himself admits that it is not clear how his analysis of PS any would help here. In this paper, we are going to propose an analysis of any which applies to it on both its uses.

2. OUR PROPOSALS

2.1. Widening.

We start with a central aspect of the semantics of <u>any</u> that is common to its polarity sensitive and free choice uses. Consider examples (18) and (19).

(18) Every match I strike lights.
(19) Any match I strike lights.

There are various subtle differences between (18) and (19). Some of these have to do with the 'choiciness' of (19): its feeling of 'it doesn't matter which', the fact that it applies to 'arbitrary matches' (we will have only little to say about this aspect, we intend to discuss it at length in a separate paper). Others have to do with the feeling that maybe in (19) it is more vague what exactly is quantified over (we will come back to this in the last section of this paper). In this section we are concerned with the following, third difference between (18) and (19): there is an intuition that (19) states more strongly than (18) does that (in some sense) there are no exceptions. (19) sounds like even marginal matches, ones you may not expect to light, do light when I strike them.

Now because of the mentioned other differences between <u>every</u> and <u>any</u>, this intuition is necessarily rather subtle (and we will spend a large part of the last section of this paper in determining exactly in what sense (19) states more strongly that there are no exceptions). Nevertheless, if one goes through the examples in the literature of FC and PS <u>any</u>, then this same intuition emerges case after case as one of the most prominent feelings about <u>any</u>: the examples with <u>any</u>, in comparison with related examples without <u>any</u>, express more prominently that there are no exceptions.

The comparison between (18) and (19) is useful to get a first grip on finding out in what sense <u>any</u> expresses this 'no exception' feature more strongly. Since there is universal quantification in both (18) and (19), the difference between (18) and (19) cannot lie simply in the quantificational force of <u>every</u> and <u>any</u>: already (18) does not allow any exceptions. Therefore, the stronger suggestion of 'no exceptions' in (19) can only mean that the quantification in (19) is over more objects than that in (18). It's over objects that include even marginal matches. In other words, the set of objects that satisfy the restriction <u>match I strike</u> is bigger in (19) than it is in the case of (18).

Let's consider (18). The context would normally select an interpretation for <u>match I strike</u> that excludes all sorts of matches, for example, used matches and wet matches. For that reason, you can accept (18) as true even if you don't think that wet matches will light. This is of course traditionally discussed in terms of domain selection: all quantification is restricted in the context to a domain of relevant objects; the context selects a domain which does not contain irrelevant things like wet or used matches.

We think that the effect of _any_ in (19) is to widen the contextually given domain of quantification, which it does by extending the interpretation of _match I strike_. With _any_, the extension of _match I strike_ is extended to include matches that wouldn't normally be taken into account in the given context.

Take the following example. Suppose you state that if you take a dry match and strike it, it lights.

In such a context, I can support this statement from my own experience by saying (18): _Yeah, right, every match I strike lights!_. In this context, the domain of quantification of _every match_ in (18) has been restricted to dry matches.

Suppose that in the same context, I say (19): _Any match I strike lights!_. While (18) is normally used to **confirm** your earlier statement (taking over the restricted domain of quantification), (19) rather seems to modify it: (19) would be interpreted as meaning that when I strike matches, they light, no matter whether they are wet or dry.

What the example illustrates is the following:
(i) _any_ indicates that the interpretation of _match I strike_ is intended as wider;
(ii) this widening is done along a particular contextual 'dimension', in this case, the dimension 'wet vs. dry'.

Given the dimension, (19) states that wet matches are no exception, that is, they are included in the domain of quantification. Note that in the same context, (19) **does** allow other kinds of matches to be exceptions, for example, used matches may continue to be legitimate exceptions: widening is usually not total, but restricted by a salient opposition in the context. This is what we call the dimension.

So, the first ingredient of our analysis is (A):

(A) WIDENING
In an NP of the form _any_ CN, _any_ widens the interpretation of the common noun phrase CN along a contextual dimension.

We believe that this widening is found with FC and PS _any_ alike. In the next sections we will concentrate on PS _any_; we will return to FC _any_ in the last section.

2.2. **_Any_ is an indefinite.**

The literature provides strong evidence that PS _any_ is existential (see especially Ladusaw 1979). We assume that an NP with _any_ should be regarded semantically as an indefinite NP with some additional characteristics contributed by _any_. For example, _any_ contributes widening. The NP with _any_ has the usual semantic properties of an indefinite. We won't make a choice here concerning the proper way of treating indefinite NPs. If indefinite NPs are best regarded as existential quantifiers, then so is the any NP; if indefinites are best treated as new variables (Heim 1982), then also the any NP is a new variable. From this assumption the existential behavior of PS any follows without problems. However, note that, since we will assume that any is **not** ambiguous, we will also treat FC any as an indefinite. We will show how this is compatible with the behavior of FC any in the last section.

Basically what we are assuming is that any CN is just the indefinite NP a CN with some additional features contributed by any.

This is the second ingredient of our analysis:

(B) any CN = the corresponding indefinite NP a CN with additional semantic/pragmatic characteristics (widening, strengthening) contributed by any

2.3. Strengthening.

Let us now come to the licensing of any. We have above made some assumptions about the meaning of any (any is an indefinite and induces widening), we will now make an assumption about the **function** of any. We propose the following. The **function** of any (and of NPIs in general) is to **strengthen** the statement that it occurs in. That is, the widening that any induces makes the statement it's in stronger than it would be without the widening. Also this aspect of any and NPIs jumps out as a very prominent feature if you go through the examples in the literature. We could say if John reads a newspaper, he reads the New York Times, but we say if John reads any newspaper, he reads the New York Times; we could say if you move I hit you, but we say if you budge an inch I hit you. In both cases, the use of the NPI strengthens the statement.

So we assume strengthening as a second feature of any. We define, for assertions, strengthening in terms of entailment. (As will become clear, this will make our strengthening requirement into a restricted form of downward entailment.) Strengthening, we assume, is a **licensing** feature of any. The point is this. We know that the meaning of any induces widening. The function of any is to make a stronger statement. We assume that any is only licensed in contexts where with the meaning that it has it can perform its function. Thus, any is licensed only if the widened statement strengthens the statement before widening, i.e. if the statement on the wider interpretation entails this same statement on the narrower interpretation.

This is the third ingredient of our analysis, we assume that a lexical property of any is a strengthening requirement:

(C) STRENGTHENING
<u>any</u> is licensed only if the widening that it induces creates a stronger statement, i.e. if:
the statement on the wide interpretation ==>
the statement on the narrow interpretation

Let us look at an example. Consider (21).

(20) We don't have potatoes.
(21) We don't have any potatoes.

It is clear that, at least if the <u>any</u> is emphatically stressed, we observe the same 'no exceptions' effect that we saw for FC <u>any</u>. (21) sounds stronger than (20), it sounds like we don't even have marginal potatoes. We conclude that PS <u>any</u> also induces widening. (We will assume that the same holds if <u>any</u> is destressed, although in the particular case of plurals under negation the difference is minimal (presumably because <u>some</u> is not an alternative here).)

Both (20) and (21) can be represented as something like (22).

(22) $\neg\exists$ x [Potato(x) & We have(x)]

In context, we can understand 'potato' to mean 'cooking potato'. Then (22), and hence (20), will mean that we have no cooking potatoes. The effect of <u>any</u> is to widen the interpretation of 'potato', for example, to also include decorative potted potatoes. So (21) can mean that we don't even have potted potatoes.

Now if we check whether <u>any</u> is licensed in (22), we see that this is indeed the case, because the strengthening requirement is satisfied: the statement on the wide interpretation entails the statement on the narrow interpretation:

wide: We don't have potatoes, cooking or other.
==> narrow: We don't have cooking potatoes.

Let us compare this with a case where <u>any</u> is not good:

(23) *We have any potatoes.

We assume that the same widening is involved here. Now for strengthening we have to check the following pattern:

wide: We have potatoes of SOME kind (cooking or other).
=/=> narrow:We have cooking potatoes.

We see that here strengthening is not satisfied, so indeed <u>any</u> is not licensed in (23).

Another example is (24).

(24) Every man who has any matches is happy.

The effect of any is to widen the interpretation of matches, to include, say, wet matches. Obviously, this in turn widens the interpretation of man who has matches. The result is that every quantifies over men who have dry and wet matches alike. Clearly, this example satisfies strengthening: If every man who has matches on the wide interpretation is happy, it follows that every man who has matches on a narrower interpretation is happy.

This contrasts with (25).

(25) *Every boy has any potatoes.

Widening will not strengthen this statement: if every boy has potatoes of SOME kind, it does not follow that every boy has cooking potatoes. Hence any is not licensed.

2.4. Locality.

Our analysis has one more ingredient, a locality constraint, whose function (and thr problems there are with it) we can here only indicate briefly.

We have mentioned that widening has to strengthen the statement that any is in. But what is the statement that any is in? We will follow Linebarger 1987 by assuming a locality constraint here:

(D) LOCALITY
Strengthening is to be satisfied by the 'local' proposition

Defining 'local' precisely is problematic. Roughly the idea is that the local proposition of any is the proposition at the level of the smallest operator that any is in the scope of. We think of this in terms of Heim 1982's notion of operator: Heim's operators are functions that create subordinated anaphoric domains, that is, things like universal quantifiers, negation, conditionals, propositional attitude verbs.

The main reason for this locality constraint is examples like (26):

(26) *It's not the case that every boy has any potatoes.

This example is generally regarded as infelicitous (also by Ladusaw 1983). However, Ladusaw's theory predicts that it is OK. Not only is any in scope of a DE operator (the negation), but even stronger, the whole context It's not the case that every boy--- is DE.

Given our locality constraint, strengthening has to be checked at the level of the local proposition. The smallest operator that any is in the scope of is every boy, so what has to be checked is whether every boy has any potatoes satisfies the strengthening requirement. This is (25), hence (26) is out for the same reason as (25).

The problems of locality are very subtle, and what we have presented above is not more than a crude first stab at them. Let us mention one problem. Propositional attitude verbs are operators, but we have to assume that they are weaker operators than the other ones, in that sometimes, if the strengthening requirement is not satisfied at their level, the sentence may still be OK if strengthening is satisfied by the next operator up. The reason is that, though (27) is infelicitous (as predicted: at the level of admitted strengthening is not satisfied), (28) is OK:

(27) *Everybody admitted that Bill had seen anything.
(28) Nobody admitted that Bill had seen anything.

Finally, let us mention one more example where locality plays a role in our theory. Linebarger 1987 mentions examples like (29) as a problem for Ladusaw's theory:

(29) Only people who had seen anything were questioned.

The problem is that only seems not to be DE on its first argument, as Linebarger observes:

Only people who have had a terrible disease know what suffering is. =/=>
Only people who have had polio know what suffering is.

Now, part of the problem is the presuppositional part of only. Ladusaw 1979 argues that to check for the DE pattern, you should look at the semantic part of the meaning of the sentence, i.e. at the meaning minus its presuppositions. Thus (30) should semantically be represented as (31), with presupposition (32):

(30) Only John came.
(31) Everybody who isn't John didn't come.
(32) John came.

Thus for (29) the DE pattern is only required to hold of its semantic representation (33):

(33) $\forall x$ [¬ saw(x,anything) --> ¬ Questioned(x)]

The problem for Ladusaw is that (33) still is not DE (everybody who doesn't move doesn't come does not entail everybody who doesn't fly doesn't come).

With locality, the strengthening requirement is to be satisfied at the level of the smallest operator that any is in the scope of in (33), which is the negation: if x didn't see something in the wide domain, then surely x didn't see something in the narrow domain, so strengthening is satisfied and any is OK in (29).

2.5. Comparison with Ladusaw.

Our analysis of PS any is close to Ladusaw's. Like

Ladusaw, we claim that there is one uniform condition that constrains any. Also, our strengthening requirement is a requirement of an inference pattern that reverses the direction of entailment, like Ladusaw's DE. The difference is that we check for a particular instance of the DE pattern: the inference from the wide to the narrow interpretation of the same statement. Still, by and large, DE contexts are typically contexts in which our strengthening is satisfied.

However, there are also several differences between the two analyses. By positing widening and strengthening instead of DE, we achieve a number of things:

1. First of all, something that we see as central to our whole enterprise: our analysis does explain **why** it is that any should be licensed only in a certain kind of context. We have suggested a connection between the function and the semantic effect of any (the strengthening and widening) and the contexts in which any is licensed. Whereas Ladusaw's DE only captures the distribution of any, we present a rationale for this distribution based on the meaning and function of any: any occurs in these contexts because those are the only contexts in which it can both mean what it should mean (widening) and do what it should do (strengthening).

2. Our analysis accounts for the intuition that any suggests particularly strongly that there are no exceptions.

3. Our analysis also has empirical advantages concerning the distribution of any. This will be illustrated shortly.

4. We will argue that our analysis extends to FC any, and constitutes a unified analysis of any on its two uses.

In the next sections, we are going to show how we deal with a couple of problematic cases with PS any that are discussed in Linebarger 1987. We can only go through a few of Linebarger's problems. Moreover, though we think that Linebarger's own analysis runs in to very serious problems, we have no space here to compare our theory with hers. It is good to point out here that for most of the problem cases (the ones we discuss and the ones we don't discuss) the solution does not **simply** follow from our analysis. In most of Linebarger's examples there is interference with other factors, and our solution derives partly from our analysis, partly from specific features of the particular type of example.

3. MORE OFTEN

In this section we will briefly look at a case where any is unacceptable, though it occurs in a DE context. Consider Linebarger's (34):

(34) ?The sun rises more often than John visits any relatives.

The problem is that (34) is weird, although the context is

DE (as argued in Hoeksema 1983). Also on our theory strengthening is guaranteed, so _any_ should be OK. Indeed, in the very similar example (35) (also from Linebarger) _any_ is licensed . Why then is (34) weird?

(35) Cows fly more often than John visits any relatives.

We think that (34) sounds weird because it doesn't seem to make pragmatic or conversational sense.

Note first that (36), the counterpart of (34) without _any_, already sounds odd.

(36) ?The sun rises more often than John visits relatives.

Obviously, the reason is that it is not clear what the point would be of saying such a thing: the sun rises quite often, so (36) is unlikely to provide relevant information about whether John visits relatives often or seldom.

Now it seems that the sentences sound even worse with the _any_. We think that with the idea of widening, we have the means of explaining this fact. We think that the _any_ makes (34) particularly strange because it's very hard to imagine how the widening induced by _any_ would make pragmatic sense here.

When does widening make sense pragmatically? For example, in (35): The point of this sentence is that John visits relatives very seldom, in fact, never. Here widening makes sense: The use of widening on the word _relatives_ indicates that we are lenient about what counts as a relative. One might have thought that if you are lenient enough, you'd be able to adopt the view that John visits relatives often. The speaker is saying that that is not the case: no matter how lenient we are about what counts as a relative, John still doesn't visit relatives.

Similar reasoning does not work with (34). As we said before, this sentence doesn't tell us if John visit relative often or seldom. Therefore, what's the point of trying to be lenient? No view is expressed about the frequency of visits, so no view could be changed by taking a lenient approach.

So the point we can make here is this: Because our analysis specifies what it is that _any_ does, it allows us to identify and explain examples where _any_ sounds odd because there is no pragmatic sense to doing what _any_ is supposed to do.

4. _SORRY_ AND _GLAD_

As noted in the literature, adversative predicates like _surprised_ and _sorry_ license NPIs, contrasting with predicates like _sure_ and _glad_. This is illustrated in (37) and (38).

(37) I'm surprised/sorry that he ever said anything.
(38) *I'm sure/glad that I ever met him.
*I'm sure/glad I said anything.

Are the adversative predicates DE? Take surprised - does (39) entail (40)?

(39) I'm surprised he bought a car. Honda ==> car
(40) I'm surprised he bought a Honda.

We have to be careful: as in the earlier case of only, surprised has a factive presupposition, which interferes with the entailment judgments. Ladusaw assumes that the DE pattern should hold of the sentence minus its factive presupposition. So what we have to check is whether (39) entails (40) on the assumption that he bought a Honda. Linebarger argues that even so, (39) does not **logically** entail (40): it is possible that the fact of him buying a car is surprising, but, given that he is buying a car, the choice of car is not. Linebarger discusses the possibility that a weaker relation, 'psychological DE', does hold, but she concludes that such a relation would fail to distinguish surprised from glad: i.e. either both surprised and glad or neither are 'psychologically DE'.

There are then two related questions to answer:
(i) Why do adversative predicates license NPIs?
(ii) How do they differ from predicates like glad?

To this we wish to add the following problem:
(iii) Glad **does** in some cases license NPIs. For example, in (41).

(41) A: But these tickets are terrible!
B: Be glad we got ANY tickets!

We believe, with Linebarger, that neither sorry nor glad is generally, or logically, DE, or strengthening. If you just look at their logical properties, neither one is going to satisfy strengthening. We will argue that the licensing of any under these predicates depends on extra factors which create the required strengthening.

We start with the adversative predicates. Without getting into details, we just note the following: When you try to find a difference in the DE behavior of glad and sorry, you often get the impression that maybe there is some kind of slippery intuition that sorry is 'more DE' than glad. At the same time, we agree with Linebarger that when you start looking 'logically' at the DE pattern, you can't seem to find any difference between sorry and glad. We will propose an explanation for this.

Consider example (42).

(42) I'm sorry that anybody hates me.

Consider the widening that any induces here. The context would give us a relevant set of people that count as 'somebody'. For example, 'semanticists'. Any widens this set, so that more people count as 'somebody'. For example, linguists in general.

Given this widening, (42) (the wide interpretation) says what is expressed in (a): I'm sorry that the set of linguists who hate me is non-empty.

(a) sorry that: {x : Linguist who hates me(x) } $\neq \phi$ WIDE

Now, it seems in some sense to be part of the meaning of sorry that if I am **sorry** that a set is not empty, then I **want** that set to be empty. With this, from (a) we can conclude (b).

(b) want that: {x : Linguist who hates me(x) } $= \phi$

But I cannot want a set to be empty without wanting all its subsets to be empty. One of these subsets is the set of semanticists, so we get (c).

(c) want that: {x : Semanticist who hates me(x) } $= \phi$

Using once more the relation between sorry and want, this means that I should be sorry if the last set is not empty, so we get (d).

(d) sorry that: {x : Semanticist who hates me(x) } $\neq \phi$ NARROW

But (d) is the narrow interpretation, so with use of this argument (42) satisfies strengthening after all: from the wide interpretation in (a) we can infer the narrow interpretation in (d).

Recall the slippery intuition that sorry is somehow DE. We think that when you judge sorry to be DE, you do that because of the reasoning that we just went through. We propose that sorry licenses any because, based on this reasoning, sorry is taken to satisfy strengthening.

When we look at glad, we see that the same kind of reasoning cannot create a DE pattern here. Take (43). Given the same widening as before, (43) says (e).

(43) *I'm glad I saw anybody.
(e) glad that: {x : Linguist that I saw(x) } $\neq \phi$

Also for glad there is a connection between glad and want, but this time a positive one: (e) implies (f).

(f) want that: {x : Linguist that I saw(x) } $\neq \phi$

But wanting a set to have members does not entail that you want each particular subset to have members: So no entailment from the wide set to the narrow set goes through. This explains why glad differs from sorry and does not freely license NPIs.

Now that we have created a difference between sorry and glad, we have to explain why in special cases like (41) any is licensed under glad. Look again at (41) and a similar example (44).

(41) A: But these tickets are terrible!
B: Be glad we got ANY tickets!
(44) I'm glad ANYBODY likes me!

We assume that glad is not DE, and we know that it does not freely license NPIs. Then why is any licensed in (41) and (44)?

There is an additional fact to be explained: Examples like (41-B) and (44) have a negative implicature. (41-B) suggests that we didn't get any decent tickets; (44) suggests that nobody who really counts likes me. And this implicature is not an unrelated phenomenon, because without it, any is not licensed in (41) and (44).

Let us concentrate on (44). In general, there is no guarantee that this example satisfies strengthening. Suppose as before that the set of people that count as 'somebody' is widened from semanticists to linguists:

wide : I'm glad that some linguist likes me.
=/=> narrow: I 'm glad that some semanticist likes me.

Then, as we argued before, strengthening doesn't hold: There is no guarantee that my being glad that somebody in the wider set likes me entails that I am glad that somebody in the narrow set likes me. I can be glad that there are linguists who like me while preferring not to be liked by any semanticist.

Now let us ask: under what circumstances **would** strengthening be guaranteed? Suppose that the **narrow** interpretation, the smaller set includes all and only the people whose liking me would make me glad enough to count in the given context and that any widens this set, to also include people that would not make me glad: i.e. the following context:

(45) narrow: the people who count as 'somebody' = all and only the people whose liking me would make me glad enough to count
wide: also includes people whose liking me would not make me glad enough to count

In this context strengthening is guaranteed: (44) on the wide interpretation entails (44) on the narrow interpretation (apart from the factive presupposition). Why? Because if it makes me glad that somebody who is 'less gladdening' likes me, it follows that it would make me glad if somebody 'more gladdening' liked me. Therefore, if the interpretation is chosen in this particular way, any is licensed.

Now, we're only halfway through the problem, but let us make one observation here. In the case of example (44), the selection of contextual domain (45) is very natural given the linguistic context: given glad I would - for normal contextual reasons - not include in the domain of the 'somebodies' irrelevant people, people that wouldn't gladden me anyhow. The fact that this choice of context is so natural for glad plays a central role in the licensing of (44). For instance, if we substitute in (44) an otherwise similar predicate sure, the same argument would not work. The choice of the narrow set as the set of all and only the people about whose liking me I would be sure enough for them to count is ridiculous. That is, though it is perfectly natural to assume that in I am glad that somebody likes me, somebody means somebody that would make me glad enough, it is ridiculous to assume that in I'm sure that somebody likes me, somebody means somebody that would make me sure enough. There just doesn't seem to be any point to the latter domain selection. So this choice of context would not work for sure, and (46) is not licensed:

(46) *I'm sure ANYBODY likes me!

Coming back to (44), we have argued that strengthening is satisfied in the context given, but we run into a problem of a different sort now: after widening, (44) now means that I'm glad that someone likes me in a wide set, which we got by adding people whose liking me would not make me glad enough to count. This is problematic. Presumably glad means glad enough to count in the present context. But the added persons are such that I cannot be glad enough about any of them for them to count. So it seems that the widening we have here couldn't possibly add anybody that I could be glad about. So this widening should be pointless. The problem, thus, is that even though technically strengthening is satisfied, the use of any would be pragmatically pointless, so (44) should still be out for that reason.

Why isn't it pointless? We think that there is another factor involved which eliminates this problem.

We note that there is a strong similarity in meaning between sentences like (41-B) and (44) and the reading of (B) in (47).

(47) A: Couldn't you get any tickets better than this!?
B: I'm glad we even got THESE tickets!

(47-B) says something like this: These tickets are not ones that you would expect me to be glad about, but in fact I am glad about them. We suggest that what is going on is the following. Associated with the sentence is a scale that orders tickets with respect to how glad they would make us. On the intended reading of (47-B), even indicates something about the location of 'these tickets' with respect to the gladdening scale. Even implies the following:
(i) 'these tickets' are normally not on the gladdening scale (or they have a negative value on it);
(ii) we are now being told to **reset** our standards for getting glad about tickets, so that even these tickets get

a positive value on the gladdening scale.

We think that example (44) involves the same kind of resetting of standards. We are told to reset our standards for gladdening so that even the people whose liking me wouldn't normally gladden me now get a positive value on the gladdening scale.

So what we think is going on is this: besides its standard neutral interpretation, glad can sometimes have an implicit 'even'-quality, where the 'even' tells us to reset our standards. Any is allowed only when glad has this 'even'-quality.

Note that this 'even'-quality of glad is possible also when there is no any involved, as in (48):

(48) I know, he should have apologized, but you know how he is, I'm glad he SPEAKS to me.

Furthermore, note that any is **not** allowed when glad doesn't have an 'even'-quality, which is for instance the case when it has an 'at least'-quality, as in (49):

(49) *I'm glad that at least we got ANY tickets.

In our example (44) glad has an 'even'-quality; the resetting of standards that this induces does away with the problem we mentioned before. Although the people added by the effect of any are not 'gladdening enough' by our usual standards, they do become gladdening enough when we reset our standards. So the added persons' liking me can make me glad after all and (44) is informative after all.

Also, the resetting of standards accounts for the fact that (44) has (to have) a negative implicature. (44) implicates that nobody likes me who would normally count as sufficiently gladdening. The reason for that is that if somebody did like me who would normally count as sufficiently gladdening, then there would be no point in resetting the standards for gladdening. So any requires an 'even'-quality; this 'even'-quality requires the implicature, so indeed, we not only account for the fact that the implicature is there, but also for the fact that it has to be there for any to be licensed.

To summarize, we have explained the difference between glad and sorry, as well as the existence and special flavor of examples where glad licenses any. As announced earlier, the solution of the problems involved with glad and sorry does not follow straightforwardly from our analysis of any. Nor should it, because, as we have indicated, various features of context are involved in the particular examples that show up in other phenomena that have to be accounted for as well, like certain implicatures. Note, however, that our general analysis of any **does** play a central role in the explanations: the account of the licensing of any under glad is based on the widening idea, as it involves specific choices of the narrower and wider sets.

5. QUESTIONS

As is well known, questions license NPIs, as in (50).

(50) Does Sue have any potatoes?

But it is not clear in what sense questions are DE. Take the standard notion of entailment between questions which is given in (51) (see, e.g. Karttunen 1977, Groenendijk and Stokhof 1982):

(51) Question A entails question B iff every true answer to A entails a true answer to B.

Given this notion, questions are not DE. Look at questions (52) and (54). (52) can be answered by (53), without this providing an answer to (54). So (52) doesn't entail (54) - so we don't see a DE pattern here. Why is it then that questions license NPIs?

(52) Who moves? run ==> move
(53) John, Bill and Sue are the ones who move.
(54) Who runs?

Before we answer this, we note another point about the facts. It is stated in the literature that questions with NPIs are associated with an expectation that the answer be negative, i.e. a question like (50) is asked when it is expected that Sue doesn't have potatoes. We would like to refine this observation. We think that the expectation associated with (50) is **not** that Sue doesn't have any potatoes whatsoever. Rather, the expectation is that Sue doesn't have potatoes within the set that potatoes would normally refer to in the given context. For example, if in the present context potatoes would normally mean 'cooking potatoes', then the negative expectation (or negative suggestion) is that Sue doesn't have cooking potatoes.

Now the analysis. According to our analysis of any, what (50) means is the question expressed by (55), plus widening and strengthening.

(55) Does Sue have potatoes?

Any widens the interpretation of potatoes, for example from cooking potatoes only to both cooking and decorative potatoes.

To be licensed, any has to satisfy strengthening. The strengthening requirement is a requirement that a certain relation hold between the unit that contains any on a narrow interpretation and the same unit on a wide interpretation. In the case of declaratives, the narrow reading has to be **entailed** by the wide reading. The idea behind this is, of course, that the wider statement should provide more information than the narrow statement. We think that in the case of questions, strengthening should be based on an inverse relation: the stronger question is the one that **asks** more information. Of course, we have to state what it means to ask more information. We will base our definition on a very weak relation between questions (i.e. not a partial order), that however does form a real comparative relation (a partial order) between the question-pairs that we are interested in (questions that ask the same thing about a set and an superset); this relation is given in (56):

(56) Q' strengthens Q iff when question Q is already answered, question Q' is still unanswered.

Given this, the strengthening requirement takes the following form for questions:

(57) <u>any</u> is licensed in a question only if when the question on the narrow interpretation is already answered, the question on the wide interpretation is still open.

Now look at (50) - why is <u>any</u> licensed here? If we just look at (50) itself, we see that it does not satisfy the requirement in (57). If question (55) on the narrower interpretation is answered positively, then the question on the wider interpretation is already answered as well. If Sue has cooking potatoes, then of course she has cooking or decorative potatoes.

This is where the negative expectation or suggestion comes in. If the context contains the assumption that the question on the narrower interpretation has a **negative** answer, then even given this answer to this question, the question on the wider interpretation is still unanswered. If Sue doesn't have cooking potatoes, it is still not known whether she has cooking or decorative potatoes. We see then that the requirement in (57) is only satisfied in contexts where the question on the narrower interpretation is assumed to have a negative answer.

To summarize, we have proposed a relation between questions which plays the same role that entailment played in the case of declaratives. This relation allows us to explain why <u>any</u> is licensed in questions, and why it must be accompanied there by a negative expectation or suggestion.

Let us make one more remark here. In questions, this negative implication can at times be very weak, up to non-existence, as in the standard question by an attentive shop-attendant:

(58) Is there anything I can do for you?

We think that this is a consequence of conventionalization. If it has become conventionalized to ask a question in a certain form in a certain kind of situation, then that situation may be sufficient to license that question, even without its original implicatures. Yet, even in (58), the negative implicature has not completely disappeared: (58)

is a more polite question than (59)

(59) Is there something I can do for you?

and this politeness is explained by the negative implicature. To exaggerate a bit, in asking (58) it is as if the speaker says: 'I know that I won't be able to help you with your real problems, but maybe, besides that, there is some small issue that I may be able to help you with.'

6. FREE CHOICE ANY

6.1. The proposal.

We start by noting some central properties of FC any.

1. FC any behaves like a universal quantifier. This seems clear from simply considering the meaning of example (60). Example (61) shows that any can be modified by almost, just like the true universal quantifiers every and no.

(60) Any owl hunts mice.
(61) Almost any owl hunts mice.

2. FC any is in certain ways similar to generic indefinites. Carlson 1981 observes that the contexts where FC any is allowed are to a large extent typically the same contexts where generics are allowed. Also, FC any, like generics, has a modal nature: statements like (60) are 'law-like', in that they have counterfactual entailments. (60) entails roughly that if you were an owl, you would hunt mice.

We take the similarity with generics very seriously, and propose the following analysis of FC any.

We would like to claim that the analysis of any that was presented in section 2 applies exactly as it is to examples with FC any. The NP with any is an indefinite NP with widening and strengthening. We propose that FC any is what you get when the indefinite NP is interpreted **generically**:

Free choice any CN = the corresponding indefinite NP (a CN) **interpreted generically** + widening and strengthening

6.2. Generics.

We are claiming that a FC any NP is a generic NP. So we have to make some assumptions about generics. Two aspects of the semantics of generics are relevant here:

(62) An owl hunts mice.

1. Generics allow exceptions. For example, sentence (62) can be true even in a situation where there are, say, baby owls that don't hunt mice.
2. The modal, 'law-like' nature of generics: Generic statements have counterfactual entailments. For example, (62) entails roughly that if you were an owl (and not an exception) you would hunt mice.

We won't say much about the modal nature of generics here. We think that determining its exact nature is crucial for the understanding of many aspects of the semantics of FC any and we want to deal with it in a separate paper. For the present paper, we will just assume that the quantification involved in generics is modal quantification. On the other hand, we **do** want to focus on the issue of exceptions. This is of course a central issue in the semantics of generics: How can you specify the truth conditions for generics in such a way that the quantification is sort of universal but nevertheless allows exceptions? We are well aware that we cannot in the space of this section solve the central problem of generics. Rather we want to make some minimal suggestions, that, to us, seem compatible with several approaches to generics, like for instance, non-monotonic analyses like circumscription.

Consider example (62). We assume that generics like an owl are formed from the normal indefinite with a **generic operator**. However, we take this generic operator to be basically a (modal) **universal quantifier**. We suggest, furthermore, that this quantifier is restricted by a **contextually given set of properties**, properties which determine, roughly, what sort of owls example (62) is about. One can think about them as the properties that 'normal' owls, or 'standard' owls, or just 'owls that count for the present purposes' have. So we propose that (62) is interpreted as something like (63), where X_{owl} is this set of contextually given properties.

(63) $\forall x \sim X_{owl}$ [Owl(x) --> Hunt mice(x)]

(63) means something like this: For every possible object which has all the properties in X_{owl}, if it's an owl, it hunts mice.

(63) accounts for the intuition that certain owls that don't hunt mice may be regarded not as refuting (62), but rather as legitimate exceptions. For example, if ADULT is one of the properties in X_{owl}, then the quantification is over adult owls only, and a baby owl that doesn't hunt mice would not refute the statement.

But this cannot be the whole story. If the variable X_{owl} simply gets as its value from the context an actual set of properties, then that would mean that generic quantification is just contextually restricted universal quantification. This can't be right, since all quantification is restricted in this way, and this would fail to explain why generics differ from regular universal quantifiers with respect to tolerating exceptions. So we need a further assumption.

What we would like to claim is that it is part of the nature of the generic that the value of X_{owl} is deliberately left **vague** (and the vagueness of X_{owl} induces vagueness in the truth conditions of the generic statement). The context does not tell you what the properties are that are the members of this set.

We think that this fits the intuition about generics. We think that when you use a generic, you are not trying to be precise. It's not supposed to be clear to your hearers exactly what owls are supposed to actually hunt mice.

In our view, if you say an owl hunts mice, that's just like saying 'every owl with the right properties hunts mice', while, **crucially**, not committing yourself to what the right properties are. You are not just saying that there are some properties and you don't know what they are: the vagueness is an integral part of what you say. In short, we claim that (62) means something like (64).

(64) 'all normal owls hunt mice',
where what counts as normal is inherently vague.

Note that we are **not** claiming that (62) should be reanalyzed in terms of the meaning of the sentence all normal owls hunt mice: the latter is a normal universal sentence, which does not have the vagueness that we are talking about: although the adjective normal may be vague, a speaker uttering the latter sentence may be taken to commit herself to some way of making it precise; in the generic statement there is no such commitment. This is what we mean when we say that the vagueness is an integral part of the utterance.

To summarize: Generic statements allow exceptions in a way that regular universal statements do not. We have sketched an analysis that attributes this feature to inherent vagueness in the generic quantification.

6.3. Generics are not universal.

We will argue now that this exception-allowing vagueness explains the fact illustrated in (65), namely, that generics cannot be modified by almost, because they are not universal.

(65) *Almost an owl hunts mice.

The quantifiers that can be modified by almost are the universal ones, like every and no. Universal quantifiers can be defined as the quantifiers that do not allow exceptions, in the sense given in (66).

(66) A generalized quantifier Q(A) **does not allow exceptions** iff for any B and for any d in A, d is either a confirming instance or a refuting instance for the statement Q(A)(B).

For instance, every owl does not allow exceptions in the sense of (66): If an owl is an owl that hunts mice, it is a confirming instance of the statement every owl hunts mice, if it is an owl that doesn't hunt mice, it refutes the statement. So every is a true universal, and that's why we can say almost every owl. The same holds for no owl. On the other hand some owl, for example, does allow exceptions in the sense of (66): an owl that doesn't hunt mice is not a confirming instance, but neither a refuting instance for the statement Some owl hunts mice, and that's why we cannot say almost some owl.

Generics allow exceptions in the sense of (66). Take the statement an owl hunts mice and a sick owl d that doesn't hunt mice. It is possible that d neither confirms nor refutes the statement. This owl certainly is no confirmation. Now is it a counterexample? Not necessarily. Take the vague set X of properties that define normality for owls. It may very well be possible to make X more precise in such a way that it will include the property HEALTHY. This means that stating the vague generalization 'an owl hunts mice' may very well allow for the possibility that it's supposed to apply to healthy owls only. Since this is possible, our sick owl need not count as a counterexample.

To sum up, generics allow exceptions in the sense of (66), so they are not true universals, so they are not compatible with almost.

6.4. Licensing of FC any.

We now return to any and to example (60). We have proposed that any owl is just an owl with widening and strengthening. It follows that (60) has the same representation as (62), but with widening applied to it.

(63) $\forall x \sim X_{owl}$ [Owl(x) --> Hunt mice(x)]

Widening might be along the dimension 'healthy vs. sick'. If someone has said that a healthy owl hunts mice, you can respond with ANY owl hunts mice, meaning that healthy and sick owls alike hunt mice. Let us call the widened interpretation of owl 'owl, healthy or sick'.

Before we go into the details of widening and strengthening, let us briefly summarize what we've got now. Our analysis of FC any, reduces the difference between PS and FC any to the difference between non-generic and generic indefinites. This analysis allows for a unified treatment of any in its two uses, as an element that contributes widening and strengthening. In addition, it accounts for the properties of FC any which it shares with regular generic indefinites: being sort of universal, being modal, occurring in the same contexts.

What is left to do is to see why FC any is licensed, and to complete our account of those properties of FC any that it does not share with generics.

We return to the widening process. In the representation in (63), the predicate owl occurs twice: once in the antecedent of the conditional, and once as an index on the vague set X. Widening will have to apply to both occurrences, so the result of widening can be represented as in (67).

(67) $\forall x \sim X_{\text{owl, healthy or sick}}$ [owl, healthy or sick(x) --> hm(x)]

Thus crucially, in the case of FC any, widening gives us a new contextual vague set of properties. How does this new set differ from the old one? Before widening, the vague set was the set of properties that define normality for owls. At that point, it was possible that the property HEALTHY was one of the properties in X, or that on some precisification of the vague X, HEALTHY would turn out to be one of the properties in it. After widening, we get the set of properties that define normality for the predicate 'owl, healthy or sick'. Clearly, it should not be possible for HEALTHY to turn out to be one of the properties in this new set, because normality for sick and healthy owls alike should not be restricted to healthy owls only. Rather, normality should now be compatible with being sick. Therefore, we propose the definition in (68).

(68) $X_{\text{owl, healthy or sick}}$ is the result of minimally changing X_{owl} so as to make both HEALTHY and SICK compatible with it and with its precisifications.

Given this, it is clear that $X_{\text{owl, healthy or sick}}$ and its precisifications can't contain HEALTHY.

Let us summarize: Before widening, it was still possible for HEALTHY to be in X or in its precisifications. Widening makes SICK compatible with X and its precisifications, which means that if HEALTHY was in there, after widening it must be taken out.

We can now consider the licensing of FC any. We note that any is OK in (60), because strengthening is satisfied. The interpretations of (60) before and after widening are roughly as in (69), and the required entailment clearly holds.

(69) wide: Every owl (healthy or sick) with the 'normality' properties (which are compatible with HEALTHY and with SICK) hunts mice.

==> narrow: Every owl with the 'normality' properties (which include perhaps HEALTHY) hunts mice.

6.5. Any is a quantifier that is universal along a dimension.

We continue with the effect of widening. Consider again the sick owl that doesn't hunt mice. Before widening, our sick owl can constitute a legitimate exception to (60), because of the possibility that HEALTHY is one of the properties in X_{owl}. After widening, it is guaranteed that HEALTHY is not in X_{owl} or its precisifications. So it is no longer possible that only healthy owls are supposed to hunt mice. Now, does this mean that our sick owl automatically ceases to be a legitimate exception and becomes a counterexample?

Well, no. It is still possible that our owl has some other property that makes it exceptional, for example being very young. Yet one thing **has** changed: Being sick can no longer be the reason why our owl is a legitimate exception. Again: after widening, our sick owl may happen to be a legitimate exception, but not because it is sick.

We describe this situation by saying that after widening, the generic quantifier does not allow exceptions along the dimension 'healthy vs. sick'. We mean by this that the quantifier does not allow an owl to be an exception just because it is sick (or healthy, for that matter). This is the case if the property HEALTHY is **irrelevant** for the truth of the generic statement.

The property HEALTHY is irrelevant for the truth of the generic statement if the following holds: if the statement is true with HEALTHY being one of the properties determining normality for owls, it **stays** true if we remove HEALTHY. If that is the case, then the truth of the statement could not have had anything to do with the property HEALTHY. This is made precise in (70):

(70) A vague quantifier $Q \sim X_A$ (A) **allows no exceptions along the dimension <H vs. S>** iff

for any B and any precisification $p(X_A)$ of X_A such that H is in $p(X_A)$ or S is in $p(X_A)$,

if $Q \sim p(X_A)$ (A)(B) then $Q \sim p(X_{A,\ H\ or\ S})$ (A)(B).

What follows, thus, is that the effect of widening on the generic quantifier is to turn it into a quantifier which allows no exceptions along a certain dimension, the dimension associated with the corresponding <u>any</u>. This is the sense in which <u>any owl</u> is universal: it is universal along the associated dimension healthy vs. sick.

This fits very well with our intuitions about the effect of FC <u>any</u> (for instance, the 'no exceptions' feature), and also explains the compatibility of FC <u>any</u> with <u>almost</u>.

The natural generalization of <u>almost</u> to vague quantification is:

(71) <u>Almost</u> is an operator that turns a quantifier that allows no exceptions, or no exceptions along its associated dimension into a quantifier that allows **almost** no exceptions, or **almost** no exceptions along its associated dimension.

Given this, it follows that although <u>almost</u> does not apply to generic indefinites, it **does** apply to FC <u>any</u>.

To summarize: we have made the assumption that FC <u>any</u> is a generic indefinite with widening and strengthening. We have made some assumptions about generic indefinites and we have given some details about how widening operates in the case of FC <u>any</u>. We have argued that on this account strengthening is satisfied, so we have shown why FC <u>any</u> is licensed. In the case of FC <u>any</u> widening does not just concern the interpretation of the common noun, but also of the contextually given set of properties. This analysis accounts for the properties of FC <u>any</u> that distinguish it both from PS <u>any</u> and from regular generics: The universality of FC-<u>any</u> comes from the generic operator and from the effect of widening: the combined effect is that FC <u>any</u> does not allow exceptions along its dimension. This effect in turn explains the compatibility of FC <u>any</u> with <u>almost</u>.

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To summarize, we have seen that, contrary to [illegible] generic indefinite with [illegible] of stereotyping. We have made some [illegible] about generic indefinites and we have given some [illegible] in the case of TP [illegible] this account [illegible] is [illegible] licensed. In the case of [illegible] concern the interpretation of the common noun, but [illegible] the consecutively [illegible] of predication. This [illegible] accounts for the [illegible] both from FS and [illegible] universality of [illegible] from the effect on [illegible] any does not [illegible] effect in [illegible] of TP are with generics.

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D.I.R.T.: An Overview

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0. Introduction

The shift of focus from sentence to discourse meaning, which has characterized many semantic theories in this decade, has serious consequences on the entire conceptual framework of modern semantics. Both Church's type theory incorporated in Montague Grammar and Frege's ideas underlying it distinguish names and sentences as the minimal and maximal units of linguistic expressions. The general interest in discourse is justified by the fact that certain phenomena, such as inter-sentential connections, cannot be treated in those classical theories. On the other hand, those current semantic theories which assign truth conditions to pieces of discourse (or 'contexts') rather than to individual sentences [**Note 1**: Some examples] systematically fail to account for the intuition that sentences are somehow distinguished units rather than just arbitrary packages of information put together for purely syntactic reasons.

One of the aims of the present paper is to propose a solution to this dilemma. For reasons of space limitations, we will concentrate on the central concepts of our approach and certain ways of applying them to phenomena like implicit information, existential entailments vs. existential presuppositions, or 'scope ambiguity'. The paper is split into two parts. **Section 1** puts forth background assumptions, some empirical motivation, and an informal outline of the theory, called **D.I.R.T.** for Deferred Information Representation Theory. In **Section 2**, we introduce a quantifier-free version of the syntax and semantics of the theory's language (**Section 2.1**), which we will extend in order to deal with plural expressions and quantification (**Section 2.3**). We chose such a two-step presentation in order to separate the technicalities inherent in quantification from the core of the theory. In both cases, the formal construction is followed by examples which illustrate the application of the theory and its explanatory power (**sections 2.2** and **2.4**).

1. Informal Outline

1.1. Sentence and Discourse Meaning

The limitations of a strictly Fregean/Montagovian approach stem from the view that sentences are (quantificationally) closed formulae, which means that they are semantically independent from each other. Therefore, such a theory is bound to exclude from semantics all connections between the sentences of a piece of discourse and between sentences and external contexts, i.e., all that makes a piece of discourse more than just a conglomerate of sentences. In this way, disparate phenomena such as cross-sentential anaphoric links, indexical expressions, stylistic coherence and the knowledge used for syntactic or lexical disambiguation are all relegated to a poorly understood domain considered 'pragmatic'.

As opposed to this approach, we believe that there are at least two mechanisms which are to be captured in strictly semantic terms within this vast domain of phenomena:

(A) **Anchoring.** Many expressions can only be used in contexts which meet certain requirements. For example, anaphoric expressions need antecedents, ellipses need material which can fill them, and definite descriptions need contexts in which their presuppositions can be accomodated. What all these expressions share is that they are to be connected to specific parts of the previous context. [**Note 2**: Cataphors are irrelevant here.] The mechanism which builds such connections will be referred to as **anchoring.**

(B) **Specification.** It is obvious that natural languages provide partial, or vague, information. The underspecified character of meanings can gradually diminish owing to subsequent discourse. [**Note 3**: True ambiguities are different.] The mechanism responsible for this effect will be called **specification.**

We claim that the *mechanisms* in (A–B) above cannot be pragmatic, because they work in regular context-independent ways, even though the *information* they operate on may have pragmatic sources. In contradistinction, indexical reference, stylistic evaluation or disambiguation are influenced by extra-linguistic factors.

In what follows, we will concentrate on various aspects of the mechanisms thus selected. In particular, we will address the following problems:

1. *How do the mechanisms in (A) and (B) relate to each other?* We will answer this question in the end of **Section 1.2**, after introducing a simple model of information flow which will yield our basic terminology.
2. The importance of anchoring connections has led certain theories (cf. **Note 1**) to conceive of a piece of discourse as a kind of huge sentence. Now, *is it an accidental fact that human languages are based on sentences?* Or: Is it possible to obtain a meaning-preserving procedure to translate pieces of discourse into sentences? We will answer this question in **Section 1.3**, after outlining our semantic theory.
3. The assumption that the meanings of natural language expressions can be specified by subsequent context implies that not all semantic decisions related to an expression are to be taken immediately. The question arises

what possibilities can be left open if a given expression admits several interpretations. The answer to this question will be provided by **Section 1.4.**

1.2. A Simple Model of Information Flow

In this section, we will put forward a view of *sentences as instructions.* We will introduce terms to denote the necessary parts of such instructions and illustrate them by some examples. We will outline the relationship of sentences as instructions to anchoring and specification.

According to a well-known slogan, the role of a new information unit (a sentence) is to update previous information (a context). If we take this seriously, then we arrive at a view of sentences regarded as **instructions.** [**Note 4**: Apparent predecessors and 'hearer's meaning'.] In order to perform an updating instruction, the addressee has to know at least the following two things: (1) *which part* of the context to modify; and (2) *what change* to effect. In the simplest case, the sentence itself specifies these two things and only these; at any rate, it encodes what change to carry out. This way, the Fregean idea of the 'completeness' of a sentence is preserved: *sentences are the smallest expressions corresponding to complete instructions.* On the other hand, a sentence will contain at least two parts, one that specifies what target to look for and one that tells you what change to effect. This presupposes two corresponding mechanisms: one to look targets up (this is what we have called the **anchoring** mechanism) and one to update information (this is the **predication** mechanism). The process of carrying out an instruction embodied in a sentence will be called the **incorporation** of the sentence into the given context.

> **Incorporation, anchoring, and predication**
> The **incorporation** of a sentence in a context, i.e., carrying out the instruction embodied in the sentence, is based on an **anchoring** mechanism, which retrieves the part of the context to be modified, and on a **predication** mechanism, which effects a modification of the context.

As a simple example, let us look at the way of describing chess games. As a matter of course, only entire states of the play can be evaluated (e.g., in terms of winning chances), so these play the role of 'pieces of discourse' or 'contexts'. On the other hand, the 'sentences' of a game description correspond to one move each. For example, "B: Q $e4$—$e7$" represents a complete instruction. Obviously, "B: Q" ('black queen') refers to the part of the context to be changed, whereas "$e7$" denotes the field to which the black queen moves, i.e., it refers to the change to be performed. These two parts of the instruction correspond to two ways in which the move at issue can fail. First, it may be the case that the black queen has been taken already; second, it may be the case that the black queen cannot move to $e7$ (because this would be illegal). (In this example, we have assumed that there is only one black queen, i.e., "$e4$" does not contribute to finding the figure to be moved. On the other hand, it can be argued that the move can fail in a third way as well, namely, if the black queen happens not to be on field $e4$. We will come back to this problem in **sections 1.3** and **1.4.**)

As can be seen, "B: Q" belongs to the *presupposition* of our 'sentence'. Nevertheless, from now on, we will use the term **anchor** for expressions that refer to the target of an instruction, for not all that is presuppositional necessarily belongs to the anchor (in the example above, "$e4$" is also presuppositional, but we have assumed that it does not belong to the anchor). The part of the instruction which corresponds to the change effected (which is the non-presuppositional part, i.e., "$e7$" in our example) will be referred to as the **claim** of the instruction or sentence.

> **Anchor and claim**
> The two basic parts of a context-updating instruction are its **anchor** and its **claim**. The anchor carries information about which part of the context is to be changed or anchored to the context, and the claim specified what change to effect.

In fact, we believe that natural language sentences work in a similar way. Let us illustrate this with a series of examples, of increasing complexity, from English:

(1) *Joe was walking.* He *whistled.*
(2) *Joe was walking.* The fool *whistled.*
(3) *A couple entered.* The woman *smiled.*
(4) *Joe got married yesterday.* The priest *spoke harshly.*
(5) *Joe got married yesterday.* A man *behaved impolitely.*

The anchor of the second sentence in (1) is *he*; the anchoring mechanism has to find the antecedent of this pronominal anaphor in the previous context. The claim of the sentence, embodied by *whistled*, can only be predicated after a successful anchoring. In (2), the anchor is a definite noun phrase in an anaphoric function. In (3), the anchor (*the woman*) will not be *identified* with the antecedent which licenses it (i.e., *the couple*), and a good deal of lexical knowledge is involved. The situation in (4) is even more complex: the connection which anchors *the priest* (to 'marriage') seems to involve encyclopedic knowledge as well. In (5), we observe that, if the context is restricted to the first sentence, the anchor (i.e., *a man*) can only refer to someone in the audience of the marriage ceremony. That is, even though an indefinite NP corresponds to a 'new' participant in a piece of discourse, it is still to be licensed by the context when it is the anchor of a sentence. [**Note 5**: *A couple* in (3) is different.]

Let us now discuss the sentences in (4) in more detail. Obviously, the statement 'there was no church ceremony' would be just fine after the first sentence, but it cannot be true after the second one has been uttered, although the second sentence does not entail in itself that there was such a ceremony. In other words, the anchoring of *the priest* in the second sentence *requires that the hearer infer* that there was a church ceremony, though that is not a valid inference on the basis of the first sentence. That is, establishing anchoring connections can trigger inferences or additional **specification.** [**Note 6**: Open possibilities.] At this point, we would like to suggest a radical answer to our first question in **Section 1.1**, namely, that all implicit knowledge that is part of a context is due to the need for some anchoring connection:

> **Implicit information**
> A piece of information ι is **implicit** in a sequence of sentences $S_1 S_2$ if and only if there are readings R_1 and R_2 of S_1 and S_2, respectively, such that neither R_1 nor R_2 entails ι, and all readings of the sequence $S_1 S_2$ do entail ι.

1.3. Outline of the Semantics

The opposition between the anchor vs. the claim of a sentence is probably reminiscent of the topic—comment or subject—predicate distinction. Instead of going into detailed comparison, we will simply outline the formal interpretation of pieces of discourse as they arise from sentences as instructions. Furthermore, we complete the picture of sentences as instructions with a third, optional, part.

The essence of the distinction between anchors and claims, reduced to the terminology of truth-conditional semantics, is as follows. It is the claim of a sentence that can be true or false in the traditional sense of the word; the impossibility of anchoring simply makes predication impossible (cf. our chess example in **Section 1.2**). Consequently, the translation of a sentence will not correspond to an atomic formula, but to a special kind of complex formula, which will be splitted into two parts, namely, a set of atomic formulae (called **anchor conditions**), and a possibly complex **predicate condition**. [**Note 7**: The relational view of sentences.]

> **Anchor and predicate conditions**
> The formal representation of sentences as instructions are formulae splitted into two parts, namely, a set of **anchor conditions** and a (possibly complex) **predicate condition**.

To implement the above ideas, we will say that pieces of discourse are structured in the same way. The 'falsity' of anchor conditions leads to a **truth value gap** (represented by the truth value 2); a complex formula can be true or false (depending on its predicate condition) only if its anchor conditions are true. The full syntax and semantics of our formal language will be presented in **Section 2**.

Let us now illustrate the points made so far by three examples.

(6) *A man walked and talked.*

The formula corresponding to this one-sentence piece of discourse (if *a man* is an indefinite to be anchored, like the one in (5), cf. **Note 5**) is

$$\big[\textsc{man}(m)\big]\,\big[\lambda m'[\ ][\textsc{walk}(m') \wedge \textsc{talk}(m')](m)\big],$$

where the condition in the first pair of brackets is the single anchor condition of the formula, whereas the one in the second pair is the predicate condition. The predicate condition is a λ-abstraction applied to the argument m. Note that the body of the abstraction is itself like a formula, i.e., it has two pairs of brackets. The first pair is not always empty, e.g., the formula corresponding to *A man walked a dog* can be

$$\big[\textsc{man}(m)\big]\,\big[\lambda m'[\textsc{dog}(d)][\textsc{walk}(m', d)](m)\big].$$

Assuming that *a man* in (6) has been anchored somehow (i.e., it corresponds to a referent which has an established value), the truth conditions of this formula are

$$\begin{cases} 1 & \text{if } [\![\textsc{man}(m)]\!] = 1 \ \&\ [\![\textsc{walk}(m)]\!] = [\![\textsc{talk}(m)]\!] = 1 \\ 0 & \text{if } [\![\textsc{man}(m)]\!] = 1 \ \&\ ([\![\textsc{walk}(m)]\!] \neq 1 \vee [\![\textsc{talk}(m)]\!] \neq 1) \\ 2 & \text{otherwise.} \end{cases}$$

The next example is

(7) *A man who walked talked.*

$$\big[\textsc{man}(m), \textsc{walk}(m)\big]\,\big[\lambda m'[\ ][\textsc{talk}(m')](m)\big]$$

Truth conditions:

$$\begin{cases} 1 & \text{if } [\![\text{MAN}(m)]\!] = [\![\text{WALK}(m)]\!] = 1 \ \& \ [\![\text{TALK}(m)]\!] = 1 \\ 0 & \text{if } [\![\text{MAN}(m)]\!] = [\![\text{WALK}(m)]\!] = 1 \ \& \ [\![\text{TALK}(m)]\!] \neq 1 \\ 2 & \text{otherwise.} \end{cases}$$

We emphasize that the truth conditions of (6) and (7) are essentially different: if there are no walking objects in the universe, then (6) is false, whereas (7) has no truth value. That is, the condition "WALK(m)" is **presuppositional** in (7) but not in (6). On the other hand, there is linguistic evidence to the effect that *walk* belongs to the **predicate** in (6), but not in (7):

(6′) *A man walked and talked. Joe did too.*
'Joe walked and talked, too'

(7′) *A man who walked talked. Joe did too.*
'Joe talked, too'

(We assume that the verb phrase anaphor *did* refers to the predicate of the previous sentence.)

Our third example is a sequence of two sentences with identical subjects:

(8) *A man walked. He talked.*

Intuitively, the meaning of (8) is very similar to those of (6) or (7). But what is it precisely? In order to answer this question, let us conceive of (8) as if it was a single instruction. Is this instruction the same as either (6) or (7)? Clearly, (6), (7) and (8) contain the same set of conditions, i.e., "MAN(m)", "WALK(m)" and "TALK(m)". As in (6) and (7), "MAN(m)" is an anchor condition in (8). To decide where the other two conditions belong, we will examine whether they qualify as presuppositional in (8) and/or belong to the predicate of (8).

Presuppositional character: It seems obvious that "TALK(m)" is not presuppositional in (8). On the other hand, we would like to argue that "WALK(m)" *is* presuppositional in that piece of discourse. We believe this is a way of making sense out of pieces of discourse as *sequences* of sentences. In a sense, when a speaker utters a new sentence, (s)he presupposes the truth of whatever (s)he has said before. As a consequence, the truth conditions of (8) are exactly as those of (7).

VP-ellipsis:

(8′) *A man walked. He talked. Joe did too.*
'Joe talked' or 'Joe walked and talked'

(As a matter of course, not all sentences of the type shown in (8′) have several readings. What is important here is that they *may*, or so we judge, when the two consecutive predicates are 'about the same situation'. The reason why this is so is immaterial for the present discussion.)

According to the way in which we conceive of the meaning of *did*, the fact that the second interpretation above is also possible shows that "WALK(m)" belongs to the predicate condition of (8). On the other hand, as the VP-ellipsis test above shows, "TALK(m)" certainly belongs to the predicate condition of (8), which is necessarily so since it is the claim of (8).

Consequently, it is possible for a condition ("WALK(m)" in the example (8)) to belong to a predicate without being the claim of the instruction. (The fact that "WALK(m)" is such a condition is also responsible for the fact that (8′) has several meanings; cf. **Section 1.4**.)

Floating conditions (first approximation)
Presuppositional conditions which belong to the predicate condition of a representation are called **floating conditions.** The truth of floating conditions is not a prerequisite of performing predication.

Taking up the chess description example (cf. **Section 1.2**), we can consider "*e*4—*e*7" to be the predicate of "*B*: *Q* *e*4—*e*7", in which "*e*4" corresponds to a floating condition. We have seen in (8) that the claim of a sentence may become a floating condition as soon as the piece of discourse is continued; we will argue in the next section that floating conditions occur within single sentences as well.

In this section, we have examined the discourse structure that sentences as instructions give rise to. From a *truth-conditional* point of view, everything is presuppositional except the claim of the last sentence. On the other hand, there is evidence to the effect that predicates as *structural constituents* of sentences are preserved in discourse structure to a certain extent. Under the assumption that sentences are instructions, we have introduced a third, *instructional*, level of sentence analysis. According to this, a sentence contains an anchor and a claim, plus optional floating conditions. Schematically:

<table>
<tr><td>interpretation:</td><td colspan="2">presupposition</td><td rowspan="2">claim</td></tr>
<tr><td>instruction:</td><td rowspan="2">anchor</td><td>(floating)</td></tr>
<tr><td>constituents:</td><td colspan="2">predicate</td></tr>
</table>

1.4. Deferred Information

In the first part of this section, we will give a precise structural characterization of floating conditions. Then we will put forward an hypothesis about the relationship of such conditions to underspecified meanings. Finally, we will propose a distinction between underspecification and ambiguity using that hypothesis.

As we have mentioned in the previous section, bodies of λ-abstractions consist of anchor and predicate conditions just like formulae. We will stipulate that the precidate condition of a complex predicate belongs to the claim rather than the presupposition. But there may be anchor conditions in a λ-abstraction which also belong to the claim. For example, in the sentence

(9) *A man saw a unicorn*,

the fact that the object seen is a unicorn is claimed rather than presupposed although, obviously, it is to be represented by an internal anchor condition:

$$[\text{MAN}(m)]\,[\lambda m'[\text{UNICORN}(u)][\text{SEE}(m',u)](m)].$$

The peculiarity of the condition "UNICORN(u)" is that it contains no λ-variable, i.e., it does not make reference to any variable related to the anchor. [**Note 8**: The closedness of λ-abstractions.] On the other hand, we have mentioned that the instruction "*B*: *Q* *e*4—*e*7" can be analyzed in such a way that the condition corresponding to *e*4 is floating (cf. **sections 1.2 and 1.3**):

$$[B{:}\,Q(q)]\,[\lambda q'[e4(q')][-e7(q')](q)].$$

The internal anchor condition "$e4(q')$" qualifies as floating because it makes reference to a λ-variable, i.e., something that is present in the external context. Accordingly, the piece of discourse in (8) (cf. **Section 1.3**) is to be represented as follows:

$$[\text{MAN}(m)]\,[\lambda m'[\text{WALK}(m')][\text{TALK}(m')](m)].$$

This representation is not equivalent to that of (7) given in **Section 1.3**, although its truth conditions are the same. Therefore, our answer to the second question in **Section 1.1** is on the negative: It is not always possible to obtain a meaning-preserving translation from pieces of discourse into sentences.

The structural characterization of floating conditions given above is in perfect accordance with their semantic behaviour. That is, a condition which refers to objects already introduced by the context without updating information about them is necessarily presuppositional. Accordingly, such a condition represents extra information, independent from the claim itself, about some argument of the predicate. (The exact definition will be given in **Section 2.1**).

> **Floating conditions** (second approximation)
> A **floating condition** is an anchor condition in a λ-abstraction which represents information about an argument of the λ-abstraction independently from the claim of the λ-abstraction.

It is easy to find now natural language examples for instructions containing floating conditions:

(10) *A man loved his cat.*
'a man loved the cat he owned'
$[\text{MAN}(m)]\,[\lambda m'[\text{CAT}(c), \text{OWN}(m', c)][\text{LOVE}(m', c)](m)]$

Obviously, the condition "$\text{OWN}(m'c)$" represents information about m' which is independent from the claim "$\text{LOVE}(m', c)$". Therefore, it is a floating condition. [**Note 9**: "$\text{CAT}(c)$" is also floating.] Note the similarity between the above formula and the representation of (8). This suggests that the meaning of (10) will be identical to that of the following piece of discourse (for details cf. **Section 2.2**):

(11) *A man owned a cat. He loved it.*

Let us now turn back to the problem of open possibilities, mentioned in **sections 1.1** and **1.2**. The problem of open possibilities for truth-conditional semantics is that they must be available for establishing anchoring connections, but must not contribute to truth conditions before they are 'activated' by an anchoring process. Therefore, we will say that the pieces of information in question constitute **deferred information** in the piece of discourse. Since we will stay within the limits of truth-conditional semantics throughout this paper, our only aim here is to point out which pieces of information in our representations of sentences can be deferred at all.

In our view, deferment means delaying the interpretation of certain conditions. Now, it is obvious that conditions which immediately belong to the anchor or the claim of a sentence cannot be deferred, because they have anchoring and updating functions, respectively, and these parts of an instruction have to be carried out. Consequently, only floating conditions (at any level of embedding) can be deferred. Such conditions play no role in either finding the target in context or updating it; their only function is that they are to be *checked* when the target is found. We assume that, under certain conditions [**Note 10**: Under what conditions?], this checking can be postponed.

As a matter of fact, neither one of the floating conditions seen so far (i.e. those in (8) and (10)) is deferred. Nevertheless, both (8) and (10) behave in a peculiar way under our VP-ellipsis test:

(8′) *A man walked. He talked. Joe did too.*
'Joe talked' or 'Joe walked and talked'

(10′) *A man loved his cat. Joe did too.*
'Joe loved the man's cat, too' or 'Joe loved the cat his own cat, too'

The difference between the two readings is the same in both (8′) and (10′), i.e., it lies in whether the information corresponding to the floating condition of the antecedent predicate (i.e., "WALK(x)" and "OWNS(x, c)", respectively) is interpreted as 'a man' or 'Joe' in the sentence containing the VP anaphor.

According to what has been said so far, if the above examples involve deferred information, then the possible readings of (8′) and (10′) constitute 'open possibilities'. Therefore, it must be possible to disambiguate them by later specification:

(10′) *A man loved his cat. Joe did too.*

a. ... *The cat was black.*
'the cat the man owned and both the man and Joe loved was black'

b. ... *Joe's cat was black.*

Assuming that *the cat* in (10′a) is anaphoric and wants a unique antecedent, the continuation (10′a) triggers the so-called 'strict reading' of (10′), whereas (10′b) triggers the so-called 'sloppy reading'.

Let us assume that, indeed, the fact that (8′) and (10′) have several readings is due to the deferred character of certain floating conditions in their representations. Recall that floating conditions are idle when you carry out the instruction in which they occur. Accordingly, we propose the following definition:

Underspecified meaning and ambiguity
An expression has multiple readings owing to its **underspecified meaning** if and only if its different readings stem from different possibilities of interpreting some of its floating conditions. The expression is said **ambiguous** otherwise.

In other words, an expression is ambiguous if it may correspond to several (mutually incompatible) instructions, which coincide on the surface, as it were.

In terms of this definition, pieces of discourse like (10′) above qualify as semantically underspecified rather than ambiguous, since they consist of two unambiguous instructions. In particular, in the case of (10′), we have assumed that *his* refers to *a man*, and *did* in the second sentence refers to the predicate of the first (i.e., *loved his cat*). (The case of (8′) is similar although, seemingly, *did* could there has two potential antecedents. Nevertheless, we have assumed that its unique antecedent is the predicate of the entire preceding discourse which, as we have shown, contains both 'walked' and 'talked'.)

Our approach to VP-ellipsis is certainly non-standard. Most linguists and logicians would consider (10′) to be ambiguous rather than underspecified, and would argue that the first sentence of (4) in **Section 1.2** (i.e., *Joe got married yesterday*) and (10′) are radically different. This seems intuitively plausible, but it would be rather cumbersome to formulate the difference precisely. What comes closest to an empirical test of 'true' ambiguity is the following (this test was called into our attention by B.H. Partee):

> **Ambiguity test**
> An expression that has several meanings qualifies as ambiguous if and only if the speaker is not entitled to be uncertain about the particular reading of the expression (s)he has in mind when uttering it. The expression has an underspecified meaning otherwise.

That is, the speaker does not necessarily know whether Joe's marriage was accompanied by a church ceremony when (s)he utters (4), whereas (s)he must know whose cat Joe loves when (s)he utters (10′).

What the above test fails to capture is the difference between ambiguity and underspecification from the hearer's point of view. As we have suggested in connection with (4) and (10′a–b), all underspecified meanings can be further specified using implicit information, which is not necessarily true for ambiguous sentences. In the theory that we propose, this follows from the fact that underspecified meanings are due to deferment.

In this section, we have argued for semantic differences between formulae which do not necessarily correspond to differences of truth conditions. But the fact that deferred information can license presuppositions shows that these differences do have indirect truth-conditional effects. [**Note 11**: We will not go into these here.]

2. Some Details

In this section, we will elaborate on certain details of D.I.R.T. In **Section 2.1**, a simple version of D.I.R.T.'s language, allowing for 'singular' entities only, will be presented. In **Section 2.2**, some cases of incorporation will be shown and informally explained. **Section 2.3** introduces a possible extension of the language to 'plural' objects, and **Section 2.4** offers examples of using the extended version for describing 'quantification' in natural languages.

2.1. D.I.R.T.'s Language

In this section, we present the syntax and semantics of a 'singular' version of D.I.R.T.'s representation language, $\mathcal{L}$. $\mathcal{L} = \langle \mathrm{LC}, \mathrm{Con}, \mathrm{Var}, \mathrm{Pred}, \mathrm{Cond}, \mathrm{Form} \rangle$, where $\mathrm{LC} = \{(,),[,],\lambda,\neg,\wedge,\ ,\}$ is the set of **logical constants**, $\mathrm{Con} = \mathrm{Con}_{\mathrm{pred}} \cup \mathrm{Con}_{\mathrm{ind}}$ is the set of **non-logical constants**, namely, $\mathrm{Con}_{\mathrm{pred}} = \bigcup_{i<\omega} \mathrm{Con}^{i}_{\mathrm{pred}}$ is the set of **predicate constants**, in which each $\mathrm{Con}^{i}_{\mathrm{pred}}$ is the set of i-ary predicates, and $\mathrm{Con}_{\mathrm{ind}} = \{b_i\}_{i<\omega}$ is the set of **individual constants**. $\mathrm{Var} = \{v_i\}_{i<\omega}$ is the set of **variables**. LC, each $\mathrm{Con}^{i}_{\mathrm{pred}} \in \mathrm{Con}_{\mathrm{pred}}$, $\mathrm{Con}_{\mathrm{ind}}$ and Var are to be pairwise disjoint sets. Pred (the set of **predicates**), Cond (the set of **conditions**) and Form (the set of **formulae**) are the expressions of D.I.R.T., to be defined by simultaneous induction in clauses (i–vii) below.

$I = \langle \mathcal{U}, \varrho \rangle$ is an interpretation of the expressions of $\mathcal{L}$; $\mathcal{U}$ is the **universe** $(0, 1, 2 \notin \mathcal{U})$, and ϱ is the **interpretation function** ($\text{Dom}(\varrho) = \text{Con}$). In the case of an n-place predicate constant, the value of ϱ is an ordered pair consisting of two disjoint sets, the **extension** and the **anti-extension** of the predicate in question. Both extensions and anti-extensions are sets of ordered n-tuples whose components are in $\mathcal{U}$:

$$\text{Pr}^n \in \text{Con}^n_{\text{pred}} \Rightarrow \varrho(\text{Pr}^n) \in \{\langle \Phi, \Psi \rangle : \Phi, \Psi \in \mathcal{P}(\mathcal{U}^n) \;\&\; \Phi \cap \Psi = \emptyset\}.$$

$\Phi \cup \Psi$ is not necessarily identical to $\mathcal{U}^n$, so a relation may be 'undefined' for an n-tuple. If $b \in \text{Con}_{\text{ind}}$, then $\varrho(b) \in \mathcal{U}$. The semantic value function is $[\![\cdot]\!]^I_g$, where I is an interpretation and g is an assignment such that $g\colon \text{Var} \cup \text{Con}_{\text{ind}} \to \mathcal{U}$. That is, we extend the assignment function to individual constants. This has purely technical reasons. We stipulate that for all $b \in \text{Con}_{\text{ind}}$, $g(b) = \varrho(b)$.

Let us now turn to the parallel definition of the syntax and semantics of $\mathcal{L}$. Note that the function $\text{Free}\colon \text{Cond} \to \mathcal{P}(\text{Var} \cup \text{Con}_{\text{ind}})$, which determines the set of variables and individual constants occurring free in a condition, is defined in parallel.

(i) Predicate constants are predicates.

$$\text{Pr}^n \in \text{Con}^n_{\text{pred}} \Rightarrow \text{Pr}^n \in \text{Pred}^n;$$
$$[\![\text{Pr}^n]\!]^I_g = \varrho(\text{Pr}^n).$$

(ii) Predicate negation.

$$P^n \in \text{Pred}^n \Rightarrow \neg P^n \in \text{Pred}^n.$$
$$[\![P^n]\!]^I_g = \langle \Phi, \Psi \rangle \Rightarrow [\![\neg P^n]\!]^I_g = \langle \Psi, \Phi \rangle.$$

(iii) Predicate application yields conditions.

$$P^n \in \text{Pred}^n \;\&\; x_1, \ldots, x_n \in \text{Var} \cup \text{Con}_{\text{ind}} \Rightarrow P^n(x_1, \ldots, x_n) \in \text{Cond}.$$
$$\text{Free}(P^n(x_1, \ldots, x_n)) := \{x_1, \ldots, x_n\}.$$
$$[\![P^n]\!]^I_g = \langle \Phi, \Psi \rangle \Rightarrow [\![P^n(x_1, \ldots, x_n)]\!]^I_g = \begin{cases} 1 & \text{if } \langle g(x_1), \ldots, g(x_n) \rangle \in \Phi; \\ 0 & \text{if } \langle g(x_1), \ldots, g(x_n) \rangle \in \Psi; \\ 2 & \text{otherwise.} \end{cases}$$

(iv) Conjunction of conditions.

$$c_1, c_2 \in \text{Cond} \Rightarrow c_1 \wedge c_2 \in \text{Cond};$$
$$\text{Free}(c_1 \wedge c_2) := \text{Free}(c_1) \cup \text{Free}(c_2);$$
$$[\![c_1 \wedge c_2]\!]^I_g = \begin{cases} 1 & \text{if } [\![c_1]\!]^I_g = [\![c_2]\!]^I_g = 1; \\ 2 & \text{if } ([\![c_1]\!]^I_g = 2) \vee ([\![c_2]\!]^I_g = 2); \\ 0 & \text{otherwise.} \end{cases}$$

(v) The only way of recursively embedding a D.I.R.T. representation into another one is to apply a **λ-abstraction.** λ-abstractions are like formulae, but contain a non-empty set $V = \{v_1, \ldots, v_n\}$ of n **λ-variables** ($V \subseteq \text{Var}$) and qualify as n-ary predicates. As will be the case in formulae, a λ-abstraction contains a set $A = \{a_1, \ldots, a_k\}$ of **anchor conditions** ($A \subseteq \text{Cond}$) and a **predicate condition** p ($p \in \text{Cond}$; $P = \{p\}$). The set $R \subseteq \text{Var}$ is the set of **referents** of the box (disjoint from the set of λ-variables). Each of these must occur in at least one anchor condition, i.e., $R = \bigcup_{c \in A} \text{Free}(c) - V$. Notice that individual constants are not allowed to occur as referents in λ-abstractions. We distinguish the subset $F \subseteq A$ of **floating conditions**, and those referents which occur in at least one floating condition (**floating referents**, R_F). Both are defined as the smallest sets satisfying the following conditions:

(1) $c \in A \;\&\; \bigvee_{v \in V} [v \in \text{Free}(c)] \Rightarrow c \in F$;
(2) $r \in R \;\&\; \bigvee_{c \in F} [r \in \text{Free}(c)] \Rightarrow r \in R_F$; and
(3) $c \in A \;\&\; \bigvee_{r \in R_F} [r \in \text{Free}(c)] \Rightarrow c \in F$.

(In the meta-language, "$\bigvee$" is the existential quantifier, and "$\bigwedge$" is the universal quantifier.)

The non-floating referents (R_E) are called **existential:** $R_E = R - R_F$. We define the graph $\text{Graph}(V \cup R) = \langle V \cup R, E \rangle$, with $V \cup R$ as vertices, where an edge runs between two vertices ($(r_1, r_2) \in E$) iff both occur free in some condition ($\bigvee_{c \in A \cup P}[r_1, r_2 \in \text{Free}(c)]$). $\text{Graph}(V \cup R)$ is to be connected. If all these requirements are met, then

$$\lambda v_1 \ldots v_n[a_1, \ldots, a_k][p] \in \text{Pred}^n.$$

In the corresponding semantic rule, we use the notational convention

$$\mathbf{V} = \langle v_1, \ldots, v_n \rangle \in \text{Var}^n,\ \mathbf{u} = \langle u_1, \ldots, u_n \rangle \in \mathcal{U}^n \Rightarrow$$
$$\Rightarrow g[\mathbf{V} : \mathbf{u}] := g[v_1 : u_1] \ldots [v_n : u_n]$$

(note that we use $\mathbf{V}$, $\mathbf{R}$, $\mathbf{R}_E$ and $\mathbf{R}_F$ for arbitrary orderings of V, R, R_E and R_F, respectively, and we implicitly assume that

$$\mathbf{v} \in \mathcal{U}^n; \quad \mathbf{f} \in \mathcal{U}^{|R_F|}; \quad \mathbf{e} \in \mathcal{U}^{|R_E|};$$
$$f \in F; \quad q \in (A - F) \cup P; \quad g^* = g[\mathbf{V} : \mathbf{v}][\mathbf{R}_F : \mathbf{f}][\mathbf{R}_E : \mathbf{e}].$$

The semantic rule is

$$[\![\lambda v_1 \ldots v_n[a_1, \ldots, a_k][p]]\!]^I_g = \langle \Phi, \Psi \rangle,$$

where the extension Φ is the set of n-tuples from $\mathcal{U}$ for which the conditions in the λ-abstraction hold true if we take the referents to be existentially quantified:

$$\Phi = \left\{ \mathbf{v} : \bigvee_{\mathbf{f}} \bigvee_{\mathbf{e}} \bigwedge_{f} \bigwedge_{q} [\![f]\!]^{I}_{g^*} = [\![q]\!]^{I}_{g^*} = 1 \right\}$$

The calculation of the anti-extension Ψ is more complex. Floating conditions are presuppositional, and the existence of floating referents is presupposed. So floating conditions are to be true (with the floating referents understood as existentially quantified; existential referents do not occur in floating conditions) for a given n-tuple to be in the anti-extension. On the other hand, the existence of existential referents is *claimed* by the predicate, so no corresponding objects for which the predicate condition holds true must exist:

$$\Psi = \left\{ \mathbf{v} : \bigvee_{\mathbf{f}} \bigwedge_{\mathbf{e}} \left[\left(\bigwedge_{f} [\![f]\!]^{I}_{g^*} = 1 \right) \& \left(\bigvee_{q} [\![q]\!]^{I}_{g^*} \neq 1 \right) \right] \right\}.$$

Notice that all referents are interpreted as quantified *within* the λ-abstraction. This is what we called the Principle of Referential Privacy in **Note 8**.

(vi) The formulae of $\mathcal{L}$ contain the sets $A = \{a_1, \ldots, a_k\}$ and $P = \{p\}$ as in (v) above, and a set $R \subseteq \mathrm{Var} \cup \mathrm{Con}_{\mathrm{ind}}$ such that $R = \bigcup_{c \in A \cup P} \mathrm{Free}(c)$. The graph $\mathrm{Graph}(R) = \langle R, E \rangle$, defined in an analogous way to $\mathrm{Graph}(V \cup R)$ in (iv), is to be connected. Then

$$[a_1, \ldots, a_k][p] \in \mathrm{Form}.$$

In a formula, anchor conditions are presuppositional, and predicate conditions are claimed. Accordingly,

$$[\![[a_1, \ldots, a_k][p]]\!]^{I}_{g} = \begin{cases} 1 & \text{if } \bigwedge_{c \in A \cup P} \left[[\![c]\!]^{I}_{g} = 1 \right]; \\ 2 & \text{if } \left(\bigvee_{c \in A} \left[[\![c]\!]^{I}_{g} \neq 1 \right] \right) \vee \left([\![p]\!]^{I}_{g} = 2 \right); \\ 0 & \text{otherwise.} \end{cases}$$

(vii) The only well-formed expressions of D.I.R.T. are those produced by finite sequences of applications of (i–vi) above.

2.2. Two Types of Incorporation

We consider the language described in the previous section to be the basic variant of D.I.R.T.'s representation language, which we will only augment in a conservative way, although we can imagine non-standard versions of its semantics, as we pointed out in **Note 11** (in the end of **Section 1.4**), in order to treat deferred information. On the other hand, the mechanisms producing D.I.R.T. formulae will be covered only partially in this paper. In what follows, we will only touch upon the mechanism called **incorporation**, which is responsible for building complex meaning representations from simpler ones. In a language like that of chess game descriptions, incorporation is very straightforward. In natural language, however, the syntax-driven incorporation processes which build sentence representations and the (possibly universal) system of 'discourse syntax' (which we imagine along the lines of Scha and Polanyi 1988) are extremely complex.

In this section, we will treat two simple sub-cases of incorporation only, to illustrate the basic ideas and to introduce the **unpacking** operation, often involved in incorporation. (We will touch upon a third sub-kind of incorporation in **Section 2.4.**) We will ignore most details of the anchoring step of incorporation. That is, we do not treat antecedent retrieval algorithms [**Note 12**: Antecedent retrieval in D.I.R.T.], and we assume that referents corresponding to indefinites can always be introduced freely.

According to what has been explained so far, incorporation is to take care of the fact that all discourse preceding a new claim is presuppositional. That is, in the following piece of discourse:

(11) *A man is walking. A woman is sleeping,*

the first sentence becomes presuppositional as soon as the second is processed. If (11a) and (11b) below are the meaning representations of these two sentences,

(11) a. $[\text{MAN}(m)]\,[\lambda m'[\;][\text{WALK}(m')](m)]$;

b. $[\text{WOMAN}(w)]\,[\lambda w'[\;][\text{SLEEP}(w')](w)]$,

and (11a) is also the representation of the context in which (11b) is to be incorporated, then the result of incorporation should be (11′):

(11′) $[\text{MAN}(m), \text{WOMAN}(w), \lambda m'[\;][\text{WALK}(m')]]\,[\lambda w'[\;][\text{SLEEP}(w')](w)]$.

The incorporation yielding (11′) represents our first sub-case, i.e., when two consecutive sentences have **different subjects**.

The second sub-case corresponds to a sequence of sentences with **identical subjects**, e.g.,

(8) *A man walked. He talked.*

with (8a) and (8b) representing the two sentences that constitute it:

(8) a. $[\text{MAN}(m)]\,[\lambda m'[\;][\text{WALK}(m')](m)]$;

b. $[\text{HE}(x)]\,[\lambda x'[\;][\text{TALK}(x')](x)]$.

We assume that $\text{HE}(x)$ in (2b) encodes that the subject of the second sentence is anaphoric, and that the incorporation mechanism is somehow able to retrieve m (i.e., the man) as its antecedent. We have seen in **Section 1.4** that incorporation has to yield a complex predicate assigned to m:

$$[\text{MAN}(m)]\,[\lambda m'[\text{WALK}(m')][\text{TALK}](m)].$$

The rule is that the incorporation of a sentence whose subject is identical to that of the previous one consists in incorporating the new predicate into the predicate condition of the previous context. On the other hand, the general principle of the context being presuppositional must hold true for such cases as well. Therefore, what used to be the embedded predicate becomes an anchor condition during incorporation.

The mechanism just described automatically accounts for the fact that, as we mentioned in **Section 1.4**, the following two pieces of discourse have the same meaning:

(10) *A man loved his cat.*
'a man loved the cat he owned'
$[\text{MAN}(m)]\,[\lambda m'[\text{CAT}(c), \text{OWN}(m', c)][\text{LOVE}(m', c)](m)]$

(12) *A man owned a cat.* *He loved it.*
$[\text{MAN}(m)]\,[\lambda m'[\text{CAT}(c)][\text{OWN}(m', c)](m)]$ $[\text{HE}(x), \text{IT}(y)][\text{LOVE}(x, y)]$
$[\text{MAN}(m)]\,[\lambda m'[\text{IT}(y), \text{CAT}(c), \text{OWN}(m', c)][\text{LOVE}(m', y)](m)]$
$[\text{MAN}(m)]\,[\lambda m'[\text{CAT}(c), \text{OWN}(m', c)][\text{LOVE}(m', c)](m)]$

The last question that we will examine here is the following. What happens if the antecedent of an anaphoric expression is not to be found immediately within the formula or abstraction where the new sentence is incorporated? Consider:

(13) *A farmer saw a unicorn.* It *was black.*
$[\text{FARMER}(f)]\,[\lambda f'[\text{UNICORN}(u)][\text{SEE}(f', u)](f)]$ $[\text{IT}(x)][\text{BLACK}(x)]$

(13) is a simple example of subject change between two sentences. However, the antecedent of *it*, i.e., u, is to be found in an abstraction embedded in the representation of the first sentence. Assume that the incorporation mechanism can retrieve that antecedent. What happens next? The following representation, which would result from the blind application of the principles mentioned so far, is ill-formed:

(*13′) $*[\text{FARMER}(f), \lambda f'[\text{UNICORN}(u)][\text{SEE}(f', u)](f)]\,[\text{BLACK}(\underline{u})]$

In terms of the Principle of Referential Privacy, the underlined occurrence of u is independent from the non-underlined occurrences of the same variable, since they belong to different abstractions. As a consequence, underlined u is a referent which does not occur in any anchor condition, which makes (*13′) ill-formed.

To solve this problem, we stipulate that, whenever the antecedent of an anaphor is found in an embedded abstraction, it has to be **unpacked** from there by the incorporation mechanism in such a way that it belongs to the λ-abstraction or formula in which the anaphoric referent resides. Before we go into the details of unpacking, let us show the correct counterpart of (*13′):

(13′) $[\text{FARMER}(f), \text{UNICORN}(u), \lambda f'u'[\text{UNICORN}(u')][\text{SEE}(f', u')](f, u)]$
$[\text{BLACK}(u)]$

What happened here is the following. First, the anchor condition "$\text{UNICORN}(u)$" has been copied into the anchor part of the formula. [**Note 13**: Why this redundancy?] The internal occurrences of u have been replaced by a 'fresh' variable u' for better readability (variable names are irrelevant in D.I.R.T.); these occurrences have become λ-bound. This stage of the incorporation is represented in (13″) below:

(13″) *A farmer saw a unicorn$_i$.*
$[\text{FARMER}(f), \text{UNICORN}(u)]\,[\lambda f'u'[\text{UNICORN}(u')][\text{SEE}(f', u')](f, u)]$
It$_i$ was black.
$[\text{IT}(u)][\text{BLACK}(u)]$

From here, everything is analogous to the case of (11) (i.e., that of subject shift). The result is (13′) above.

In the definition of unpacking, we will assume that X is an abstraction with variables $V_V = \{v_i\}_{1 \le i \le n}$, anchor conditions $A = \{a_i\}_{1 \le i \le k}$, predicate condition p $(P = \{p\})$ and referents R. We need the following ancillary definition:

Def. If $x \in R$, then the set Dep_X^x of **the referents depending on** x in X is defined as follows. Let Indep_X^x be the smallest set satisfying the following conditions:

(1) $y \in V \Rightarrow y \in \mathrm{Indep}_X^x$;

(2) $y \in \mathrm{Indep}_X^x \;\&\; c \in A \cup P \;\&\; y, z \in \mathrm{Free}(c) \;\&\; z \neq x \Rightarrow z \in \mathrm{Indep}_X^x$.

Then $\mathrm{Dep}_X^x = R - \mathrm{Indep}_X^x$.

The abstraction X' will be the modification of X which results from unpacking the referent x from it. The modification is $V' = V \cup \mathrm{Dep}_X^x$ (for some $x \in R$). Let $D = \{a \in A : \mathrm{Free}(a) \subseteq \mathrm{Dep}_X^x\}$. Furthermore, let Y be an abstraction or a formula with conditions C such that for some $c \in C$, $c = X(x_1, \ldots, x_n)$. Assume $\mathrm{Dep}_X^x = \{d_i\}_{1 \leq i \leq l}$.

Def. Y' is the result of **unpacking** x from c iff Y' differs from Y only in its conditions C':
$C' = (C - \{c\}) \cup D \cup \{X'(x_1, \ldots, x_n, d_1, \ldots, d_l)\}$.

2.3. Extension of D.I.R.T.'s Language

In this section, we will extend D.I.R.T.'s representation language to plural objects. Instead of arguing for the particular solutions adopted, we will summarize the extension of the syntax and semantics given in **Section 2**. In what follows, all symbols not explicitly redefined will be understood as in that section.

We add "λ^+" and "λ^-" to LC. So we have three kinds of λ-operators now, which will not co-occur in λ-abstractions. We add $\mathrm{Con}_{\mathrm{Pl}} = \{B_i\}_{i<\omega}$ (the set of **plural individual constants**) to $\mathrm{Con}_{\mathrm{ind}}$ and $\mathrm{Con}_Q = \{\alpha, \ldots\}$ (the set of **quantifying constants**) to Con (α is the quantifying constant corresponding to *all*). $\mathrm{Var} = \mathrm{Var}_V \cup \mathrm{Var}_A \cup \mathrm{Var}_N$, where Var_V is what Var used to be, $\mathrm{Var}_A = \{\dot{x}_i\}_{i<\omega}$ is the set of **arbitrary variables** (an arbitrary variable stands for any member of a plural object), and $\mathrm{Var}_N = \{\bar{x}_i\}_{i<\omega}$ is the set of **negative variables** (a negative variable stands for no member of a plural object). Notice that we do not distinguish 'plural variables'. All the sets mentioned are pairwise disjoint unless stated otherwise. The new set $\mathrm{Cond}_Q \subseteq \mathrm{Cond}$ of **quantifying conditions** will be defined in clause (iiia) below.

In the extended semantics, we will use a universe with a semi-lattice structure similar to the one proposed by Landman (1989). We add the set $\mathcal{A}$ of atomic objects to the interpretation. The universe $\mathcal{U}$ is a $\mathcal{H}_h$ $(h < \omega)$ where

(1) $\mathcal{H}_0 = \mathcal{A}$;

(2) $\mathcal{H}_{n+1} = \mathcal{P}\left(\bigcup_{0 \leq i \leq n} \mathcal{H}_i\right) - \{X : |X| \leq 1\}$ $(0 \leq n)$.

If $0 < m \leq h$, then $\mathcal{H}_m$ is the set of **plural objects** (of degree m). For each $m < \omega$ we distinguish the set $\mathcal{H}_m^*$ of **pure plural objects** (of degree m), defined as follows:

(1) $\mathcal{H}_0^* = \mathcal{H}_1$;

(2) $\mathcal{H}_{n+1}^* = \mathcal{P}(\mathcal{H}_n^*) - \{X : |X| \leq 1\}$ $(0 \leq n)$.

Obviously, $\mathcal{H}_m^* \subseteq \mathcal{H}_{m+1}$ for all $m < h$. Note that the elements of $\mathcal{U} - \mathcal{H}_0$ (together with the partial order $\subseteq$ and the join $\cup$) form a free complete atomic join semi-lattice generated by the base $\mathcal{H}_1$. If $b \in \mathrm{Con}_{\mathrm{ind}} - \mathrm{Con}_{\mathrm{Pl}}$, then $\varrho(b) \in \mathcal{A}$; if

$B \in \mathrm{Con_{Pl}}$, then $\varrho(B) \in \mathcal{U} - \mathcal{A}$. If $Q \in \mathrm{Con}_Q$, $\Phi \in \mathcal{P}(\mathcal{H}_m)$ for some m, and $\Psi \in \mathcal{P}(\mathcal{U})$ (where $\Phi \cap \Psi = \emptyset$), then

$$\varrho(Q)(\langle\Phi, \Psi\rangle) \in \{\langle\Phi', \Psi'\rangle : \Phi' \in \mathcal{P}(\mathcal{H}^*_m) \;\&\; \Phi' \cap \Psi' = \emptyset\}.$$

That is, the members of Con_Q are pluralizing functions from one-place predicates to one-place predicates. For example, the quantifying constant $\alpha \in \mathrm{Con}_Q$ is defined in such a way that $\varrho(\alpha)(\langle\Phi, \Psi\rangle) = \langle\Phi', \Psi'\rangle$, where $\Phi' = \{\Phi\}$ and $\Psi' = \mathcal{P}(\Phi) - \{\Phi\}$.

(iii) **a.** If $Q \in \mathrm{Con}_Q$ is a quantifying constant, $c \in \mathrm{Cond} - \mathrm{Cond}_Q$ is a non-quantifying condition, then Q can introduce a 'plural variable' $x \in \mathrm{Var}$ ($x \notin \mathrm{Free}(c)$) and bind $y \in \mathrm{Free}(c)$ in a quantifying condition:

$$Q(x, y)(c) \in \mathrm{Cond}_Q.$$
$$\mathrm{Free}(Q(x, y)(c)) := \{x\} \cup (\mathrm{Free}(c) - \{y\}).$$

Let $\Phi = \{u \in \mathcal{U} : [\![c]\!]^I_{g[y:u]} = 1\}$, $\Psi = \{u \in \mathcal{U} : [\![c]\!]^I_{g[y:u]} = 0\}$ and $\varrho(Q)(\langle\Phi, \Psi\rangle) = \langle\Phi', \Psi'\rangle$. Then

$$[\![Q(x, y)(c)]\!]^I_g = \begin{cases} 1 & \text{if } x \in \Phi'; \\ 0 & \text{if } x \in \Psi'; \\ 2 & \text{otherwise.} \end{cases}$$

In (va–b), we will strictly respect the following notational conventions:

1. If $\mathbf{o}, \mathbf{O} \in \mathcal{U}^m$, then $\mathbf{o}\mathcal{E}\mathbf{O} \Leftrightarrow_{\mathrm{def}} o_1 \in O_1 \;\&\; \ldots \;\&\; o_m \in O_m$.
2. If $\mathbf{v} = \langle v_1, \ldots, v_n\rangle \in \mathcal{U}^n$ and $\mathbf{O} = \langle o_1, \ldots, o_m\rangle \in \mathcal{U}^m$, then $\mathbf{v} \oplus \mathbf{O} := \langle u_1, \ldots, u_{n+m}\rangle$ such that, if $1 \le k \le n+m$, then we define the characteristic function $\chi(k) = \begin{cases} 1 & \text{if } x_k \in V_V; \\ 0 & \text{otherwise,} \end{cases}$ and we set $c_k = \begin{cases} v_s & \text{if } \chi(k) = 1; \\ o_s & \text{otherwise,} \end{cases}$ where $s = |\{j < k : \chi(j) = \chi(k)\}| + 1$.

Moreover, we implicitly assume:

$$\mathbf{O} \in \mathcal{U}^m; \qquad \mathbf{o} \in \mathcal{U}^m \;\&\; \mathbf{o}\mathcal{E}\mathbf{O};$$
$$g^* = g[\mathbf{V}_V : \mathbf{v}][\mathbf{V}_A : \mathbf{o}][\mathbf{R}_F : \mathbf{f}][\mathbf{R}_E : \mathbf{e}];$$
$$g^\circ = g[\mathbf{V}_V : \mathbf{v}][\mathbf{V}_N : \mathbf{o}][\mathbf{R}_F : \mathbf{f}][\mathbf{R}_E : \mathbf{e}]$$

(where $\mathbf{V}_V$, $\mathbf{V}_A$, $\mathbf{V}_N$ are orderings of V_V, V_A and V_N, respectively).

(v) **a.** λ^+-abstractions. The difference from (v) is that $V = V_V \cup V_A = \{x_i\}_{1 \le i \le n+m}$, where $V_V = \{v_1, \ldots, v_n\}$ is the set of **proper** variables ($V_V \subseteq \mathrm{Var}_V$), and $V_A = \{\dot{v}_1, \ldots, \dot{v}_m\}$ is the set of **arbitrary** variables ($V_A \subseteq \mathrm{Var}_A$).

$$\lambda^+ x_1 \ldots x_{n+m}[a_1, \ldots, a_k][p] \in \mathrm{Pred}^{n+m};$$
$$[\![\lambda^+ x_1 \ldots x_{n+m}[a_1, \ldots, a_k][p]]\!]^I_g = \langle\Phi, \Psi\rangle,$$

where Φ and Ψ are calculated as follows:

$$\Phi = \left\{ \mathbf{v} \oplus \mathbf{O} : \bigvee_{\mathbf{o}} \& \bigwedge_{\mathbf{o}} \bigvee_{\mathbf{f}} \bigvee_{\mathbf{e}} \bigwedge_{f} \bigwedge_{q} [\![f]\!]^I_{g^\bullet} = [\![q]\!]^I_{g^\bullet} = 1 \right\};$$

$$\Psi = \left\{ \mathbf{v} \oplus \mathbf{O} : \bigvee_{\mathbf{o}} \bigvee_{\mathbf{f}} \bigwedge_{\mathbf{e}} \left[\left(\bigwedge_{f} [\![f]\!]^I_{g^\bullet} = 1 \right) \& \left(\bigvee_{q} [\![q]\!]^I_{g^\bullet} \neq 1 \right) \right] \right\}.$$

b. λ^--abstractions. Differences from (va): $V = V_V \cup V_N = \{x_1, \ldots, x_{n+m}\}$, where $V_N = \{\bar{v}_1, \ldots, \bar{v}_m\}$ is the set of **negative** variables ($V_N \subseteq \mathrm{Var}_N$).

$\lambda^- x_1 \ldots x_{n+m}[a_1, \ldots, a_k][p] \in \mathrm{Pred}^{n+m}$;
$[\![\lambda^- x_1 \ldots x_{n+m}[a_1, \ldots, a_k][p]]\!]^I_g = \langle \Phi, \Psi \rangle$,

where Φ and Ψ are calculated as follows:

$$\Phi = \left\{ \mathbf{v} \oplus \mathbf{O} : \bigvee_{\mathbf{o}} \& \bigwedge_{\mathbf{o}} \bigvee_{\mathbf{f}} \bigwedge_{\mathbf{e}} \left[\left(\bigwedge_{f} [\![f]\!]^I_{g^\circ} = 1 \right) \& \left(\bigvee_{q} [\![q]\!]^I_{g^\circ} \neq 1 \right) \right] \right\};$$

$$\Psi = \left\{ \mathbf{v} \oplus \mathbf{O} : \bigvee_{\mathbf{o}} \bigvee_{\mathbf{f}} \bigvee_{\mathbf{e}} \bigwedge_{f} \bigwedge_{q} [\![f]\!]^I_{g^\circ} = [\![q]\!]^I_{g^\circ} = 1 \right\}.$$

2.4. Quantification in D.I.R.T.

In the previous section, we have introduced two new devices, related to plural objects. First, we can build λ-abstractions which distributively predicate about each (λ^+-abstractions) or none (λ^--abstractions) of the members of a plural argument. Second, we can construct predicates characterizing plural objects from one-place predicate extensions using generalized quantifiers. For example, 'all my friends' can be represented by "$\alpha(F, f)$MY-FRIEND(f)". The latter device will play a minor role only in what follows.

Let us illustrate these devices by the following examples:

(14) *Every man stroked a cat.*
$\left[\alpha(M, m)\text{MAN}(m)\right] \left[\lambda^+ \dot{m}[\,]\,[\lambda m'[\text{CAT}(c)][\text{STROKE}(m', c)](\dot{m})](M)\right]$

(15) *No man stroked a cat.*
$\left[\alpha(M, m)\text{MAN}(m)\right] \left[\lambda^- \dot{m}[\,]\,[\lambda m'[\text{CAT}(c)][\text{STROKE}(m', c)](\dot{m})](M)\right]$

In this section, we would like to draw attention to some interesting consequences of our treatment. We will not stipulate anything special about λ^+- and λ^--abstractions. (As a matter of course, we assume that the antecedent retrieval algorithm ignores λ^--abstractions altogether, but this will play no role in the following.)

Our first example is

(16) *Every man came to the party.* They *wore coats and ties.*

This example justifies our assumption that *every*-type quantifiers introduce plural referents. [**Note 14:** Other explanations?]

On the other hand, we can also justify the assumption that *every*-type quantification involves individual-type objects (corresponding to the arbitrary members of the plural object):

(17) *Every player chooses a pawn.* He *puts it on square one.*

The assumption is that the antecedent of *he* in the second sentence is an arbitrary member of the plural object corresponding to *every player* in the first sentence. But that arbitrary member is a λ^+-variable, i.e., it belongs to an embedded abstraction:

$$\big[\alpha(P,p)\text{PLAYER}(p)\big]\,\big[\lambda^+\dot{p}[\ ][\text{CHOOSES-A-PAWN}(\dot{p})](P)\big].$$

In terms of what has been said in **Section 2.2**, embedded antecedents are to be unpacked. But, by the definition of unpacking, it is impossible to unpack a λ- (or λ^+-) variable. So incorporation cannot proceed in the way described in **Section 2.2**.

The solution is to postulate a new type of incorporation mechanism, which takes place entirely within an embedded abstraction. Assume that the embedded abstraction in this case is the λ^+-abstraction above. In that case, the anaphoric pronoun *he* can be anchored to $\dot{p}$, which is singular, and no unpacking is necessary. So the representation of (17) is

$$\big[\alpha(P,p)\text{PLAYER}(p)\big]$$
$$\big[\lambda^+\dot{p}[\ ]\,[\lambda p'[\text{CHOOSES}(p',w),\text{PAWN}(w)][\text{PUT-ON-SQ-1}(p',w)](\dot{p})](P)\big].$$

Note that *every*-phrases license singular anaphors under special circumstances only. Compare (16) to (the non-habitual reading of) (18):

(18) *Every player$_i$ chose a pawn.* *He$_i$ *put it on square one.*

The 'special circumstances' in which such sentences are good can be considered cases of **modal subordination** (cf. Roberts 1987), in which the entire new sentence is to be interpreted within the sub-structure corresponding to the previous predicate. Compare:

(19) *If a player starts to play, he chooses a pawn.* He *puts it on square one.*

Such examples provide independent justification for the sub-case of incorporation just proposed. In this example, the second sentence as a whole is to be incorporated within the sub-structure corresponding to the *then*-clause of the first sentence.

Finally, we would like to point out that the mechanisms proposed so far offer an interesting solution to some of the so-called 'scope ambiguity' phenomena. Consider:

(20) *Every man saw a unicorn.*
$$\big[\alpha(M,m)\text{MAN}(m)\big]$$
$$\big[\lambda^+\dot{m}[\text{UNICORN}(u)][\text{SEE}(\dot{m},u)](M)\big]$$

(20′) *Every man saw a unicorn. The unicorn was black.*
'the single unicorn, which they all saw, was black'
$$\big[\alpha(M,m)\text{MAN}(m),\text{UNICORN}(u'),\lambda^+\dot{m}u[\text{UNICORN}(u)][\text{SEE}(\dot{m},u)](M,u)\big]$$
$$\big[\text{BLACK}(u')\big]$$

That is, the anaphoric noun phrase *the unicorn* triggers the wide-scope existential reading of (20). So the two readings of (20) constitute 'open possibilities' rather than ambiguity, although we do not have to rely on deferment in this case.

We want to emphasize that the unpacking operation, which has a 'disambiguating' function in (20′), is not identical to 'quantifier raising', because it is a copying operation. Consider:

(21) *Every man saw exactly two unicorns.*
$[\alpha(M,m)\mathrm{MAN}(m)]$
$[\lambda^{+}\dot{m}[\mathrm{EX\text{-}2}(U,u)\mathrm{UNICORN}(u)][\mathrm{SEE}(\dot{m},U)](M)]$

(21′) *Every man saw exactly two unicorns. The two unicorns were black.*
'the exactly two unicorns, which they all saw, were black'
$[\alpha(M,m)\mathrm{MAN}(m), \mathrm{EX\text{-}2}(U',u)\mathrm{UNICORN}(u'),$
$\quad \lambda^{+}\dot{m}U[\mathrm{EX\text{-}2}(U,u)\mathrm{UNICORN}(u)][\mathrm{SEE}(\dot{m},U)](M,U')]$
$[\mathrm{BLACK}(U')]$

As can be seen, the meaning we assign to (21′) is not just

'there were exactly two unicorns which they all saw, and they were black',

which would correspond to 'quantifier raising', and would not entail the meaning of (21), but rather

'there were exactly two unicorns, which they all saw, and none of them saw other unicorns, and those two were black',

which does entail the meaning of (21). In general, we intend to define D.I.R.T.'s incorporation and unpacking mechanisms in such a way that we can only account for pieces of discourse in which the growth of information is monotonic. We believe this is a serious advantage from the points of view of both formal tractability and theoretical predictiveness.

Notes

1. **Some examples** are Kamp (1981) and Heim (1982), where only 'discourse representation structures' and 'contexts', respectively, are assigned truth conditions.

2. **Cataphors are irrelevant here.** It may be possible that certain expressions impose requirements on subsequent discourse. This possibility does not influence our argument, but we will ignore it in the following for its marginal character.

3. **True ambiguities are different.** The solution of true (syntactic and lexical) ambiguities is a different, typically pragmatic, process. It does not involve specification, because the different readings are more often than not independent from each other.

4. **Apparent predecessors and 'hearer's meaning'.** Although the theories mentioned in **Note 1** share this view in principle, our approach differs from theirs in that we take the updating function as updating the *hearer's* state of information, irrespective of what the speaker exactly has in mind. As will become clear in **Section 1.4**, the 'open possibilities' mentioned earlier are open for the hearer, but not necessarily for the speaker.

5. *A couple* **in (3) is different.** *A couple* is not the anchor of the first sentence of (3), but belongs to its predicate. We assume this is a true grammatical difference between (3) and (5). Webber (1988) would term the indefinite NP in (5) 'anaphoric', but it is not clear whether she would postulate a grammatical difference between the two sentences.
6. **Open possibilities.** The existence vs. non-existence of a church ceremony are possibilities left open by 'marriage', as opposed to, say, the existence vs. non-existence of a dog. Hence, *The dog barked* is not a good continuation of the first sentence in (4). We will come back to this in **Section 1.4**.
7. **The relational view of sentences.** The semantics proposed here is in a way similar to other attempts to reconstruct the traditional distinction between subjects and predicates, e.g., Barwise and Cooper (1981). Our approach differs from theirs in that the only relationship that we treat in this way is predication, and that we have semantic value gaps.
8. The closedness of λ-abstractions. As opposed to DRT or Situation Semantics, our theory does not allow for semantically free variables in embedded positions. Each variable occurring in a λ-abstraction is either λ-bound or existentially quantified (which is syntactically implicit). We will refer to this fact as the **Principle of Referential Privacy (PRP)**. We consider this to be an extremely restrictive property of our theory, which makes it impossible for us to adopt purely representational solutions (such as that of Kamp 1981 or Klein 1985) to binding problems.
9. **"CAT(c)" is also floating.** Because of the presence of the condition "OWN(m', c)", the condition "CAT(c)" is related to an argument independently from the claim, i.e., it is floating. Compare:

   A man loved a cat.

   A man loved his cat.

   In the second sentence, the fact that the object is a cat is presupposed.
10. **Under what conditions?** The answer to this question will be given in Kálmán and Szabó (in prep.). The precise characterization of the conditions of deferment allows us to offer an explicit structural description of phenomena like collective readings, VP-ellipsis and genericity. This does not solve the problem of interpretation, but offers important predictions on these phenomena.
11. **We will not go into these here.** In what follows, we will not intend to account for these indirect effects in the semantics of D.I.R.T. The idea of delaying effects is inherent in most approaches to parallel programming. We believe that a semantic theory applicable to parallel programmes may be used for dealing with our deferment mechanisms.
12. **Antecedent retrieval in D.I.R.T.** As opposed to other formal theories of discourse meaning, the antecedent retrieval algorithms of D.I.R.T. can use only the pieces of information explicitly present in the formula corresponding to the context. That is, many (but not all) types of information usually considered external to discourse are directly available in the meaning representation of D.I.R.T. because of the possibility of deferring information.

13. **Why this redundancy?** It is harmless in this case, and very useful in others. See **Section 2.4**.
14. **Other explanations?** Other theories are bound to assume that (15) involves some kind of inference making. According to those theories, the relationship of *they* to the first sentence is essentially the same as the relationship of *it* to the first sentence in the following example:

    I lost ten marbles and I found only nine of them. It *was under the sofa.*

    We believe that (15) is far from being analogous to this example.

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Polarity Phenomena and Alternative Semantics

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1. Introduction

The theory of negative polarity elements (NPIs) as developed by Ladusaw (1979, 1983) and in related analyses (Fauconnier 1975a, 1975b, 1978; Zwarts 1981; Hoeksema 1986) can be seen as a fruitful application of formal semantics to explain a wide range of natural-language phenomena. However, there are some serious problems with this approach. In this paper, I try to develop a refined theory to overcome some of these problems.

2. Problems with downward-entailingness

According to Ladusaw's analysis, NPIs occur in the scope of DOWNWARD-ENTAILING (DE) operators. An operator is DE if an expression which occurs in its scope can be replaced by a semantically "stronger" (that is, more restricted) expression without "weakening" the complex expression. The relation of semantic strength can be defined by set inclusion; an expression α is STRONGER THAN OR EQUALLY STRONG AS an expression β iff $[\![\alpha]\!] \subseteq [\![\beta]\!]$. (If we take propositions to be set of possible worlds, then a proposition α will be stronger than or equally strong as a proposition β if α entails β.) For instance, the following example shows that negation is a DE operator, and therefore can license NPIs like *any* or *a red cent* in its scope:

1. a) $[\![$*a red car*$]\!] \subseteq [\![$*a car*$]\!]$, hence
   $[\![$*Mary doesn't have a car*$]\!] \subseteq [\![$*Mary doesn't have a red car*$]\!]$
   b) *Mary doesn't have any car.*
   c) *Mary doesn't have a red cent.*

This principle seems to cover quite a few NPI contexts. For instance, we can explain why NPIs occur in the scope of monone decreasing quantifiers such as *less than three persons*, in the scope of anti-persistent determiners such as *every*, and in the scope of the temporal operator *before*.

However, it also has to face some serious problems. One is that some operators which license NPIs are not really DE, as shown by Linebarger (1980, 1987), Heim (1984) and Jacobs (1985). They explicitly discuss the protasis of conditionals, which fails to be a DE context but nevertheless may license NPIs:

2. a) ⟦*You go to Yemen and get sick there*⟧ ⊆ ⟦*You go to Yemen*⟧
   ¬ ⟦*If you go to Yemen, you will enjoy it*⟧ ⊆ ⟦*If you go to Yemen and get sick there, you will enjoy it*⟧
   b) *If you* ***ever*** *go to Yemen, you will enjoy it.*

3. a) ⟦*a poisoned fruit*⟧ ⊆ ⟦*a fruit*⟧
   ¬ ⟦*If you eat a fruit, you will feel better*⟧ ⊆ ⟦*If you eat a poisoned fruit, you will feel better*⟧
   b) *If you eat* ***any*** *fruit, you will feel better.*

Heim (1984) sketches a promising solution to this problem. According to her, we cannot choose any old strenghthenings of the protasis, but only those which are induced by ALTERNATIVE ITEMS in the position of the NPI. Heim calls this "limited DEness". The licensing conditions for NPIs in the protasis of conditional clauses are given as follows; note that they take the background conditions c into account, which in general are part of the truth conditions of conditional sentences.

4) Let *if* Φ *then* Ψ be a conditional where Φ contains the NPI-occurrence Φ. Let $\Phi[\alpha/\beta]$ be just like Φ, except with α replaced by β. Let c be the set of presupposed background conditions. Then α is licensed in *if* Φ *then* Ψ iff for any β of the appropriate type (= any alternative to α):
   [c & [$\Phi[\alpha/\beta] \rightarrow \Phi$] & *if* Φ *then* Ψ] $\rightarrow$ *if* $\Phi[\alpha/\beta]$ *then* Ψ.

To see how this proposal works, let's apply it to example (2). Heim analyzes the NPI *ever* as *at least once*, and takes as alternatives adverbials like *twice*, *several times* etc. Note that these alternatives are semantically stronger than the NPI. If a speaker asserts a conditional sentence with an NPI in the protasis on a set of background conditions c, he claims that the sentence will be true if we substitute the NPI by its alternatives, given the conditions c. For example, a speaker who claims (2.b) should be ready to accept the truth of *If you go to Yemen twice, you will enjoy it.*

The idea that NPIs relate to alternatives can be traced back to the early works on NPIs by Fauconnier (1975a,b, 1978). In Fauconnier's theory, NPIs are related to SCALES, or ordered sets, and denote minimal elements in these sets; these sets can be considered as the alternatives of an NPI. Note that Fauconnier tried to develop a general theory of NPIs, not only for NPIs in the protasis of conditionals; so it seems promising to extend the treatment in terms of alternatives of NPIs in general.

Another problem of Ladusaw's approach is that the notion of DEness, as Ladusaw presents it, is restricted to set inclusion. However, set inclusion is not always a good model for the alternatives of NPIs. As an example, consider the sentence frame *Mary doesn't have X*, which should be DE in X. It surely is; from *Mary doesn't have a car* follows: *Mary doesn't have a red car*. However, the alternatives which seem to matter in this case are amounts of money. For example, from *Mary doesn't have five dimes* we can conclude *Mary doesn't have ten dimes*. But the extensions of *five dimes* and *ten dimes* are not related to each other by set inclusion -- they are disjunct, as no object to which *five dimes* applies is such that *ten dimes* applies to it, and vice versa.

Some additional problems for Ladusaw's theory: There are some NPI contexts for which Ladusaw lacks a convincing treatment, most notably questions. Furthermore, he does not offer a treatment for every occurrence of the NPI determiner *any*: he has to assume an NPI *any* and a non-NPI, "free choice" *any*. And finally, although Ladusaw may have offered a good semantic characterization of NPI contexts, he does not give any reason WHY NPIs are licensed in the scope of DE operators.

3. Polarity Items in Alternative Semantics

We have seen that NPI contexts cannot be explained by general DEness. Following Heim, we have to refine the original notion of DEness in two respects: First, we have to specify what it means that an operator is DE with respect to a certain constituent (the NPI); second, we have to specify the class of alternative expressions.

For the first task, we must be able to recover an NPI representation even if it occurs embedded in a complex semantic representation. There are two frameworks in which this can be done, namely STRUCTURED SEMANTIC REPRESENTATIONS as developed by Cresswell & von Stechow (1982) and Jacobs (1983), or ALTERNATIVE SEMANTICS, as developed by Rooth (1985). (See von Stechow 1988 for a comparison). Here I will follow the approach of alternative semantics.

Alternative semantics was developed to treat focus-sensitive operators such as *only*. According to this theory, a semantic representation in focus is related to a set of alternatives. While more complex representations are built up, these alternatives generate alternatives for the more complex representations as well. Focus-sensitive expressions do not operate only on semantic representations proper, but also on their alternatives.

This idea can be carried out as follows: Assume that $\mathbb{I}$ is the interpretation function for syntactically well-formed expressions, then $\mathbb{I}$ does not only give us the meaning proper of an expression, but also specifies its alternatives, and perhaps other things as well. Let us assume that, if α is a well-formed expression, then $\mathbb{I}(\alpha)$ is the INTERPRETATION of α, and $\mathbb{M}(\mathbb{I}(\alpha))$ and $\mathbb{A}(\mathbb{I}(\alpha))$ are the MEANING and the ALTERNATIVES of (the interpretation of) α, respectively. In general, if the meaning of an interpretation is of a type σ, its alternatives are of type $\langle\sigma,t\rangle$ (that is, sets of objects of type σ), and the meaning of an interpretation is an element of its alternatives. The interpretation of a complex expression is derived compositionally from the interpretation of its immediate parts and the syntactic rule of their combination. That is, if S is a syntactic rule which combines two expressions α, β to an expression $S(\alpha,\beta)$, then $\mathbb{I}(S(\alpha,\beta))$ will be $\mathbb{C}(S, \mathbb{I}(\alpha), \mathbb{I}(\beta))$, where $\mathbb{C}$ is a general semantic combination function, driven by the syntactic rule S and possibly also by the types of $\mathbb{I}(\alpha)$ and $\mathbb{I}(\beta)$. If $S(\alpha,\beta)$ does not contain focus-sensitive operators, then its meaning will be a function of S and the meanings of α, β, and its alternative set will be a function of S and the alternative sets of α, β. Moreover, the combination function of the alternative sets can be derived from the combination function for meanings, in the following way: Let **c** be the combination function for meanings, that is, $\mathbb{M}(\mathbb{I}(S(\alpha,\beta))) = \mathbf{c}(S, \mathbb{M}(\mathbb{I}(\alpha)), \mathbb{M}(\mathbb{I}(\beta)))$, then the combination function for alternatives is given by $\mathbb{A}(\mathbb{I}(S(\alpha,\beta))) = \{X: \exists Y,Z[Y\in \mathbb{A}(\mathbb{I}(\alpha))$ & $Z\in \mathbb{A}(\mathbb{I}(\beta))$ & $X = \mathbf{c}(S,Y,Z)]\}$, where X, Y, Z are variables of the appropriate type not occurring free in $\mathbb{A}(\mathbb{I}(\alpha))$ or $\mathbb{A}(\mathbb{I}(\beta))$. Thus **c**, the combination for meanings, completely determines $\mathbb{C}$, the combination for interpretations.

As an example, consider (5). I have listed the syntactic tree, the meaning proper, and the alternatives, of the English expression *Jim saw SUE*, where capitalization means that *Sue* bears sentence accent and hence is in focus. Expressions in focus have "proper" alternatives; non-focused expressions have as alternatives the singleton set with its meaning as the only element. In the example, the expression in focus, *Sue*, is taken to have two alternatives, **s**, which is $\mathbb{M}(\mathbb{I}(\textit{Sue}))$, and **m** (Mary). These alternatives are given by the context. In general, I will specify meanings of expressions simply by boldface. I use variables x,y,z for individuals, P,Q for properties, R for relations-in-intensions, p,q,r for propositions, and i for possible worlds. The S_n are syntactic rules; neither these rules nor the corresponding semantic combinations $\mathbf{c}(S_n,...)$ will be treated explicitly here. As semantic representation language, I use a version of intensional logic with explicit quantification over possible worlds (that is, a version of two-sorted type theory instead of Montague's IL).

5) *Jim saw SUE.*

SYNTACTIC TREE	MEANING	ALTERNATIVE SET
SUE	$\mathbb{M}(\mathbb{I}(SUE))$ $= \mathbf{s}$	$\mathbb{A}(\mathbb{I}(SUE))$ $= \{\mathbf{s}, \mathbf{m}\}$
saw	$\mathbb{M}(\mathbb{I}(saw))$ $= \mathbf{saw}$ $(= \lambda i,x,y[\mathbf{saw}(i)(y,x)])$	$\mathbb{A}(\mathbb{I}(saw))$ $= \{\mathbf{saw}\}$
S_1(*SUE, saw*) = *saw SUE*	$c(S_1, \mathbf{s}, \mathbf{saw}),$ $= \lambda i,x[\mathbf{saw}(i)(x,\mathbf{s})]$	$\{P: \exists x,R[x \in \{\mathbf{s}, \mathbf{m}\}$ & $R \in \{\mathbf{saw}\}$ & $P = c(S_1, x, R)]\}$ $= \{\lambda i,x[\mathbf{saw}(i)(x,\mathbf{s})], \lambda i,x[\mathbf{saw}(i)(x,\mathbf{m})]\}$
Jim	$\mathbb{M}(\mathbb{I}(Jim))$ $= \mathbf{j}$	$\mathbb{A}(\mathbb{I}(Jim))$ $= \{\mathbf{j}\}$
S_2(*Jim, saw SUE*) = *Jim saw SUE*	$c(S_2, \mathbf{j}, \lambda i,x[\mathbf{saw}(i)(x,\mathbf{s})])$ $= \lambda i[\mathbf{saw}(i)(\mathbf{j},\mathbf{s})]$	$\{p: \exists x,P[x \in \{\mathbf{j}\}$ & $P \in \{\lambda i,x[\mathbf{saw}(i)(x,\mathbf{s})],$ $\lambda i,x[\mathbf{saw}(i)(x,\mathbf{m})]\}$ & $p = c(S_2, x, P)]\}$ $= \{\lambda i[\mathbf{saw}(i)(\mathbf{j},\mathbf{s})], \lambda i[\mathbf{saw}(i)(\mathbf{j},\mathbf{m})]\}$

The meaning of *Jim saw SUE*, then, is the set of worlds in which Jim saw Sue, and its alternatives are the set consisting of the set of worlds in which Jim saw Sue and the set of worlds in which Jim saw Mary.

The combination rules for focus-sensitive operators are dependent on the meanings of subexpressions and on their alternatives. Take as an example *only*. If Φ is a sentence which can be combined with the sentential adverbial *only* by a rule S_3, then the meaning of *only* Φ, $\mathbb{M}(\mathbb{I}(S_3(only, \Phi)))$, can be given as $\lambda i[\mathbb{M}(\mathbb{I}(\Phi))(i)$ & $\forall p[p \in \mathbb{A}(\mathbb{I}(\Phi))$ & $p \neq \mathbb{M}(\mathbb{I}(\Phi)) \rightarrow \neg p(i)]]$. In our example, if we assume that the sentence *Jim only saw SUE* has the syntactic form S_3(*only, Jim saw SUE*), then it has the meaning $\lambda i[\mathbf{saw}(i)(\mathbf{j},\mathbf{s})$ & $\forall p[p \in \{\lambda i[\mathbf{saw}(i)(\mathbf{j},\mathbf{s})], \lambda i[\mathbf{saw}(i)(\mathbf{j},\mathbf{m})]\}$ & $p \neq \lambda i[\mathbf{saw}(i)(\mathbf{j},\mathbf{s})] \rightarrow \neg p(i)]$, which is $\lambda i[\mathbf{saw}(i)(\mathbf{j}, \mathbf{s})$ & $\neg\mathbf{saw}(i)(\mathbf{j}, \mathbf{m})]$, the set of worlds in which Jim saw Sue, but didn't see Mary. As alternatives, we can simply take the singleton set of that proposition.

Polarity items are similar to expressions in focus insofar as they come with a set of alternatives. This opens a way of explaining why they typically are accented; however, I will not elaborate on this here. There are two major differences to simple focus constituents: First, whereas the alternative set of constituents typically is given by the context, the alternative set of polarity items is often given by the lexical knowledge of the language users. For example, the alternatives of *a red dime* are amounts of money, and the alternatives of *lift a finger* are acts of labor. Second, whereas the alternative set of focus constituents may be unordered, the alternatives of NPIs are ordered. For example, the alternatives of *a red dime* are ordered by their monetary value, and the alternatives of *lift a finger* are ordered by the effort they involve. The meaning proper of the NPI denotes the bottom element of these orders.

These ordered alternative sets are closely related to Fauconnier's notion of a scale. Contrary to what Fauconnier suggests, however, we need not assume that the alternatives are ordered linearly; it is sufficient to claim a preorder with a unique bottom element. Let us introduce the notion of a NEGATIVE POLARITY STRUCTURE:

6. $\langle A_\alpha, a_\alpha, \leq_\alpha \rangle$ is an NPI STRUCTURE with the POLARITY ITEM REPRESENTATION a_α, the POLARITY SORT A_α, and the POLARITY ORDERING $\leq_\alpha$ iff:
   a) if a_α is of type σ, then A_α is of type $\langle\sigma,t\rangle$ (sets of σ-representations);
   b) $\leq_\alpha$ is a preorder relation on A_α (that is, it is reflexive and transitive);
   c) $a_\alpha \in A_\alpha$, and there is at least one additional element $X \in A_\alpha$ with $X \neq a_\alpha$;
   d) a_α is the unique element $Y \in A_\alpha$ such that for every $X \in A_\alpha$, $Y \leq_\alpha X$.

We can introduce the converse notion of a POSITIVE POLARITY STRUCTURE (PPI STRUCTURE) which is like an NPI structure with the exception that the polarity item representation is the unique greatest element of the polarity ordering (that is, we replace 6.d by: a_α is the unique element $Y \in A_\alpha$ such that for every $X \in A_\alpha$, $X \leq_\alpha Y$). One example of a PPI is *bags of money*. It can be argued that the polarity sort and the polarity ordering of the PPI structure of *bags of money* are the same as for the NPI *a red dime*: *bags of money* denotes an arbitrary high amount of money and has as alternatives amounts of money which are less in monetary value.

In the following, I assume that negative (positive) polarity items in general are interpreted by negative (positive) polarity structures. That is, for any polarity item α, there is a polarity structure $\langle A_\alpha, a_\alpha, \leq_\alpha \rangle$ of the same polarity such that $\mathbb{M}(\mathbb{I}(\alpha)) = a_\alpha$ and $\mathbb{A}(\mathbb{I}(\alpha)) = A_\alpha$. I take $\mathbb{O}$ to be a function which gives us the order relation of (the interpretation of) a polarity item: $\mathbb{O}(\mathbb{I}(\alpha)) = \leq_\alpha$.

Before I go into some more examples of polarity structures, I will reveal the plot of the next sections. The idea is that basic polarity items like ***a red dime*** generate complex polarity items, via the semantic combination rules. That is, the polarity structure of a basic polarity item induces polarity structures of more complex polarity items. Depending on the semantic combinations which are involved in building up complex polarity items, the polarity may be preserved, or it may be switched to the opposite (for example, by combination with a negation). Finally we have rules which say, for example, that we should not end up with a sentential polarity item as the basis for an assertion.

4. Basic Polarity Items

There are different types of basic polarity items which can be chararcterized by properties of their polarity structures. Here I will look only at some characteristic examples (see Krifka 1990a for more).

First, there are IDIOMATIC POLARITY ITEMS like *lift a finger, bags of money, a peep, bat an eyelash, a drop of (wine etc.)*, and the like. It is part of the lexical knowledge of language users that they are related to some polarity sort and polarity order. For example, *a peep* , as in *he didn't utter a peep*, is related to utterances and their elaborateness, *bat an eyelash* is related to reactions to disturbing stimuli and the strength of these reactions, and *a drop of wine* is related to quantities of wine and their size.

To be more precise, take the last example. The polarity structure of the NPI *a drop of wine* can be rendered as $\langle A_{a.drop.of.wine}, a_{a.drop.of.wine}, \leq_{a.drop.of.wine}\rangle$. Here, the polarity sort $A_{a.drop.of.wine}$ is a set of properties which apply to wine of a specific amount, that is, if $P \in A_{a.drop.of.wine}$ then $\forall i,x[P(i)(x) \rightarrow \mathbf{wine}(i)(x) \ \& \ \forall y[P(i)(y) \rightarrow y \approx_{amount} x]]$, where **wine** is a property which applies to quantities of wine, and $\approx_{amount}$ is an equivalence relation which holds between quantities of stuff of the same amount. In addition, we should claim that $A_{a.drop.of.wine}$ should be exhaustive in the sense that every quantitiy of wine is in the extension of some alternative, that is, $\forall i,x[\mathbf{wine}(i)(x) \rightarrow \exists P[P \in A_{a.drop.of.wine} \ \& \ P(i)(x)]]$. The polarity ordering $\leq_{a.drop.of.wine}$ is such that, if $P \leq_{a.drop.of.wine} Q$, then the quanties of wine to which P applies are smaller than or equal to the quantities of wine to which Q applies. That is, $P \leq_{a.drop.of.wine} Q \ \& \ P(i)(x) \ \& \ Q(i)(y) \rightarrow x \leq_{amount} y$, where $\leq_{amount}$ is an order relation between quantities of stuff such that $x \leq_{amount} y$ is true if x is an amount of stuff smaller than or equal to y. And the polarity item representation $a_{a.drop.of.wine}$ is that element of $A_{a.drop.of.wine}$ which applies to the smallest quantities of wine. These conditions can be spelled out in such a way that $A_{a.drop.of.wine}$ is a partitioning of all the wine into equivalence classes; then $\leq_{a.drop.of.wine}$ would be a linear order. However, we cannot plausibly find such linear orders for many other polarity items, such as *lift a finger* (as acts of labor cannot generally be compared as to the amount of labour they involve). The definition of polarity structures in (6) is deliberately kept quite weak, so it can capture cases like that as well.

Although the ordering relations of idiomatic polarity items seems to be quite idiosyncratic, they typically are in tune with a very general ordering relation, namely the part relation $\leq_p$ on objects and events (cf. Bach 1986, Krifka 1987, 1990b). For example, if x is a quantity of wine and y is a part of it, $y \leq_p x$, then y cannot be more wine than x. Or, to take the less obvious example *bat an eyelash*, if x is a reaction to a disturbing stimulus and y is a part of x, $y \leq_p x$, then y cannot be a stronger reaction than x. In general, we can assume that idiomatic polarity structures are COMPATIBLE with the part relation for objects and events. For polarity items of the property type, this notion can be defined as follows:

7) A polarity structure $\langle A_\alpha, a_\alpha, \leq_\alpha \rangle$, where a_α, a property of type $\langle s,\langle e,t\rangle\rangle$, is COMPATIBLE WITH THE PART RELATION $\leq_p$ iff:
if $P,Q \in A_\alpha$, $P(i)(x)$, $Q(i)(y)$ and $x \leq_p y$, then $P \leq_\alpha Q$.

Another class of polarity items denote the MOST GENERAL CONCEPT of a given sort. Examples are *sound*, as in *he didn't hear a sound*, and *thing*, as in *he doesn't know a thing*. If *a sound* is analyzed as a property of acoustic events, then the polarity sort contains properties which characterize acoustic events more specifically, for example **laughter**. In this case, the polarity ordering is the subproperty ordering, restricted to subproperties of **sound**. The subproperty notion, in turn, can be defined through a generalization of the inclusion relation:

8) Generalized inclusion relation $\subseteq$:
If α, β are of type t, then $\alpha \subseteq \beta$ iff $\alpha \rightarrow \beta$;
if α, β are of a t-based type $\langle \sigma, t\rangle$, then $\alpha \subseteq \beta$ iff $\forall u[\alpha(u) \subseteq \beta(u)]$,
where u is a variable of type σ and free for α, β.

One example is the subproperty relation: If P, Q are properties (type $\langle s,\langle e,t\rangle\rangle$), then P is a subproperty of Q iff $P \subseteq Q$, that is, iff $\forall i[P(i) \subseteq Q(i)]$, that is, iff $\forall i,x[P(i)(x) \rightarrow Q(i)(x)]$ (where i, x are free for P, Q).

Polarity structures in which the structure ordering is related to the general inclusion relation will be called INCLUSION STRUCTURES. In an NPI inclusion structure, the NPI will denote the most inclusive element. Let us introduce a symbol, ≤, for inverse general inclusion; it holds that $\alpha \leq \beta$ iff $\beta \subseteq \alpha$. NPI and PPI inclusion structures then can be defined as follows:

9) Let $\langle A_\alpha, a_\alpha, \leq_\alpha \rangle$ be a NPI (PPI) structure. Then it is called a NPI (PPI) INCLUSION STRUCTURE iff $\leq_\alpha$ is a restriction of ≤ to the elements of A_α.

As their ordering relation is determined by the structure sort, inclusion structures can be simply rendered as a pair, $\langle A_\alpha, a_\alpha \rangle$. The polarity structure of *a sound* then can be given as ⟨λP[**sound**≤P], **sound**⟩. As the general set inclusion is a partial order, inclusion structures are not only preorders, but more specifically partial orders.

Another case of inclusion structures are noun phrases with the determiner *any*. Let us assume that a noun phrase with an indefinite article, like *a boy*, denotes a property, in this case **boy**, the property of being a boy (we do not care for the semantics of the indefinite article, for the sake of simplicity). Then we can analyze *any boy* as follows: Its meaning is the same as the meaning of *a boy*, but in contrast with *a boy* it is related to a polarity sort, namely the set ob subproperties of *a boy*. The polarity ordering is inverse subproperty relation, that is, we have an NPI inclusion structure ⟨λP[**boy**≤P], **boy**⟩

Finally, let us have a look at the adverbial NPI *ever*. As it is well known, sentences are normally interpreted with respect to some time which may be specified by linguistic means or by the non-linguistic context (cf. Hall-Partee 1973, Bäuerle 1979). Now, *ever* makes the temporal interpretation of a sentence independent of a possible reference time. This becomes obvious with examples like *When I left home yesterday, I didn't (*ever) remember to close the window*. Here, the *when*-clause explicitly specifies a reference time for the following clause, but *ever* prevents this clause from picking up that reference time, and therefore the sentence is anomalous. If sentences are analyzed as containing a free variable to pick up the reference time (cf. Hinrichs 1981, Partee 1984), then *ever* can be rendered as an operator which introduces a narrow-scope existential quantifier that binds that variable, thus making in inaccessible for the reference time parameter of the context. What is, then, the polarity sort of a clause containing *ever*? The simplest assumption is that it consists of the representations of the clause containing *ever* and the same clause without *ever*. The latter, of course, depends on the current reference time, and thus the polarity structure as a whole depends on the context. As an example, the polarity structure of *John drank ever wine* can be given as ⟨{λi[**John.drank.wine** at t_r in i], λi∃t[**John.drank.wine** at t in i]}, λi[**John.drank.wine** at t_r in i]⟩, where t_r is a time variable which picks up the current reference time. This is a proper inclusion structure, as we have for any t_r, λi∃t[**John.drank.wine** at t in i] ≤ λi[**John.drank.wine** at t_r in i]. Its polarity sort consists only of two elements; let us call polarity structures of this type PAIR STRUCTURES.

Note that we did not specify polarity structures for *any* and *ever*, but only for phrases containing these expressions. We could have done so, however. To see this, look at the example *any boy*. We assume that *any* and *boy* are combined by a rule S_4 to S_4(*any, boy*) = *any boy*. The following tree shows which meaning and which alternative set we have to propose for *any* and *boy* to arrive at the intended interpretation of *any boy*, using functional application as the semantic combination corresponding to S_4. (M should be a variable of the type of property modifiers, $\langle\langle s,\langle e,t\rangle\rangle,\langle s,\langle e,t\rangle\rangle\rangle$.)

10)

any	$M(\mathbb{I}(any))$ = λPP	$A(\mathbb{I}(any))$ {M: M=λP,i,x$\exists$Q[P$\leq$Q & Q(i)(x)]}
boy	$M(\mathbb{I}(boy))$ = **boy**	$A(\mathbb{I}(boy))$ = {**boy**}
S_4(*any, boy*) = *any boy*	c(S_4, λPP, **boy**) = **boy**	{P: $\exists$M,Q[Q$\in$ {**boy**} & M$\in$ {M: M= λP,i,x$\exists$Q[P$\leq$Q&Q(i)(x)]}&P=c(S_4,M,Q)]} = {P: **boy** $\leq$ P}

The essential function of *any* (and of *ever*, *a drop of*, etc.) is to yield polarity structures for more complex categories. In the next section, we will see that this is indeed a general property of polarity items.

5. The projection of polarity properties

As indicated above, the general semantic combination rules may lead to new polarity structures, and so we get a recursive definition of polarity items. There are two interesting types of combinations with polarity item interpretations, which will be called UPWARD-ENTAILING (UE) and DOWNWARD-ENTAILING (DE) with respect to the polarity item interpretation:

11) Let α be a polarity item which is interpreted by $\langle A_\alpha, a_\alpha, \leq_\alpha \rangle$, that is, we have $\mathbb{M}(\mathbb{I}(\alpha)) = a_\alpha$, $\mathbb{A}(\mathbb{I}(\alpha)) = A_\alpha$, and $\mathbb{O}(\mathbb{I}(\alpha)) = \leq_\alpha$. Let $c[a_\alpha]$ be some semantic combination of meanings containing a_α, that is, $c(S,a_\alpha,b)$ or $c(S,b,a_\alpha)$, where S is a syntactic rule and **b** is some meaning $\mathbb{M}(\mathbb{I}(\beta))$, then $c[a_\alpha]$ will be called
   a) UPWARD ENTAILING WITH RESPECT TO a_α iff for every $X \in A_\alpha$, if $a_\alpha <_\alpha X$ then $c[a_\alpha] < c[X]$, and if $X <_\alpha a_\alpha$ then $c[X] < c[a_\alpha]$;
   b) DOWNWARD ENTAILING WITH RESPECT TO a_α iff for every $X \in A_\alpha$, if $a_\alpha <_\alpha X$ then $c[X] < c[a_\alpha]$ and if $X <_\alpha a_\alpha$ then $c[a_\alpha] < c[X]$.

   Here, $<_\alpha$ is the strict version of $\leq_\alpha$ (that is, $X <_\alpha Y$ iff $X <_\alpha Y$ and $\neg Y <_\alpha X$), and $<$ is the strict version of $\leq$ (inverse proper set inclusion, $X<Y$ iff $Y \subset X$).

When the combination is neither UE nor DE with respect to a polarity item representation α, I will call it NEUTRAL. These definitions are weaker than the ones proposed in Krifka (1990a), where I assumed that the respective relations should hold between any two elements of the alternative sets. It is sufficient for our sake, however, to claim (11).

With UE and DE combinations, a polarity structure will create a more complex polarity structure of the inclusion type. Consider the example of a NPI α and a semantic combination $c(S, \mathbb{M}(\mathbb{I}(\alpha)), \mathbb{M}(\mathbb{I}(\beta)))$ which is UE with respect to $\mathbb{M}(\mathbb{I}(\alpha))$. By the general rules for semantic combinations, we have $\mathbb{M}(\mathbb{I}(S(\alpha,\beta))) = c(S, \mathbb{M}(\mathbb{I}(\alpha)), \mathbb{M}(\mathbb{I}(\beta)))$, and $\mathbb{A}(\mathbb{I}(S(\alpha,\beta))) = \{X: \exists Y,Z[Y \in \mathbb{A}(\mathbb{I}(\alpha)) \,\&\, Z \in \mathbb{A}(\mathbb{I}(\beta)) \,\&\, X = c(S, Y, Z)]\}$, which is (under the assumption that β has no proper alternatives) $\{X: \exists Y[Y \in \mathbb{A}(\mathbb{I}(\alpha)) \,\&\, X = c(S, Y, \mathbb{M}(\mathbb{I}(\beta)))]\}$. By the general property for negative polarity structures (6) we know that $\mathbb{M}(\mathbb{I}(\alpha)) \in \mathbb{A}(\mathbb{I}(\alpha))$, and that there is at least one $Y \in \mathbb{A}(\mathbb{I}(\alpha))$ such that $\mathbb{M}(\mathbb{I}(\alpha)) <_\alpha Y$. By the UE property we know that for all such Y, $c(S, \mathbb{M}(\mathbb{I}(\alpha)), \mathbb{M}(\mathbb{I}(\beta))) < c(S, Y, \mathbb{M}(\mathbb{I}(\beta)))$. Hence $\langle \{X: \exists Y[Y \in \mathbb{A}(\mathbb{I}(\alpha)) \,\&\, X = c(S, Y, \mathbb{M}(\mathbb{I}(\beta)))]\}, c(S, \mathbb{M}(\mathbb{I}(\alpha)), \mathbb{M}(\mathbb{I}(\beta)))\rangle$ is a negative polarity structure, and $S(\alpha,\beta)$ is a NPI. By similar reasoning, it can be shown that NPIs generate PPIs under DE combinations, that PPIs generate PPIs under UE combinations, and that PPIs generate NPIs under DE combinations. The combination of TWO polarity items will typically (depending on the specific type of combination) yield a polarity item if the two basic items were of the same polarity. I will call polarity items which are generated in these ways DERIVED POLARITY ITEMS.

Let us look at some examples. We start with the UE case, namely the combination of the NPI *any boy* with the verb *came*. As we have analyzed *any boy* as denoting a property, and *came* denotes a property as well, **c** cannot be functional application; I assume that it gives the set of worlds for which the two properties have overlapping extensions.

12)

any boy	M(𝕀(*any boy*)) = **boy**	A(𝕀(*any boy*)) = {P: **boy** ≤ P}
came	M(𝕀(*came*)) = **came**	A(𝕀(*came*)) = {**came**}
S_2(*any boy, came*) = *any boy came*	c(S_2, **boy, came**) = λi∃x[**came**(i)(x) & **boy**(i)(x)]	{p: ∃P,Q[P∈ {P: **boy** ≤ P} & Q∈ {**came**} & p=c(S_2, P, Q)]} = {p: ∃P[**boy** ≤ P & p = λi∃x[**came**(i)(x) & P(i)(x)]]}

To see that this combination is UE with respect to **boy**, consider a proper subproperty of **boy**, say **red-haired.boy**. If we assume that the set of possible worlds is "modally complete", that is, every possible state of affairs is realized in some possible world, then it is obvious that the set of worlds in which a red-haired boy came is a proper subset of the set of worlds in which a boy came. Thus, the semantic combination of the meanings of *any boy* and *came* is upward-entailing with respect to the semantic representation of *any boy*. This guarantees that *any boy came* is a negative polarity item.

Now let us look at a DE semantic combination, the combination of *any boy came* with a sentential negation *it is not the case that* :

13)

..	..	..
it is not the case that	M(𝕀(*it is not*...)) = λp,i[¬p(i)]	A(𝕀(*it is not*...)) {λp,i[¬p(i)]}
S_5(*any boy came, it is not*...) = *it is not the case that any boy came*	c(S_5, λi∃x[**came**(i)(x) & **boy**(i)(x)], λp,i[¬p(i)]) = λi¬∃x[**came**(i)(x) & **boy**(i)(x)]	{p: ∃q,M[q∈ A(𝕀(*any boy came*)) & M∈ A(𝕀(*it is not*...)) & p=c(S_5,q,M)]} = {p: ∃P[**boy** ≤ P & p = λi¬∃x[**came**(i)(x) & P(i)(x)]]}

To see that this combination is DE, consider again a subproperty of **boy**. If the set of possible worlds is modally complete, then the set of worlds in which it is not the case that a red-haired boy came is a proper superset of the set of worlds in which it is not the case that a boy came. This is because negation is complementation on sets of worlds, and complementation reverses set inclusion. Thus, the semantic combination of the interpretations of *any boy came* and *it is not the case that* is DE with respect to the interpretation of *any boy came*. Consequently, *it is not the case that any boy came* is a positive polarity item.

In these examples, we started with an inclusion structure (the polarity structure of *any boy* and *any boy came*, respectively). Let us look now at a construction which contains a non-inclusion structure. To keep things simple, we take again the combination of a subject NP with a verb phrase, namely *a drop of wine was drunk*:

14)

a drop of wine	$\mathbf{M}(\mathbb{I}(\textit{a drop of wine}))$ $= \mathbf{a}_{\mathbf{a.drop.of.wine}}$	$\mathbf{A}(\mathbb{I}(\textit{a drop of wine}))$ $= \mathbf{A}_{\mathbf{a.drop.of.wine}}$
was drunk	$\mathbf{M}(\mathbb{I}(\textit{was drunk}))$ $= \mathbf{was.drunk}$	$\mathbf{A}(\mathbb{I}(\textit{was drunk}))$ $= \{\mathbf{was.drunk}\}$
S_2(*a drop of wine, was drunk*) = *a drop of wine was drunk*	$\mathbf{c}(S_2, \mathbf{a}_{\mathbf{a.drop.of.wine}}, \mathbf{was.drunk})$ $= \lambda i\exists x[\mathbf{was.drunk}(i)(x) \;\&\; \mathbf{a}_{\mathbf{a.drop.of.wine}}(i)(x)]$	$\{p: \exists P,Q[P\in \mathbf{A}_{\mathbf{a.drop.of.wine}} \;\&\; Q\in\{\mathbf{was.drunk}\} \;\&\; p=\mathbf{c}(S_2,P,Q)]\}$ $= \{p: \exists P[P\in \mathbf{A}_{\mathbf{a.drop.of.wine}} \;\&\; p=\lambda i[\mathbf{was.drunk}(i)(x) \;\&\; P(i)(x)]]\}$

We have to show that this combination is UE with respect to $\mathbf{a}_{\mathbf{a.drop.of.wine}}$, that is, for every $P\in \mathbf{A}_{\mathbf{a.drop.of.wine}}$ with $\mathbf{a}_{\mathbf{a.drop.of.wine}} <_{\mathbf{a.drop.of.wine}} P$ it holds that $\lambda i\exists x[\mathbf{was.drunk}(i)(x) \;\&\; \mathbf{a}_{\mathbf{a.drop.of.wine}}(i)(x)] < \lambda i\exists x[\mathbf{was.drunk}(i)(x) \;\&\; P(i)(x)]$. This is so because both the property **was.drunk** and the polarity structure are related to the part relation $\leq_p$. For the property **was.drunk** we can assume DIVISIVITY, that is, for every x, y, if $\mathbf{was.drunk}(i)(x)$ and $y\leq_p x$, then $\mathbf{was.drunk}(y)$. For the polarity structure, we can assume that every quantity of wine has a part which is a minimal quantity of wine, that is, if $P\in \mathbf{A}_{\mathbf{a.drop.of.wine}}$ and $P(i)(x)$, then there is a y, $y\leq_p x$ and $\mathbf{a}_{\mathbf{a.drop.of.wine}}(i)(y)$. Therefore we can conclude that $\lambda i\exists x[\mathbf{was.drunk}(i)(x) \;\&\; \mathbf{a}_{\mathbf{a.drop.of.wine}}(i)(x)] \leq \lambda i\exists x[\mathbf{was.drunk}(i)(x) \;\&\; P(i)(x)]$. If we assume that the set of possible worlds is modally complete, then there is a set of possible worlds in which only a minimal amount of wine was drunk, and therefore we can strengthen "≤" to "<". That means, the semantic combination (14) is indeed UE with respect to $\mathbf{a}_{\mathbf{a.drop.of.wine}}$, that we get a negative inclusion structure, and that *a drop of wine was drunk* is a derived NPI.

6. Polarity items in assertions, questions and directives

In Krifka (1990a) I presented an explanation why propositional NPIs don't make good assertions, and why questions and directives based on them have special connotations (e.g. why questions tend to be understood as rhetorical or inquisitive). Following Zaefferer (1983) and Jacobs (1984), I assumed that assertions, questions and directives are represented by illocutionary operators, and that these operators are sensitive to alternatives. The general rule is that whenever a speaker makes an assertion, asks a question, or gives a directive on the basis of a proposition, he can be assumed to have reasons not to do that on the basis of an alternative of that proposition. This has consequences for the illocutionary use of sentential polarity items. For example, an assertion based on a sentential negative polarity item will be bad, as the speaker does not have good reasons not to assert any of the alternatives -- after all, if the NPI meaning is true, some of the alternatives will be true as well, and they are more specific, hence more informative and preferable to the NPI representation. On the other hand, an assertion based on a sentential positive polarity item will be good, as the alternatives are less specific, hence less informative, than the NPI representation.

Here, I will develop a framework in which this explanation can be cast in a formal way. Let us assume a dynamic view of communication, where sentences are used to change common background assumptions of speaker and hearer (the CONTEXT). Let us represent the context by a set of possible worlds (an oversimplification in some respects, but sufficient for our purposes). Then the assertion of a sentence with respect to a context can be rendered formally as an operator which maps the interpretation of that sentence and the context (which may also be called the INPUT CONTEXT) to another context (the OUTPUT CONTEXT). Following Jacobs (1984), we can assume that the assertion operator is focus-sensitive, hence cannot be specified solely as a function of the meaning of the sentence. However, we can assume that the semantics of assertion can be rendered in terms of the meaning alone; the alternatives are related to the pragmatic conditions of use. Thus, the assertion operator for sentence interpretations can be spelled out by an assertion operator for sentence meanings (that is, propositions) and some additional pragmatic conditions (some of which will be discussed below):

15) $\mathrm{ASSERT}(s, h, c, \mathbb{I}(\Phi)) = \mathrm{ASS}(s, h, c, \mathbb{M}(\mathbb{I}(\Phi)))$
+ pragmatic felicity conditions which relate to $\mathbb{A}(\mathbb{I}(\Phi))$

Here, s and h stand for the speaker and the hearer, c for the input context, and ASS is the assertion operation for propositions. This assertion operation can be taken to restrict the input context to those worlds which are compatible with that proposition. Formally, the assertion operator for propositions maps the input context c and a proposition p to the intersection of c and p as the output context. We should, in addition, assume some felicity conditions for ASS, among them that Φ is possible with respect to c in the first place (that is, that this intersection is non-empty).

16) Assertion rule: $ASS(s, h, c, \mathbb{M}(\mathbb{I}(\Phi))) = c \cap \mathbb{M}(\mathbb{I}(\Phi))$
Felicity condition: $ASS(s, h, c, \mathbb{M}(\mathbb{I}(\Phi))) \neq \emptyset$

There are other felicty conditions; for example, that the assertion is relevant, that is, changes the input context, $ASS(s, h, c, \mathbb{M}(\mathbb{I}(\Phi))) \neq c$.

How should the pragmatic felicity conditions in (15) be spelled out? First, we should assume that at least some proper alternatives are compatible with the input context c. If all the alternatives were excluded to begin with, then there would be no reason for the speaker to construct them. Second, if the assertion of an alternative with respect to the input context would be felicitous, then its assertion should lead to a different output context than the assertion of the meaning itself; otherwise there would be no reason to construct alternatives to begin with. Third, we can assume that the speaker has reasons not to base his assertion on one of the proper alternatives which would be felicitous; otherwise, he would have done so. And there might be more conditions, for example, that the speaker says nothing which he does not believe to be true (that is, set of worlds which might be the real world according to the speaker's believe, BEL(s), is a subset of the output context):

17. Pragmatic felicity conditions for $ASSERT(s, h, c, \mathbb{I}(\Phi))$:
   a) There are p, $p \in \mathbb{A}(\mathbb{I}(\Phi))$, $p \neq \mathbb{M}(\mathbb{I}(\Phi))$, such that $p \cap c \neq \emptyset$;
   b) for every such p, $ASS(s, h, c, p) \neq ASS(s, h, c, \mathbb{M}(\mathbb{I}(\Phi)))$;
   c) for every such p, s has reasons for not ASS(s, h, c, p).
   d) $BEL(s) \subseteq ASSERT(s, h, c, \mathbb{I}(\Phi))$

Take as an example the sentence *Jim saw SUE*, as analyzed in (5). The assertion of that sentence, ASSERT(s, h, c, $\mathbb{I}$(*Jim saw SUE*)), is $c \cap \lambda i[saw(i)(j, s)]$, abbreviated as c', under the following conditions for the input context c: First, $c \cap c' \neq \emptyset$, that is, c' was not excluded to begin with. Second, $BEL(s) \subseteq c'$, that is, c' is believed by the speaker to be true. Third, the only alternative, $\lambda i[saw(i)(j, m)]$, must not be excluded by c, that is, $c \cap \lambda i[saw(i)(j, m)] \neq \emptyset$. And s must have reasons not to assert $\lambda i[saw(i)(j, m)]$. There might be various reasons for that -- for example, s may lack evidence whether it is true, or s knows that it is not true. Assertions may be quite vague in that respect.

Note that all these felicity conditions for a context c may be used to change contexts by ACCOMODATION, in the sense of Lewis (1979). For example, if s says *Dolphins are fish* and h thinks they aren't, h can reconstruct s's concept of the shared context as one which allows for the possibility that dolphins are fish, and in addition that s's believe set is a subset of that propostion. And if s says *Dolphins aren't fish,* h may reconstruct a context in which this is not obvious to begin with (but excluded by that very assertion). Of course, this extraordinary context change by accomodation should be as conservative as possible. That is, h has to accomodate a context which is as close to the old one as possible, but allows for worlds which are or are not in $\mathbb{M}(\mathbb{I}(\Phi))$.

This rudimentary theory of assertion can be used to explain the distribution of polarity items when we take the following condition as a consequence of (17.c): There should not be a proper alternative which is believed by the speaker to be true and whose assertion with respect to the context c would be MORE INFORMATIVE than the assertion of the meaning is. Given a pragmatic maxim to maximize informativity (which follows from the maxim of Quantity of Grice 1975), the speaker should then have asserted this alternative in the first place. We obviously cannot claim that the speaker uses the most informative proposition possible in general. But given that the alternatives are taken as propositions which the speaker CONSTRUCTS as alternatives to what he is in fact saying, this seems to be a sound rule.

18) More specific pragmatic felicity condition for ASSERT(s, h, c, $\mathbb{I}(\Phi)$):
c is such that for no p, $p \in \mathbb{A}(\mathbb{I}(\Phi))$ with $BEL(s) \subseteq p$ and $p \neq \mathbb{M}(\mathbb{I}(\Phi))$:
$ASS(s, h, c, \mathbb{M}(\mathbb{I}(\Phi))) \leq ASS(s, h, c, p)$.

From these conditions, it follows that an assertion on the basis of a sentential NPI will be bad with respect to every input context. To see this, take Φ to be a sentential NPI such that its meaning is believed by s, $BEL(s) \subseteq \mathbb{M}(\mathbb{I}(\Phi))$. First, as Φ is an NPI (cf. 6), it holds that there are proper alternatives of Φ, that is, $\exists p[p \in \mathbb{A}(\mathbb{I}(\Phi))\ \&\ p \neq \mathbb{M}(\mathbb{I}(\Phi))]$, and from the felicity conditions for assertions (17), we know that some of these proper alternatives satisfy the conditons for assertion, or more specifically, $\exists p[p \in \mathbb{A}(\mathbb{I}(\Phi))\ \&\ p \neq \mathbb{M}(\mathbb{I}(\Phi))\ \&\ c \cap p \neq \varnothing]$. Second, as Φ is an NPI (cf. 6), it holds that every proper alternative of Φ is more specific than Φ, that is, $\forall p[p \in \mathbb{A}(\mathbb{I}(\Phi))\ \&\ p \neq \mathbb{M}(\mathbb{I}(\Phi)) \rightarrow \mathbb{M}(\mathbb{I}(\Phi)) < p]$. Furthermore, we can assume that if the speaker believes Φ, then he believes at least some of the more specific alternatives of Φ. With the first result and the set-theoretic fact that $q < p \rightarrow [c \cap q \leq c \cap p]$ we can derive that $\exists p[p \in \mathbb{A}(\mathbb{I}(\Phi))\ \&\ p \neq \mathbb{M}(\mathbb{I}(\Phi))\ \&\ c \cap p \neq \varnothing\ \&\ BEL(s) \subseteq P\ \&\ [c \cap \mathbb{M}(\mathbb{I}(\Phi)) \leq c \cap p]]$, where "$\leq$" can be strenthened to "$<$" by condition (17.b). However, by the pragmatic felicity conditions for assertions (18), we know that there is no proper alternative which satisfies the conditions for assertion and whose assertion will be more specific than the assertion of the meaning itself, that is, $\neg\exists p[p \in A(\mathbb{I}(\Phi))\ \&\ p \neq M(\mathbb{I}(\Phi))\ \&\ c \cap p \neq \emptyset\ \&\ BEL(s) \subseteq P\ \&\ [c \cap M(\mathbb{I}(\Phi)) < c \cap p]]$. Thus, there is no admissible context for the assertion of a sentential NPI.

Now take the assertion of a sentential PPI. As in this case the alternatives are less specific than the meaning, we should expect to find admissible contexts. Furthermore, by the properties of PPIs and felicity condition (17.b) we can derive an interesting property of these contexts, namely that the assertion of every proper alternative for which the assertion is defined is a proper superset of the assertion of the meaning itself, that is, $\forall p[p \in A(\mathbb{I}(\Phi))\ \&\ p \neq M(\mathbb{I}(\Phi))\ \&\ c \cap p \neq \emptyset \rightarrow [c \cap p < c \cap M(\mathbb{I}(\Phi))]]$. That is, the assertion of the meaning itself is indeed the strongest assertion with respect to the input context. This is corroborated by the fact that the use of sentential PPIs suggests that the relevant alternatives would be "more likely" than the PPI meaning. For example, a proper context for the assertion of *Jim didn't see anyone* is such that an alternative, say "Jim didn't see Mary (but he saw Sue)" would be considered as more likely. And a proper context for the assertion of *John didn't lift a finger* is such that an alternative, say "John didn't sweep the floor (but he mowed the lawn)" is considered as more likely.

Now take QUESTIONS, or more specifically, dialectic (that is, yes/no-) questions. Similar to assertions, they modify the input context, but the way in which they do that will depend on the answer: If a question is based on a sentence Φ, then a positive answer will restrict the input context to the worlds which are compatible with the meaning of Φ, and a negative answer will restrict the input context to the worlds which are compatible with the complement of the meaning of Φ. If we assume that the question operator QUEST has an additional argument a for the answer, we can define it like follows:

19. a) $QUEST(s, h, c, \mathbb{I}(\Phi), a) = QU(s, h, c, M(\mathbb{I}(\Phi)), a)$
+ pragmatic felicity conditions which relate to $A(\mathbb{I}(\Phi))$

b) $QU(s, h, c, M(\mathbb{I}(\Phi)), yes) = c \cap M(\mathbb{I}(\Phi))$,
$QU(s, h, c, M(\mathbb{I}(\Phi)), no) = c \cap \neg M(\mathbb{I}(\Phi))$,
+ pragmatic felicity conditions

Questions which are answered by yes are quite similar to assertions. However, questions which are answered by no behave differently. In particular, if the question is based on a sentential NPI, then we should expect that the answer is no, because only with that answer can we construct an admissible input context.

To see this, take a sentential NPI Φ and the answered question QUEST(s, h, $\mathbb{I}(\Phi)$, **no**). As Φ is a NPI, it has proper alternatives. The pragmatic felicity condition (17.a), which should hold for questions as well, requires that some of these proper alternatives are compatible with the context c, that is, overlap with c. The pragmatic condition (18) requires that no such alternative yields a more specific output context, that is, there is no p with $p \in \mathbb{A}(\mathbb{I}(\Phi))$ & $p \neq \mathbb{M}(\mathbb{I}(\Phi))$ & $c \cap p \neq \emptyset$ for which QU(s, h, c, $\mathbb{M}(\mathbb{I}(\Phi))$, **no**) < QU(s, h, c, p, **no**), that is, for which $c \cap \neg \mathbb{M}(\mathbb{I}(\Phi)) < c \cap \neg p$. As the proper alternatives of the NPI are all stronger than its meaning, and hence their complements are weaker than the complement of the meaning, this condition can be easily satisfied. In addition, we can conclude, similar to the case of PPI assertions, that the meaning of the NPI question should indeed be the strongest relevant alternative.

Of course, when asking a question the speaker typically does not know the answer in advance, and so he does not know how the input context c will be restricted, and thus he does not know whether a question based on an NPI is appropriate. However, there are pragmatic conditions where the speaker can be assumed to be biased towards a negative answer, and hence put a question based on a NPI:

The first case are RHETORICAL QUESTIONS, and they are a well-known context for NPIs; cf. e.g. *Did you ever lift a finger to help me?* By choosing an NPI, the speaker suggests a negative answer to the hearer, because only then the choice of an NPI is appropriate.

But sentential NPIs also occur as real information questions, e.g. *Have you ever smoked Marihuana?*, or *Was any person injured in the accident?* I will call questions of this type INQUISITIVE QUESTIONS as they urge the hearer to report any instance which could lead to a positive answer. The occurrence of NPIs in inquisitive questions can be traced back to a particular question strategy. There are two strategies with dialectic questions: (i) The speaker may base his question on a rather strong proposition; if the answer is affirmative, he will be highly informed, and if the answer is negative, he will remain relatively uninformed. (ii) The speaker may base his question on a rather weak proposition; if the answer is affirmative, he will remain relatively uninformed, and if the answer is negative, he will gain much information. As an example, consider a case where the hearer draws a card from a deck of cards, and the speaker has to find out which one it is with as few questions as possible. According to the first stragegy, he would ask: *Is it the seven of diamonds?*, *Is it the eight of diamonds?*, etc. According to the second strategy, he would ask: *Is it a diamonds?*, *Is it a seven?*, etc. In this setting, the second strategy will obviously be more efficient. The second strategy will be used in general if the speaker has relatively little background information or if he does not want to give the hearer the possibility of an evasive answer. This strategy is more biased towards a negative answer. As the hearer assumes that the speaker wants to get as much information as possible out of his question, he can conclude that a negative answer would be highly informative for the speaker. Hence the hearer may accomodate an input context which is fairly wide (that is, weak), because this enhances the possiblity that a negative answer is informative.

The framework developed so far can also be applied to explain the occurrence of NPIs in DIRECTIVES. There are two kinds of directives, COMMANDS and OFFERS, and at least one type of NPIs, the one based on *any*, may occur in both of them, cf. *Confiscate any alcohol you can find!* and *Take any cookie you like!* Of course, directives cannot be treated in terms of informativity; the speaker expresses by them the states of the world which the speaker should bring about. By using a directive based on a NPI, the speaker suggests to interpret his wish as wide as possible. In the case of a command, this means that the comman must be taken as particularly strict. In the case of an offer, this means that the speaker wants to leave as much freedom to the hearer as possible. In Krifka (1990a), I argue that so-called "free choice" readings of *any* occur in these contexts.

7. Conditionals

Let us finally have a look at conditional sentences and why they allow for NPI protases. The explanation of Heim (1984) should follow from the semantics we propose from conditionals and the general rules for the assertion of sentences.

The simplest semantics of conditional sentences, which is too simplistic in some respect but sufficient for our sake, is to assume that a conditional sentence like *if* Φ *then* Ψ is false just in those worlds in which Φ is true but Ψ is false, reflecting the propositional rule $A \rightarrow B \Leftrightarrow \neg A \vee B$. That is, the meaning of *if* Φ *then* Ψ is the union of the meaning of Ψ and the complement of the meaning of Φ. Let us assume that conditional sentences are formed by a syntactic rule S_6 which takes two sentences, and let us assume a corresponding semantic combination rule $\mathfrak{c}(S_6, ., .)$, then we can formulate the interpretation of conditional sentences as follows (let $\neg$ be the complement function on possible worlds):

20. a) $S_6(\Phi, \Psi) = if\ \Phi\ then\ \Psi$.
    b) $\mathbb{M}(\mathbb{I}(if\ \Phi\ then\ \Psi)) = \mathfrak{c}(S_6, \mathbb{M}(\mathbb{I}(\Phi)), \mathbb{M}(\mathbb{I}(\Psi))) = \neg\mathbb{M}(\mathbb{I}(\Phi)) \cup \mathbb{M}(\mathbb{I}(\Psi))$,
    c) $\mathbb{A}(\mathbb{I}(if\ \Phi\ then\ \Psi)) = \{r: \exists p,q[p \in \mathbb{A}(\mathbb{I}(\Phi))\ \&\ q \in \mathbb{A}(\mathbb{I}(\Psi))\ \&\ r = \mathfrak{c}(S_6, p, q)]\}$

As we find NPIs in the protasis of asserted conditionals, we should expect that the semantic combination $\mathfrak{c}(S_6, *, .)$ is DE with respect to *, because then the conditional *if* Φ *then* Ψ is guaranteed to be a PPI, and hence would make a good assertion. However, we can only show that $c(S_6, *, .)$ is "nearly" DE with respect to *: Assume that Φ is a NPI based on a negative inclusion structure $\langle A(\mathbb{I}(\Phi)), M(\mathbb{I}(\Phi))\rangle$, and assume for the sake of simplicity that Ψ has no proper alternatives. We can prove that every p, with $p \in A(\mathbb{I}(\Phi))$ and $p \neq M(\mathbb{I}(\Phi))$, is such that $M(\mathbb{I}(\Phi)) < p$ holds (as Φ is a NPI), and for every such p, it will hold that $c(S_6, p, M(\mathbb{I}(\Psi))) \leq c(S_6, M(\mathbb{I}(\Phi)), M(\mathbb{I}(\Psi)))$, that is, $\neg p \cup M(\mathbb{I}(\Phi)) \leq \neg M(\mathbb{I}(\Phi)) \cup M(\mathbb{I}(\Psi))$ (this is a set theoretic fact: if $p' \supset p$, then $\neg p \cup q \supseteq \neg p' \cup q$). But we cannot strengthen "$\leq$" to "<", as the definition of DE-ness (6) requires us to do. The reason is that we might well have proper subsets p of $M(\mathbb{I}(\Phi))$ such that $\neg p \cup M(\mathbb{I}(\Psi)) = \neg M(\mathbb{I}(\Phi)) \cup M(\mathbb{I}(\Psi))$.

However, we may assume a pragmatic principle which restricts the contexts under consideration such that semantic combination is indeed DE. The most plausible principle of that sort is the CONSERVATION PRINCIPLE OF ALTERNATIVES which says that under semantic combinations, distinct alternatives must create distinct alternatives:

21) If $\mathbb{C}[\mathbb{I}(\alpha), \mathbb{I}(\beta)]$ is a semantic combination containing the interpretations $\mathbb{I}(\alpha)$ and $\mathbb{I}(\beta)$, and if $c[M(\mathbb{I}(\alpha)), M(\mathbb{I}(\beta))]$ is the corresponding meaning combination, then for all α–alternatives X, $Y \in A(\mathbb{I}(\alpha))$ for which $X \neq Y$ there are alternatives X', $Y' \in A(\mathbb{C}[\mathbb{I}(\alpha), \mathbb{I}(\beta)])$ sucht that $X' \neq Y'$ and there are Z, $Z' \in A(\mathbb{I}(\beta))$ such that $X' = c[X,Z]$ and $Y' = c[Y,Z']$.

This principle is motivated pragmatically: The introduction of proper alternatives adds complexity to the semantic interpretation. This additional complexity can be exploited only at the level of focus-sensitive operators such as *only* or the illocutionary operators. Therefore the alternatives have to be kept distinct until they are evaluated by a focus-sensitive operator.

For the semantic combination with polarity items, the conservation principle tells us that the meaning of the complex polarity item is generated only by the meaning of the embedded polarity item, and that the proper alternatives of the complex polarity item are generated only by the proper alternatives of the embedded polarity item. Note that this is part of the definition of UE/DE semantic combinations (cf. 6). It is this consequence of the conservation principle which is sufficient for our purpose.

The conservation principle can be formulated as a restriction of the input contexts: only those input contexts are admitted which satisfy the conservation principle. In the case of a polarity item, the input context must allow for the "modal completeness" alluded to in section (5): The combination of *any boy* and *came* (cf. 12) would not be licensed with respect to such input contexts which contain only worlds in which every boy came, because in this type of context the meaning of the polarity item would not be a proper superset of its alternatives.

In order to carry out this idea, we have to assume that the input context c is accessible not only for illocutionary operators, but also for lower-level semantic combinations. This is, in fact, a standard assumption in the theory of context-sensitive expressions (cf. Kaplan 1977). It obviously applies also to the indices of speaker and hearer, which determine the meanings of the pronouns *I* and *you*. So we should assume that the rules for an illocutionary operator like ASSERT have the format ASSERT(s,h,c, $\mathbb{I}^{s,h,c}(\alpha)$), where the indices s, h, c are passed down to the interpretation function. Semantic combination rules pass the indices s, h, c to their subconstituents, and therefore have the rule format $\mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(S(\alpha,\beta))) = \mathbf{c}(S, \mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(\alpha)), \mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(\beta)))$. The indices may be evaluated in rules like $\mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(\mathit{I})) = s$ and $\mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(\mathit{you})) = h$. As for the context c, we can assume that it restricts the choice of possible worlds in properties and propositions. For example, $\mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(\mathit{boy}))$ will be the property **boy** restricted to the worlds in c, that is, $\lambda i,x[\mathbf{boy}(i)(x)\ \&\ c(i)]$.

It is reasonable to assume that not only the illocutionary operators, but also other, lower-level operators impose felicity conditions on the input context. An obvious example are (non-anaphoric) definite NPs. Take the sentence *The king of Swaziland died.* The felicity conditions for *the king of Swaziland* are such that the context must provide for exactly one entity in the extension of *king of Swaziland.* If S_7 is the syntactic rule which combines the definite article with a noun, then we have $\mathbf{c}(S_7, \mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(\mathit{the})), \mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(\alpha))) = \lambda i\, \iota x[\mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(\alpha))(i)(x)]$, with the felicity condition that for every world in c, α applies to exactly one entity. In this way, a definite NP may force an accommodation of the context c, and this accomodation is passed through the semantic combinations to the illocutionary operator. Therefore, ASSERT(s,h,c, $\mathbb{I}^{s,h,c}$(*The king of Swaziland died*)) will be felicitous for such c that every world in c provides for exactly one king of Swaziland.

Now let us apply these refinements to the analysis of a conditional sentence with a NPI protasis, example (3), *if you eat any fruit, you will feel better*. Given that the context c restricts the choice of possible worlds down to the basic expressions, the protasis of this sentence will have the following interpretation:

22)

any	$\mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(\mathit{any}))$ $= \lambda PP$	$\mathbb{A}^{s,h,c}(\mathbb{I}^{s,h,c}(\mathit{any}))$ $\{M: M=\lambda P,i,x\exists Q[P\leq Q\ \&\ Q(i)(x)]\}$
fruit	$\mathbb{M}^{s,h,c}(\mathbb{I}^{s,h,c}(\mathit{fruit}))$ $= \lambda i,x[\mathbf{fruit}(i)(x)\&c(i)]$	$\mathbb{A}^{s,h,c}(\mathbb{I}^{s,h,c}(\mathit{fruit}))$ $= \{\lambda i,x[\mathbf{fruit}(i)(x)\&c(i)]\}$

S_4(*any, fruit*) = *any fruit*	c(S_4, . . , .) = **λi,x[fruit(i)(x)&c(i)]**	{P: **λi,x[fruit(i)(x)&c(i)]** ≤ P}
eat	$M^{s,h,c}(\mathbb{I}^{s,h,c}$(*eat*)) = **λi,y,x[eat(i)(x,y)&c(i)]**	$A^{s,h,c}(\mathbb{I}^{s,h,c}$(*eat*)) = {**λi,y,x[eat(i)(x,y)&c(i)]**}
S_2(*any fruit, eat*) = *eat any fruit*	c(S_2, . . , .) = **λi,x∃y[eat(i)(x,y) & fruit(i)(y) & c(i)]**	{Q: **∃P[λi,x[fruit(i)(x)&c(i)] ≤ P & Q = λi,x∃y[eat(i)(x,y) & P(i)(y) & c(i)]**}
you	$M^{s,h,c}(\mathbb{I}^{s,h,c}$(*you*)) = **h**	$A^{s,h,c}(\mathbb{I}^{s,h,c}$(*you*)) = {**h**}
S_1(*you, eat any fruit*) = *you eat any fruit*	c(S_1, . . , .) = **λi∃y[eat(i)(h,y) & fruit(i)(y) & c(i)]**	{p: **∃P[λi,x[fruit(i)(x)&c(i)] ≤ P & p = λi∃y[eat(i)(h,y) & P(i)(y) & c(i)]**}

In these semantic combinations, the conservation principle for alternatives imposes restrictions on admissible input contexts c. They guarantee, in particular, that the meaning of *you eat any fruit* is the only proposition in the set of alternatives of *you eat any fruit* that is generated by the meaning of *any fruit*. This means, for example, that a context c is excluded which contains only such worlds i in which h eats every fruit in i, because then the alternatives of *you eat any fruit* would collapse to a singleton.

The following analysis shows the construction of the conditional sentence (I assume that the interpretation of *you will feel better* is as shown).

23)

..	..	..
you will feel better	**λi[feel.better(i)(h)&c(i)]**	{**λi[feel.better(i)(h)&c(i)]**}
S_6(*you eat any fruit, you feel better*) = *if you eat any fruit, you will feel better*	c(S_6, . . , .) = **¬λi∃y[eat(i)(h,y) & fruit(i)(y) & c(i)] ∪ λi[feel.better(i)(h)&c(i)]** = **λi[[¬∃y[eat(i)(h,y) & fruit(i)(y)] ∨ feel.better(i)(h)] &c(i)]**	{p: **∃P[λi,x[fruit(i)(x)&c(i)] ≤ P & p = λi[[¬∃y[eat(i)(h,y) & & P(i)(y)] ∨ feel.better(i)(h)] & c(i)]]**}

The meaning of that interpretation is the set of all context-worlds in which the hearer either eats no fruit or will feel better. The alternatives are sets of (context-)worlds in which the hearer either eats nothing which falls under a certain subconcept of *fruit* or in which he will feel better. The conservation principle forces the context to be such that the meaning is the most specific proposition of all the alternatives. Intuitively speaking, the context must be such that it suggests that it is "less likely" that you feel better after eating just any old fruit than after eating some more narrowly specified fruit.

Note that not only the meanings, but also the other alternatives are restricted by the context. This follows from our general procedure of alternative construction. Therefore, we can explain why *if you eat any fruit, you will feel better* does not imply *if you eat a poisoned fruit, you will feel better* (cf. section 2). The reason is that poisoned fruits, although they are characterized by a subconcept of **fruit**, need not be part of the context. If the context captures the background knowledge, then it will be accomodated as to exclude worlds in which there are poisoned fruits, because then the condtional would obviously be false.

The rules for conditionals as formulated here are close to Heim's condition (cf. 4), which can be rephrased as follows: The NPI protasis Φ is licensed in *if* Φ *then* Ψ in case for every alternative $p \in A(\mathbb{I}(\Phi))$ in which $\mathbb{M}(\mathbb{I}(\Phi)) \leq p$ it holds that for all contexts c in which $ASS(s,h,c, \mathbb{M}(\mathbb{I}(S_6(\Phi,\Psi))))$ is defined, we have $ASS(s,h,c, \mathbb{M}(\mathbb{I}(S_6(\Phi,\Psi)))) \subseteq ASS(s,h,c, p)$. In the representation developed here, we could strengthen $\leq$ to $<$ and $\subseteq$ to $\subset$.

This explains why NPIs occur in the protasis of conditionals and why they require input contexts of a specific type. However, this is not the whole story of condtionals and polarity items. Heim (1984) suggests that there might be different sorts of polarity items which require different types of background conditions. And Hoeksema (1986) observes that we also find PPIs in the protasis of conditionals (he took this as an argument against a recursive notion of polarity items):

24. a) *If John doesn't know anything about logic, he doesn't know Modus Ponens.*
    b) *If I had bags of money, I would (still) spend my vacations at the Wolfgangssee.*

In (24.a), an example of Hoeksema, we have the sentential PPI *John doesn't know anything about logic* as a protasis, and in (24.b) we have the sentential PPI *I had bags of money*. I think that conditional clauses of this type require a special interpreation. Sentence (24.a) can be interpreted in two ways: First, it may suggests that John will know Modus Ponens even with a limited knowledge of logic. This reading is preferred with stress on *anything*. Note that the example becomes odd when we change it to *If John doesn't know ANYthing about logic, he will not be able to prove the completeness theorem*. A good paraphrase of that reading is *Only if John doesn't know ANYthing about logic, he doesn't know Modus Ponens*. So we might assume that the NPI *anything* actually is itself the focus of an operator similar to *only*, possibly the assertion operator. But this is equivalent to *If John doesn't know Modus Ponens, he doesn't know anything about logic*. Here, the sentential NPI is in the apodosis, and this is a position where we should expect it to occur.

In (24.b), we also have an implicit focus-sensitive operator, namely *even*; the sentence may be paraphrased as *Even if I had bags of money, I would still spend my vacations at the Wolfgangssee*. This sentence suggests a context in which it is less likely to spend the vacations at the Wolfgangssee, the more money one has. With respect to that context, the sentence expresses a strong restriction.

In a second interpretation, which seems possible for (24.a), the polarity item is unstressed. In this case, the proposition representend by the protasis must have been made salient in the preceding discourse. This could be rendered formally by claiming that the input context is already a part of the protasis proposition. Then weakening the protasis has no effect on the context-change function anymore.

8. Conclusion

In concluding, I want to stress the main points of this paper. First, I have shown that we can apply alternative semantics to explain polarity phenomena. They can be described as expresssions whose interpretations have alternatives which stand in some order relation. These alternatives and their order relation are projected to more complex expressions. In this projection, we find that idiosyncratic orderings change to more regular orderings based on set inclusion, a process which is mediated by the part relation between individuals. Second, we can assume that illocutionary operators are sensitive to alternatives, so that the appropriateness of an NPI or PPI in the scope of an illocutionary operator depends on specific pragmatic conditions of this operator. Finally, we referred to other pragmatic felicity conditions to explain the use of polarity items, namely that the polarity property is conserved under semantic combinations. By this we could explain why polarity items typically suggest special contexts, or background assumptions.

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ENGLISH ASPECTUAL VERBS AS GENERALIZED QUANTIFIERS

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1.Introduction

Linguistic intuitions support a fundamental three-fold distinction of the aspectual verbs into
1) the verbs of initiation: *start* ,*begin* , *commence* , *initiate* , *resume*
2) the verbs of cessation: *stop* ,*finish* , *complete* , *terminate* , *halt* , *cease, end*
3) the verbs of continuation: *continue* , *keep* , *go on.*
A second traditional division of aspectual verbs between causative and non-causative verbs concerns, as I will explain later, their control-structure. The non-causative verbs require a bound subject-argument internal to the event in the scope of the aspectual verb, for causatives this argument remains free.
Thirdly, there is the syntactic observation that English aspectual verbs either take only gerunds (e.g. *resume, stop, keep*) or they take both gerunds and infinitival clauses (e.g. *start, begin, cease, continue*).
Taking these three basic divisions as my starting-point, I will argue that the aspectual verbs are essentially quantificational in nature, and provide a uniform analysis of their syntax and semantics which will account for a number of interesting inferences and the projection of their presuppositions. From this viewpoint the three divisions can be seen as different aspects of a coherent and integrated theory of quantificational structure in natural language.
As for their syntactic structure I assume a universal base-generated structure as in (1) with a VP-internal subject-position in Spec, espousing Heim's dynamic theory of quantification and reference as background semantic framework (i.e. VP = nuclear scope).

1) $_{IP}[$ NP $_{I'}[$ V_{aspect} TENSE $_{VP}[$ Spec V']]]

My approach here is semantically motivated, though it has important consequences for syntax too. I utilize principles and results of the theory of generalized quantifiers to explain the properties characteristic of the different classes of aspectual verbs. The aspectual verbs are semantically represented as relations, called *aspectual constraints*, between an antecedently determined reference-time t as restrictive term and the event-type E described by the complement in nuclear scope, as in (2).[1]

2) $V_{aspect}(t, E)$

This relational approach opens up the powerful tool-kit of GQ-theory, which proves a very fruitful and comprehensive theory of semantic structure, if applied to aspectual quantification. The presuppositions and entailments of aspectual verbs are my central explananda, but in the background there is a more general concern with dynamic interpretation and context-dependence which I cannot spell out in any detail here. In addition this analysis of aspectual verbs embodies a theory of events which clarifies significantly how events are identified and individuated. Just like an individual can have many different properties, in my theory an event can simultaneously be of many different event-types. The theory provides a systematic distinction between aspectual verbs which concern the internal structure of an event, quantifying over stages of one and the same event, and aspectual verbs which govern the external temporal relations between events, quantifying over distinct occurrences of the same type of event. I will discuss how thematic structure mediates between syntactic argument structure and the structure of events, arguing that thematic role assignment should occur prior to anaphoric linking. Furthermore, this analysis of aspectual quantification can be related very naturally to the account of the aspectual classes of states, activities, accomplishments and achievements developed in ter Meulen (forthcoming Bradford Books, MIT Press).

2. Events and event-types

It has already often been argued that the semantic interpretation of complements of aspectual verbs are dependent events, supported by their lack of full clausal complements as in (3), VP-ellipsis in (4), and anaphora in (5).[2]

3) * John started that he read a novel

4) a. John read a novel and Mary read too, but not a novel.

b. John started/ finished a novel and Mary started/ finished too, *but not a novel.

5) a. *John kept reading [a novel]$_i$ and Mary finished it $_i$

b. *John kept [reading a novel]$_i$ and Mary finished it $_i$

c. John kept baking [a cake/cakes/two cakes]$_i$ and Mary finished it/them

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[1] Löbner (1987) suggests a similar quantificational account of aspectual verbs in the context of a relational theory of temporal adverbials, treating them as relations between propositions and moments of time. E.g. *stop* (p, t) is true iff. t is the last point of the period during which p is true. A basic difference with my account is that he relies on intervalsemantics and propositions, where my analysis is essentially more dynamic, based on events and contextually determined reference-times. The present paper provides a much more detailed and comprehensive theory of aspectual quantification.

[2] Note that verbs of creation seem to allow anaphoric dependencies, e.g. *John started baking a cake and Mary finished it* is much better than 5 in both cases.

d. ?John kept [baking a cake]$_i$ and Mary finished it $_i$

Some aspectual quantifiers like *keep* create a dependency of the event-type in the complement on the reference-time *t* in the restrictive term, just like the subordination in quantificational NPs. Due to this subordination, the complement event-type constitutes an inaccessible domain for nominal or temporal anaphora in the main event-structure as in (5). As we see in (5c) and (5d), verbs of creation typically do allow NP anaphora to the indefinite internal argument in nuclear scope with an iterative interpretation, but they don't allow event anaphora quite so easily.[3] The tense-inflection of aspectual verbs will normally depend on the contextually determined reference-time, which explains why aspectual verbs can be used to describe the internal structure of an event otherwise represented as atomic or indivisible (accomplishments or achievements). An event anaphor complementing a second aspectual verb with a subject coreferential to the first subject is unproblematic, as in (6).

6) John kept [reading a novel]$_i$, but left before he finished it $_i$

If the pronoun occurs at the same level of subordination as its antecedent event, it can corefer, if thematic role assignments agree, as is satisfied in (6) but not in (5). These facts concerning the subordination of the complement event hold only for gerundive complements, as the infinitival complements seem to admit this binding more freely.

7) a. John continued to bake [a cake]$_i$, but Mary eventually finished it$_i$

b. John continued [to bake a cake]$_i$, but Mary eventually finished it$_i$

This suggests a fundamental distinction between located event-types, as interpretations of gerunds, as unlocated event-types, interpreting infinitives. The basic distinction between an event-type, which has at least one free argument-indeterminate, and an event, which is a saturated event-type, is fundamental to Situation Semantics and in the theory of events in ter Meulen (forthcoming), which allows an event to be of or to 'realize' different event-types simultaneously.[4] The observations in (7) show that infinitival complements of aspectual verbs can quantify over distinct events or occurrences of the same event-type, allowing temporal gaps. The gerundive complements, however, are quantifying over stages of one and the same event, no gaps allowed and continuity of location is assumed. The anaphoric reference to the event-type in (7b) forces uniqueness of the cake-baking event. But otherwise the infinitival complements can get an interpretation which is temporally divided or 'iterative'. E.g. (8a) describes a situation in which Mary's singing is not interrupted by noise, whereas for (8b) she may stop singing, listen and then continue.

8) a. Mary kept singing, when she heard the noise

b. Mary continued to sing, when she heard the noise

The aspectual verbs *stop, resume, keep, end* , which presuppose that some of the complement event has already been going on, i.e. there are already some stages of it realized, take only gerundive complements. Most other aspectual verbs take either gerundive or infinitival complements, and if they carry a presupposition, like *finish* and *continue* do, it is that there are prior realizations of the same type of event, which do not necessarily belong to the same event.

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3 Making the argument definite seems to help, for obvious reasons.

4 This is an important difference with a theory of events developed by Link and Krifka, where each predicate defines a distinct event, and locations of events have to account for their coincidence. In my 1989 paper I have argued against such a fine-grained conception of events.

3. Aspectual verbs as generalized quantifiers

The lack of presuppositions in verbs of initiation account for the fact that they occur in English with expletive subjects (*it ,there*), whereas the other two classes of aspectual verbs do not.

9) a. There started a reading of a novel
b. * There continued / finished a reading of a novel
10) a. There began a story
b. * There went on / stopped/resumed a story

There appears to be quite some cross-linguistic variation in the acceptability of expletive subjects. In Dutch the expletive *er* with aspectual verbs occurs liberally, as shown in (11).

11) a. Er begonnen twee films te draaien
There started two films to play (litt. turn)
b. Er bleven twee films draaien
There remained two films play
c. Er gingen twee films door met draaien
There went two films on with play
d. Er eindigden twee films om tien uur
There finished two films at ten o'clock

The French data, partly from Lamiroy (1987, p. 280), indicate the opposite judgements, where only real arguments (including weather-arguments) rather than expletives are acceptable with aspectual verbs.

12) a. Il cesse de pleuvoir
It stops raining
b. Il commence de avoir besoin d'une drogue
He begins to need a drug
c. *Il continue à falloir partir
It continues to be necessary to leave
d. *Il commence à s'agir de travailler
It begins to stir him to work

An explanation of this cross-linguistic variation is beyond the scope of the present paper, but the cross-linguistic variation is not really surprising, since definiteness effects are exhibited in all languages, but in a wide variety of forms (see Reuland & ter Meulen (eds.), 1987).

The English data in (9) and (10) justify the hypothesis that *start* and *begin* are indefinite 'existential' aspectual verbs. There is another 'existential' verb *resume* , which asserts that there is a new positive stage of the event, but presupposes that there is already a prior positive and negative stage of the same event in the representation. You can't resume anything unless you have started and stopped it first. So *resume* describes an event-internal change, represented by a new positive stage of an event of which an earlier stage is already realized and a negative stage was going on. Hence *resume* takes only gerundive complements, as in (13).

13) a. Mary started singing/to sing, when John entered
b. Mary resumed singing/*to sing, when John entered

Generalized quantifier theory provides a neat formalization of such presuppositions, discussed in section 5, as computational procedures corresponding to GQs.

Let's continue on this path and wonder which aspectual verb may correspond to the universal quantifier. The universal quantifier in a first order logic is definable as the external negation of the internal negation of the existential quantifier. This will prove a good heuristic: if it is not the case that a negative event-type (with polarity 0) starts, then it must mean that the positive event-type continues. So *continue* is the dual of *start*. . The facts on anaphoric subordination in (5) support this too, as the existential aspectual verbs do allow such binding into arguments of the event-type in nuclear scope.

Just like *start* and *resume* , the presuppositional universal aspectual verb which describes event-internal continuation is *keep* , taking only gerundive complements.

14) a. Jane kept coughing/*to cough during the concert

b. Jane continued coughing/to cough during the concert

From (14a) we infer that during the concert Jane was coughing, and due to the fact that *cough* is an achievement verb, we know its not one prolonged cough but rather repetitive coughing. But from (14b) we infer that Jane may have been coughing before the concert already. Although you cannot continue to do something, if you have not already done some of it, the part already done does not necessarily belong to the same event. The presupposition of *continue* requires that there are distinct realizations of the same event-type, which are not necessarily located as stages of the one and the same event, as *keep* requires. This is an important difference between presuppositions regarding the event-external structure and presuppositions which concern event-internal structure.

Now look at the verbs of cessation : *stop* , *finish* , *complete* , *terminate* , *halt* , *cease*, and *end* . They indicate that the polarity of the event-type is negative; they mark the transition from a positive stage to a negative stage. We see that *finish* , *cease* and *complete* are the internal negation of *start* and the external negation of *continue* . The others *stop, terminate* , *halt* , and *end* are event-internal, and take gerundive complements only. If they do occur with infinitival clauses as in (15), these are purpose clauses - the analysis of which is beyond my present concern.[5]

15) Jane stopped to eat

The verb *stop* is importantly different from the other cessation-verbs, because it entails the event may be resumed. Often we use it to indicate that there is only a temporary event-internal interruption, rather than a complete break or last stage reached. In fact, polarity reversal in the complement of *stop* gives us exactly the complement of *resume* , see (16) : to stop not reading is to resume reading.

16) << *stop* , *t* , <<*read* , *x* , *y* >*0* > *1* > <=>

<< *resume* , *t* , <<*read* , *x* , *y* >*1* > *1* >

The other presuppositional cessation verbs,*terminate* , *halt* , and *end* indicate that a final stage is reached; hence any later realizations of the same event-type are considered parts of other events of the same event-type.

Now I have established the correspondence of the three aspectual classes to the existential, the universal and the internal negation of the existential (or equivalently the external negation of the universal). The traditional square of oppositions has a fourth corner, the external negation of the existential which is equivalent to the internal negation of the universal. In English NPs this is lexicalized by the determiner *no* , but English does not seem to lexicalize the corresponding aspectual verb, having only the complex *not start* , or *continue not to V.* [6] In the logical quantifier square the internally negated existential is not lexicalized in English, although the corresponding aspectual verb *finish* exists. It is remarkable that both

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[5] I advocate an analysis of purpose clauses as VP-adjuncts, cf. Chierchia (1989).

[6] It has been suggested to me that *refuse* or *refrain from* may lexicalize this position, but I hesitate to include these verbs among the aspectual ones.

squares have one, but a different position which is not lexicalized in English. This deserves a further explanation, but I don't yet have one to offer.

Much of the older literature on aspectual verbs, especially in the syntactic tradition, has focussed on the fact that some aspectual verbs seem to behave like 'raising'-verbs, but others like 'control'-verbs. A controversy revolved around the question whether one is more basic than the other, and hence a transformational derivation accounts for their difference or whether a lexically governed coindexing strategy can explain it all.[7] As I indicated already I assume a universal base-generated structure where lexically induced thematic constraints of a semantic nature put the requirement onto the non-causative verbs that the agent be the same as the agent of the complement event - this I call the constraint on thematic role transparency. The generalized quantifier perspective here proves to pay off again: the causatives *stop* , *start* and *keep* make up a square of oppositions together with the non-lexicalized *non-start*. . The non-causatives *finish, resume, end ,cease, terminate, halt,* and *continue* constitute another such square. Since the event-internal gerundive-only verbs *stop, resume* , *end/terminate/halt* and *keep* are also related by external and internal negations, time is ripe to draw some diagrams of these squares.

4. Squares of aspectual quantifiers

The square of opposition of NPs as generalized quantifiers is as in figure 1. where the four corners are related by internal negation and external negation and two quantifiers are each others duals when they are related by composition of internal and external negation.[8]

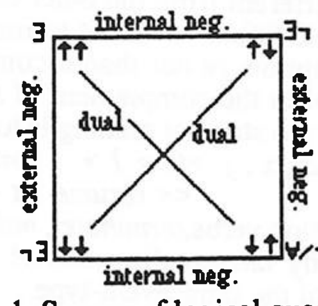

Fig. 1. Square of logical quantifiers

The arrows indicate the monotonicity direction in respectively the interpretation of the CN in the subject NP and the interpretation of the VP, which are the left and right argument of the determiner regarded as a relation D between sets A and B. Internal negation reverses the direction of monotonicity in the right argument, external negation reverses it in the left argument. The concepts and tests are illustrated in (17).

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[7] See Lamiroy (1987) for a lucid account of the arguments on the three alternatives.

[8] According to van Eijck (1984, p. 5) the Aristotelian square of oppositions looks slightly different. It has external negations exchanged with duals of my fig. 1. I don't think any of the semantic properties are affected by this permutation. See also Löbner (1986, p. 54 -56) for diagrams resembling mine more closely.

17) a. existential - *a, some* - left and right increasing
a beautiful woman sings => a woman sings
a woman sings an aria => a woman sings
b. universal - *every, all* - left decreasing, right increasing
every woman sings => every beautiful woman sings
every woman sings an aria => every woman sings
c. internally negated existential - left increasing, right decreasing
a beautiful woman is not singing => a woman is not singing
a woman is not singing => a woman is not singing an aria
d. externally negated existential - *no* - left and right decreasing
no woman is singing => no beautiful woman is singing
no woman is singing => no woman is singing an aria

Applying this basic square to the event-external aspectual verbs, we get fig. 2.

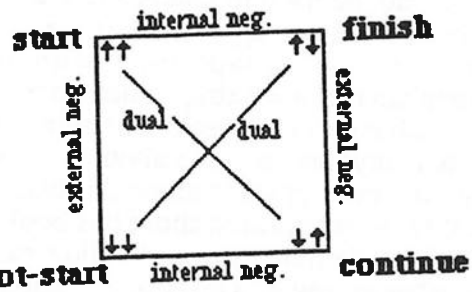

Figure 2. Square of event-external aspectual verbs

Let's check some facts now (e0/1 means e with positive polarity 1 or e with negative polarity 0 respectively):

18) a. *start* (t, e0/1) <=> *finish* (t, e1/0)
b. *not start* (t, e0/1) <=> *continue* (t, e1/0)
c. *continue* (t, e1/0) <=> *not finish* (t, e1/0)
d. *start* (t, e0/1) <=> *not not start* (t, e0/1)

The monotonicity properties for the aspectual verbs as GQs V_A (*t*, *E*) are interpreted as follows. First, the Conservativity of NP-quantifiers is reformulated as requiring that only realizations of the event-type which temporally overlap with the reference-time are to be considered, i.e.

CONSERVATIVITY : $V(t, E) = V(t, (t \circ E))$

The reference-times which constitute the left argument constitute a set partially ordered by the temporal precedence relation $\leq$, and the realizations of the event-type which constitute the right argument are partially ordered by a 'part-of' relation restricted to realizations of the same event-type, i.e. having the same relation, arguments and polarity. (e' and t' are implicitly universally quantified). If at *t* an event *e* is started, then at any later *t'* that event has been started (note how the shift in reference time affects the change of simple past to present perfect) : *start* is left increasing. If at *t* an event *e* is started, and we take a larger realization of the same event-type *e'* which covers *e*, then at *t e'* is started too : *start* is right-increasing. This means that larger parts of the same event cannot ever get located before *t*, i.e. *t* is the starting-point of the event, i.e. *e* can only 'grow' into the future of reference time (19a). Similarly, if you assume that an event is finished at *t*, and take a later time *t'*, then the event certainly has been finished at *t'* too: *finish* is left-increasing. And with the same assumption taking a smaller part of the event, that part is finished at *t* too: *finish* is right-decreasing. In other words, an event which is finished cannot grow into the future of reference time (19b). And again, for *continue*, the event was continued before the reference time, and a larger part continued at the reference time, i.e. you're 'amidst' of the event.

19) a. *start* - left and right increasing
start (t, e) , t ≤ t' => *start* (t', e)
start (t, e) , e is part of e' => *start* (t, e')
b. *finish* - left increasing, right decreasing
finish (t, e) , t ≤ t' => *finish* (t', e)
finish (t, e) , e' is part of e => *finish* (t, e')
c. *continue* - left decreasing, right increasing
continue (t, e) , t' ≤ t => *continue* (t', e)
continue (t, e) , e is part of e' => *continue* (t, e')

The right-decreasing character of *stop* and *cease* is further attested in their triggering negative polarity NPs like *anything*.[9]

(20) John stopped/ceased/*continued/*started doing anything else

Monotonicity properties can be used to characterize a larger class of verbs (incl. aspectual verbs) as indefinite (e.g. *appear*, *exist*) and definite (e.g. *disappear*, *discontinue*). Parallel to the extraposition with indefinite NPs in (21), indefiniteness of VPs explain the new extraposition data in (22).

(21) Only NPs with indefinite (weak) dets allow extraposition
a. many/most reviews about this book appeared
b. many reviews appeared about this book
c. *most reviews appeared about this book

(22) Only VPs with indefinite main verbs allow extraposition
a.*many reviews disappeared about this book.
b. a lecture just started/?continued/?*ended/*discontinued

These event-external aspectual verbs appear to be restricted as to their object NP, taking preferably singular event-denoting ones. We see in (23) that they do not, at least not easily, take a predicative bare mass term or plural.

23) a. John started/finished/continued the concert/the conversation/his affair
b. John started/finished/continued his book
c. ? John started/finished/continued poetry/books

In fact, (23c) is often interpreted generically, as if John started some kind of action, rather than doing something with any particular amount of poetry or set of books. There is an interesting connection here to the fact that these verbs take either an infinitival or a gerundive complement (with the exception of *finish*). Infinitives, as I argued, are event-types where the aspectual verb quantifies over distinct realizations of the same unlocated event-type not necessarily packaged into the same event, but gerunds denote events where the aspectual verb quantifies over internal stages of one and the same event requiring continuity of location. The reason why *finish* does not take infinitives is that the verb means there are no future realizations of the event-type to be part of the same event, which is equivalent to saying that the representation contains the last stage of that event. Hence the meaning of the verb rules out that further realizations that belong to same event can still be forthcoming, as one would expect if *finish* took an infinitival complement.

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[9] I owe this supporting observation to Jack Hoeksema. The explanation of why *finish* does not take such negative polarity complements needs to appeal to the fact that *finish* takes only count-term arguments, and *anything* is a mass term.

Let's look at the event-internal aspectual verbs in a diagram, fig. 3. Note that all four positions are lexically realized, if *end* is taken in the sense of *not resuming*. Of course, *resume* means *begin again*, and begin could be in the same position, if we drop the presupposition of the prior realization of a negative stage of the same event-type.

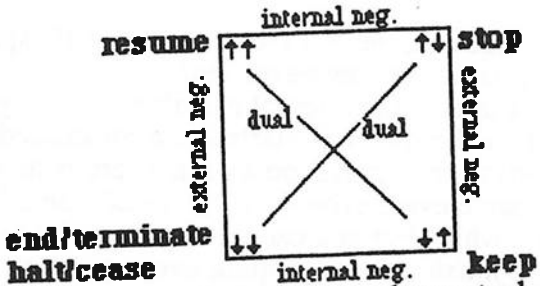

Figure 3. Square of event-internal aspectual verbs

Exactly the same tests can be applied to this square for equivalences and entailments, as I have specified in (24) and (25).

24) a. *resume* (t, e0/1) <=> *stop* (t, e1/0)
b. *not resume* (t, e0/1) <=> *keep* (t, e1/0)
c. *keep* (t, e1/0) <=> *not stop* (t, e1/0)
d. *resume* (t, e0/1) <=> *not end* (t, e0/1)

25) a. *resume* - left and right increasing
resume (t, e), t ≤ t' => *resume* (t', e)
resume (t, e), e is part of e' => *resume* (t, e')
b. *stop* - left increasing, right decreasing
stop (t, e), t ≤ t' => *stop* (t', e)
stop (t, e), e' is part of e => *stop* (t, e')
c. *keep* - left decreasing, right increasing
keep (t, e), t' ≤ t => *keep* (t', e)
keep (t, e), e is part of e' => *keep* (t, e')
d. *end/cease/terminate/halt* - left decreasing, right decreasing (achievement)
end (t, e), t' ≤ t => *end* (t', e)
end (t, e), e' is part of e => *end* (t, e')

Note again that the event-internal aspectual verbs take only gerundive complements, and if they occur with infinitival clauses they must be purpose clauses in Adjunct position to VP.
Another noteworthy feature of these verbs is that they cannot take ordinary count NPs, unless they are commonly understood as temporally extended, changing over time and hence divisible objects. A concert is such a temporally extended object, though it is a real count noun, but a book or an apple is not temporally extended in this sense. There are lots of nouns which may be taken in either way, e.g. a program is either a static description of a set of actions or a procedure a computer can execute. The two senses are clearly related in a way that could be spelled out along the lines developed here. Strangely, the universal *keep* seems to be an exception to this. It takes any count NP in object position of the main verb, but it is also the only event-internal aspectual verb which has an obligatory gerundive. This must be indicative of a meaningful relation between these two exceptional properties of *keep*, but I have not been able to formulate an insightful explanation yet. Bare

predicative plurals and mass terms are also acceptable object NPs for the event-internal aspectual verbs, being divisible, as illustrated in (26).

(26) a. John resumed pot/the concert/*the apple
b. John kept drinking milk/ a glass of milk/playing the sonata
c. John stopped (with) poetry/ * a poem
d. John ended/terminated/halted the affair/*the apple/the music

Now we have two squares, one for the event-external aspectual verbs, one for the event-internal ones. How can they be related?
We have already seen that *stop* is the internal negation of *start*, just as *finish*, but that *stop* presupposes that the event is started and an immediately preceding positive stage exists. *Finish* only presupposes that there is at least one distinct earlier realization of the same event-type. You can finish something even though you have not done it for a while, but you cannot stop doing something unless you are right there doing it. So *finish* admits gapping over time, *stop* does not. If *stop* is the internal negation of *start* with the additional presupposition that the event is started, we can relate the two squares in a three-dimensional diagram by flipping the event-internal diagram along its vertical axis, as in figure 4.

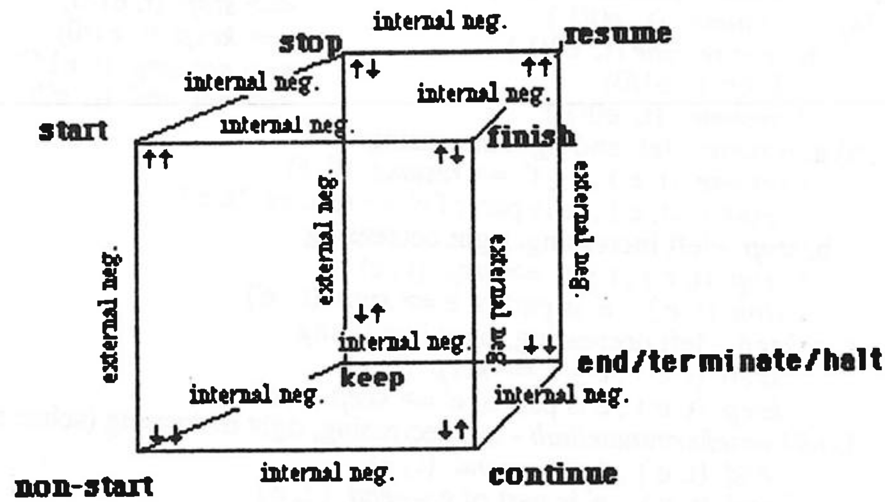

Figure 4. Three-dimensional square of aspectual quantifiers

It is now easy to check that the other internal negations work as polarity reversal.

(27) a. *stop* (t, e1/0) <=> *start* (t, e0/1)
b. *resume* (t, e1/0) <=> *finish* (t, e0/1)
c. *not resume* (t, e1/0) <=> *continue* (t, e0/1)
d. *keep* (t, e1/0) <=> *not start* (t, e0/1)

Furthermore a diagonal across the top from *start* to *resume* or *begin again* is equivalent to going from *start* to *stop* to *resume*, and if it did not go through *stop*, it would simply be *start* entailing *begin*. Similarly *continue* entails *keep* - the entailments across the diagonal over the top and the bottom are from the front event-external quantification to the back event-internal quantification.
It is immediate now that to resume a positive event presupposes you start it and stop it first. Similarly, to finish an event presupposes to start it and perhaps stop and resume it first. Or you end an event, by first starting it, then non-starting the negative counterpart, which is equivalent to keeping the positive event going on, and then you end it. Any transition through the diagram represents a valid

inference, if we make sure to enter in *start* and switch polarity of the embedded event when passing an internal negation and exit in *finish* or *end*.
To appreciate the full explanatory power of this diagram observe that the left side, consisting of *start, stop, keep* (and *non-start*), is the square of exactly the causative aspectual verbs with free agent arguments in the event-type. The right side, consisting of *finish* , *resume* , *end/terminate/halt* , *continue* is the square of just the non-causative ones which require that the agent of the embedded event-type be referentially dependent upon the agent of the aspectual verb. Such constraints on thematic role assignments are needed prior to determining anaphoric linking.

A last remark on these squares. Note that in the case of an achievement verb, like *knock, jump, arrive* , intuively denoting atomic, i.e. indivisible events within the given perspective, the entire structure collapses into a topological point just like proper names do in the case of NPs as generalized quantifiers, i.e they are both self-dual quantifiers. This accounts naturally for the fact that achievement-verbs are either blocked from complementing aspectual verbs or force an iterative or accomplishment- interpretation as in (28).

(28) a. *John/ ?the Pope began to arrive
b. People began to arrive
c. John finished jumping

5. Semantic automata for aspectual quantifiers

The semantic automata designed by Johan van Benthem for NPs (see van Benthem 1986) as GQs can be applied to the aspectual verbs. It shows how the internal-external duality of the figure 4 can be represented by corresponding finite state automata. The main result here is that the presuppositions of the event-internal aspectual verbs are compositionally obtained from the simple machines for the external ones, accounting neatly for the projections of presuppositions of aspectual verbs.

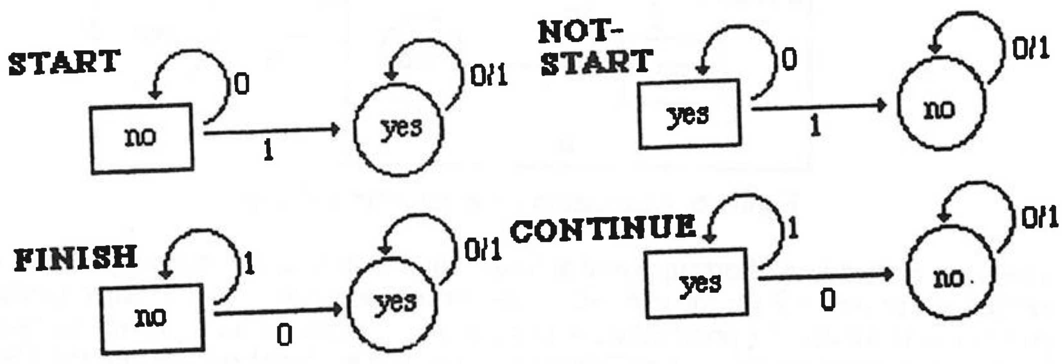

Figure 5. Automata for event-external aspectual verbs

In fig. 5 the starting-state for all machines is the square on the left, and the halting state the circle on the right. 'Yes' indicates the accepting state and 'No' the rejecting state. For the START-automaton you scan the event with negative polarity, rejecting until you find the first instance of the event with a positive polarity and you go into the accepting halting state. It is easy to see that if we scan individuals rather than polarities of events, the same machine would work, for what matters to an existential quantifier is that you find at least one positive instance of the predicate you are testing for. The other automata for event-external aspectual verbs are easy to construct once you know that internal negation switches the 0 and 1 on the arrows, and external negation switches accepting and rejecting state. So the automaton for *finish* starts with checking positive instances and keeps rejecting until you get a negative instance, which means the event has come to an end. The presupposition of *finish* , i.e. that there is at least one earlier stage of the same event-type in the representation, is indirectly captured by the fact that the from its starting state you reject as long as the event polarity remains positive. It is not necessary to require that there must be at least one realization of the event with positive polarity, because we do use *finish* sometimes in contexts where the event is clearly not started. For instance, I ask my three-year old son whether he has finished his breakfast, even when it is clear to both of us that he has not touched it, but has had ample opportunity to do so. The semantic characteristic of *finish* (and internally negated existentials) is that if there are positive instances the verb or quantifier is rejected (falsified), but as soon as there is a negative instance, the verb (or quantifier) is accepted. Note that the automaton for *continue* does require at least one positive instance of the event, since it does not accept or reject until you've passed the 1-loop once. In that precise sense *continue* has a stronger presupposition than *finish*. .

Creating an automaton for an event-internal aspectual verb is by constructing the automata for each node you pass from *start* until you reach the intended node in the backwall of the three-dimensional square. The automata for prior nodes then get 'pushed down' into a stack of sub-machines. For example, we can get the automaton for *stop* by embedding the *start* - machine into the starting-state of the machine for *finish* as in fig. 6 (*finish* and *stop* have the same monotonicity properties, fig. 4.).

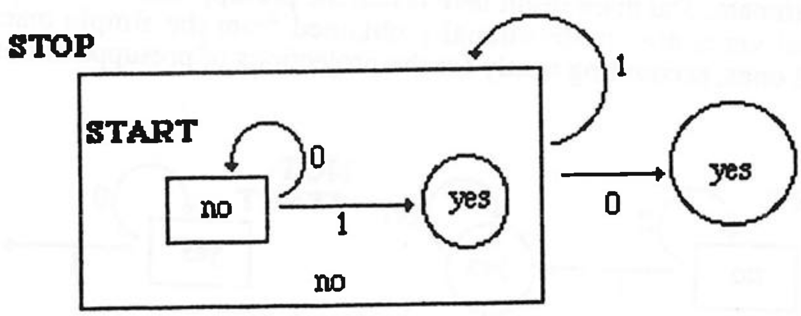

Figure 6. Automaton for event-internal *stop*

The same embedding of presupposed automata by compiling the four simple ones is carried out in figure 7 for the event-internal *resume* as *resume* (t, e) = *start* (*finish* (*start* (t, e))). Hence the procedure for designing automata for all aspectual verbs is algorithmic, once we have established the three-dimensional square and the four elementary machines.

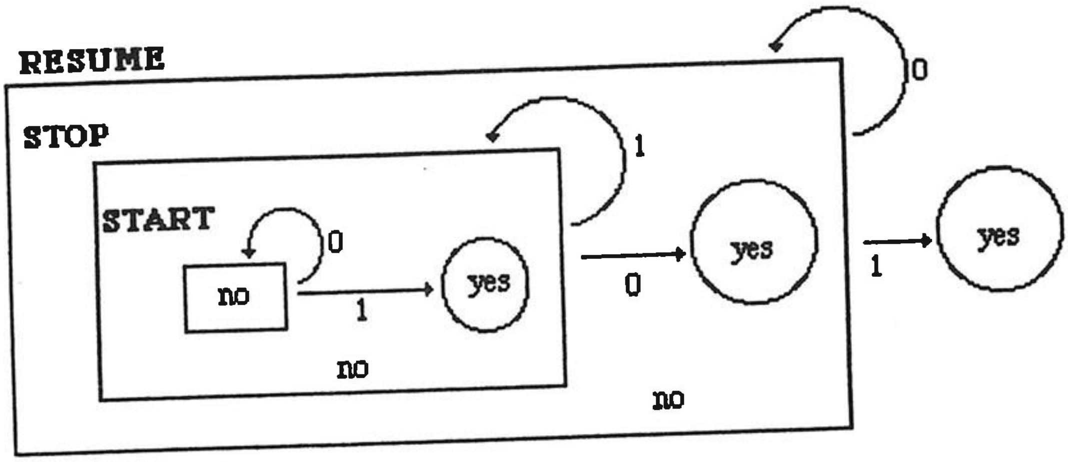

Figure 7. Automaton for event-internal *resume*

More speculatively I may conclude here suggesting that this study of aspectual verbs throws some new light on NPs as generalized quantifiers, on the relations between individuals and the quantities of stuff that constitutes them, and hence on the deep questions of identity and individuation. For more the mathematically minded this analysis may provide an interesting view of the interaction between continuous domains and discrete ones, and further a measure of complexity of quantificational structures, determined by the degree of embedding of submachines.

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AN ANALYSIS OF THE YALE SHOOTING PROBLEM BY MEANS OF DYNAMIC EPISTEMIC LOGIC

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Abstract

We analyse the well-known Yale Shooting Problem by means of a dynamic version of epistemic logic in the sense that besides epistemic operators also operators are available that can express a change by means of an action.

0. INTRODUCTION

In AI literature ([HMcD]) the so-called Yale Shooting Problem has caused problems of formalisation in such a way that intuitively incorrect inferences can be made. A version of this problem reads as follows:

> *John loads his gun;*
> *John waits one second;*
> *John fires his gun aiming at Fred;*
> *Fred is dead.*

In order to give a formal representation of this situation we need to explicitate facts that are presupposed in it: we need to say that loading a gun results in a state in which the gun is (known to be) loaded. Moreover, when a gun is fired (or rather when the trigger is pulled) in a loaded state, Fred will be hit and die (and the gun will be unloaded again). Of course, it is also presupposed that Fred is alive at the beginning of the story, although it does not say so explicitly. But finally, and most importantly, it is understood that by default an action does not change the truth value of an assertion unless it is given otherwise specifically. Semantically speaking, this is to say that an action does not change an aspect of a state unless it is specified otherwise. This way of representing knowledge concerning the effect of actions is efficient (it saves a lot of work in specifying explicitly all possible effects of all possible actions), but introduces a form of nonmonotonic reasoning originating from dealing with uncertainties: by default, an action is not supposed to change an aspect of a state. So it is assumed that this is not the case until we run across information (knowledge) that implies the opposite.

This default nature of the situation causes the problems in the formalisation. Hanks & McDermott have shown in [HMcD] that by using the circumscription formalism of McCarthy [McC] one obtains two models of the above situation: either one gets the intended model in which the gun is still loaded after the second's wait and then the firing of the gun (i.e. pulling the trigger) results in Fred's death, or one gets another unintended model in which the gun mysteriously gets unloaded after waiting for a second, and then firing the gun (i.e. pulling the trigger) does *not* result in the death of Fred. On the other hand, the default nature of the problem is imperative, since in these situations we cannot or won't specify all *non*-effects of all actions in all possible states under consideration. This issue is known in AI as the Frame Problem (cf. e.g. [B]).

In the present paper we shall analyse the Yale Shooting Problem by the use of a dynamic version of an epistemic logic, and see that in this way we have a better way to represent its common-sense reading. Moreover, we shall see that in this approach we are able to represent the problem in such a way that no confusion arises concerning the intended model. That *epistemic* logic is useful in the treatment of this problem may be argued as follows: the Yale Shooting Problem is - as is the Frame Problem in general - a problem concerning the availability (or rather lack of) information (knowledge) concerning the situations at hand. Due to the need for an efficient representation of the situation to be described not all information is represented in order to render its interpretation unambiguous. This is in fact the problem's default nature again, and the lack of information results in unintended (nonstandard) models of the representation as is the case in the representation by means of circumscription. So we need to keep track of the *knowledge* (or *belief*) available in order to decide on the problem's solution.

The paper is organised as folows: In Section 1 we shall define our version of dynamic epistemic logic, together with its interpretation. In Section 2 we shall discuss briefly how we represent defaults in this logic. Then in Section 3 we shall discuss how to represent the Yale Shooting Problem in dynamic epistemic logic without running into the problems we described above.

1. DYNAMIC EPISTEMIC LOGIC

By epistemic logic we mean a variant of modal logic that is concerned with the notions of knowledge (and belief) (Cf. [Hin], [HM]). Recently, the epistemic logic approach has become rather popular amongst computer scientists and AI researchers for describing the knowledge of agents in distributed and intelligent systems. Dynamic logic is a logic of changes by actions, developed originally in the context of program verification (cf. [Ha]). In [Mo] Bob Moore has combined both epistemic and a version of dynamic logic into what we may call *dynamic epistemic logic*. In this logic it is possible to describe both the effect of actions on the knowledge an agent possesses and the effect that knowledge has on the actions of an agent (see [Mo]).

In [MH] we have introduced an epistemic logic to treat defaults. To do so we took an S5-like modal language extended with a special modal operator *P* denoting *preference* or *practical belief*. (The logic resembles auto-epistemic logic (AEL) of [Mo2], but has a much simpler interpretation.)

Assume a fixed collection **P** of primitive propositions and a fixed collection **A** of elementary actions. The language **L** we shall use is the minimal set of formulas closed under the following rules: (We use the notation of dynamic logic ([Ha]) rather than Moore's notation in [Mo].)

(i) $\mathbf{L} \supseteq \mathbf{P}$.

(ii) if $\varphi, \psi \in \mathbf{L}$, then $\neg\varphi, \varphi \wedge \psi, \varphi \vee \psi, \varphi \supset \psi, \varphi \equiv \psi \in \mathbf{L}$.

(iii) if $\varphi \in \mathbf{L}$, then $L\varphi$, $M\varphi$, $P_i\varphi \in \mathbf{L}$ $(i=1,...,n)$.

(iv) if $\varphi \in \mathbf{L}$ and $\alpha \in \mathbf{A}$, then $[\alpha]\varphi \in \mathbf{L}$.

Informally, $L\varphi$ is read as "φ is certain", $M\varphi$ as "φ is (considered) possible", $P\varphi$ as "φ is preferred (a practical belief)", and $[\alpha]\varphi$ as "performance of α results in a state in which φ holds". Formally, expressions of **L** are interpreted by Kripke-structures of the form $(\Sigma,\pi,\Sigma_1,...,\Sigma_n,\wp,\wp_1,...,\wp_n)$, where Σ is a collection of possible worlds, π: $\mathbf{P} \to \Sigma \to \{t,f\}$ is a truth assignment to the primitive propositions per world, $\Sigma_i \subseteq \Sigma$ are sets (clusters) of preferred worlds, $\wp = \Sigma \times \Sigma$ (and $\wp$ is hence reflexive, transitive and euclidean), and $\wp_i = \Sigma \times \Sigma_i \subseteq \wp$ is transitive and euclidean. We let **S5P** denote the collection of Kripke-structures of the above form (the **P** in **S5P** referring to preferences).

Let $\mathbf{S5P}^+$ denote the collection of pairs $(\mathbb{M},s)$ with $\mathbb{M} = (\Sigma,\pi,\Sigma_1,...,\Sigma_n,\wp,\wp_1,...,\wp_n) \in \mathbf{S5P}$ and $s \in \Sigma$. Elements of $\mathbf{S5P}^+$ are referred to as *states*. Actions $\alpha \in \mathbf{A}$ (may) change states. Formally, we assume that the interpretation $[\![\alpha]\!]$ is a function $\mathbf{S5P}^+ \to \mathbf{S5P}^+$.

<u>Remark:</u> By interpreting α by a *function*, we assume that the action α is *deterministic* (and always defined). This is sufficient for our present purposes, but it is possible to relax this requirement by stipulating that the interpretation $[\![\alpha]\!]$ of α is a *relation*, i.e. a subset of $\mathbf{S5P}^+ \times \mathbf{S5P}^+$. However, it is not clear whether we need completely

arbitrary subsets of $S5P^+ \times S5P^+$, or whether we can restrict ourselves in this case to special relations R such that if $((\mathbb{M},s),(\mathbb{M}',s')) \in R$, $(\mathbb{M},\mathbb{M}') \in R_\alpha$ for some relation $R_\alpha \subseteq S5P \times S5P$ (not involving the actual world s in which the evaluation takes place). An action α is called *serial* if the relation R_α associated with it is serial, i.e. for all states $(\mathbb{M},s)$ there is a state $(\mathbb{M}',s')$ such that $((\mathbb{M},s),(\mathbb{M}',s')) \in R_\alpha$. Note that a deterministic action such as introduced above is serial, since in this case R_α is the graph of a function.

The formal interpretation of the language now reads:

for $\mathbb{M}=(\Sigma,\pi,\Sigma_1,\ldots,\Sigma_n,\wp,\wp_1,\ldots,\wp_n)$,

$(\mathbb{M},s) \models p$ iff $\pi(p)(s)=\mathbf{t}$

$(\mathbb{M},s) \models \neg\varphi$ iff not $(\mathbb{M},s) \models \varphi$

$(\mathbb{M},s) \models \varphi \wedge \psi$ iff $(\mathbb{M},s) \models \varphi$ and $(\mathbb{M},s) \models \psi$

$(\mathbb{M},s) \models \varphi \vee \psi$ iff $(\mathbb{M},s) \models \varphi$ or $(\mathbb{M},s) \models \psi$

$(\mathbb{M},s) \models \varphi \supset \psi$ iff $(\mathbb{M},s) \models \varphi$ implies $(\mathbb{M},s) \models \psi$

$(\mathbb{M},s) \models \varphi \equiv \psi$ iff $[(\mathbb{M},s) \models \varphi$ iff $(\mathbb{M},s) \models \psi]$

$(\mathbb{M},s) \models L\varphi$ iff $(\mathbb{M},t) \models \varphi$ for all $t \in \Sigma$.

$(\mathbb{M},s) \models M\varphi$ iff $(\mathbb{M},t) \models \varphi$ for some $t \in \Sigma$.

$(\mathbb{M},s) \models P_i\varphi$ iff $(\mathbb{M},t) \models \varphi$ for all $t \in \Sigma_i$.

$(\mathbb{M},s) \models [\alpha]\varphi$ iff $[\alpha]((\mathbb{M},s)) \models \varphi$

and $\mathbb{M} \models \varphi$ iff $(\mathbb{M},s) \models \varphi$ for all $s \in \Sigma$, as usual.

Although it is also possible to pose a fine structure on the actions (as is done in e.g. dynamic logic [Ha]), we shall not do this here, since we do not need this fine structure for the treatment of the Yale Shooting Problem.

It is possible to give an axiomatisation of the modal logic above: take the S5 system for the modality L (and dual M) and use K45 for the P-modalities, together with the relating axioms $L\varphi \supset P_i\varphi$, $LP_i\varphi \equiv P_i\varphi$, $\neg P_i$ **false** $\supset (P_iP_j\varphi \equiv P_j\varphi)$, $\neg P_i$ **false** $\supset (P_iL\varphi \equiv L\varphi)$; furthermore for the $[\alpha]$-modality we can employ a K-axiomatisation.

In the language **L** we express defaults of the form $\varphi : \psi / \chi$ (using Reiter's notation) as

$$\varphi \wedge M\psi \supset P\chi.$$

The reading of such a formula is "if φ is true and ψ is (considered) possible, then χ is preferred".

Multiple defaults $\{ \varphi_i : \psi_i / \chi_i \}_i$ are represented by formulas

$$\varphi_i \wedge M\psi_i \supset P_i\chi_i.$$

In [MH] we have shown several examples how to use this formalism for defaults. In this paper we shall show that it can also be used for the representation of the Yale Shooting Problem.

2. THE YALE SHOOTING PROBLEM

The Yale Shooting Problem as stated in the introduction is now represented in **L** by:

1. [load] L loaded
2. $\varphi \wedge M\,[\alpha]\varphi \supset P\,[\alpha]\varphi$
3. loaded $\supset$ [fire] ($\neg$alive $\wedge$ $\neg$loaded)

From this representation, letting φ_0 stand for alive $\wedge$ M[load] alive, we can derive the following assertions:

4. P [load] L loaded (from 1, using the necessitation rule[1]).
5. $\varphi_0 \supset$ P [load] alive (from 2).
6. $\varphi_0 \supset$ P [load] (alive $\wedge$ L loaded) (from 4 and 5, using properties of necessity-like modalities).
7. L loaded $\wedge$ M [wait] L loaded $\supset$ P [wait] L loaded (from 2).
8. P [load] (L loaded $\wedge$ M [wait] L loaded $\supset$ P [wait] L loaded) (from 7).

[1] In this paper we adopt the approach in which the necessitation rule is applicable to theorems derivable in S5 together with the additional axioms 1 - 4, cf. [MH].

9. P [load] L loaded $\wedge$ P [load] M [wait] L loaded $\supset$ P [load] P [wait] L loaded (from 8).

10. P [load] M [wait] L loaded $\supset$ P [load] P [wait] L loaded (from 4 and 9).

Analogously,

11. P [load] alive $\wedge$ P [load] M [wait] alive $\supset$ P [load] P [wait] alive (from 2).

12. $\varphi_0 \wedge$ P [load] M [wait] alive $\supset$ P [load] P [wait] alive (from 5 and 11).

13. L (loaded $\supset$ [fire] $\neg$alive) (from 3).

14. L loaded $\supset$ L [fire] $\neg$alive (from 13).

15. L loaded $\supset$ L $\neg$ [fire] alive, since $[\alpha]\neg\varphi \supset \neg[\alpha]\varphi$ for serial actions α.

16. L loaded $\supset \neg$ M [fire] alive (from 15).

17. P [load] P [wait] L loaded $\supset$ P [load] P [wait] $\neg$ M [fire] alive (from 16).

Note that–unless P [load] P false holds, in which (uninteresting) case the cluster associated with the modality P [load] P is empty and every assertion of the form P [load] Pφ is trivially true–17 blocks the application of the derived (generalised) default 20 below:

18. alive $\wedge$ M [fire] alive $\supset$ P [fire] alive (from 2).

19. P [load] P [wait] alive $\wedge$ P [load] P [wait] M [fire] alive $\supset$ P [load] P [wait] P [fire] alive (from 18).

20. $\varphi_0 \wedge$ P [load] M [wait] alive $\wedge$ P [load] P [wait] M [fire] alive $\supset$ P [load] P [wait] P [fire] alive (from 12 and 19).

On the other hand, we can derive:

21. P [load] P [wait] L loaded $\supset$ P [load] P [wait] L [fire] ($\neg$alive) (from 3).

22. P [load] M [wait] L loaded $\supset$ P [load] P [wait] L [fire] ($\neg$alive) (from 10 and 21).

23. P [load] M [wait] L loaded $\supset$ P [load] P [wait] P [fire] ($\neg$alive) (from 22), reading (admittedly sloppily) that the desired result of Fred's being dead after the sequence load ; wait ; fire is indeed preferred (if possible).

There are, however, some subtleties to be noted in this.treatment of the Yale Shooting Problem.

First of all, please note that in our derivation we use "L loaded" rather than just "loaded": this is first encountered in 1. Although it is also true that

1'. [load] loaded,

this is essentially weaker, since in 1 also the *epistemic* aspect is stated that it is not only the case that the gun is loaded after loading, but also that this *known*.[2] Moreover, this epistemic assertion L loaded is carried through the argument, and is essential in the blocking of the default 20. If we would give the derivation with "loaded" instead of "L loaded", we cannot derive that the default is blocked by showing that one of its premisses is not true. In this case we obtain that if both P [load] M [wait] alive $\wedge$ P [load] P [wait] M [fire] alive and P [load] M [wait] loaded hold then there is a preferred cluster for Fred's being alive (P [load] P [wait] P [fire] alive), although it holds as well that P [load] P [wait] [fire] $\neg$alive. This is not unreasonable, since due to *ignorance* it is not known what is really the case: note the difference between P [α] φ and P [α] Pφ; the former expresses that it is believed (preferred) that the performance of α leads to a state in which φ, whereas the latter states that it is believed (preferred) that if α is performed one is in a state in which φ is (still) believed (preferred).[3] (Under certain conditions - by imposing certain restrictions - it is perhaps possible and desirable that nested occurrences of P-operators as in P [load] P [wait] P [fire] φ collapse, so that simple expressions of the form P [α] [β] [γ] ... can be used, but we have not chosen for this option here in order to maintain generality.) In the above situation it holds that although it is believed (preferred) after loading that waiting leads to a state where firing results in Fred's being dead, it is nevertheless the case that in the (preferred) state after loading *and subsequent waiting* it is believed (preferred) that firing does *not* result in Fred's death. Although this is not inconsistent, it is not the intended meaning of the

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[2]Note, that, on the contrary, the use of L-assertions in normal 'static' defaults $\alpha \wedge M\beta \supset P\beta$ is nil: for instance, for β=Lp, $\alpha \wedge M\beta \supset P\beta$ becomes $\alpha \wedge MLp \supset PLp$, which is equivalent to the tautology $\alpha \wedge Lp \supset Lp$.

[3]This is similar to the difference in [KL] between the epistemic temporal expressions BOφ and BOBφ: the former states that it is believed that tomorrow φ holds, whereas the latter expresses that it is believed that tomorrow it is (still) believed tha φ.

situation. Here we see the need for an *epistemic* approach: using "L loaded" does not yield this strange situation.

Also note that although, in principle, it is possible and allowed to use different P-modalities (say P_α) for every instance of α in formula 2, it is not needed in our analysis of the Yale Shooting, and it would, in fact, only lead to cumbersome notation in this case.

3. CONCLUSION

In this paper we have seen how a dynamic version of epistemic logic can be used to give an adequate representation of the infamous Yale Shooting Problem. We used epistemic logic for expressing preferred beliefs that correspond to the biases in defaults, and we needed a dynamic version of this logic to be able to express the results of performing actions. Moreover, we stress that this way of representing defaults with respect to actions is a very general one, which is a direct generalisation of the way defaults in the 'static' (i.e. non-dynamic) case are represented by means of epistemic logic (with preferred beliefs, see [MH]), and is not to be considered as a mere *ad hoc* solution of the particular problem of the Yale shooting.

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Extended Partiality in Intensional logic*

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1. Syntax and Semantics of Partial Intensional Logic

One of the most important aim of the semantic investigations of natural language is to associate the expressions of natural language with their meanings in a systematic way. Some of these investigations use logical tools and so they may have a special logical relevance.[1] It means, on one hand, that by the help of these results we can treat some of the logical properties expressed by the meanings of expressions, and, on the other hand, that we can compose logical laws explicitly.

I think one of the most important task of a logician dealing with modelling a representative fragment of natural language may be the following: Two logically relevant components of the mentioned investigations, i.e. the syntactic algebra of a logical language and meanings for this logical language, should be defined in such a way that two conditions are satisfied:

(i) they should be flexible enough to represent linguistical properties of natural language;

(ii) they should be inflexible enough to satisfy the fundamental requirements of a logical system.

As it is well known there are some solutions of this problem. For example type theoretical intensional logic is a tested tool in the logical reconstruction of a representative fragment of natural language. According to Theo Janssen and Johan van Benthem the language of intensional logic is especially suitable for representing the intended meanings because — as Benthem said — it "wears its interpretation upon its sleeves".[2] The application of type theoretical intensional logic implies accepting two important presuppositions: In syntax we need a system of categories corresponding to the main classes of well formed expressions of a natural language and in semantics we need a system of semantic values.

In order to define possible categories of logical language we have to distinguish complete and incomplete expressions where the latter ones contain one or more unfilled place. For example, "John runs" and "the present King of France" are complete expressions whereas "... run" and "the present king of ..." are incomplete ones. If an incomplete expression is filled in, the (final) result is a complete one. There are some sorts of complete expressions; for example: affirmative sentences, interrogative sentences, individual terms, etc. In type theoretical logic there are two categories of complete expressions which are considered as fundamental ones: the category of affirmative sentences and the category of individual terms.

In order to introduce possible categories of incomplete expressions — called functors — we have to take into consideration two principles:

(1) according to Frege any meaningful part of a meaningful expression may be qualified as an argument of the functor expressed by the remained part of the expressions;

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[1] *See for example Theo Janssen: Foundation and Application of Montague Grammar p.27, where Theo Janssen gives the general structure of the main class of mentioned investigations.*

[2] *Theo Janssen: Foundation and Application of Montague Grammar p. 96*

(2) an argument of a functor is always replaceable by other expressions belonging to the same category.

It means that from a logical point of view an incomplete expression i.e. a functor can be characterized in syntax by the category of its input and the category of its output. But as it is well known in natural language functors do not behave in the same regulated manner. For example we can distinguish extensional and intensional functors. To do this the notion of semantic values has to be introduced.

Semantic values characterize the possible connections between expressions and world. Expressions may take place in communication by means of their semantic value or values. Every meaningful expression except proper names has a meaning. So meaning is the most fundamental semantic value. In this paper we shall use the notion of meaning in a strict logical sense and henceforth, the term "*intension*" will be mentioned instead of "meaning". An individual term may have — beside its intension — another semantic value, the concrete or abstract object which is denoted by the individual term. A sentence may have — beside its intension — a truth value too. In the case of individual terms and sentences, the connection between semantic values is given by the following principles: The *intension of a name* is the class of conditions which determine its denotatum. The *intension of a sentence* is the class of conditions which determine its truth value.

The intension of a functor is a basic notion of semantics. A functor is rightly conceived if the intension of its output can be determined from the intension of its input. Therefore the *intension of a functor* is a rule which determines the intension of its output from the intension of its input.

The truth value of a sentence and the denotatum of an individual term might be called the *factual value*[3] of the sentence and of the individual term respectively. A functor is said to be an *extensional* one if the factual value of its output can be determined by the factual value of its input. If this condition does not hold, the functor is said to be an *intensional* one. The *factual value of an extensional functor* is a rule which determines the factual value of its output from the factual value of its input.

If we take into consideration the system of categories and semantic values, then the following question may arise: Do we have to extend the following definition of the system of types to represent extensional and intensional functors?

Definition 1

The set $TYPE$ is the smallest set of symbols such that

(1) $o, \iota \in TYPE$;

(2) $\alpha, \beta \in TYPE \Rightarrow \alpha(\beta) \in TYPE$.

In this definition, o and ι are the types of sentences and individual terms respectively, $\alpha(\beta)$ is the type of functors which, when they are filled in by an argument of type β, yield an expression of type α.

As it is well known Montague extended extensional type theory by introducing the symbol s in the definition of his intensional logic. But there are some problems connected with this extension. Expressions of type $\langle \alpha, s \rangle$ may be called names of intensions of type α.[4] The translation (into IL)[5]

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[3] *We shall use "factual value" instead of "extension" following Ruzsa's original terminology.*

[4] *In fact, Montague wrote $\langle s, \alpha \rangle$. We use current reverse notation. In general, we use $\langle \alpha, \beta \rangle$ for denoting the type of functors which when they are filled in by an argument of type β, yield an expression of type α.*

[5] *For the sake of brevity we shall use "IL", "IL/G" and "PIL" for denoting Montague's inten-*

of a functor F of English must belong — according to Montague — to some type of form $\langle\alpha,\langle\beta,s\rangle\rangle$ where β is the type of the translations of the possible arguments of F. Following Montague's approach we are in conflict with the practice of natural language for in most cases the argument of a functor is an expression rather than name of an intension. Further, the iterated and multiply embedded occurrences of the type symbol s yield types such that the objects of their semantic domains are highly incomprehensible without a difficult platonic ontology.[6]

There is another way to treat intensional functors conforming with the practice of natural language. Carnap distinguished extensional and intensional occurrences of expressions.[7] According to his distinction, an extensional functor operates on the factual value of its argument and an intensional one operates on the intension of its argument. Ruzsa generalizes this principle. Following Carnap's distinction between extensional and intensional occurrences of expressions, Imre Ruzsa, in intensional logic permitting semantic value gaps, extends extensional type theory by introducing extensional and intensional functor types. In his type theory, extensional and intensional functor–types are distinguished, and there is no need to introduce types for the names of intensions.[8] Hence the mentioned problem of iterated and embedded intensions does not appear.

Compare Montague's and Ruzsa's definitions:
Montague:

(i) $o,\iota \in TYPE_{IL}$;
(ii) $\alpha,\beta \in TYPE_{IL} \Rightarrow \langle\alpha,\beta\rangle \in TYPE_{IL}$;
(iii) $\alpha \in TYPE_{IL} \Rightarrow \langle\alpha,s\rangle \in TYPE_{IL}$.

Ruzsa:

(i) $o,\iota \in EXTY$;
(ii) $\alpha,\beta \in EXTY \Rightarrow (\alpha.\beta) \in EXTY$;
(iii) $\alpha,\beta \in EXTY \Rightarrow (\alpha;\beta) \in OPTY$;
(iv) $\tau \in OPTY, \beta \in EXTY \Rightarrow (\tau;\beta) \in OPTY$;
(v) $TYPE_{IL/G} = EXTY \cup OPTY$.

According to Montague's and Ruzsa's extensions the difference between extensional and intensional functors already appears on the level of type theory, that is to say on the level of syntax. So these features do not depend on interpretation and assignment in spite of the following facts:

(1) there is only a semantic difference between extensional and intensional functors;
(2) most of the nonlogical functors of natural language are used sometimes as extensional and sometimes as intensional ones.

The system of partial intensional logic outlined in this paper may be considered as a generalization of Montague's intensional logic and the intensional logic permitting semantic value gaps worked out by Ruzsa. But in partial intensional logic a functor is only characterized on the level of syntax by the categories of its input and output, and the type of a functor does not entail whether it is an extensional or an intensional one. It means that the definition of type theory used by partial intensional logic may be definition 1 (see above).

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sional logic, Ruzsa's intensional logic permitting semantic value gap and partial intensional logic respectively. See Montague's and Ruzsa's publications mentioned in references.

[6] *See I. Ruzsa: An Approach to Intensional Logic*

[7] *See R. Carnap: Meaning and Necessity p. 125*

[8] *A single exception: he admits names of sentence intensions, i.e. expressions corresponding to "that p" where p is a sentence.*

Beside the rules of using logical constants the syntax of partial intensional logic only contains two fundamental syntactic operations:
(1) the filling of a functor by an argument;
(2) the construction of a functor from a well formed expression.

Definition 2

By a *partial intensional type theoretical language* let us mean a quadruple

$$L^{int} = \langle LC, Var, Con, Cat \rangle$$

satisfying the following conditions:

(1) $LC = \{\lambda, \equiv, =, \mathcal{N}, \mathcal{I}, \hat{}, \vdash, \dashv, (,)\}$ [LC is the set of logical constants.]

(2) $Var = \bigcup_{\alpha \in TYPE}(Var^{ext}(\alpha) \cup Var^{int}(\alpha) \cup Var^{op}(\alpha))$, where $Var^{ext}(\alpha)$, $Var^{int}(\alpha)$ and $Var^{op}(\alpha)$ are denumerably infinite sets of symbols ($\alpha \in TYPE$). [$Var^{ext}(\alpha)$, $Var^{int}(\alpha)$ and $Var^{op}(\alpha)$ are the sets of extensional, intensional and operator variables of the type α respectively.]
$Var(\alpha) = Var^{ext}(\alpha) \cup Var^{int}(\alpha) \cup Var^{op}(\alpha)\ (\alpha \in TYPE)$

(3) $Con = \bigcup_{\alpha \in TYPE} Con(\alpha)$, where $Con(\alpha)$ is a denumerably set of symbols ($\alpha \in TYPE$) [Con is the set of nonlogical symbols of L^{int}.]
All mentioned sets of symbols are assumed to be pairwise disjoint ones.

(4) $Cat = \bigcup_{\alpha \in TYPE} Cat(\alpha)$ [Cat is the set of all well-formed expressions of L^{int}. The set $Cat(\alpha)$ is the α-category of L^{int} ($\alpha \in TYPE$) where the sets $Cat(\alpha)$ are defined by the inductive rules (4.1)...(4.8) as follows:

(4.1) $\alpha \in TYPE \Rightarrow Var(\alpha) \cup Con(\alpha) \subseteq Cat(\alpha)$;
(4.2) $\alpha, \beta \in TYPE,\ C \in Cat(\alpha(\beta)),\ B \in Cat(\beta) \Rightarrow$ "$C(B)$" $\in Cat(\alpha)$;
(4.3) $\alpha, \beta \in TYPE,\ A \in Cat(\alpha),\ \tau \in Var(\beta) \Rightarrow$ "$(\lambda\tau A)$" $\in Cat(\alpha(\beta))$;
(4.4) $\alpha \in TYPE,\ A, B \in Cat(\alpha) \Rightarrow$ "$(A = B)$" $\in Cat(o)$, "$(A \equiv B)$" $\in Cat(o)$;
(4.5) $A \in Cat(o) \Rightarrow$ "$\hat{}A$" $\in Cat(\iota)$;
(4.6) $A \in Cat(o(\iota)) \Rightarrow$ "$\mathcal{I}(A)$" $\in Cat(\iota)$;
(4.7) $A \in Cat(o) \Rightarrow$ "$\mathcal{N}(A)$" $\in Cat(o)$;
(4.8) $A, B \in Cat(o) \Rightarrow$ "$(A \vdash B)$" $\in Cat(o)$, "$(A \dashv B)$" $\in Cat(o)$.

In this definition "λ" is the usual lambda operator; " $\equiv$ " expresses the identity of intensions; " $=$ " expresses the identity of factual values; "$\mathcal{I}$" is the descriptor; "$\hat{}$" is the intensor. In order to express temporal structures we use two operators: "A since B" and "A till B" ($\vdash$ and $\dashv$ respectively). "$\mathcal{N}$" corresponds to the verbal expression "it is not true that". Only sentences are permitted as the arguments of intensor. There is no need to introduce as primitives the logical constants of classical propositional logic, the quantifiers, the modal operators and the usual past and future operators for they can be introduced via definitions.

In every type there are three sorts of variables: extensional, intensional and operator ones. Extensional, intensional and operator variables behave syntactically in the same way.

The next step in the construction of our semantic logical system is the definition of type theoretical structures which serve as basis for the interpretations. Of course, from a logical point of view we have to define only the notion of possible interpretation. Similarly to IL and IL/G. the semantics of partial intensional logic is a special possible word semantics. It means, first of all that the set of indices is used to define the domains of possible intensions. But there are some characteristic differences among type theoretical structures used by these kinds of intensional logic.

The first difference is in the definition of the function *Int* which gives the set of possible intensions for all types. You can see it as follows:
in IL:

$$Int_{IL}(\alpha) = {}^{I}D_{IL}(\alpha) \text{ for all } \alpha \in TYPE_{IL};^{9}$$

in IL/G:

(i) $Int_{IL/G}(\alpha) = {}^{I}D_{IL/G}(\alpha)$ for all $\alpha \in EXTY$;

(ii) $Int_{IL/G}((\tau;\beta)) = {}^{Int_{IL/G}(\beta)}Int_{IL/G}(\tau)$ for all $(\tau;\beta) \in OPTY$;

in PIL:

(i) $Int_{PIL}(o) = {}^{I}D_{PIL}(o)$, $Int_{PIL}(\iota) = {}^{I}D_{PIL}(\iota)$;

(ii) $Int_{PIL}(\alpha(\beta)) = {}^{Int_{PIL}(\beta)}Int_{PIL}(\alpha)$ for all $\alpha(\beta) \in TYPE_{PIL}$

where the function D gives the corresponding sets of factual values.

The second difference is that in a partial type theoretical structure, the partiality of intensions is universally permitted. It has two main reasons:

1. The following requirements can be realized correctly:
   (i) on the level of syntax, a functor is characterized only by the categories of its input and its output; the type of a functor does not entail whether it is an extensional or an intensional one;
   (ii) extensionality or intensionality of a functor depends on the interpretation and assignment;
   (iii) the formal construction reflects the non-formal notion of intension of a functor according to which the intension of a functor is a rule which determines the intension of its output from the intension of its input;
   (iv) Frege's following principle holds universally: any well-formed part of a well-formed expression can be considered as an argument and its remaining part as a functor.
2. The problem connected with the well-formedness constraint gets a conforming solution.

   Following Barbara Partee, the well-formedness constraint can be composed as follows:

   "Each syntactic rule operates on well-formed expressions of specified categories to produce a well-formed expression of a specified category."[10]

According to the well-formedness constraint the employment of a logical language in modelling a representative fragment of a natural language involves serious consequences. It cannot be avoided that expressions which are well-formed in syntax but which may be semantically meaningless in an interpretation occur in a formal language. For example, according to Montague's PTQ the category of intransitive verb phrases (t/e) differs from the category of common noun phrases ($t//e$), but in Montague's intensional logic the translations of expressions of these categories belong to the same category of formal language. It means that on the level of logic theory the difference between these expressions cannot be treated, i.e. from a logical point of view there is no difference between expressions of such categories. There are two ways to solve this problem:

1. by restricting the set of well-formed expressions;
2. by distinguishing well-formedness from semantic meaning-fulness.

Partial intensional logic follows the second way.

Partiality of intensions of functors corresponds to the non-formal notion of intensions of functors according to which the intension of a functor is a rule which determines the intension

[9] *In what follows, ${}^{A}B$ denotes the set of functions from A into B (where A, B are sets).*

[10] *B. Partee: Constraining Transformational Montague Grammar: A Framework and a Fragment p. 55*

of its output from the intension of its input. If a functor intension is handled as a partial function, it gives not only the way of correspondence (between its domain and counterdomain) but also determines the set of its permitted arguments.

In intensional logic permitting semantic value gaps Imre Ruzsa treats the factual values of extensional expressions as partial functions. So in his system, the notion of semantic value gaps represents the lack of factual values. To model intension as a partial function in partial intensional logic we should have another sort of value gap, called intensional value gap which represents the lack of intension.

How can we model partiality?

If f is a partial function from A to B (A and B are sets), then we can represent f by a total function f^* from A to B^* where $B^* = B \cup \{\theta\}$ and

(i) $f^*(x) = f(x)$ for all $x \in Dom(f) (\subseteq A)$;

(ii) $f^*(x) = \theta$ for all $x \in A \setminus Dom(f)$.

The member θ may be called the zero entity of the set B^*. In introducing θ, two conditions are to be satisfied: First, this artificial object should be distinguishable from the "normal" object of the set B^* that is the members of the set B, secondly it must be definable via set theoretical notions by using the members belonging to B.

Therefore in formal semantics, intensional value gaps can be filled in by special objects called intensional zero entities. There are two ways to define intensional zero entities:

(i) by using extensional zero entities which correspond to factual value gaps;

(ii) by introducing new zero entities which are independent from extensional zero entities (so in this case intensional zero entities cannot be defined by means of extensional zero entities).

Both possibilities can be realized formally. The main difference between the two ways lies in the intended non-formal meaning of partiality and of intensional value gaps. In the formal system this difference appears only in the definition of intensional zero entities of the two basic categories (sentences and individual terms) for in both cases the intensional zero entities can be defined inductively in functor categories.

In the case of (i) the following two statements hold:

a. if an expression has no intension then it cannot have a factual value;

b. if an extensional expression has factual value "nowhere" then it has no intension.

The last statement does not hold in the case of (ii).

In the case of (i) uselessness can be represented by means of intensional value gaps. It is shown by the following expressions:

(1) what is not equal to itself

(2) the green prime number

(3) What is not equal to itself is equal to itself.

(4) The Empire State Building is ruminant.

(5) The smallest prime number is a dog.

(6) The present king of USA is clever.

(7) The smallest positive real number is irrational.

According to a common interpretation and the meaning of identity and of definite description these expressions have no intensions. Of course they are useless for denoting an object or expressing an unambiguous information. We can realize that some of them [(1), (3), (6), (7)] are useless because of the state of affairs, facts, and that the others are useless for containing semantically incompatible expressions. So uselessness may derive from meanings of expressions too. We may consider expressions (1), (3), (6), (7) as meaningful ones but this cannot be said about the rest.

Therefore semantic meaningfulness does not guarantee usefulness. In the case of (i), usefulness and meaningfulness cannot be distinguished; in the case of (ii), meaningfulness which is not equal to usefulness can be expressed by means of intensional value gap. For the sake of simplicity, the partial intensional logic presented here follows the first way in defining intensional zero entities.

In order to represent extensional expressions in the semantics of partial intensional logic using partial type theoretical structure, we have to define not only the set of possible intensions of every type but the set of possible intensions and the set of possible factual values of extensional expressions of every type. We use the functions $Int.ext$ and D for this purpose. Of course $Int.ext(\alpha) \subseteq Int(\alpha)$ holds for all $\alpha \in TYPE$.

In the definition of semantic rules concerning extensional expressions we need a special function m which connects the intensions of extensional expressions to the corresponding determination rules. The definitions of functions $Int.ext$, D and m can be found in the definition of partial type theoretical structure.

Definition 3

By a *partial type theoretical structure* let us mean an ordered pair

$$PS = \langle S, F \rangle$$

where

(1) S is an intensional type theoretical structure [that is $S = \langle U, W, T, \leq, d \rangle$ where U, W, T are nonempty sets, $\leq$ is a linear ordering on T and D is a function such that $Dom(d) = W$ and the following condition is satisfied: if $w \in W$, then ${}^{I}3 \subseteq d(w) \subseteq {}^{I}3 \cup U$ $(I = W \times T)$]

(2) $F = \langle Int, Int.ext, D, \theta_{int}, \theta_{ext}, m \rangle$ and the following conditions are satisfied:

(2.1) $Dom(Int) = Dom(Int.ext) = Dom(D) = Dom(\theta_{int}) = Dom(\theta_{ext}) = TYPE$;

(2.2) $Int(o) = {}^{I}D(o)$, $Int(\iota) = {}^{I}D(\iota)$;

(2.3) $Int(\alpha(\beta)) = {}^{Int(\beta)}Int(\alpha)$;

(2.4) $D(o) = \{0, 1, 2\}\ (= 3)$; $\theta_{ext}(o) = 2$;

(2.5) $D(\iota) = U \cup {}^{I}3 \cup \{U\}$; $\theta_{ext}(\iota) = U$;

(2.6) $D(\alpha(\beta)) = \{\phi : \phi \in {}^{D(\beta)}D(\alpha),\ \phi(\theta_{ext}(\beta)) = \theta_{ext}(\alpha)\}$;

(2.8) $\theta_{ext}(\alpha(\beta)) = \psi$ where $\psi(b) = \theta_{ext}(\alpha)$ for all $b \in D(\beta)$;

(2.9) $\theta_{int}(\iota) = \phi$ where $\phi(i) = \theta_{ext}(\iota)$ for all $i \in I$;

(2.10) $\theta_{int}(o) = \psi$ where $\psi(i) = \theta_{ext}(o)$ for all $i \in I$;

(2.11) $\theta_{int}(\alpha(\beta)) = \varphi$ where $\varphi(b) = \theta_{int}(\alpha)$ for all $b \in Int(\beta)$;

(2.12) $Int.ext(\alpha) = Q\{{}^{I}D(\alpha)\}$ $(\alpha \in TYPE)$ where Q satisfies following conditions:

(2.12.1) $Dom(Q) = \bigcup_{\alpha \in TYPE} {}^{I}D(\alpha)$;

(2.12.2) $b \in {}^{I}D(o) \cup {}^{I}D(\iota) \Rightarrow Q(b) = b$;

(2.12.3) if $f \in {}^{I}D(\alpha(\beta))$ then $Q(f) = g$ where $g \in Int(\alpha(\beta))$ and for all $b \in Int(\beta)$

$$g(b) = \begin{cases} Q(s) & \text{if } b \in Int.ext(\beta) \text{ and } s \in {}^{I}D(\alpha) \text{ such that } s(i) = f(i)(b^{*}(i)) \\ & \text{for all } i \in I, \text{ where } b = Q(b^{*}); \\ \theta_{int}(\alpha) & \text{otherwise.} \end{cases}$$

(2.13) $Dom(m) = \bigcup_{\alpha \in TYPE} Int.ext(\alpha)$ and $m(s) = Q^{-1}(s)$ for all $s \in \bigcup_{\alpha \in TYPE} Int.ext(\alpha)$.

Remark 1

(1) If $PS = \langle S, F\rangle$ is a partial type theoretical structure, $S = \langle U, W, T, \leq, d\rangle$ and $F = \langle Int, Int.ext, D, \theta_{int}, \theta_{ext}, m\rangle$ then U is the set of possible individuals; W is the set of possible worlds; T is the set of points of time; d is a function which determines the set of actual individuals of the world $w (w \in W)$; Int gives the set of possible intensions for all types; $Int.ext$ gives the set of possible intensions of extensional expressions for all types; D gives the set of possible factual values of extensional expressions for all types; θ_{int} and θ_{ext} give the intensional and the extensional zero entities of type α respectively; m connects intensions of extensional expression to corresponding determination rules.

(2) It is easy to show by means of structural induction with respect to types that

$$\alpha \in TYPE,\ \phi_1\phi_2 \in Ext(\alpha),\ \phi_1 \neq \phi_2 \quad \Rightarrow \quad Q(\phi_1) \neq Q(\phi_2),$$

where $Ext(\alpha) = {}^{I}D(\alpha)$ $(\alpha \in TYPE)$, and $I = W \times T$. So the function m is a one-to-one correspondence between the sets $\bigcup_{\alpha \in TYPE} Int.ext(\alpha)$ and $\bigcup_{\alpha \in TYPE} Ext(\alpha)$ such that $m(\varphi) \in Ext(\alpha)$ if and only if $\varphi \in Int.ext(\alpha)$.

(3) $\phi \in Int.ext(o) \cup Int.ext(\iota) \Rightarrow m(\phi) = \phi$;
If $\alpha, \beta \in TYPE$, $\phi \in Int.ext(\alpha(\beta))$, $\psi \in Ext(\alpha(\beta))$ then

$$m(\phi) = \psi \quad \Longleftrightarrow \quad m(\phi(c))(i) = \psi(i)(m(c)(i))$$

for all $c \in Int.ext(\beta)$ and $i \in I$.

(4) $\theta_{int}(\alpha) \in Int.ext(\alpha)$ for all $\alpha \in TYPE$ and $m(\theta_{int}(\alpha))(i) = \theta_{ext}(\alpha)$ for all $i \in I$.

(5) If S is an intensional type theoretical structure and $PS = \langle S, F\rangle$ then F is uniquely determined. So S can be considered as the base of PS.

By means of partial type theoretical structure we can define the notions of interpretations and assignments as usually. With their help we can give the definition of intension of expressions according to a given interpretation and assignment.

In the following let L^{int} be a partial intensional type theoretical language, $PS = \langle S, F\rangle$ be a partial type theoretical structure.

Definition 4

(1) By an *interpreting function* of the language L^{int} into a partial type theoretical structure PS let us mean a function σ such that
  (1.1) $Dom(\sigma) = Con$;
  (1.2) $c \in Con(\iota) \Rightarrow \sigma(c) \in U \quad (\subseteq D(\iota))$;
  (1.3) $\alpha \in TYPE$, $\alpha \neq \iota$, $c \in Con(\alpha) \Rightarrow \sigma(c) \in Int(\alpha)$.

(2) By an *assignment* of the variables of the language L^{int} into the partial type theoretical structure PS let us mean a function v such that
  (2.1) $Dom(v) = Var$;
  (2.2) $\alpha \in TYPE$, $x \in Var^{ext}(\alpha)$, $\xi \in Var^{int}(\alpha)$, $\tau \in Var^{op}(\alpha) \Rightarrow v(x) \in D(\alpha)$, $v(\xi) \in Int.ext(\alpha)$, $v(\tau) \in Int(\alpha)$.

(3) $V(PS) = \{v : v \text{ an assignment from } Var \text{ into } PS\}$
If $v \in V(PS)$, $\alpha \in TYPE$, $x \in Var^{ext}(\alpha)$, $a \in D(\alpha)$, $\xi \in Var^{int}(\alpha)$, $f \in Int.ext(\alpha)$, $\tau \in Var^{op}(\alpha)$, $g \in Int(\alpha)$ then

$$v[x:a](y) = \begin{cases} a & \text{if } x = y; \\ v(y) & \text{otherwise.} \end{cases} ; \quad v[\xi:f](y) = \begin{cases} f & \text{if } \xi = y; \\ v(y) & \text{otherwise.} \end{cases} ;$$

$$v[\tau : g](y) = \begin{cases} g & \text{if } \tau = y; \\ v(y) & \text{otherwise} . \end{cases}$$

In definition 4, the difference among extensional, intensional and operator variables is perceptible. Extensional variables denote factual values as objects; intensional variables denote intensions of extensional expressions; operator variables denote intensions of arbitrary expressions.

Definition 5

Let σ be an interpreting function of the language L^{int} into a partial type theoretical structure PS and $v \in V(PS)$. For all $A \in Cat$ let $int_{\sigma v}^{PS}(A)$ be defined by the following recursive clauses[11]:

(1.1.1) If $x \in Var^{ext}(\iota)$, then

$$i \in I \quad \Rightarrow \quad int_{\sigma v}^{PS}(x)(i) = \begin{cases} \theta_{ext}(\iota) & \text{if } v(x) \in D(\iota) \setminus d(i_1); \\ v(x) & \text{otherwise} \end{cases}$$

for all $i \in I \quad (= W \times T,\ i = \langle i_1, i_2 \rangle)$.

(1.1.2) If $\alpha \in TYPE$, $\alpha \neq \iota$, $x \in Var^{ext}(\alpha)$, then $int_{\sigma v}^{PS}(x) = g$ where $g \in Int.ext(\alpha)$ and $m(g)(i) = v(x)$ for all $i \in I$.

(1.2.1) If $\zeta \in Var^{int}(\iota) \cup Var^{op}(\iota)$,then

$$i \in I \quad \Rightarrow \quad int_{\sigma v}^{PS}(\zeta)(i) = \begin{cases} \theta_{ext}(\iota) & \text{if } v(\zeta)(i) \in D(\iota) \setminus d(i_1); \\ v(\zeta)(i) & \text{otherwise} \end{cases} .$$

(1.2.2) If $\alpha \in TYPE$, $\alpha \neq \iota$, $\zeta \in Var^{int}(\alpha) \cup Var^{op}(\alpha)$, then $int_{\sigma v}^{PS}(\zeta) = v(\zeta)$.

(1.3.1) If $c \in Con(\iota)$, then $int_{\sigma v}^{PS}(c) = \phi$ where $\phi \in {}^{I}D(\iota)$ and

$$i \in I \quad \Rightarrow \quad \phi(i) = \begin{cases} \theta_{ext}(\iota) & \text{if } \sigma(c) \in D(\iota) \setminus d(i_1); \\ \sigma(c) & \text{otherwise} \end{cases} .$$

(1.3.2) If $\alpha \in TYPE$, $\alpha \neq \iota$, $c \in Con(\alpha)$, then $int_{\sigma v}^{PS}(c) = \sigma(c)$.

(2.1) If $\beta \in TYPE$, $C \in Cat(\iota(\beta))$, $B \in Cat(\beta)$, then $int_{\sigma v}^{PS}(\text{“}C(B)\text{”}) = \phi$ where $\phi \in {}^{I}D(\iota)$ and

$$i \in I \quad \Rightarrow \quad \phi(i) = \begin{cases} \theta_{ext}(\iota) & \text{if } int_{\sigma v}^{PS}(C)(int_{\sigma v}^{PS}(B)) \in D(\iota) \setminus d(i_1); \\ int_{\sigma v}^{PS}(C)(int_{\sigma v}^{PS}(B))(i) & \text{otherwise.} \end{cases}$$

(2.2) If $\alpha, \beta \in TYPE$, $\alpha \neq \iota$, $C \in Cat(\alpha(\beta))$, $B \in Cat(\beta)$, then

$$int_{\sigma v}^{PS}(\text{“}C(B)\text{”}) = int_{\sigma v}^{PS}(C)(int_{\sigma v}^{PS}(B)).$$

(3.1) If $\alpha, \beta \in TYPE$, $A \in Cat(\alpha)$, $x \in Var^{ext}(\beta)$, then $int_{\sigma v}^{PS}(\text{“}(\lambda x A)\text{”}) = \phi$ where $\phi \in Int.ext(\alpha(\beta))$ and for all $i \in I$ and $b \in D(\beta)$

$$m(\phi)(i)(b) = \begin{cases} m(int_{\sigma v[x:b]}^{PS}(A))(i) & \text{if } b \neq \theta_{ext}(\beta) \text{ and } int_{\sigma v[x:b]}^{PS}(A) \in Int.ext(\alpha); \\ \theta_{ext}(\alpha) & \text{otherwise.} \end{cases}$$

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[11] *$int_{\sigma v}^{PS}(A)$ denotes the intension of the expression A with respect to $\langle PS, \sigma \rangle$ and v.*

(3.2) If $\alpha, \beta \in TYPE$, $A \in Cat(\alpha)$, $\xi \in Var^{int}(\beta)$, then for all $f \in Int(\beta)$

$$int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)\text{”}) = \begin{cases} int^{PS}_{\sigma v[\xi:f]}(A) & \text{if } f \in Int.ext(\beta); \\ \theta_{int}(\alpha) & \text{otherwise.} \end{cases}$$

(3.3) If $\alpha, \beta \in TYPE$, $A \in Cat(\alpha)$, $\tau \in Var^{op}(\beta)$,then $int^{PS}_{\sigma v}(\text{“}(\lambda\tau A)\text{”}) = \phi$ where

$$f \in Int(\beta) \quad \Rightarrow \quad \phi(f) = int^{PS}_{\sigma v[\tau:f]}(A).$$

(4) If $\alpha \in TYPE$, $A, B \in Cat(\alpha)$, then $int^{PS}_{\sigma v}(\text{“}(A = B)\text{”}) = \phi$ where (for all $i \in I$)

$$\phi(i) = \begin{cases} 2 & \text{if } m(int^{PS}_{\sigma v}(A))(i) = \theta_{ext}(\alpha) \text{ or } m(int^{PS}_{\sigma v}(B))(i) = \theta_{ext}(\alpha); \\ 1 & \text{if } m(int^{PS}_{\sigma v}(A))(i) = m(int^{PS}_{\sigma v}(B))(i) \neq \theta_{ext}(\alpha); \\ 0 & \text{otherwise;} \end{cases}$$

if $int^{PS}_{\sigma v}(A), int^{PS}_{\sigma v}(B) \in Int.ext(\alpha)$;
and $\phi(i) = \theta_{ext}(o)$ (for all $i \in I$) if $int^{PS}_{\sigma v}(A) \notin Int.ext(\alpha)$ or $int^{PS}_{\sigma v}(B) \notin Int.ext(\alpha)$.
If $\alpha \in TYPE$, $A, B \in Cat(\alpha)$, then $int^{PS}_{\sigma v}(\text{“}(A \equiv B)\text{”}) = \psi$ where

$$\psi(i) = \begin{cases} 1 & \text{if } int^{PS}_{\sigma v}(A) = int^{PS}_{\sigma v}(B); \\ 0 & \text{otherwise.} \end{cases}$$

(5) If $A \in Cat(o)$, then $int^{PS}_{\sigma v}({}^{\wedge}A)(i) = int^{PS}_{\sigma v}(A)$ for all $i \in I$.
(6) If $A \in Cat(o(\iota))$, then
$int^{PS}_{\sigma v}(A) \notin Int.ext(o(\iota)) \Rightarrow int^{PS}_{\sigma v}(\text{“}\mathcal{I}(A)\text{”}) = \theta_{int}(\iota)$;
$int^{PS}_{\sigma v}(A) \in Int.ext(o(\iota)) \Rightarrow int^{PS}_{\sigma v}(\text{“}\mathcal{I}(A)\text{”}) = \phi$ where for all $i \in I$

$$\phi(i) = \begin{cases} u_0 & \text{if } \{u \in d(i_1) : m(int^{PS}_{\sigma v}(A))(i)(u) = 1\} = \{u_0\}; \\ \theta_{ext}(\iota) & \text{otherwise.} \end{cases}$$

(7) If $A \in Cat(o)$, then $int^{PS}_{\sigma v}(\text{“}\mathcal{N}(A)\text{”}) = \phi$ where

$$i \in I \quad \Rightarrow \quad \phi(i) = \begin{cases} 0 & \text{if } int^{PS}_{\sigma v}(A)(i) = 1; \\ 1 & \text{otherwise.} \end{cases}$$

(8) If $A, B \in Cat(o)$, then $int^{PS}_{\sigma v}(\text{“}A \vdash\!\!- B)\text{”}) = \phi$, $int^{PS}_{\sigma v}(\text{“}(B -\!\!\dashv A)\text{”}) = \psi$ where

$$\phi(i) = \begin{cases} 2 & \text{if for all } t \leq i_2\ int^{PS}_{\sigma v}(A)(\langle i_1, t\rangle) = 2 \text{ or } int^{PS}_{\sigma v}(B)(\langle i_1, t\rangle) = 2; \\ 1 & \text{if there is a } t \leq i_2 \text{ such that } int^{PS}_{\sigma v}(A)(\langle i_1, t\rangle) = 1 \text{ and} \\ & int^{PS}_{\sigma v}(B)(\langle i_1, t^*\rangle) = 1 \text{ for all } t^* \text{ such that } t \leq t^* \leq i_2; \\ 0 & \text{otherwise.} \end{cases}$$

$$\psi(i) = \begin{cases} 2 & \text{if for all } t \geq i_2\ int^{PS}_{\sigma v}(A)(\langle i_1, t\rangle) = 2 \text{ or } int^{PS}_{\sigma v}(B)(\langle i_1, t\rangle) = 2; \\ 1 & \text{if there is a } t \geq i_2 \text{ such that } int^{PS}_{\sigma v}(A)(\langle i_1, t\rangle) = 1 \text{ and} \\ & int^{PS}_{\sigma v}(B)(\langle i_1, t^*\rangle) = 1 \text{ for all } t^* \text{ such that } i_2 \leq t^* \leq t; \\ 0 & \text{otherwise.} \end{cases}$$

Definition 6

(1) By an *interpretation* of a partial intensional type theoretical language L^{int} let us mean an ordered pair $\langle PS, \sigma\rangle$ where PS is a partial type theoretical structure and σ is an interpreting function of the language L^{int} into PS.

(2) If $\langle PS, \sigma\rangle$ is an interpretation of L^{int}, $A \in Cat$, and $v \in V(PS)$ then

(i) by the *contextual intension* of A with respect to $\langle PS, \sigma\rangle$ and v let us mean $int^{PS}_{\sigma v}(A)$;

(ii) a function ϕ is said to be the *absolute intension* of A $[\|A\|^{PS}_{\sigma}]$ if $Dom(\phi) = V(PS)$ and $\phi(v) = int^{PS}_{\sigma v}(A)$ for all $v \in V(PS)$;

(iii) A is said to be *extensional* with respect to $\langle PS, \sigma\rangle$ and v if $int^{PS}_{\sigma v}(A) \in \bigcup_{\alpha \in TYPE} Int.ext(\alpha)$;

(iv) A is said to be *intensional* with respect to $\langle PS, \sigma\rangle$ and v if $int^{PS}_{\sigma v}(A) \notin \bigcup_{\alpha \in TYPE} Int.ext(\alpha)$;

(v) $Cat.ext^{PS}_{\sigma v}(\alpha) = \{A : A \in Cat(\alpha)$ and A is extensional with respect to $\langle PS, \sigma\rangle$ and $v\}$, $Cat.ext^{PS}_{\sigma v} = \bigcup_{\alpha \in TYPE} Cat.ext^{PS}_{\sigma v}(\alpha)$;

(vi) $Cat.int^{PS}_{\sigma v}(\alpha) = \{A : A \in Cat(\alpha)$ and A is intensional with respect to $\langle PS, \sigma\rangle$ and $v\}$ $Cat.int^{PS}_{\sigma v} = \bigcup_{\alpha \in TYPE} Cat.int^{PS}_{\sigma v}(\alpha)$;

(vii) A is said to be *meaningful* with respect to $\langle PS, \sigma\rangle$ and v if $A \in Cat.mean^{PS}_{\sigma v}$ where $Cat.mean^{PS}_{\sigma v} = \bigcup_{\alpha \in TYPE}\{A : A \in Cat(\alpha)$ and $int^{PS}_{\sigma v}(A) \neq \theta_{int}(\alpha)\}$;

(viii) if $A \in Cat.ext^{PS}_{\sigma v}$ then by the *factual value* of A in i $(i \in I)$ $[|A|^{PS}_{\sigma v i}]$ let us mean $m(int^{PS}_{\sigma v}(A))(i)$.

By means of our formal notions, the central notions of semantics (such as semantic representation, factual representation, factual model, satisfiability, validity, some sorts of consequence relations) can be defined. Because of existing intensional and factual value gaps their meanings differ from the usual ones. Thus we have to take into consideration that in partial intensional logic "A is well-formed" does not even imply "A is meaningful". Furthermore if, according to a given interpretation and assignment, A is a meaningful sentence, then "A is nowhere false" does not imply "A is everywhere true".

In spite of the possibility of extensional and intensional value gaps, the semantics of partial intensional logic accepts the principle of tertium non datur. But this principle holds for meaningful statements rather than for sentences. The semantics of partial intensional logic differs from classical semantics (without permitting value gaps) in accepting that some sentences may be meaningless according to an interpretation and an assignment and also accepting that some meaningful sentences do not express statements in some worlds.

Definition 7

(1) An ordered triple $\langle PS, \sigma, v\rangle$ is said to be a *semantic representation* of the set $\Gamma \subsetneq Cat$ if $\langle PS, \sigma\rangle$ is an interpretation of L^{int}, $v \in V(PS)$ and $\Gamma \subsetneq Cat.mean^{PS}_{\sigma v}$.

(2) An ordered quadruple $\langle PS, \sigma, v, i\rangle$ is said to be a *factual representation* of the set $\Gamma \subsetneq Cat$ if $\langle PS, \sigma, v\rangle$ is a semantic representation of Γ and

$$\alpha \in TYPE,\ A \in \Gamma \cap Cat.ext^{PS}_{\sigma v}(\alpha) \quad \Rightarrow \quad |A|^{PS}_{\sigma v i} \neq \theta_{ext}(\alpha).$$

(3) An ordered quadruple $\langle PS, \sigma, v, i\rangle$ is said to be a *factual model* of the set $\Gamma \subsetneq Cat$ if $\langle PS, \sigma, v, i\rangle$ is a factual representation of Γ and

$$A \in \Gamma \cap Cat(o) \quad \Rightarrow \quad |A|^{PS}_{\sigma v i} = 1.$$

(4) Let $\mathcal{K}$ be an arbitrary class of interpretations of L^{int}. Then

(4.1) the set $\Gamma \subseteq Cat$ is said to be *semantically $\mathcal{K}$-representable* if there is a semantic representation $\langle PS, \sigma v\rangle$ of Γ such that $\langle PS, \sigma\rangle \in \mathcal{K}$;

(4.2) the set $\Gamma \subseteq Cat$ is said to be *factually $\mathcal{K}$-representable* if there is a factual representation $\langle PS, \sigma, v, i\rangle$ of Γ such that $\langle PS, \sigma\rangle \in \mathcal{K}$;

(4.3) the set $\Gamma \subseteq Cat$ is said to be *semantically $\mathcal{K}$-meaningful* if for all $\langle PS, \sigma\rangle \in \mathcal{K}$ there is $v \in V(PS)$ such that $\langle PS, \sigma, v\rangle$ is a semantic representation of Γ;

(4.4) the set $\Gamma \subseteq Cat(o)$ is said to be *$\mathcal{K}$-satisfiable* if there is a factual $\mathcal{K}$-model of Γ (that is there is a factual model $\langle PS, \sigma, v, i\rangle$ of Γ such that $\langle PS, \sigma\rangle \in \mathcal{K}$);

(4.5) a sentence A $(A \in Cat(o))$ is said to be a *strong $\mathcal{K}$-consequence* of $\Gamma \subseteq Cat(o)$ $[\Gamma \models^{\mathcal{K}}_{s} A]$ if

(i) Γ is factually $\mathcal{K}$-representable,

(ii) every semantic $\mathcal{K}$-representation of Γ is a semantic representation of $\{A\}$,

(iii) every factual $\mathcal{K}$-representation of Γ is a factual representation of $\{A\}$,

(iv) every factual $\mathcal{K}$-model of Γ is a factual model of $\{A\}$;

(4.6) a sentence A $(A \in Cat(o))$ is said to be a *strong relevant $\mathcal{K}$-consequence* of $\Gamma \subseteq Cat(o)$ $[\Gamma \models^{\mathcal{K}}_{sr} A]$ if

(i) Γ is semantically $\mathcal{K}$-representable,

(ii) every semantic $\mathcal{K}$-representation of Γ is a semantic representation of $\{A\}$,

(iii) every factual $\mathcal{K}$-model of Γ is a factual model of $\{A\}$;

(4.7) a sentence A $(A \in Cat(o))$ is said to be a *weak relevant $\mathcal{K}$-consequence* of $\Gamma \subseteq Cat(o)$ $[\Gamma \models^{\mathcal{K}}_{wr} A]$ if

(i) Γ is semantically $\mathcal{K}$-representable,

(ii) every semantic $\mathcal{K}$-representation of Γ is a semantic representation of $\{A\}$,

(iii) A is false in no $\mathcal{K}$-models of Γ;

(4.8) a sentence A $(A \in Cat(o))$ is said to be a *weak $\mathcal{K}$-consequence* of $\Gamma \subseteq Cat(o)$ $[\Gamma \models^{\mathcal{K}}_{w} A]$ if A is false in no factual $\mathcal{K}$-models of Γ;

(4.9) a sentence A $(A \in Cat(o))$ is said to be *$\mathcal{K}$-valid* if $\emptyset \models^{\mathcal{K}}_{s} A$;

(4.10) a sentence A $(A \in Cat(o))$ is said to be *semantically $\mathcal{K}$-irrefutable* if $\emptyset \models^{\mathcal{K}}_{wr} A$;

(4.11) a sentence A $(A \in Cat(o))$ is said to be *factually $\mathcal{K}$-irrefutable* if $\emptyset \models^{\mathcal{K}}_{w} A$;

(4.12) the expression A and B are said to be *$\mathcal{K}$-synonymous* $[A||{=}||_{\mathcal{K}} B]$ if they belong to the same category and for all $\langle PS, \sigma\rangle \in \mathcal{K}$ $||A||^{PS}_{\sigma} = ||A||^{PS}_{\sigma}$.

Definition 8

(1) A sentence A $(A \in Cat(o))$ is said to be *partially $\mathcal{K}$-valid* if

(i) $\{A\}$ is semantically K-representable,

(ii) if $\langle PS, \sigma, v\rangle$ is a semantic $\mathcal{K}$-representation of $\{A\}$, then $\langle PS, \sigma, v, i\rangle$ is a factual model of $\{A\}$ for all $i \in I$.

(2) A sentence A $(A \in Cat(o))$ is said to be *partially $\mathcal{K}$-irrefutable* if

(i) $\{A\}$ is semantically $\mathcal{K}$-representable,

(ii) if $\langle PS, \sigma, v\rangle$ is a semantic $\mathcal{K}$-representation of $\{A\}$, then $|A|^{PS}_{\sigma v i} \neq 0$ for all $i \in I$.

2. Semantic Metatheorems

Theorem 1

(1) If $A \in Cat$, then $int^{PS}_{\sigma v}(A)$ is uniquely determined by definition 5.

(2) If $A \in Cat(\alpha)$, then $int^{PS}_{\sigma v}(A) \in Int(\alpha)$.

(3) If $A \in Cat.ext^{PS}_{\sigma v}$, then $|A|^{PS}_{\sigma vi}$ is uniquely determined for all $i \in I$;
if $A \in Cat.ext^{PS}_{\sigma v}(\alpha)$, then $|A|^{PS}_{\sigma vi} \in D(\alpha)$;
if $A \in Cat(\iota)$ and $|A|^{PS}_{\sigma vi} \notin d(i_1)$ for some $i \in I$, then $|A|^{PS}_{\sigma vi} = \theta_{ext}(\iota)$.

(4) If $A \in Cat.ext^{PS}_{\sigma v}(\alpha(\beta))$, then $|A(B)|^{PS}_{\sigma vi} = |A|^{PS}_{\sigma vi}(|B|)^{PS}_{\sigma vi}$.

Definition 9

Assume that $A \in Cat$.

(1) $Var(A) = \{\tau : \tau \in Var$ and τ is a free variable of $A\}$.

(2) B is *substitutable for the variable τ in A* if B and τ belong to the same category and no free variables of B become bound by the substitution. Let us denote A^B_τ the term obtained from A by replacing all free occurrences of τ by B.

Theorem 2

Suppose that
$A \in Cat$, $v_1, v_2 \in V(PS)$ such that $v_1(\tau) = v_2(\tau)$ for all $\tau \in Var(A)$.
Then
$$int^{PS}_{\sigma v_1}(A) = int^{PS}_{\sigma v_2}(A).$$

Corollary 1

If $A \in Cat$ is a closed expression, then $int^{PS}_{\sigma v}(A)$ is independent from v. So the absolute intension of A is a constant function in that case.

Lemma 1

Suppose that
$A \in Cat$; $\alpha \in TYPE$; $B, C \in Cat(\alpha)$;
$A[C//B]$ is the expression obtained from A by replacing an occurrence of B not preceded immediately by 'λ' by C;
$\mathcal{K}$ is an arbitrary class of interpretations of L^{int}.
Then
$$B \,||\!=\!||_{\mathcal{K}}\, C \quad \Rightarrow \quad A \,||\!=\!||_{\mathcal{K}}\, A[C//B].$$

Lemma 2

Suppose that
$A \in Cat$, $\tau \in Var^{op}(\alpha)$, $B \in Cat(\alpha)$, and B is substitutable for τ in A;
$Ip = \langle PS, \sigma \rangle$ is an interpretation of L^{int}, $v \in V(PS)$, and $int^{PS}_{\sigma v}(B) = f$.
Then
$$int^{PS}_{\sigma v}(A^B_\tau) = int^{PS}_{\sigma v[\tau : f]}(A).$$

Proof

If τ has no free occurence in A, then $A^B_\tau = A$. Therefore $v(\chi) = v[\tau : f](\chi)$ for all $\chi \in Var(A)$ and so $int^{PS}_{\sigma v}(A) = int^{PS}_{\sigma v[\tau : f]}(A)$.

Suppose that τ has at least one free occurence in A. We can prove Lemma 2 by means of structural induction on well-formed expressions (i.e. the members of the set Cat).

(i) If $A = \tau$, then $A_\tau^B = B$.

If $A \in Cat(\iota)$, then $\tau \in Var^{op}(\iota)$, and $B \in Cat(\iota)$. Since $int_{\sigma v}^{PS}(B) = f$, if $f(i) \notin d(i_1)$ for some $i \in I$, then $f(i) = \theta_{ext}(\iota)$ (see Theorem 1.3). So

$$int_{\sigma v[\tau:f]}^{PS}(\tau)(i) = \begin{cases} \theta_{ext}(\iota) & \text{if } f(i) = v[\tau : f](i) \in D(\iota) \setminus d(i_1); \\ v[\tau : f](\tau)(i) = f(i) & \text{otherwise.} \end{cases}$$

Therefore

$$int_{\sigma v}^{PS}(A_\tau^B) = int_{\sigma v}^{PS}(B) = f = int_{\sigma v[\tau:f]}^{PS}(\tau) = int_{\sigma v[\tau:f]}^{PS}(A).$$

If $A \in Cat(\alpha)$, $\alpha \neq \iota$, then the last statement holds trivially.

(ii) Suppose that $A =$ "$A_1(A_2)$" and so $A_\tau^B =$ "$[A_1]_\tau^B([A_2]_\tau^B)$".

If $A_1 \in Cat(\iota(\beta))$, then

$$int_{\sigma v}^{PS}(\text{“}[A_1]_\tau^B([A_2]_\tau^B)\text{”})(i) = \begin{cases} \theta_{ext}(\iota) & \text{if } int_{\sigma v}^{PS}([A_1]_\tau^B)(int_{\sigma v}^{PS}([A_2]_\tau^B))(i) \\ & \in D(\iota) \setminus d(i_1); \\ int_{\sigma v}^{PS}([A_1]_\tau^B)(int_{\sigma v}^{PS}([A_2]_\tau^B))(i) & \text{otherwise.} \end{cases}$$

$$int_{\sigma v[\tau:f]}^{PS}(\text{“}A_1(A_2)\text{”})(i) = \begin{cases} \theta_{ext}(\iota) & \text{if } int_{\sigma v[\tau:f]}^{PS}(A_1)(int_{\sigma v[\tau:f]}^{PS}(A_2))(i) \\ & \in D(\iota) \setminus d(i_1); \\ int_{\sigma v[\tau:f]}^{PS}(A_1)(int_{\sigma v[\tau:f]}^{PS}(A_2))(i) & \text{otherwise.} \end{cases}$$

Therefore

$$int_{\sigma v}^{PS}(A_\tau^B)(i) = int_{\sigma v[\tau:f]}^{PS}(A)(i)$$

for all $i \in I$.

If $A \in Cat(\alpha(\beta))$ $\alpha \neq \iota$, then

$$int_{\sigma v}^{PS}(A_\tau^B) = int_{\sigma v}^{PS}(\text{“}[A_1]_\tau^B([A_2]_\tau^B)\text{”}) = int_{\sigma v}^{PS}([A_1]_\tau^B)(int_{\sigma v}^{PS}([A_2]_\tau^B)) =$$

$$= int_{\sigma v[\tau:f]}^{PS}(A_1)(int_{\sigma v[\tau:f]}^{PS}(A_2)) = int_{\sigma v[\tau:f]}^{PS}(\text{“}A_1(A_2)\text{”}) = int_{\sigma v[\tau:f]}^{PS}(A)$$

(iii) If $A =$ "$\lambda\zeta A_1$", where $\zeta \in Var(\beta)$, $A_1 \in Cat(\gamma)$, then, since τ has at least one free occurence in A, $\tau \neq \zeta$. Of course, since B is substitutable for τ in A, $\zeta \notin Var(B)$, and

$$\text{“}[\lambda\zeta A_1]_\tau^B\text{”} = \text{“}(\lambda\zeta[A_1]_\tau^B)\text{”}.$$

We have to take into consideration, that in L^{int} there are three sorts of variables: extensional, intensional and operator ones.

(1) Suppose that $\zeta \in Var^{ext}(\beta)$.According to Theorem 2

$$int_{\sigma v[\zeta:b]}^{PS}(B) = f$$

for all $b \in D(\beta)$. If we use ϕ_1 and ϕ_2 to denote $int^{PS}_{\sigma v[r:f]}(\text{"}(\lambda\zeta A_1)\text{"})$ and $int^{PS}_{\sigma v}(\text{"}(\lambda\zeta[A_1]^B_r)\text{"})$ respectively, then

$$m(\phi_1)(i)(b) = \begin{cases} m(int^{PS}_{\sigma v[r:f][\zeta:b]}(A_1))(i) & \text{if } b \neq \theta_{ext} \text{ and } int^{PS}_{\sigma v[r:f][\zeta:b]}(A_1) \in Int.ext(\gamma); \\ \theta_{ext}(\alpha) & \text{otherwise.} \end{cases}$$

$$m(\phi_2)(i)(b) = \begin{cases} m(int^{PS}_{\sigma v[\zeta:b]}([A_1]^B_r))(i) & \text{if } b \neq \theta_{ext} \text{ and } int^{PS}_{\sigma v[\zeta:b]}([A_1]^B_r) \in Int.ext(\gamma); \\ \theta_{ext}(\alpha) & \text{otherwise.} \end{cases}$$

where $i \in I$ and $b \in D(\beta)$.
Finally, since

$$int^{PS}_{\sigma v[\zeta:b]}([A_1]^B_r) = int^{PS}_{\sigma v[\zeta:b][r:f]}(A_1)$$

and therefore

$$m(\phi_1)(i)(b) = m(\phi_2)(i)(b)$$

for all $i \in I$ and $b \in D(\beta)$, it follows that

$$m(\phi_1) = m(\phi_2) \quad \text{and so} \quad \phi_1 = \phi_2$$

(see Remark 1.2).

(2) Suppose that $\zeta \in Var^{int}(\beta)$.According to Theorem 2

$$int^{PS}_{\sigma v[\zeta:g]}(B) = f$$

for all $g \in Int.ext(\beta)$. It means that

$$int^{PS}_{\sigma v[r:f]}(\text{"}(\lambda\zeta(A_1)\text{"})(g) = int^{PS}_{\sigma v[r:f][\zeta:g]}(A_1) = int^{PS}_{\sigma v[\zeta:g][r:f]}(A_1) = int^{PS}_{\sigma v[\zeta:g]}([A_1]^B_r) =$$

$$= int^{PS}_{\sigma v}(\text{"}(\lambda\zeta[A_1]^B_r)\text{"})(g) = int^{PS}_{\sigma v}(\text{"}[(\lambda\zeta A_1)]^B_r\text{"})(g)$$

for all $g \in Int.ext(\beta)$.
If $g \in Int(\beta) \setminus Int.ext(\beta)$, then

$$int^{PS}_{\sigma v[r:f]}(A)(g) = \theta_{int}(\gamma) = int^{PS}_{\sigma v}(A^B_r).$$

Therefore

$$int^{PS}_{\sigma v[r:f]}(A) = int^{PS}_{\sigma v}(A^B_r).$$

(3) Suppose that $\zeta \in Var^{op}(\beta)$.According to Theorem 2

$$int^{PS}_{\sigma v[\zeta:g]}(B) = f$$

for all $g \in Int(\beta)$. It means that

$$int^{PS}_{\sigma v[r:f]}(\text{"}(\lambda\zeta(A_1)\text{"})(g) = int^{PS}_{\sigma v[r:f][\zeta:g]}(A_1) = int^{PS}_{\sigma v[\zeta:g][r:f]}(A_1) = int^{PS}_{\sigma v[\zeta:g]}([A_1]^B_r) =$$

$$= int^{PS}_{\sigma v}(``(\lambda\zeta[A_1]^B_\tau)")(g) = int^{PS}_{\sigma v}(``[(\lambda\zeta A_1)]^{B}_\tau")(g)$$

for all $g \in Int(\beta)$. Therefore

$$int^{PS}_{\sigma v[\tau:f]}(A) = int^{PS}_{\sigma v}(A^B_\tau).$$

In the other cases the proof is trivial.

Theorem 3 (Lambda–conversion law for operator variables)

Suppose that

$A \in Cat$, $\tau \in Var^{op}(\beta)$, $B \in Cat(\beta)$ and B is substitutable for τ in A.

Then

$$(\lambda\tau A)(B) \,||{=}||\, A^B_\tau.$$

Proof

Since $int^{PS}_{\sigma v}(A^B_\tau) = int^{PS}_{\sigma v[\tau:f]}(A)$ (see Lemma 2), we have only to prove that

$$int^{PS}_{\sigma v}(``(\lambda\tau A)(B)") = int^{PS}_{\sigma v[\tau:f]}(A)$$

where $Ip = \langle PS, \sigma\rangle$ is an arbitrary interpretation of L^{int}, $v \in V(PS)$ and $int^{PS}_{\sigma v}(B) = f$.

(i) If $A \notin Cat(\iota)$, then

$$int^{PS}_{\sigma v}(``(\lambda\tau A)(B)") = int^{PS}_{\sigma v}(``(\lambda\tau A)")(int^{PS}_{\sigma v}(B)) = int^{PS}_{\sigma v}(``(\lambda\tau A)")(f) = int^{PS}_{\sigma v[\tau:f]}(A).$$

(ii) If $A \in Cat(\iota)$ then

$$int^{PS}_{\sigma v}(``(\lambda\tau A)(B)")(i) = \begin{cases} \theta_{ext}(\iota) & \text{if } int^{PS}_{\sigma v}(``(\lambda\tau A)")(f)(i) \in D(\iota) \setminus d(i_1); \\ int^{PS}_{\sigma v}(``(\lambda\tau A)")(f)(i) & \text{otherwise.} \end{cases}$$

Since $int^{PS}_{\sigma v}(B) = f$,

$$int^{PS}_{\sigma v}(``(\lambda\tau A)")(int^{PS}_{\sigma v}(B))(i) = int^{PS}_{\sigma v[\tau:f]}(A)(i)$$

for all $i \in I$, and according to Theorem 1.3

$$int^{PS}_{\sigma v[\tau:f]}(A)(i) \notin d(i_1) \quad \Rightarrow \quad int^{PS}_{\sigma v[\tau:f]}(A)(i) = \theta_{ext}(\iota)$$

Therefore

$$int^{PS}_{\sigma v}(``(\lambda\tau A)(B)")(i) = int^{PS}_{\sigma v[\tau:f]}(A)(i)$$

for all $i \in I$, and so

$$int^{PS}_{\sigma v}(``(\lambda\tau A)(B)") = int^{PS}_{\sigma v[\tau:f]}(A)$$

which was to be proved.

Lemma 3

Suppose that

$A \in Cat$, $\xi \in Var^{int}(\alpha)$, $B \in Cat(\alpha)$, B is substitutable for ξ in A;

$Ip = \langle PS, \sigma\rangle$ is an interpretation and $v \in V(PS)$ is an assignment such that $B \in Cat.ext^{PS}_{\sigma v}$, and $int^{PS}_{\sigma v}(B) = f$.

Then

$$int^{PS}_{\sigma v}(A^B_\xi) = int^{PS}_{\sigma v[\xi:f]}(A).$$

Theorem 4 (Lambda–conversion law for intensional variables)

Suppose that

$A \in Cat$, $\xi \in Var^{int}(\alpha)$, $B \in Cat(\alpha)$, B is substitutable for ξ in A;

$Ip = \langle PS, \sigma\rangle$ is an interpretation and $v \in V(PS)$ is an assignment such that $B \in Cat.ext^{PS}_{\sigma v}$.

Then

$$int^{PS}_{\sigma v}((\lambda\xi A)(B)) = int^{PS}_{\sigma v}(A^B_\xi).$$

Proof

According to Lemma 3 it will be sufficent to show that

$$int^{PS}_{\sigma v}(B) = f\ (\in Int.ext(\alpha)) \quad\Rightarrow\quad int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)(B)\text{”}) = int^{PS}_{\sigma v[\xi:f]}(A).$$

(i) If $A \notin Cat(\iota)$, then

$$int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)(B)\text{”}) = int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)\text{”})(int^{PS}_{\sigma v}(B)) = int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)\text{”})(f) = int^{PS}_{\sigma v[\xi:f]}(A).$$

(ii) If $A \in Cat(\iota)$ then

$$int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)(B)\text{”})(i) = \begin{cases} \theta_{ext}(\iota) & \text{if } int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)\text{”})(f)(i) \in D(\iota)\setminus d(i_1); \\ int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)\text{”})(f)(i) & \text{otherwise.} \end{cases}$$

Since $int^{PS}_{\sigma v}(B) = f \in Int.ext(\alpha)$,

$$int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)\text{”})(int^{PS}_{\sigma v}(B))(i) = int^{PS}_{\sigma v[\xi:f]}(A)(i)$$

for all $i \in I$, and according to Theorem 1.3

$$int^{PS}_{\sigma v[\xi:f]}(A)(i) \notin d(i_1) \quad\Rightarrow\quad int^{PS}_{\sigma v[\xi:f]}(A)(i) = \theta_{ext}(\iota).$$

Therefore

$$int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)(B)\text{”})(i) = int^{PS}_{\sigma v[\xi:f]}(A)(i)$$

for all $i \in I$, and so

$$int^{PS}_{\sigma v}(\text{“}(\lambda\xi A)(B)\text{”}) = int^{PS}_{\sigma v[\xi:f]}(A)$$

which was to be proved.

Corollary 2

Suppose that

$A \in Cat$, $\xi \in Var^{int}(\alpha)$, $B \in Cat(\alpha)$, B is closed;

$\mathcal{K} = \{Ip\colon Ip = \langle PS, \sigma\rangle$ is an interpretation of L^{int}, and $B \in Cat.ext^{PS}_{\sigma v}$ for all $v \in V(PS)\}$

Then

$$(\lambda\xi A)(B) \,||\!=\!||_K\, A^B_\xi .$$

Definition 10

Let us say that the variable x *occurs in extensional context in* A according to $IP (= \langle PS, \sigma\rangle)$ and $v\,(\in V(PS))$ iff

$$(\lambda\tau A^\tau_x) \in Cat.ext^{PS}_{\sigma v}$$

where $A \in Cat$, $x \in Var^{ext}(\alpha)$, $x \in Var(A)$, $\tau \in Var^{int}(\alpha)$, $\tau \notin Var(A)$, and τ is substitutable for x in A.

Lemma 4

Suppose that

$A \in Cat$, $x \in Var^{ext}(\alpha)$, $x \in Var(A)$, $B \in Cat(\alpha)$ and B is substitutable for x in A;

$IP = \langle PS, \sigma\rangle$ is an interpretation, $v \in V(PS)$ is an assignment such that $A, B \in Cat.ext^{PS}_{\sigma v}$, and x occurs in extensional context in A according to IP, and v;

$|B|^{PS}_{\sigma vi} = b\,(\in D(\alpha))$.

Then

$$A^B_x \in Cat.ext^{PS}_{\sigma v}, \quad A \in Cat.ext^{PS}_{\sigma v[x:b]},$$

$$|A^B_x|^{PS}_{\sigma vi} = |A|^{PS}_{\sigma v[x:b]i}.$$

Remark 2

The assertion of lemma 3 is trivial when $x \notin Var(A)$.

Theorem 5

Suppose that

$C \in Cat(\alpha)$, $x \in Var^{ext}(\beta)$, $x \in Var(C)$, $B \in Cat(\beta)$, B is substitutable for x in C;

$Ip = \langle PS, \sigma\rangle$ is an interpretation, $v \in V(PS)$ is an assignment such that $C, B \in Cat.ext^{PS}_{\sigma v}$, and x occurs in extensional context in C according to Ip, and v; $A = C^B_x$.

Then

$|B|^{PS}_{\sigma vi} = \theta_{ext}(\beta)$ for some $i \in I \quad \Rightarrow \quad |A|^{PS}_{\sigma vi} = \theta_{ext}(\alpha)$;

$int^{PS}_{\sigma v}(B) = \theta_{int}(\beta) \quad \Rightarrow \quad int^{PS}_{\sigma v}(A) = \theta_{int}(\alpha)$.

Theorem 6 (Lambda-conversion law for extensional variables)

Suppose that

$A \in Cat(\alpha)$, $x \in Var^{ext}(\beta)$, $x \in Var(A)$, $B \in Cat(\beta)$, B is substitutable for x in A;

$Ip = \langle PS, \sigma\rangle$ is an interpretation, $v \in V(PS)$ is an assignment such that $A, B \in Cat.ext^{PS}_{\sigma v}$, and x occurs in extensional context in A according to Ip, and v.

Then

$$int^{PS}_{\sigma v}((\lambda x A)(B)) = int^{PS}_{\sigma v}(A^B_x).$$

Proof

It suffices to prove that

$$|(\lambda x A)(B)|^{PS}_{\sigma vi} = |A^B_x|^{PS}_{\sigma vi}$$

for an arbitrary $i \in I$. According to Theorem 1.4

$$|(\lambda x A)(B)|^{PS}_{\sigma vi} = |(\lambda x A)|^{PS}_{\sigma vi}(|B|^{PS}_{\sigma vi}).$$

If $b = |B|^{PS}_{\sigma v i}$, $b \neq \theta_{ext}(\beta)$, then according to Definition 5.3.1, by means of Lemma 4 we can get the following:

$$|(\lambda x A)(B)|^{PS}_{\sigma v i} = |A|^{PS}_{\sigma v[x:b]i}, \quad |A|^{PS}_{\sigma v[x:b]i} = |A^B_x|^{PS}_{\sigma v i}.$$

Therefore $|(\lambda x A)(B)|^{PS}_{\sigma v i} = |A^B_x|^{PS}_{\sigma v i}$.

If $b = \theta_{ext}(\beta)$, then according to the corresponding semantic rule

$$|(\lambda x A)(B)|^{PS}_{\sigma v i} = \theta_{ext}(\alpha).$$

But according to Theorem 5

$$|A^B_x|^{PS}_{\sigma v i} = \theta_{ext}(\alpha).$$

According to the properties of function m (see Remark 1) it means that the theorem is proved.

3. Definitions of Classical Logical Connectives and Operators

The classical logical constants may be introduced into PIL via definitions. The symbols '$\uparrow$', '$\downarrow$', and '$\sim$' (Verum, Falsum, and Negation, respectively) are to be introduced as follows:

$$\uparrow =_{def} \text{“}(\lambda pp) = (\lambda pp)\text{”}; \quad \downarrow =_{def} \text{“}(\lambda pp) = (\lambda p \uparrow)\text{”}; \quad \sim =_{def} \text{“}\lambda p(p = \downarrow)\text{”}$$

where $p \in Var^{ext}(o)$.

In introducing universal quantification into PIL we have to take into consideration that the domain of quantification may be either the set of possible factual values of extensional expressions of type α or the set of possible intensions of expressions of type α, and that different truth conditions may be stipulated in the two cases. It may be a requirement that the quantified functor must be true for all members — except the zero entity — of the domain of quantification or that it must be true for some members of the domain and it must be false none of them. So we can obtain four sorts of quantification: strong or weak and extensional or intensional ones.

Suppose that $F \in Cat(o(\alpha))$, $\alpha \neq \iota$, $x \in Var^{ext}(\alpha)$, then

$$int^{PS}_{\sigma v}(\text{“}(F = \lambda x \uparrow)\text{”})(i) = \begin{cases} 2 & \text{if } int^{PS}_{\sigma v}(F) \notin Int.ext(o(\alpha)), \text{ or } m(int^{PS}_{\sigma v}(F))(i) = \theta_{ext}(o(\alpha)); \\ 1 & \text{if } m(int^{PS}_{\sigma v}(F))(i) = m(int^{PS}_{\sigma v}((\lambda x \uparrow))(i); \\ 0 & \text{otherwise.} \end{cases}$$

Therefore strong extensional universal quantification (for all α, $\alpha \in TYPE$, $\alpha \neq \iota$) can be defined as follows:

$$\forall_s(F) =_{def} (F = \lambda x \uparrow) \quad \text{or} \quad \forall_s = (\lambda y(y = \lambda x \uparrow))$$

where $x \in Var^{ext}(\alpha)$, $y \in Var^{ext}(o(\alpha))$.

In the case of type ι the domain of quantification is the set of actual individuals of world w that is the set $d(w)$. Suppose that F is an extensional expression of type $o(\iota)$ with respect to an interpretation and assignment, $x \in Var^{ext}(\iota)$, and x does not occur in F. If $\phi = m(int^{PS}_{\sigma v}(\text{“}((\lambda x F(x)) = (\lambda y(\lambda x \uparrow)(y)))\text{”})$ where $y \in Var^{ext}(\iota)$, then

$$\phi(i) = \begin{cases} 2 & \text{if } |(\lambda x F(x))|^{PS}_{\sigma v i} = \theta_{ext}(o(\iota)); \\ 0 & \text{if there is a } u \in d(i_1) \text{ such that } |F|^{PS}_{\sigma v i} \neq 1; \\ 1 & \text{otherwise.} \end{cases}$$

Therefore in the case of type ι

$$\forall_s =_{def} (\lambda P((\lambda x P(x)) = (\lambda y(\lambda x \uparrow)(y))))$$

where $x, y \in Var^{ext}(\iota)$, $P \in Var^{ext}(o(\iota))$.

Let the function Rel be the following:

(i) $Rel(\iota)(i) =_{def} d(i_1)$;

(ii) $Rel(\alpha)(i) =_{def} D(\alpha) \setminus \{\theta_{ext}(\alpha)\}$.

Using function Rel, which gives the set of relevant object of type α at the index i, truth conditions of strong extensional universal quantification (for all types) can be composed as follows:

$$|\forall_s(F)|^{PS}_{\sigma v i} = \begin{cases} 2 & \text{if } F \notin Cat.ext^{PS}_{\sigma v}(o(\iota)) \text{ or } |(\lambda x F(x))|^{PS}_{\sigma v i} = \theta_{ext}(o(\iota)); \\ 0 & \text{if there is a } u \in Rel(\alpha)(i) \text{ such that } |F|^{PS}_{\sigma v i}(u) \neq 1; \\ 1 & \text{otherwise.} \end{cases}$$

where $F \in Cat(o(\alpha))$, $x \in Var^{ext}(\alpha)$, x does not occur in F, and $i \in I$.

Of course in the definition of the weak extensional universal quantification the functor "$(\lambda x \uparrow)$" cannot be used. Instead of this we can use the functor "$(\lambda x(F(x) = F(x)))$" where F is an extensional functor of type $o(\alpha)$ with respect to an interpretation and assignment, $x \in Var^{ext}(\alpha)$ such that x does not occur in F. So the definition of weak extensional universal quantification may be the following:

$$\forall =_{def} (\lambda P((\lambda x P(x)) = (\lambda x(P(x) = P(x)))))$$

where $P \in Var^{ext}(o(\iota))$, $x \in Var^{ext}(\alpha)$.

Truth conditions of weak extensional universal quantification (for all types) can be composed as follows:

$$|\forall(F)|^{PS}_{\sigma v i} = \begin{cases} 2 & \text{if } F \notin Cat.ext^{PS}_{\sigma v}(o(\iota)) \text{ or } |(\lambda x F(x))|^{PS}_{\sigma v i} = \theta_{ext}(o(\iota)); \\ 0 & \text{if there is a } u \in Rel(\alpha)(i) \text{ such that } |F|^{PS}_{\sigma v i}(u) = 0; \\ 1 & \text{otherwise.} \end{cases}$$

where $F \in Cat(o(\alpha))$, $x \in Var^{ext}(\alpha)$, x does not occur in F, and $i \in I$.

Of course our strong and weak extensional universal quantifiers are extensional functors of type $o(o(\alpha))$ with respect to all interpretations and assignments.

Using the two sorts of extensional universal quantification different extensional existential quantifications can be defined:

$$\exists =_{def} (\lambda P(\sim \forall_s(\lambda x \sim P(x)))),\ \exists_s =_{def} (\lambda P(\sim \forall(\lambda x \sim P(x))))$$

where $x \in Var^{ext}(\alpha)$, $P \in Var^{ext}(o(\iota))$.

Truth conditions of weak and strong extensional existential quantifications (for all types) can be composed as follows:

$$|\exists(F)|^{PS}_{\sigma v i} = \begin{cases} 2 & \text{if } F \notin Cat.ext^{PS}_{\sigma v}(o(\iota)) \text{ or } |(\lambda x F(x))|^{PS}_{\sigma v i} = \theta_{ext}(o(\iota)); \\ 1 & \text{if there is a } u \in Rel(\alpha)(i) \text{ such that } |F|^{PS}_{\sigma v i}(u) \neq 0; \\ 0 & \text{otherwise.} \end{cases}$$

$$|\exists_s(F)|^{PS}_{\sigma v i} = \begin{cases} 2 & \text{if } F \notin Cat.ext^{PS}_{\sigma v}(o(\iota)) \text{ or } |(\lambda x F(x))|^{PS}_{\sigma v i} = \theta_{ext}(o(\iota)); \\ 1 & \text{if there is a } u \in Rel(\alpha)(i) \text{ such that } |F|^{PS}_{\sigma v i}(u) = 1; \\ 0 & \text{otherwise.} \end{cases}$$

where $F \in Cat(o(\alpha))$, $x \in Var^{ext}(\alpha)$, x does not occur in F, and $i \in I$.

Different intuitive meanings of four sorts of extensional quantification are the following:

$\forall_s(F)$: F is true for all (relevant) objects;

$\forall(F)$: F is false for no (relevant) objects, and F is true at least for an object;

$\exists_s(F)$: F is true for an (relevant) object;

$\exists(F)$: F is false for no (relevant) object.

Connection among four sorts of extensional quantification is shown by the following:

$$\forall_s(F) \models_s \forall(F); \quad \forall_s(F) \models_s \exists_s(F); \quad \forall(F) \models_s \exists_s(F); \quad \exists_s(F) \models_s \exists(F); \quad \forall(F) \models_s \exists(F).$$

Similarly to the extensional universal quantification, strong ($\forall^{\iota}_s$) and weak ($\forall^{\iota}$) intensional universal quantifications can be introduced:

$$\forall^{\iota}_s =_{def} (\lambda P(P \equiv \lambda \tau \uparrow)),$$

$$\forall^{\iota} =_{def} (\lambda P(P \equiv \lambda \tau(P(\tau) = P(\tau))))$$

where $P \in Var^{op}(o(\alpha))$, $\tau \in Var^{op}(\alpha)$.

Truth conditions of strong and weak intensional universal quantifications (for all types) can be composed as follows:

$$|\forall^{\iota}_s(F)|^{PS}_{\sigma v i} = \begin{cases} 0 & \text{if there is a } b \in Int(\alpha) \text{ such that } int^{PS}_{\sigma v}(F)(b)(j) \neq 1 \text{ for some } j \in I; \\ 1 & \text{otherwise.} \end{cases}$$

$$|\forall^{\iota}(F)|^{PS}_{\sigma v i} = \begin{cases} 0 & \text{if there is a } b \in Int(\alpha) \text{ such that } int^{PS}_{\sigma v}(F)(b)(j) = 0 \text{ for some } j \in I; \\ 1 & \text{otherwise.} \end{cases}$$

By means of the two sorts of intensional universal quantification, different intensional existential quantifications can be defined:

$$\exists^{\iota} =_{def} (\lambda P(\sim \forall^{\iota}_s(\lambda \tau \sim P(\tau)))), \quad \exists^{\iota}_s =_{def} (\lambda P(\sim \forall^{\iota}(\lambda \tau \sim P(\tau))))$$

where $\tau \in Var^{op}(\alpha)$, $P \in Var^{op}(o(\alpha))$.

Truth conditions of strong and weak intensional existential quantifications (for all types) can be composed as follows:

$$|\exists^{\iota}_s(F)|^{PS}_{\sigma v i} = \begin{cases} 1 & \text{if there is a } b \in Int(\alpha) \text{ such that } int^{PS}_{\sigma v}(F)(b)(j) = 1 \text{ for some } j \in I; \\ 0 & \text{otherwise.} \end{cases}$$

$$|\exists^{\iota}(F)|^{PS}_{\sigma v i} = \begin{cases} 1 & \text{if there is a } b \in Int(\alpha) \text{ such that } int^{PS}_{\sigma v}(F)(b)(j) \neq 0 \text{ for some } j \in I; \\ 0 & \text{otherwise.} \end{cases}$$

Note that the result of an intensional quantification is always a *rigid* sentence in the sense that its truth value (0 or 1) is the same at all indices $i \in I$.

By means of the weak extensional universal quantification certain sentence functors can be defined which correspond to classical ones in some sense:

$$\& =_{def} (\lambda p(\lambda q \forall (\lambda f(p) = f(q)))))$$

where $p, q \in Var^{ext}(o)$, $f \in Var^{ext}(o(o))$.

$$(A\&B) =_{def} \&(A)(B); \quad (A \vee B) =_{def} \sim(\sim A\& \sim B); \quad (A \supset B) =_{def} \sim(A\& \sim B).$$

If $A, B \in Cat(o)$, then

$$|\&(A)(B)|^{PS}_{\sigma v i} = \begin{cases} 2 & \text{if } |A|^{PS}_{\sigma v i} = 2 \text{ or } |B|^{PS}_{\sigma v i} = 2; \\ 1 & \text{if } |A|^{PS}_{\sigma v i} = 1 = |B|^{PS}_{\sigma v i}; \\ 0 & \text{otherwise.} \end{cases} \quad |\vee(A)(B)|^{PS}_{\sigma v i} = \begin{cases} 2 & \text{if } |A|^{PS}_{\sigma v i} = 2 \text{ or } |B|^{PS}_{\sigma v i} = 2; \\ 0 & \text{if } |A|^{PS}_{\sigma v i} = 0 = |B|^{PS}_{\sigma v i}; \\ 1 & \text{otherwise.} \end{cases}$$

$$|\supset(A)(B)|^{PS}_{\sigma v i} = \begin{cases} 2 & \text{if } |A|^{PS}_{\sigma v i} = 2 \text{ or } |B|^{PS}_{\sigma v i} = 2; \\ 0 & \text{if } |A|^{PS}_{\sigma v i} = 1 \text{ and } |B|^{PS}_{\sigma v i} = 0; \\ 1 & \text{otherwise.} \end{cases}$$

In the semantics of extensional languages a function is said to be n-argument truth function if it is a member of the set ${}^{2^{(n)}}2$. These are the possible factual values of sentence functors. This notion can be generalized in a natural way as follows:

Definition 11

The members of the function set ${}^{3^{(n)}}3$ are said to be *n-argument value gap truth functions.* (For the sake of brevity we shall say "*E-function*" instead of "value gap truth function".)

The factual values of logical sentence functors introduced via definitions above are E-functions which not only inherit factual value gaps but the factual value of their output is equal to 2 if and only if the factual value of at least one of their arguments is equal to 2. For example if the factual values of negation and alternation are the functions *neg* and *altern* respectively, then

$$neg(i) = \begin{cases} 2 & \text{if } i = 2; \\ 1 - i & \text{if } i \in \{0, 1\}. \end{cases} \quad altern(i, j) = \begin{cases} 0 & \text{if } i = j = 0; \\ 2 & \text{if } i = 2 \text{ or } j = 2; \\ 1 & \text{otherwise.} \end{cases}$$

Of course this does not hold for all E-functions. Let us introduce the following definition:

Definition 12

By a *classical* E-function we mean a value gap truth function, whose output equals 2 if and only if the value of at least one of their arguments equals 2.

Obviously, every classical E-function is a member of the domain of factual values of extensional sentence functors. The following theorem has been proved:

Theorem 7

Suppose that

L^{int} is a partial type theoretical intensional language, $Ip = \langle PS, \sigma \rangle$ is an interpretation of L^{int}, $v \in V(PS)$ is an arbitrary assignment;

ϕ is a n-argument classical E-functions.

Then

there is an $F \in Cat(o(o))$ which contains only bound extensional variables of type o beside identity and lambda operator such that $F \in Cat.ext^{PS}_{\sigma v}$ and

$$|F|^{PS}_{\sigma v i} = \phi$$

for all $i \in I$.

The proof of this theorem is similar to that used in classical extensional case. The proof shows that every classical E-function can be expressed by means of functions *neg* and *altern*.

Of course not all E-functions are classical ones, but there are some non-classical E-functions which inherit factual value gap.

Definition 13

A *normal* E-function is an E-function which inherits value gap, so the output of these functions is 2 if the value of at least one of their arguments equals 2.

According to this definition normal n-argument E-functions are the members of the set $D(o(o)_n)$ (where $(o)_1 = (o)$ and $(o)_{n+1} = (o)_n(o)$). It is trivial that normal E-functions cannot be expressed by classical E-functions. We need the descriptor beside the lambda operator and identity to express normal E-functions.The following theorem holds:

Theorem 8

Suppose that

L^{int} is a partial type theoretical intensional language, $Ip = \langle PS, \sigma \rangle$ is an interpretation of L^{int}, $v \in V(PS)$ is an arbitrary assignment;

ϕ is an n-argument normal E-function.

Then

there is an $F \in Cat(o(o)_n)$ which contains only bound extensional variables of type o beside identity. descriptor and lambda operator such that $F \in Cat.ext^{PS}_{\sigma v}$ and

$$|F|^{PS}_{\sigma v i} = \phi$$

for all $i \in I$.

If we want to deal with any E-function, we have to take into consideration that non-normal E-functions do not inherit value gaps so non-normal E-function may have semantic role only in connection with intensional sentence functors.

Definition 14

Suppose that

L^{int} is a partial intensional type theoretical language, $Ip = \langle PS, \sigma \rangle$ is an interpretation of L^{int};

$F \in Cat(o(o)_n)$, ϕ is a n-argument E-function.

Then

ϕ is said to be the *computational rule* of F with respect to the given interpretation if for all $A_1, \ldots, A_n \in Cat(o)$

$$|F(A_1)\ldots(A_n)|^{PS}_{\sigma v i} = \phi(|A_1|^{PS}_{\sigma v i}, \ldots, |A_n|^{PS}_{\sigma v i})$$

for all $v \in V(PS)$ and $i \in I$.

Theorem 9

Suppose that

L^{int} is a partial type theoretical intensional language, $Ip = \langle PS, \sigma \rangle$ is an interpretation of L^{int};

ϕ is a n-argument E-functions.

Then

there is an $F \in Cat(o(o)_n)$ which contains only bound intensional variables of type o beside identity, descriptor, functor '$\mathcal{N}$' and lambda operator such that the computational rule of F is ϕ with respect to the given interpretation.

Summary

Richard Montague was the first who created a viable semantic theory of intensional logic. By introducing intensional logic with semantic value gaps, Imre Ruzsa presented a tool for logicians dealing with the logical reconstruction of natural and scientific language which does not have unacceptable logical and philosophical consequences and extends the power of logic by representing factual value gaps correctly. Joined to the intensional logic of R. Montague and to the semantic value gap theory developed by I. Ruzsa a new partial semantic theory of (tensed and typed) intensional logic is introduced. By means of extension of semantic value gap to intensions, PIL gives us a complete logical tool to model partiality of natural language expressions. This approach leads to the possibility that some syntactically well-formed expressions might be "meaningless" in certain interpretations.

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