Paul Dekker

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PART 2

EDITED BY
MARTIN STOKHOF
& LEEN TORENVLIET

ITLI, Institute for Language, Logic and Information

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Preface

From December 19-22, 1989, the Seventh Amsterdam Colloquium was held. It was organized by the Institute for Language, Logic, and Information (ITLI), which is founded by the Departments of Philosophy and Mathematics of the University of Amsterdam.

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Preface

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Contents of Part 2

Contents of Part 1

Semantic Selection and the Determination of Part Structures in the Use of Natural Language

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Entities have (or can be described with) certain types of part structures. Part structures may differ not only in what counts as a part, but also in what properties parts and wholes have. I would like to discuss some aspects of the role of part structures in natural language semantics. I maintain that certain types of part structures play a significant role in semantics. These part structures are related to, but in principle independent of, the mass-count distinction and presuppose a certain notion of part which differs from set-theoretical and mereological part relations usually employed in semantic theory (cf. Link 1983 and others), as I have argued elsewhere (Moltmann, 1989a and 1989b). In this paper, I am interested mainly in the role of part structures in semantic selectional requirements (or sortal correctness conditions) of predicates and semantic operations. To show the generality of these part structures and to clarify the general nature of the relevant notion of part, I also present basic facts about quantification that involves the parts of an entity. I propose two very general semantic selectional requirements that correlate semantically characterizable classes of predicates and semantic operations with arguments which have certain types of part structures. General considerations on the motivation of the requirements lead to certain philosophical implications with respect to the ontology (or, better, the level of semantically relevant reference objects) that natural language presupposes and with respect to the issue of relative identity.

In order to introduce the notion of part that I assume for semantic theory, I first outline the essential properties and implications of mereological part relations and clarify the issue under discussion with some general methodological remarks about the required notion of parts. In section 2, I present in an intuitive way the relevant notion of part. In section 3, I discuss the basic properties of this part relation with quantification over parts and show how part structures can be individuated at the relevant semantic level in various ways - independently of the mass-count distinction. In the main section, section 4, I introduce the two semantic selectional requirements showing how these requirements involve the same notion of part and the same means of determining part structures as quantification over parts. I conclude the discussion with some general remarks about the individuation of part structures in the context of natural language semantics.

1. Properties and applications of mereological part relations

The notion of part that I am employing differs in crucial respects from mereological part relations. It is formally a much weaker relation and lacks characteristic properties of mereological part relations. However, whether these properties hold in a given context can be determined by information given in that particular context. Since this relation can best be introduced on the background of mereological part relations, let me first outline briefly the main features of mereological part relations and their implications.

1.1. Formal properties of mereological part relations

Even though there are various mereological theories of the part the part relation, mereological part relations share essentially three formal properties that are of interest in the present context.[1] Mereological part relations, indicated by '$<$', generally are transitive, as defined in (1a), extensional (i.e. entities that have the same parts are identical), as defined in (1b), and closed under sum formation (i.e. for any set A, the sum of A, sum(A), exists, i.e. the entity whose parts are the elements of A), as defined in (1c).

(1) a. (transitivity) $x<y \;\&\; y<z \dashrightarrow x<z$

b. (extensionality) $(\forall z\,(z<x \;{<}\dashrightarrow\; z<y)) \dashrightarrow x = y$

c. (closure under sum formation) For any set A, sum(A) exists.

1.2. Consequences of mereological part relations in the application to semantic theory

The application of mereological part relations to semantic theory has two chief implications, which are consequences of the three properties listed under (1). First, mereological part relations lead to a division of the part relation into three different, though analogous, part relations, namely for the three domains of individuals (referents of singular count nouns), groups (or pluralities, i.e. referents of plurals) and quantities (or masses, i.e. referents of mass nouns). Thus, mereology requires a part relation $<_i$ specific to individuals, a part relation $<_p$ specfic to groups and a part relation $<_m$ specific to quantities.[2] The reason for this domain-distinction for the part relation lies in the properties of transitivity and extensionality. Regarding transitivity, this can be seen as follows: a member of a group is a part of the group, but no proper part of the member, under normal circumstances, counts as a part of the group. But this would have to hold if the part relation was the same for individuals and groups and a transitive relation. Regarding extensionality, the implication can be illustrated with the following example. A ring of gold and the gold in that ring arguably have the same parts. Thus, given that the part relation is the same for individuals and masses and given that it is an extensional relation, the ring and the gold in the ring should be identical. But this is against intuition and semantic evidence.[3]

Second, mereological part relations generally lead to a mereological characterization of the mass-count distinction. For this characterization of the mass-count distinction, the following mereological properties of sets are used: a set A being cumulative (any subset of A has a sum in A), divisive (any part of a member of A is in A) and atomic (any member of A is composed of parts in A that do not have any proper parts in A themselves). According to the mereological characterization of the mass-count distinction, mass noun extensions are defined as being cumulative and divisive (or 'as if' divisive, cf. Bunt 1985); singular count extensions (or the extensions of 'sortals', cf. Griffin 1977) as generally neither cumulative nor divisive; plural extensions, finally, are defined as being cumulative and atomic. Each of these mereological properties is then intended to hold only with respect to the part relation specific to the respective domain, i.e. the part relation specific to either the domain of masses, individuals or plurals.

Mereological part relations have been applied essentially only to the semantics of definite and indefinite plurals and mass NPs (cf. Sharvy 1981, Link 1983) and quantified mass NPs (cf. Roeper 1983, Bunt 1985, Loenning 1987). For these phenomena, mereological part relations are not too problematic. But the situation changes if other sorts of phenomena of natural language semantics are taken into account that arguably involve a part relation. I maintain that there are various types of phenomena that require a part relation that is uniform and does not have any of the properties in (1). Some of those phenomena will be discussed in section 3 and 4.

1.3. Methodological remarks on the part relation in semantics

Formal part relations such as lattice-theoretical and set-theoretical part relations have been construed for purposes other than semantic theory, for instance for a foundation of mathematics (though they were sometimes also intended to account for the intuitive notion of part). The part relation as a notion relevant for semantic theory, however, need neither coincide with any relation construed for other scientific purposes, nor with the intuitive notion of part, nor with the meaning of the word 'part' in English. Notice, for example that the English phrase 'is a part of' does not apply to parts of groups, as in (2b). It is, by the way, also not always transitive, but rather restricted to functional parts or parts with a strong degree of inner integrity, cf. (2c). (See also Cruse 1979 for a discussion of related phenomena in natural language.)

(2) a. This leg is a part of the chair.

b. ??John is a part of the school orchestra.

c. ? The lower half of the leg is a part of the chair.

In contrast, the phrase 'is / are part of' applies only to masses and groups. (This contrast between the two occurrences of the word *part* in English were noted by Sharvy 1983).

(3) a. These children are part of the children of this school

b. This rice is part of the food of today.

c. ?This leg is part of the chair.

2. New applications and a new picture of the part relation relevant in semantics

The new applications that lead to a revision of the part relation are basically of two kinds: quantification over parts (which is involved in a variety of constructions, cf. Moltmann 1989b) and semantic selectional requirements that are sensitive to the part structure of an argument. Part quantification requires that the part relation be a uniform relation, since part quantifiers generally apply without regard to the categories singular count, plural and mass. Partly as a consequence of uniformity, as we have seen, part quantification requires that the part relation not generally be transitive, extensional and closed under sum formation. Part structure-specific selection, as we shall see, requires types of part structures that are not characterizable by properties that can be formulated within extensional mereology.

2.1. A different picture of the part relation

The part relation that I assume is inherently neither transitive, extensional nor closed under sum formation. More important than such absolute formal properties are properties that I call integrity conditions. Integrity conditions define an entity as an integrated whole (whereby the degree of integrity of an integrated whole may vary). An entity x may, for instance, be defined as an integrated whole by having only parts that are connected in space or time with each other and that are connected with no entity that is not a part of x. A predicate in English that expresses integrity conditions in a general sense is the adverb *together*. It expresses, for instance, both integrity on the bsis of group action (*John and Mary lifted the piano together*) and spatio-temporal proximity (*John and Mary sat together*) (cf. Lasersohn 1989). The crucial point is that it depends on the presence or absence of integrity conditions in a given part structure, whether the part relation is transitive, extensional, and even closed under sum formation with respect to that part structure.

There are two conditions under which transitivity and extensionality hold on the basis of integrity condtions. First, transitivity is blocked if intermediary parts are integrated wholes. To illustrate this intuitively, consider an example from Rescher (1955): the nucleus is a part of a cell, the cell is a part of an organism; but the nucleus is not a part of the organism. The reason is that the cell is an integrated whole itself. Transitivity failure occurs also with groups and individuals and can be explained the same way. John may be a part of a group, but John's leg is certainly not a part of the group. The reason that transitivity in this case fails is that John is an integrated whole blocking transitivity.

Integrity conditions may explain also failure of extensionality. A ring is an integrated whole (because it has a certain form or gestalt); but the gold in the ring is not an integrated whole. Entities that have the same parts are identical only if they have the same essential integrity conditions (i.e. integrity conditions that individuate the entities).

Now it is clear that if the part relation is not taken to be inherently transitive and extensional, it can be taken as a uniform relation applying to all three domains, individuals, groups and masses.

Integrity conditions also allow for a new characterization of the mass-count distinction. This characterization need not make reference to absolute mereological properties. According to this characterization, mass nouns are those nouns that do not express integrity conditions holding of their referents as wholes or of parts of their referents; singular count nouns, in contrast, are those nouns that express integrity conditions that hold of their referents as wholes, and plurals are those nouns that express integrity conditions that hold (simplifying) of the parts (or subparts) of their referents (namely the integrity conditions expressed by the corresponding singular count nouns).

This characterization is superior to the mereological characterization, because it is the case that some of the mereological conditions stated above for the characterization of singular count, plural and mass noun extensions may systematically be violated in certain contexts. In particular, mass nouns need not be cumulative and divisive in all contexts, and neither need plurals be cumulative in all contexts. We shall see examples of this in the next section. Also singular count nouns need not always satisfy the mereological properties that were taken to be charatceristic of them. Singular count noun extensions may locally be cumulative or divisive. Two chains linked together yield again a chain, satifying local cumulativity. Any part of a chain, if it is connected and consists of sufficiently many links, may again count as a chain, satisfying local divisivity. (For more exceptions to these merological conditions see, for instance, Griffin 1977).

The reason for the potential violations of mereological characteristics of mass noun extensions is that count nouns are not the only means of providing integrity conditions. Integrity conditions may also be provided by the nonlinguistic context or by other linguistic means than those related to the mass-count distinction. The relevant properties of the part relation applied to entities in a given context, then, depend only on the relevant integrity conditions, wherever they may come from.

There seem to be some obvious counterexamples to the characterization of count nouns given above, namely so-called 'dummy sortals', for instance *thing, entity* or *piece*. However, these nouns encode implicit integrity conditions, that is integrity conditions that must be provided by the nonlinguistic context, the intended situation. Such implicit integrity conditions thus form an indexical component of the meaning of 'dummy sortals'. Implicit integrity conditions are also found in other expressions, for instance, in determiners, as we shall see.

The notion of integrity condition raises another question in connection with extensionality. Do integrity conditions always individuate entities; that is, are integrity conditions always essential for the identity of entities? The answer is, obviously, no. One and the same entity can be described with and without integrity. To give an example, intuitively, the (lose, unsystematic) collection of papers on my desk and the papers on my desk constitute the same entity. However, in semantics, both kinds of integrity conditions, accidental and essential integrity conditions, play the same kind of role. Therefore, we have to talk about relevant integrity conditions, rather than about integrity conditions in any 'ontological' sense.

But how should semantic theory account for the status of integrity conditions as conditions that hold relative to information (given by various linguistic means) irrespective of the actual individuation of entities? I propose that integrity conditions are to be relativized to situations, where situations are to be understood roughly in the sense of Barwise / Perry (1983). We can then say that an integrity condition W holds of an entity x in a situation s and that an entity x has a certain part structure in a situation s (where a part structure in a situation s can be taken to consist of the set of parts of x in s and the integrity conditions that hold of x or any of its parts in s).

3. Semantic Operations involving parts as a diagnostics of the nature and determination of part structures

3.1. Quantification over parts: the construction quantifier - *of* - NP

Let me illustrate the nature of quantification over parts with a very simple construction, the English construction quantifier-*of*- definite NP. The first observation with this construction is that it applies to plural NPs, mass NPs as well singular count NPs, as seen in (4).

(4) a. All of the children got a gift.

b. All of the water evaporated.

c. All of the wall is blue.

The fact that this construction applies to all three categories can be taken as evidence for a uniform relation of part, a relation applying to individuals, quantities as well as groups. Notice, however, that the construction is subject to some restrictions with individuals. (5) seems less acceptable than (4c).

(5) ? All of the chair is blue.

But this is not an argument against a uniform part relation. Rather, (5) shows only that the construction disfavors strong functional wholes. That is, it is subject to a condition that is purely semantic in nature. Thus, we may assume that *all of* in (4) quantifies over all parts (in the generalized sense) of the referent of the definite NPs, the referent of *the children*, a group, the referent *the water*, a quantity and the referent of *the wall*, an individual. If this analysis is correct, then we have got a good diagnostics for part structures and properties of the part relation in the generalized sense.

First, we can observe restrictions on transitivity and sum formation of the generalized part relation. Consider (6).

(6) a. All of the students have a passport

b. All of the gold contains copper / # two ounces of copper.

In (6a) *all*, apparently, quantifies only over individual students. Therefore, the part relation applied to the group of students is not transitive (because parts of any individual student are not parts of the group), and it is not closed under sum formation (because subgroups of students, the sums of subsets of students, do not count as parts of the group of students). In (6b), however, the part relation is transitive and closed under sum formation: any subquantity and any sum of subquantities of gold is a part of the gold. Therefore, the predicate must satisfy the 'homogeneity requirement' (cf. Bunt 1985, Loenning 1987, Moltmann 1989a and b).

Restrictions on transitivity need not be due to a sortal concept such as 'student' in (6a). Transitivity may also be blocked by contextual formation of subgroups, such as in (7), if (7) is taken to be about a partitioning of the students into subgroups.

(7) All of the students found a solution.

These data, however, do not necessarily show that the part relation is itself not transitive and not closed under sum formation. Failure of transitivity and closure could also be due to a contextual restriction of the quantification domain, an option available for almost all quantifiers of natural language. Then, the quantifier *all* would have a contextually restricted domain that selects among the parts of the referent of the respective definite NP. However, one can easily see that failure of transitivity and closure under sum formation cannot be due to contextual restrictions on the quantification domain. In the construction quantifier *-of* - NP, the quantifier may also be a quantifier ranging explicitly only over individuals, for instance *everyone* or *each*, as in (8).

(8) Everyone / Each of the students found a solution.

If *everyone* or *each* in (8) had its own contextual quantification domain selecting among the parts of the students (with the additional requirement that these parts be individuals), then (8) need not quantify over all students, but only a relevant subset. But this is not the case.

Thus, we can conclude so far that the semantically relevant part relation is uniform and not generally transitive or closed under sum formation. Furthermore, there are general conditions under which transitivity and closure under sum formation hold. Compare (6a) with (6b). In (6a) transitivity does not hold because it is blocked by the integrity conditions expressed by the singular count noun *student*. Also failure of sum formation can be traced to integrity conditions: a set A need not have a sum if the elements of A are integrated wholes. Since (6b) does not explicitly involve integrity conditions, transitivity and sum formation are not blocked. These principles are formulated in (9a) and (9b). They can be comprehended in a single preference law given in (9c). 'P' denotes the part relation as intended here.

(9) a. **Condition on Transitivity**

If for a situation s, xPy in s and yPz in s, then xPz only if there is no integrity condition W such that W(y) in s.

b. **Condition on Sum Formation**

If for a set A of parts of x in a situation s, there are integrity conditions W for any element y of A such that W(y) in s, then sum(A) (with respect to P) need not exist in s.

c. **Preference Law for Designated Parts**

For a set A of mereological parts of x and a situation s, if A covers x and for every element y of A there are integrity conditions W such that W(y) in s, then the elements of A are the only parts of x in s (provided no sums or parts of elements in A that are not in A have integrity in s).

3.2 Further applications of the conditions

Not only singular count nouns may provide integrity conditions. There are other linguistic means by which an entity can be characterized explicitly or implicitly as an integrated whole. Let me mention two examples, the German determiner *manche* 'some' and German inflected weak quantifiers (more explicitly discussed in Moltmann 1989a). First, *manche* applies to both plurals and mass nouns, as in (10).

(10) a. manche Tage 'some days'

b. manches Holz 'some wood'

Furthermore, *manche* does not just mean 'some', that is, roughly speaking, 'having a few (i.e. more than two) parts' (in an appropriate contextual sense of 'part', cf. Moltmann b). It has an additional meaning component which requires that the parts be relatively distant from each other, for instance in time, cf. (9a), or in space, cf. (9b). This is confirmed by the fact that some speakers find *manche* less acceptable with predicates that imply closeness in the relevant respect (at the beginning of the process described) and perceive a contrast between (11a) and (11c). Furthermore *manche*, when applied to groups of temporal intervals like days as in (12a), implies that the intervals not be contiguous. Therefore, only in (12b), but not in (12a), it may have rained just during a continuous stretch of time.

(11) a. Manche Punkte liegen sehr weit voneinander entfernt.

'Some points lay very far from each other.'

b. ? Manche Punkte liegen sehr nahe beeinander.

'Some points lie very close to each other.'

c. Einige Punkte liegen sehr nahe beeinander.

(12) a. An manchen Tagen hat es geregnet.

'On some days it has rained.'

b. An einigen Tagen hat es geregnet.

Thus, informally, the meaning of *manche* can be described as the follows (again with reference to the generalized notion of part). If *manche* holds of an entity x whose parts satisfy a predicate Q in a situation s, then x has 'a few' parts and these parts of x are distant from each other with respect to the relevant metric. Now, the requirement of distance between certain parts constitutes integrity conditions for these parts. Thus, applying the Preference Law for Designated Parts (9c), *manche* should block transitivity and closure under sum formation. That is, an entity x that satisfies 'manche' should consist only of parts that are distant from each other and not of any sums or subparts of such parts. This prediction is borne out, since heterogeneous predicates are acceptable with *manche*, as in (13b). The property 'being to heavy to wear' is, under normal circumstances not divisive, and the property 'being too light to wear' not necessarily cumulative.

(13) a. # Aller / Einiger Schmuck war zu schwer / zu leicht zum tragen.

'Some jewelry was too heavy / too light to wear.'

b. Mancher Schmuck war zu schwer / zu leicht zum tragen.

In contrast to *manche*, the application of *einige* has to satisfy only the following condition. If *einige* holds of an entity x whose parts satisfy a mass predicate Q in a situation s, then x has 'a few' parts. Since the parts of then x normally does not have integrity conditions, x will have part structure that is closed under sum formation and under the relation 'is a parts of' and therefore requires the predicate to be homogeneous.

In certain dialects of German, the inflected quantifiers, *viele* 'many' and *wenige* 'few', also apply to both plurals and mass nouns. When applied to mass nouns (and, of course, also plurals), the meaning can be described intuitively as follows: *vieles Holz* 'many (units of) wood' holds of an entity x in a situation s iff x is wood in s and has many parts and if these parts each are integrated wholes in s, i.e. if there are integrity conditions W for any parts y of x such that W(y) in s.[4] Again, *viele* (and *wenige*) blocks transitivity and closure under sum formation, as expected. Thus, when applied to mass nouns, the homogeneity requirement does not apply, as seen in (14), where the predicate holds only of the parts of the jewelry that are designated as units in the relevant context, as required by *viele* or *wenige*:

(14) Vieler / weniger Schmuck war zu leicht / zu schwer zum tragen.

In summary, integrity conditions, which are crucial for defining properties of part structures such as transitivity and closure, can be provided by the lexical meaning of words both explicitly, as by singular count nouns (except for 'dummy sortals), and implicitly, as by German metrical and inflected quantifiers (and by 'dummy sortals'). But beside lexical meanings, there are still other linguistic means of characterizing entities or their parts as integrated wholes.

3.3. Reciprocals and the various ways of determining part structures

Reciprocal constructions involve in some way quantification over parts of the antecedent. Simplifying, *the children like each other* means: every part of the children (in the usual case every single child) likes any other part of the children. Now we can first show that the same conditions that determine part structures for quantification over parts with the Q - *of* - NP construction and German metrical and inflected quantifiers can define the part structure of the antecedent of *each other*. Then, we will see that part structures can be determined in still other ways.

The antecedent of *each other* need not be a plural NP, but can be also a mass NP with *manche, viele* or *wenige* as determiner. (Some speakers do not find mass NPs with *manche* or *viele* as antecedents of *each other* very acceptable, but they still prefer them to mass NPs with, for example, *einige* or definite determiners.)

(15) a. Maria konnte (?) manchen / # etwas / # den Wein nicht voneinander unterscheiden.
'Mary could not distinguish some / some / the wine from each other.'
b. Hans wollte (?) manches / (?) vieles / *einiges / das Material miteinander vergleichen.
'John wanted to compare some material with each other.'
c. (?) Vieles / (?) Manches / *Einiges / *Das Material aehnelte einander.
'Many (units of) / Some / Some / The material resembled each other.'

In (15) the reciprocal quantifies over the subunits of the jewelry only those subunits that are designated by the implicit integrity conditions conveyed by the metrical or inflected determiners. I will come to the question why simple mass NPs (for instance, those with *einige* or definite determiners) cannot serve as the antecedent of *each other* in section 4.2.

Now there are several other ways of defining the part structure of the antecedent of *each other* with mass NPs. First, the parts of the semantic antecedent can also be designated with conjoined definite mass NPs. In this case, the only parts of the antecedent are the referents of the conjuncts. The example (16) is given in German in order to show that conjoined mass NPs may syntactically still be mass NPs allowing for singular agreement (an option not available for English, because English has 'sense agreement', see Moltmann 1989b).

(16) Das Plastik und das Holz war / waren einander sehr aehnlich.
'The plastic and the wood was / were very similar to each other.'

In the same way, the referent of a definite plural antecedent can be characterized as having a part structure consisting of proper subgroups, namely the referents of the plural conjuncts, as in (17):

(17) a. The boys and the girls hate each other.

b. The boys and girls hate each other.

(17a) and (17b) allow for a part structure of the antecedent which consists only of the group of boys and the group of girls as the only parts. But they allow also, for instance, for a part structure which consists only of individual girls and boys (in which case the predicate must hold between individual girls and individual boys). This suggests the following principle: sums of entities that satisfy a certain natural predicate, e.g. 'plastic' or 'boy', optionally, but not obligatorily, count as integrated wholes, blocking transitivity. This yields the following principle:

(18) **Principle for Sums to Count as Integrated Wholes**

For a natural property Q and a situation s, sum(Q) may count as an integrated whole in s.

It is essential that the part designated by a conjunct be a maximal entity satisfying the property expressed by the conjunct (i.e. the sum of this property). Parts that are not maximal in this respect cannot count as integrated wholes. (19) does not allow the reading in which the relation 'hate' holds between the group of boys and the group of girls (i.e. the opposite sexes) only.

(19) All boys and girls hate each other.

The reason is that *all* in (19) quantifies over all groups consisting of boys or girls. Thus, any group not consisting of the maximal group of boys and the maximal group of girls does not meet the condition in (18). Therefore, transitivity is not blocked, and the relation 'hate' must hold between individuals of the same sex as well.

Principle (18) can apply to another case, namely to conjunctions of modifiers. Definite mass and plural NPs with conjoined modifiers exhibit the same effect of designating certain mereological parts, namely the parts that fall under the conjuncts of the modifier.[5] Again, notice singular agreement in the German example in (20a).

(20) a. Die Luft draussen und im Zimmer ist kaum voneinander zu unterscheiden.
'The air outside and in the room is barely distinguishable from each other.'

b. The people outside and in the house do not know each other.

Another way of designating parts with mass nouns is with plural modifiers or modifiers containing plurals, as in (21). Again, the mass NPs in (21) go with singular agreement in German:

(21) a. (?) Der Schmuck der verschiedenen Frauen auf der Party war kaum voneinander zu unterscheiden.
'The jewelry of the various women at the party was barely distinguishable from each other.'

b. Der Inhalt der drei Schuesseln ist voneinander zu unterscheiden.
'The content (mass noun) of the three bowls is distinguishable from each other.'

c. (?) Das Material, aus dem die drei Kleider gemacht sind, aehnelt einander.
'The material of the three clothes resembles each other.'

This suggests the following additional principle for designating parts as integrated wholes independently of the mass-count distinction:

(22) **<u>Principle for the Determination of Integrated Parts on the Basis of Relations</u>**

If for an entity x in a situation s, there is a 1-1 mapping from some mereological parts of x to some integrated wholes in s, then these mereological parts of x are themselves integrated wholes in s.

4. Part structures and part structure-specific selection

Part structures whose properties are determined on the basis of integrity conditions do not only play a role in quantification over parts. In this section, we will see that the absense and presence of integrity conditions on parts or wholes and the various linguistic means of providing these integrity conditions also play a central role in semantic selection. Here the notion of integrity is of even more direct importance than in part quantification. There are two basic semantic selectional requirements that concern part structures, an Accessibility Requirement and an Integrated Parts Requirement. Both involve predicates or the mode of application of predicates to arguments as well as semantic operations.

4.1. The Accessibility Requirement

There is a requirement that seems to distinguish between plural and mass NPs, but not singular count NPs - apparently regardless of the 'ontological identity' of the referents. This requirement holds of predicates and the mode of application of predicates, namely the application of predicates with respect parts of an argument, rather than the whole. This is illustrated in (23), where '#" not only means 'semantically unacceptable', but here also 'impossible with the interpretation that relates the meaning of the predicate to parts of the argument rather than the whole'.

(23) a. Mary compared / distinguished the clothes / # the heap of clothes.

b. John counted / enumerated / listed the stamps / #? the collection of stamps.

c. between / among the flowers / # the bunch of flowers.

d. John graded / rated / ranked the students / # the class.

e. Mary classified the people / # the group of people.

f. Bill distributed the bread / the pieces of bread / #? the loaf of bread.

The requirement also seems to hold of distributive application of predicates in the traditional sense, which is not possible in (24a) and (24b), but only in (24c).

(24) a. The group of men lifted the piano.

b. The bunch of flowers costs two dollars.

c. The men lifted the piano.

Other semantic operations subject to the requirement are those involved in reciprocals as in (23a) and (23b) and binominal *each* discussed in Safir / Stowell (1988), as in (25c):

(25) a. The pillows / # The heap of pillows do(es) not go with each other.

b. The paintings / # The collection of paintings resemble(s) each other.

c. The paintings / # The collection of paintings cost(s) one hundred dollars each.

Now the question is, is this requirement syntactic in nature, or at least specific to the syntactic distinction between singular count nouns on the one hand and plural and mass nouns on the other hand, or is this requirement a purely semantic one, which concerns only the nature of the arguments, not the category of the expressions that designate the arguments. The following argument shows that the requirement is in fact a purely semantic one.

The application of predicates or semantic operations may not only be blocked by singular count nouns, but, optionally, also by definite plural NPs in conjunctions:

(26) a. John compared / distinguished the boys and the girls.

b. between the houses and the trees

c. The stones and the chairs weigh one hundred pounds.

d. The boys and the girls hate each other.

It will be clear to the reader from section 3 that the application of predicates and semantic operations in one of the readings of the examples in (26) is blocked semantically in the same way as in (25), namely by means of integrity conditions. Both singular count nouns and definite plurals (though the latter only optionally) characterize an entity as an integrated whole. Then, if an argument is an integrated whole or has parts that are integrated wholes, the predicates or operations in question cannot apply to parts or subparts of the argument.

Now, given, on the basis of this argument, that the requirement is a purely semantic requirement, we have to answer two questions. First, how is the class of predicates and semantic operations that are subject to the requirement characterizable semantically? Second, how are the arguments of the predicates and semantic operations that meet the requirement to be characterized semantically?

To answer the first question, let us consider predicates that are not subject to the requirement, as in (27):

(27) a. The clothes / the heap of clothes are / is in the room.

b. Mary discovered the flowers / the bunch of flowers.

A brief comparison of the predicates in (27) and the predicates in (23) suggests that the difference consists in the following. The predicates in (23) all make reference to the parts of an argument, whereas the predicates in (27) involve only an argument as a whole. The same condition seems to distinguish the semantic operations subject to the requirement. Distributive interpretation certainly makes reference to parts. It can be formulated, for instance, as in (28).

(28) For a predicate Q, Q(x) is interpreted distributively iff for all y, if yPx, then Q(y).

Reciprocals, as their semantics was formulated informally above, and binominal *each* certainly are interpreted by semantic operations that make reference to parts.[6] Thus, we can tentatively say that the requirement holds of those and only those predicates, application conditions on predicates and semantic operations that make reference to the parts of an argument.

A characterization of the arguments that meet the requirement has already suggested itself. The arguments may neither be referents of singular count nouns, nor referents of definite plurals in the reading in which, according to (18), the definite plurals provide integrity. Thus, semantically, those and only those arguments that are not characterized as integrated wholes, meet the requirement, i.e. that are not integrated wholes in the relevant situation.

Notice that it does not matter whether the integrity conditions are essential for the identity of an entity or not. The integrity conditions expressed by 'collection of papers' are not necessarily essential properties. If an entity ceases to be a (loose) collection, then this does not constitute a substantial change (a change under which the entity loses its identity). This corresponds to the fact that the predicate 'does not exist anymore' is not appropriate, as seen in (29a) (in contrast to (29b) with a collective NP expressing essential integrity conditions). Nonetheless, the argument denoted by *the collection of papers* in (29b) does not meet the selectional requirement in question.

(29) a. ?? The (loose) collection of papers on my desk does not exist anymore.

b. The orchestra does not exist anymore (because all its members became soloists).

c. ?? John counted the collection of papers on his desk.

The integrity conditions blocking transitivity may be explicit as with singular count nouns (except dummy sortals) and definite plurals or implicit, as with dummy sortals, as in (30).

(30) # among the entity / the thing / the quantity of gold on the table (a collection of dishes)

But why should arguments of predicates or operations that make reference to parts not be integrated wholes? It seems that integrity conditions make part structures 'inaccessible'. Let us define an accessible part structure as follows:

(31) **Definition of Accessible Part Structure**

An entity x has an accessible part structure in a situation s iff there are no integrity conditions W such that W(x) in s.

So predicates or semantic operations that make reference to the parts of an argument require that the argument be an integrated whole. Now the requirement in question can be formulated in a preliminary way as in (32):

(32) **Accessibility Requirement (ACC), preliminary version**

A predicate, application condition on a predicate or semantic operation that makes crucial reference to the parts of an argument can apply to an entity x in a situation s only if x has an accessible part structure in s.

However, there are numerous immediate counterexamples to this putative selectional requirement. Predicates like *organize, arrange, order, in the middle of* and *throughout* certainly make reference to parts of an argument. Nonetheless, they all accept singular count NPs as syntactic arguments, as seen in (33):

(33) a. John organized the collection of papers on the desk.

b. John arranged the bunch of flowers.

c. Maria ordered the heap of jewelry on the table.

d. in the middle of the heap of wood

e. through the country

f. throughout this night

But these predicates differ in an important respect from the predicates in (23). In contrast to *count, distinguish, enumerate, between, among* etc., the predicates in (33) not only make reference to parts of an argument, but also imply a characteristic property of the argument as a whole. *Organize, arrange* and *order* describe processes that result in a certain form or gestalt of the argument, that is, in certain relations that hold between the parts of the argument. *In the middle of, through* and *throughout* make reference to parts of an argument only in relation to the spatial or temporal extension of the argument as a whole. *Between* and *among*, in contrast, only locate an entity in relation to some parts of an argument. Thus, the class of potential counterexamples to ACC seems to consist of exactly those predicates that not only make reference to parts of an argument, but also to the whole.

The difference between the two classes of predicates can be illustrated with minimal pairs of verbs with and without the particle *durch* 'through' in German. Certain verbs with the particle *durch* belong to the class of potential counterexamples to ACC. But without the particle, they are subject to ACC. Consider the following data:

(34) a. Maria zaehlte die Studenten / ?? die Klasse.

'Mary counted the students / the class.'

b. Maria zaehlte die Studenten / die Klasse durch.

'Mary counted the students / the class through.'

(35) a. Hans numerierte die Bilder / ?? die Bildersammlung.

'John numbered the pictures / the collection of pictures.'

b. Hans numerierte die Bilder / die Bildersammlung durch.

'John numbered the pictures / the collection of pictures through.'

The semantic distinction between *zaehlen* and *durchzaehlen* in (34) can be characterized as follows: *zaehlen* only describes the process of counting without necessarily implying a result (for instance, that the total number of elements have been counted); *durchzaehlen*, in contrast, describes a process of counting with a clear result, a result that holds either if all members of the argument group have exhaustively been counted or if the members of the argument group have been counted in a certain order. This characterization of the difference in lexical meaning also implies that *durchzaehlen* is a telic verb, whereas *zaehlen* is an atelic verb. The difference in telicity is independently evidenced by the fact that *durchzaehlen* does not allow for measure adverbials such as *zwei Stunden lang* 'for two hours', but allows for frame adverbials such as *in zwei Stunden* 'in two hours', whereas *zaehlen* allows for measure adverbials, but not for frame adverbials. This is illustrated in (36):

(36) a. Maria zaehlte die Studenten zwei Stunden lang / ?? in zwei Stunden.
'Mary counted the students for two hours / in two hours.'
b. Maria zaehlte die Studenten ?? zwei Stunden lang / in zwei Stunden durch.
'Mary counted the students for two hours / in two hours.'

The first sentence in (36b) is acceptable only with a repetitive reading of *durchzaehlen*, an option always available for telic verbs.

Notice that it is important to distinguish the part structure of an argument befor the process described by the predicate has started and the parts structure it has possibly as a result of the process. Compare the predicates in (37) and in (38), which all make reference to the parts of an argument and also imply a property of the argument as a whole. Relative to the relevant situation, the predicates in (37) presuppose that the argument initially not be an integrated whole and imply that it be integrated as a result of the process (notice that the general 'integrity predicate' *together* occurs as a resultative in (37a) and (37c-d)). The predicates in (38) presuppose that the argument intially be an integrated whole and may imply that it looses its integrity as a result of the process. Obviously, only the initial states are relevant for selectional requirements. Thus, in accordance with the generalization above, the predicates in (38) impose ACC, but not the predicates in (37).

(37) a. John chained together the balls / # the heap of balls.
b. Mary accumulated the gold / # the total amount of gold there is.
c. Mary stuck together the souvenirs / # the collection of souvenirs.
d. These books / # This collection of books were (was) brought together.

(38) a. John subdivided / split up the chapter.
b. John partitioned the heap of rice.

The conclusion from these observations is that the ACC has to be modified to the effect that it excludes the relevant class of potential counterexamples. The modified version of ACC is given in (39).

(39) **<u>(Modified) Accessibility Requirement (ACC)</u>**
A predicate, application condition for predicates, or semantic operation that makes

crucial reference to the parts of an argument and does not explicitly involve a property that characterizes an argument or the state of the argument as a whole at the beginning of the process described can apply to an entity x in a situation s only if x has an accessible part structure in s.

4.2. The Integrated Parts Requirement

Another selectional requirement distinguishes between plural NPs on the one hand and mass NPs on the other hand - again irrespective of the 'ontological identity' of the referents of the plural and mass NPs. Predicates or operations subject to the requirement seem to allow only plural NPs, but not mass NPs. The requirement applies to a subset of the predicates and operations that are subject to ACC. These predicates and operations do not only require that an argument not be a referent of a singular count NP, but, in first formulation, also that an argument not be a referent of a mass NP. Some of those predicates are listed in (40a) - (40b); some additional predicates are given in (40d).

(40) a. Mary compared / distinguished the clothes / ?? the clothing / # the water.

b. John counted / enumerated / listed the chairs / ?? the furniture / ? the reading material.

c. between the chairs / # the furniture / # the clothing

d. The clothes / ?? The clothing / # The water are (is) distinct / identical / similar / akin.

Notice that this requirement is not strict: mass NPs are more acceptable if they denote entities with distinguishable elements like furniture or clothing than very homogeneous entities like water.

Among semantic operations, reciprocals are also subject to the requirement, as was already mentioned:

(41) # The furniture / The clothing resembles each other.

Another semantic operation that is subject to the requirement is the one involved in the interpretation of binominal *each* (or equivalent constructions):

(42) a. The chairs weigh ten kilos each.

b. # The furniture weighs ten kilos each.

The construction with binominal *each* itself, unfortunately, does not constitute an appropriate diagnostics for the requirement, since *each* (plausibly for syntactic reasons) cannot relate to NPs other than plural NPs. There is a construction in German, however, that has the same semantic content as the construction with binominal *each* and is not subject to syntactic agreement conditions, namely an adverbial construction with *je* or *jeweils*. The adverbials *je* and *jeweils* are, unlike binominal *each*, are categorically neutral with respect to nominal categories and therefore not subject to syntactic agreement. The construction is illustrated in (43).

(43) a. Die Moebel wiegen je / jeweils zehn Kilo.

'The furniture (plural) weighs each ten kilos.'

b. # Das Mobiliar wiegt je / jeweils zehn Kilo.

'The furniture (mass noun) weighs each ten kilos.'

c. Die Kleidungsstuecke wurden je / jeweils von einem Designer entworfen.

'The clothes were each designed by a designer.'

d. # Die Kleidung wurde jeweils von einem Designer entworfen.

'The clothing was each designed by a designer.'

However, a number of the predicates and semantic operations that are subject to ACC are not subject to the requirement under consideration. Examples of predicates of this sort are given in (44). Some speakers, though, do not accept *among* with mass NPs or only with a very restricted set of mass NPs:

(44) a. among the clothing / the furniture / ? the wood

b. Bill classified / rated / graded the art in the museum.

c. Mary distributed the bread / the wine.

Also distributive interpretation does not generally fall under the requirement:

(45) The furniture is heavy / small.

Other semantic operations that do not have to meet the requirement are, of course, constructions of simple quantification over parts, as in (46a) with the Q *-of-* definite NP construction, or in (46b) with the *all* -definite NP construction, or with floated quantifier *all*, as in (46c):

(46) a. All of the water has evaporated.

b. All the water has evaporated.

c. The water has all evaporated.

Now the question is again, does the requirement really depend on the syntactic categories of plurals and mass NPs or is it purely semantic in nature. It is easy to see that the requirement is a purely semantic requirement and has only to do with the part structure of the arguments, not with syntactic categories.

If we state the requirement in purely semantic terms, then it should be the condition that the argument consist of parts that are integrated wholes, because mass nouns and plurals, as was proposed at the beginning, differ in that plurals characterize their referents as consisting of parts that are integrated wholes, whereas mass nouns do not characterize their referents with any integrity conditions at all.

The crucial point is that even mass NPs can satisfy the requirement, namely when integrity conditions for parts are provided by other linguistic means than singular count nouns. The various ways of providing integrity for parts with mass NPs correspond exactly to the ways parts of referents of mass NPs can be designated linguistically.

First, in German mass NPs with the metrical determiner *manche* and with inflected quantifiers satisfy the requirement in question (as we have already seen, by the way, with reciprocals):

(47) a. Maria konnte (?) manche / (?) viele / # einige / # die Musik nicht unterscheiden.

'Mary could not distinguish some / many (types of) / some / the music.'

b. Hans wollte (?) manches / (?) vieles / # einiges / # das Material vergleichen.

'John wanted to compare some/ many (units of) / some / the material.'

Furthermore, conjoined definite mass NPs are acceptable with the relevant predicates and semantic operations.

(48) a. Bill compared / distinguished the wine and the juice.

b. between the wood and the wool

c. Der Wein und der Saft wurde jeweils in zwei Flaschen gefuellt.

'The wine and the juice were filled into two bottles each.'

Also definite mass NPs with conjoined modifiers meet the requirement.

(49) a. Bill compared / distinguished the wine in the bottle and in the glass.

b. between the clothing on the right side and on the left side of the bed

c. Das Obst im Korb und auf dem Tisch reicht je fuer eine Torte aus.

'The fruits (mass noun) in the basket and on the table suffices each for a tart.'

Finally, the requirement is also satisfied by mass NPs with referntial plural modifiers:

(50) a. Bill compared / could not distinguish the wine in the two glasses.

b. Der Saft in den beiden Flaschen wurde aus je / jeweils zehn Apfelsinen gepresst

'The juice in the two bottles was squeezed from ten oranges each.'

So we can conclude that the arguments of the predicates and operations subject to the requirement have to consist of parts that are integrated wholes (in the relevant situation). Notice, again, that it is essential to relativize integrity to situations. Integrity conditions may be provided by linguistic expressions explicitly or implicitly (as with German metrical and inflected determiners). Integrity conditions may, however, also be provided purely contextually. (51) seems acceptable for many speakers.

(51) John went from table to table to compare the wine.

Now let us turn to the semantic characterization of the class of predicates and semantic operations subject to the requirement. A comparison of the predicates that are subject to ACC and the predicates that are subject to the selectional requirement under consideration suggests the following. What is crucial is whether a predicate involves a binary relation between distinct parts or not. Processes like comparing and distinguishing involve most obviously a binary relation between distinct parts, but a process of distributing does not (necessarily) do so. Also processes like counting. enumerating and listing arguably involve binary relations between distinct parts - though in a more complex way than comparing and distinguishing. A process of counting, enumerating or listing strictly requires that the agent be able to distinguish the parts of an argument. Predicates like *distinct, different, similar* and *akin*, of course, express directly relations that hold between distinct parts of an argument. Predicates like *identical* arguably presuppose that one and the same part of an argument has been considered as two distinct entities or modes of presentation of two distinct entities. Thus, again, a relation between distinct parts is involved - though the distinctness condition holds at some level other than the level of actual reference.

Two semantically almost minimal pairs display this distinction clearly. The prepositions *between* and *among* differ in semantic selection in that *between*, but not *among* requires that the argument consist of parts that are integrated wholes. *between* obviously involves a binary relation between parts. 'x is between y' means 'x is located on the straight line connecting the two parts of y'. In contrast, 'x is among y' means something like 'x is a part of y or x is similar to the parts of y and located in the space occupied by the neighborhood of y'. Thus, *among* only makes reference to parts, but not to a relation between distinct parts.

Other minimal pairs can be made up with predicates of evaluation, namely *classify, grade* and *rate* on the one hand and *rank* on the other hand. The first group of predicates does not require arguments with integrated parts; only *rank* does. Compare (52a) with (52b):

(52) a. Mary classified / graded / rated the art in the museum.

b. ?? Mary ranked the art in the museum.

The crucial difference between *classify, grade* and *rate* on the one hand and *rank* on the other hand is the following. *Classify, grade,* and *rate* express absolute evaluation, evaluation with respect to some given measure. But *rank* expresses relative evaluation, which involves a comparison of the parts of an argument with each other, rather than an independent measure of evaluation. Thus, again, the first group makes reference only to the parts of an argument, but does not involve a comparison between parts, only a mapping from the parts into some independent values. By contrast, *rank* involves a binary relation between parts, namely the relation of qualitative comparison.

The same distinction differentiates between the semantic operations that are subject to the requirement in question and those that are not subject to it. Let us compare the semantic operation involved in reciprocal constructions and distributive interpretation. Reciprocals, obviously, involve a relation between distinct members of the antecedent, namely simply the relation of distinctness. Distributive interpretation as formulated in (25), however, involves parts only individually and does not mention a relation between parts. Now, let us examine the construction with binominal *each* as in (53a) and German *je / jeweils*. as in (53b). It is instructive to compare this construction with simple part quantification and with *all* as a floated quantifier, as in (54).

(53) a. The three critics praised two books each.

b. Die drei Kritiker lobten je zwei Buecher.

'The three critics praised two books each.'

(54) a. All of the critics praised two books.

b. All the critics praised two books.

c. The critics all praised two books.

Safir / Stowell (1988) (in attribution to an observation by J. Higginbotham) claim that binominal *each* must map the members of the referent of the antecedent (= the R-NP). The same would hold for *je / jweils* . Thus, the three critics in (53a-b) would have to be mapped into distinct referents of the NP immediately preceding *each* (= the D-NP), i.e. into distinct groups of two books. Such a requirement then could explain the difference in selection between binominal *je / jeweils* and simple part quantification, since then *je / jeweils* would require the relation of distinctness among the referent parts of the R-NP and therefore impose IP. However, the requirement does not seem to be as strict as it is claimed. (53a-b) may be true if the three critics happen to praise the same three books. But something seems to be true about this requirement. First, *je / jeweils* always requires the following indefinite NP to have narrow scope (whereas simple part quantifiers like *all* allow a following indefinite NP to take wide scope). Second, *je / jeweils* excludes an NP with *same*, even though it allows an NP with *different*:

(55) Die drei Kritiker lobten je ein anderes / # dasselbe Buch.

'The three critics praised each a different / the same book.'

So, maybe binominal *je / jeweils* and *each* carry some implication that the referents of the D-term are associated with distinct parts of the referent of the R-term be (if not distinct) at least not known to be the same. Since such an implication explicitly involves a binary relation between parts of the antecedent group (namely the relation that holds of two parts x and y iff x and y are not mapped into the same referents of the D-NP), it is correctly predicted that the *je / jeweils* construction requires that the semantic antecedent have a part structure with integrated parts. Simple part quantification as in (54), in contrast, involves parts of the semantic antecedent only individually. Therefore, simple part quantification should allow for any mass NPs as antecedents, and this prediction is borne out, as we have seen.

Further evidence for this peculiarity of *je / jeweils* comes from a comparison of *je / jeweils* with events as antecedents and another event-related binominal construction in English, namely the construction Q - *at a time*, where Q is an indefinite quantifier. Event-related *je / jeweils* is illustrated in (56), binominal *at a time* in (57).

(56) a. Hans las und schrieb je ein Buch.

'John read and wrote adv one book.'

b. Hans las am Morgen und am Abend je ein Buch.

'John read adv a book in the morning and in the evening.'

(57) a. John wrote the book one chapter at a time.

b. Mary ate the cake a little bit at a time / several pieces at a time.

Event-related *je / jeweils*, like NP-related *je / jeweils* requires that the event have integrated parts, which may be determined by conjoined event predicates. If a clause contains only simple event predicates (and no plural arguments), *je / jeweils* cannot receive an event-related interpretation:

(58) Hans las je ein Buch.

'John read adv a book.'

In (58) *je* may only have an indexical reading, relating to a set of occasions that was given previously in the context. However, binominal *at a time* is fine with simple event descriptions, as (57) illustrates. The reason for this turns out to be the same as for why the requirement of integrated parts can be satisfied by plurals as well as mass NPs with metrical determiners. It appears that the presence of *time* in (57) imposes a certain temporal part structure on the described event: the event must consist of subevents that are separated from each other in time. Since these subevents are maximal events that are connected in time, they are integrated wholes. Therefore, the event consisting of those subevents satisfies the relevant requirement. *Je / Jeweils*, by contrast, does not impose a metric on the part structure of the event. Therefore, the parts of the described event must be characterized as integrated wholes in some other way - for instance, by conjoined event predicates (or plural arguments).

In summary, we can generalize that the predicates and semantic operations that require that an argument have a part structure consisting of integrated parts are those and only those that involve a binary relation between distinct parts of an argument. This condition, which I shall call the Integrated Parts Requirement, is formulated in (59).

(59) **The Integrated Parts Requirement (IP)**

A predicate or semantic operation that explicitly involves or presupposes a binary relation between distinct parts of an argument can apply to an entity x in a situation s only if the parts of x in s are integrated wholes in s, i.e. if there are integrity conditions W for any part y of x in s such that W(y) in s.

A question that still has to be answered is, why should IP hold, i.e. what is the motivation for the connection between binary relations between distinct entities and the requirement that the entities be integrated wholes? This issue leads to a philosophical discussion, the discussion about relative identity (cf. Geach 1973, Griffin 1977, Wiggins 1980). It has been argued that the relation of identity (and thus the relation of distinctness and the process of counting) depends in some way on sortal concepts holding of the entities under consideration. The notion of a sortal in this discussion corresponds roughly to the notion of integrity condition. The only difference is that sortals must not only express integrity conditions, but also integrity conditions that are essential. (In addition, they might convey accidental integrity conditions, which may hold of an entity only temporarily. In this case the sortals are so-called phase-sortals, cf. Wiggins 1980.) That is, integrity conditions are constitutive for the identity of entities, and, therefore, form the basis for any relation of comparison and any process of counting or listing.

5. General considerations on part structures and situations

All notions involving part structures that were employed in the preceding part have been relativized to situations. The intended notion of situation now requires some further clarification.

A situation should contain that and only that information which is relevant in the context of communication. A situation should not represent all true propositions about the part structure of an entity, but only those propositions that have been communicated in the relevant context or that are presupposed in that context. A situation should constitute a level of representation of part structures that are defined just by the conveyed or presupposed information in the relevant communication situation. This information essentially consists of integrity conditions that hold of entities or their parts. As we have seen, these integrity conditions may be explicitly conveyed, by the lexical meaning of singular count nouns, implicitly by indexical components of expressions, for instance, dummy sortals or German metrical or inflected quantifiers, or, finally, constructionally, for instance, by conjunctions of definite plural or mass NPs or by plural modifiers. Integrity conditions determine important properties of the part relation when restricted to the part structure of a given entity, in particular, it determines transitivity, closure under sum formation and extensionality.

For the semantic phenomena that involve part structures, it does not matter whether given integrity conditions are essential properties or not. Part structures, when they are defined by essential and when they are defined by accidental integrity conditions, play the same role in part quantification and in semantic selectional requirements.

Notes

[1] For a general discussion of mereological part relations see Simons (1987); for the application of mereological part relations to the semantics of natural language see, for instance, Sharvy (1981), Link (1983), Roeper (1984), Bunt (1985) and Loenning (1987).

[2] Domain-specific mereological part relations for individuals, groups and quantities are assumed in Sharvy (1981), Link (1983) and Simons (1987).

[3] See Moltmann (1989a) for more discussion. This also holds for the next section.

[4] In some dialects of German, also the converse case is found, namely mass quantifier applying to plurals. It has the converse semantic effect. A plural with a mass quantifier requires a homogenous or collective predicate, whereas a plural with a plural quantifier allows for

heterogeneous predicates, predicates that hold only of the group members. This is seen in the contrast between (1a) and (1b), where *weinige* 'few' is a plural quantifier and *wenig* 'little' a mass quantifier. (2) shows a context in which plurals with mass quantifiers are acceptable.

(1) a. Hans hat zwei Gramm Salz in wenige Wasserflaschen getan.

'John has put two grams of salt in few bottles of water.'

b. # Hans hat zwei Gramm Salz in wenig Wasserflaschen getan.

(2) Im Kuehlschrank waren wenig / wenige Wasserflaschen.

'In the refridgerator were few (mass quantifier) / few (plural quantifier) bottles of water.'

Thus, mass quantifiers applied to plurals have the effect that the group referred to by the plural is perceived as a homogeneous entity where the integrity conditions expressed by the corresonding singular count noun (*bottle*) are disregarded, that is, are taken to be irrelevant in the relevant situation.

[5] Notice that mass NPs with conjoined modifiers are not, or at least need not be, elliptic constructions for conjoined NPs, which then might classify as a plural NP on syntactic grounds. That is, applied to our example, *die Luft draussen und im Zimmer* need not be elliptic for *die Luft draussen und die (Luft) im Zimmer.* The reason is simply that elliptic NPs are not formed that way. With noncoreferential singular count NPs such an ellipsis is clearly impossible (**der Mann im Garten und im Haus* 'the man in the garden and in the house' as standing for *der Mann im Garten und der(Mann) im Haus* 'the man in the garden and the (man) in the house').

[6] However, there are some exceptions to this generalization. Part quantifiers in many languages may go with singular count nouns, for instance the English construction quantifier-*of*-NP or Italian *tutto* 'all' as in *tutta la casa* 'all of the house'.

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Unambiguous proof representations for the Lambek calculus

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1. Introduction

Lambek derivations allow a semantic interpretation along the lines of the 'formulas as types' paradigm from Lambda Calculus as show in Van Benthem (1986), Buszkowski (1988) and Moortgat (1988) for different axiomatizations. In this paper we show how the effective correspondence between categorial proofs and their lambda terms can be exploited to solve the major computational tractability problem for flexible categorial systems: the problem of spurious ambiguity.

To place the discussion into its proper context, we need an enumeration procedure generating the set of possible readings for a sequent $T \Rightarrow X{:}t$ (read: 'the sequence of types T derives type X with lambda recipe t'.) We obtain such a procedure by showing that Cut Elimination preserves *strong recognizing capacity*, in the following semantic sense: modulo logical equivalence, all possible readings for **L** derivations can be obtained in cut-free fashion.

THEOREM. If $T \Rightarrow Y{:}t$ in **L**+{Cut} then $T \Rightarrow Y{:}t'$ in cut-free **L**, with $t \approx t'$

The theorem is a straightforward categorial adaptation of the proof-theoretic result that from the formulas-as-types perspective cut elimination corresponds to normalization (β-conversion) (Howard 1980). Since the number of cut-free derivations for a given sequent is finite, the standard proof search procedure for **L** effectively enumerates the set of possible readings for a sequent.

Cut elimination does not remove all sources of spurious ambiguity: the finite number of cut-free proofs for an **L** sequent may still include alternative derivations for one and the same reading. This problem can be traced to the don't-care non-determinism of the standard sequent axiomatization of **L**, which allows proofs to differ in semantically inessential application orders of the [L],[R] inferences. By means of a *partial deduction* unfolding of the [L],[R] inferences, we obtain a derived clausal form axiomatization $\mathbf{L}_C$ which removes this source of spurious non-determinism. We compare the unfolding transformation with König's (1989) normal form proof procedure, and with Roorda's (1990) proof nets for **L**.

2. Cut elimination and normalization

2.1. Lambda semantics for Gentzen proofs

In this paper sequent proofs for the product-free fragment of **L** will be associated with a reading in the form of a lambda term. Our presentation is based on the semantic interpretation procedure of Moortgat (1988). The algorithm presented there is a sequent implementation of Van Benthem's (1986) term construction algorithm for the natural deduction formulation of **L**, which is based on the correspondence between Elimination inferences and functional application, Introduction inferences and functional abstraction, and Cut and substitution[1].

In order to associate a lambda term ('reading') with the proof of a sequent $A_1,\ldots,A_n \Rightarrow B$, we associate each antecedent category A_i with a fresh variable of the appropriate type and compute the lambda term for the succedent type B by the following induction.

Axioms: $A{:}t \Rightarrow A{:}t$

The axiom scheme forces *unification* of the antecedent and succedent terms. It will simplify matters to restrict the axiom scheme to *atomic* types.

Logical rules:

Elimination: functional application

$$\frac{T \Rightarrow B{:}t' \qquad U, A{:}tt', V \Rightarrow C}{U, A/B{:}t, T, V \Rightarrow C}\ [/L]$$

$$\frac{T \Rightarrow B{:}t' \qquad U, A{:}tt', V \Rightarrow C}{U, T, B\backslash A{:}t, V \Rightarrow C}\ [\backslash L]$$

Introduction: abstraction

$$[/R]\ \frac{T, B{:}v \Rightarrow A{:}t}{T \Rightarrow A/B{:}\lambda v.t}$$

$$[\backslash R]\ \frac{B{:}v, T \Rightarrow A{:}t}{T \Rightarrow B\backslash A{:}\lambda v.t}$$

Structural rule: substitution

$$\frac{T \Rightarrow A{:}t \qquad U, A{:}t, V \Rightarrow B}{U, T, V \Rightarrow B}\ [\text{Cut}]$$

The spurious ambiguity problem for **L** now takes the following form. In the full system (*with* Cut), a derivable sequent is associated with an infinite number of proofs, hence an infinite number of lambda terms, for the number of possible readings which we know to be finite *modulo* logical equivalence (cf. Van Benthem 1986). The first step in eliminating spurious ambiguity therefore will be a demonstration of a semantic version of the Cut Elimination theorem: there is a cut-free derivation for every possible **L** reading. We undertake this task in Section 2.2. The cut-free part of **L** associates a sequent with a finite number of proofs/lambda terms. But whereas the natural deduction presentation characterizes a one-to-one correspondence between derivations and lambda terms (the so-called Curry-Howard isomorphism[2]), the sequent calculus suffers from redundancy: the same proof (lambda term) can be written in many different ways. The second step in eliminating spurious ambiguity, then, will consist in removing this 'don't care' non-determinism from sequent derivations. This is the subject of Section 3.

First, as an illustration for the lambda term construction algorithm, and an introduction to the spurious ambiguity problem, we display the different sequent derivations computing the lambda term for a type transition known as Division (or the Geach rule). The first derivation is deliberately verbose and explicitly indicates the unifying substitutions for the unknown succedent terms that have to be computed as the proof unfolds in 'backwards chaining' manner from the conclusion to the axiom sequents. The answer substitution for the completed derivation is then obtained by composition of the substitutions corresponding to each inference step. The second derivation is given in the more succinct format we will henceforth use, and performs substitutions on the fly where convenient. Note that the two sequent derivations differ in the choice of the principal type at the first [/L] inference (*b/c* or *a/b*). As can be seen from the computed lambda term, the order of application is inessential: both choices lead to the same term.

Division, derivation 1

$$
[/R]\ \dfrac{[/R]\ \dfrac{[/L]\ \dfrac{\{t_3=z\}\dfrac{\textbf{true}}{c{:}z\Rightarrow c{:}t_3} \qquad \dfrac{\{t_4=y(t_3)\}\dfrac{\textbf{true}}{b{:}y(t_3)\Rightarrow b{:}t_4} \qquad \dfrac{\textbf{true}}{a{:}x(t_4)\Rightarrow a{:}t_2}\{t_2=x(t_4)\}}{a/b{:}x,\ b{:}y(t_3)\Rightarrow a{:}t_2}[/L]}{a/b{:}x,\ b/c{:}y,\ c{:}z\Rightarrow a{:}t_2}}{a/b{:}x,\ b/c{:}y\Rightarrow a/c{:}t_1}\{t_1=\lambda z.t_2\}}{a/b{:}x\Rightarrow (a/c)/(b/c){:}t}\{t=\lambda y.t_1\}
$$

Computed answer substitution: $\{t = \lambda y \lambda z.x(yz)\}$

Division, derivation 2. Don't care non-determinism of the Gentzen proof procedure: the order of rule application (*a/b* as active type in the first branching inference, rather than *b/c*) is irrelevant for the computed lambda term.

$$
[/R]\ \dfrac{[/R]\ \dfrac{[/L]\ \dfrac{[/L]\dfrac{c{:}z\Rightarrow c{:}z \qquad b{:}yz\Rightarrow b{:}yz}{b/c{:}y, c{:}z\Rightarrow b{:}yz} \qquad a{:}x(yz)\Rightarrow a{:}x(yz)}{a/b{:}x,\ b/c{:}y,\ c{:}z\Rightarrow a{:}t''}}{a/b{:}x,\ b/c{:}y\Rightarrow a/c{:}t'}\{t'=\lambda z.t''\}}{a/b{:}x\Rightarrow (a/c)/(b/c){:}t}\{t=\lambda y.t'\}
$$

Computed answer substitution: $\{t = \lambda y \lambda z.x(yz)\}$

Compare the above proofs with a natural deduction derivation. The sequent proofs can be viewed as instructions for the construction of the corresponding natural deduction derivation (see Prawitz 1965 for discussion). But the natural deduction derivation does not suffer from the don't care non-determinism of the Gentzen proof procedure.

$$\cfrac{\cfrac{\cfrac{a/b{:}x \qquad \cfrac{[b/c{:}y]_2 \qquad [c{:}z]_1}{b{:}yz}[E/]}{a{:}x(yz)}[E/]}{a/c{:}\lambda z.x(yz)}[I/]_1}{(a/c)/(b/c){:}\lambda y\lambda z.x(yz)}[I/]_2$$

2.2. Cut Elimination and normalisation

The syntactic version of Cut Elimination (Lambek 1958) shows that the cut elimination algorithm preserves *weak recognizing capacity*, i.e. **L** and **L**+{Cut} have the same set of theorems. We give the semantic version of the theorem for **L** by demonstrating that cut elimination also preserves *strong recognizing capacity*, i.e. **L** and **L**+{Cut} associate their theorems with the same readings modulo logical equivalence.

THEOREM. If $T \Rightarrow A{:}t$ in **L**+{Cut} then $T \Rightarrow A{:}t'$ in cut-free **L**, with $t \approx t'$

For those who are surprised by this use of the term 'strong recognizing capacity': recall that the syntactic algebra of **L** is an algebra of *proofs*, not of constituent structure *trees*. It seems fair, then, to refer to the lambda term semantic recipe as the 'structure' associated with a derivable string. Ambiguous strings are associated with different proofs, hence different lambda terms, not with different constituent structure trees.

The fact that Cut-Elimination corresponds to normalisation of the associated lambda terms is well-known in the literature on the 'formulas as types' interpretation of natural deduction systems. We transfer this correspondence to our term construction algorithm for the sequent axiomatisation of **L**, following the discussion of Troelstra & Van Dalen (1988) and Girard, Lafont & Taylor (1989)[3].

Cut Inference (nodes numbered for future reference)

$$\frac{[1]\ T \Rightarrow A{:}t \qquad\qquad U,A{:}t,V \Rightarrow B\ [2]}{[0]\ U,T,V \Rightarrow B}$$

The type A that is present in the premises of the Cut inference and absent from the conclusion, is called the *cut formula*. The *degree* of a Cut inference is the sum of the degree of the parameters of the inference, i.e. $d(T)+d(U)+d(V)+d(A)+d(B)$ (where the degree of a type is the number of type-forming operators in it).

Elimination algorithm (induction on *degree* of [Cut])

Base case

Premise [1] or [2] is an axiom, i.e. the conclusion of the Cut is identical to the non-axiom premise. Remove Cut.

Recursive cases

Permutation conversions

Principal type of either [1] or [2] $\neq A$. Move Cut upwards, i.e. permute the order of application of the Cut inference and the inference that leads to this Cut.

Detour conversions

The cut formula A is the principal type of both [1] and [2], i.e. $A = A'/A''$ (or $A''\backslash A'$) . Replace Cut by *two* new cuts with cut formulae A'' and A'.

The steps of the Cut Elimination algorithm are complexity-decreasing transformations on proof trees: every step replaces a given Cut inference by one or two new Cut inferences of lower degree, until the base case is reached, where the Cut can be removed. Among the recursive cases, the crucial distinction is that between *detour* conversions and *permutation* conversions (terminology from Troelstra & Van Dalen 1988). Semantically, the permutation conversions are vacuous: permutation conversions switch the branches of a cut inference, but they leave the parameters (and the lambda terms for these parameters) untouched. The detour conversions, on the other hand, remove the original cut formula A'/A'' and replace it by two new cut formulae A' and A''. With the original cut formula A'/A'', the semantic recipe for that type disappears. The detour conversion steps therefore will have to establish the relation between the lambda terms for the new cut formulae A' and A'' and the lambda term for the original cut formula A'/A''. It is easy to check that this relation is β-conversion.

We first illustrate the semantically innocent permutation conversion case.

Left premise: principal type $A'/A'':t$ obtained from [11], [12] by [/L].

$$\dfrac{\dfrac{[11]\ T'\Rightarrow A'':t'' \qquad U',A':tt'',V'\Rightarrow A:t'\ [12]}{[1]\ U',A'/A'':t,T',V'\Rightarrow A:t'}\,[/L] \qquad U,A:t',V\Rightarrow B:s\ [2]}{[0]\ U,U',A'/A'':t,T',V',V\Rightarrow B:s}\,[Cut]$$

Elimination: permute [/L] and [Cut]. Note that the relevant parameters (i.e. $U,U',A'/A'':t,A'':t'',A':t',T',V',V$ and $B:s$) are unaffected by the transformation.

$$\dfrac{[11]\ T'\Rightarrow A'':t'' \qquad \dfrac{[12]\ U',A':tt'',V'\Rightarrow A:t' \quad U,A:t',V\Rightarrow B:s\ [2]}{U,U',A':tt'',V',V\Rightarrow B:s}\,[Cut]}{[0]\ U,U',A'/A'':t,T',V',V\Rightarrow B:s}\,[/L]$$

Next we establish the correspondence between detour conversions and normalization of the associated lambda term.

Left premise Principal type is Cut formula A'/A'' (obtained by Introduction)

$$\frac{[11]\quad T,A'':v \Rightarrow A':t'}{[1]\quad T \Rightarrow A'/A'':\lambda v.t'}[/R]$$

Right premise Principal type is Cut formula A'/A'' (obtained by Elimination)

$$\frac{[21]\quad V' \Rightarrow A'':t'' \qquad U,A':(\lambda v.t')(t''),V'' \Rightarrow B:s\quad [22]}{[2]\quad U,A'/A'':\lambda v.t',V',V'' \Rightarrow B:s}[/L]$$

Cut elimination step β-conversion

[Cut2] Cut formula A' with recipe $t'[t''/v]$ (compare [11],[11'])
[Cut1] Cut formula A'' with recipe t''

$$\frac{[21]\quad V' \Rightarrow A'':t'' \qquad \dfrac{[11']\quad T,A'':t'' \Rightarrow A':t'[t''/v] \qquad U,A':t'[t''/v],V'' \Rightarrow B:s'\quad [22']}{U,T,A'':t'',V'' \Rightarrow B:s'}[\text{Cut2}]}{U,T,V',V'' \Rightarrow B:s'}[\text{Cut1}]$$

Conclusion The conclusion of the cut elimination step replaces the original semantic recipe s by the logically equivalent s':

$$U,T,V',V'' \Rightarrow B:s', \text{ where } s' = s[t'[t''/v]/(\lambda v.t')(t'')]$$

2.3. Proof search: SLD Resolution

Since the cut-free system **L** produces all possible readings, we are in a position now to explicitly introduce the proof search procedure generating these readings. Given the fact that the Lambek-Gentzen system **L** can be interpreted as a Horn-clause axiomatization of the derivability relation '⇒', the 'backward chaining' proof search procedure that will compute the possible readings for a sequent $T \Rightarrow A:t$ is a simple categorial version of SLD-resolution, as demonstrated in Moortgat (1988). The algorithm below is identical to the standard abstract interpreter for logic programs (cf. Sterling & Shapiro 1986) but for the explicit factorization procedure (unify_factorization) which matches a goal sequent $T \Rightarrow Y$ with the head of an **L** inference where the antecedent can be a sequence of string *factors* rather than just an unanalysed string.

```
prove/2: prove(Sequent,ProofTree)

prove(true,true) .

prove(GoalSequent,(GoalSequent ← Tree)) :-
   ( Head ← Premises ) ,
   unify_factorization(GoalSequent,Head) ,
   prove(Premises,Tree) .
```

```
prove((Premise1,Premise2),(ProofTree1,ProofTree2)) :-
   prove(Premise1,ProofTree1) ,
   prove(Premise2,ProofTree2) .
```

Input: the Lambek-Gentzen calculus **L**, i.e. the logic program for the two-place derivability relation '⇒'/2 holding between an antecedent sequence of typed terms and a succedent type with the associated term; a goal sequent G of the form $A_1{:}t_1,\ldots,A_n{:}t_n \Rightarrow B{:}Term$, with the antecedent terms given (ground), the succedent Term to be computed.

Output: a proof tree for the sequent Gθ, if it is **L** derivable, failure otherwise

Algorithm

Initialize the resolvent to be the input goal sequent G.

While the resolvent is non-empty:

> Select a goal sequent Atom from the resolvent;
> Select a program clause Head ← Premises from **L** (i.e. one of the inferences axiomatizing '⇒'/2);
> Match the selected Atom sequent against the head of the program clause Head ← Premises, i.e. compute the mgu θ which makes Atom and Head equal;
> Remove Atom from and add Premises to the resolvent;
> Apply θ to the new resolvent and to G;
> Record the resolution step in the structure Tree.
>
> If the resolvent is empty output G with proof Tree, else output failure.

For those accustomed to the Definite Clause Grammar encoding it may be useful to replace our infix notation for '⇒'/2 by the more usual predicate notation. The literals in the **L** inferences then assume the form

⇒(Type:Term,String)

where *Type* represents the succedent type, *Term* the associated lambda term and *String* the antecedent, represented as a sequence of string factors. The factorized antecedent, in other words, plays the role of the (difference list) string argument in the DCG representation. For example, compare our informal notation

U,A/B:Term,T,V⇒C:Sem **if**
   T⇒B:Term1 **and**
   U,A:Term(Term1),V⇒C:Sem

with the corresponding predicate notation for '⇒'/2. ('@' is an explicit application operator.)

⇒(C:Sem,(U,[A/B:Term],T,V)) ←
    ⇒(B:Term1,T),
    ⇒(C:Sem,(U,[A:Term@Term1)],V)).

3. Normalization and partial deduction (unfolding)

The semantic version of the Cut Elimination Theorem eliminates the first, lethal, form of spurious ambiguity: the fact that in a cut-based system, there is an infinite number of proofs for the finite number of readings of an **L** sequent. But Cut Elimination does not completely solve the spurious ambiguity problem: among the cut-free derivations one may find different proofs for one and the same reading. König (1989) discusses this problem, and proposes a goal selection strategy to enforce a normal form derivation for cut-free **L** proofs. Our approach is more radical. We transform the original sequent axiomatization into a derived clausal form presentation ($\mathbf{L_C}$) obtained from **L** by means of partial execution of the [L], [R] inferences. The clausal form transformation produces König's normal form as a side effect. In addition, the transformation significantly reduces the complexity of both the proof trees and the search tree for **L** sequents.

3.1. Equivalent proofs

As an example of equivalent cut-free proofs, consider again the derivations of the Geach type transition with recipe λyλz.x(yz), repeated here for convenience.

Derivation 1:

```
                    b:yz⇒b:yz        a:x(yz)⇒a:t"
               [/L] ─────────────────────────────── {t"=x(yz)}
        c:z⇒c:z              a/b:x, b:yz⇒a:t"
   [/L] ──────────────────────────────────────
               a/b:x, b/c:y, c:z⇒a:t"
        [/R]   ─────────────────────── {t'=λz.t"}
               a/b:x, b/c:y⇒a/c:t'
        [/R]   ─────────────────────── {t=λy.t'}
               a/b:x⇒(a/c)/(b/c):t
```

Derivation 2:

```
           c:z⇒c:z           b:yz⇒b:yz
      [/L]─────────────────────────────
                b/c:y,c:z⇒b:yz        a:x(yz)⇒a:t"
          [/L]  ─────────────────────────────────── {t"=x(yz)}
                     a/b:x, b/c:y, c:z⇒a:t"
               [/R]  ─────────────────────── {t'=λz.t"}
                     a/b:x, b/c:y⇒a/c:t'
               [/R]  ─────────────────────── {t=λy.t'}
                     a/b:x⇒(a/c)/(b/c):t
```

The crucial observation is that semantically equivalent proofs are derived from the same set of Axiom leaves *A* (atomic types plus the associated semantic term). In the case of our example, the axiom set (the yield of the proof trees) is:

A = {c:z⇒c:z, b:yz⇒b:yz, a:x(yz)⇒a:x(yz)}

In order to diagnose the spurious ambiguity problem for **L**, we define the set of proof trees generated from an axiom set *A*.

Definition PROOFS(*A*), the set of proof trees generated from axiom set *A*, is the least set of trees containing all the members of *A* and closed under the inference rules of **L** viewed as tree building operations, i.e.

- For any *T1*∈PROOFS(*A*) whose root is labeled with a sequent *A*⇒*B* and for any instance of a one-premise inference rule *C*⇒*D* ← *A*⇒*B* in **L**, the tree *T* whose root is labeled with *C*⇒*D* and whose subtree *T*/*1* is equal to *T1* is in PROOFS(*A*).

- For any pair *T1*,*T2*∈PROOFS(*A*) whose roots are labeled with sequents *A*⇒*B* and *C*⇒*D* respectively, and for any instance of a two-premise inference *E*⇒*F* ← *A*⇒*B*,*C*⇒*D* in **L**, the tree *T* whose root is labeled with *E*⇒*F* and whose subtrees *T*/*1* and *T*/*2* are equal to *T1* and *T2* respectively is in PROOFS(*A*).

Consider now, for a goal sequent *T*⇒*X*:*Term*, the subset of trees in PROOFS(*A*) rooted in *T*⇒*X*:*Term*. If the proof procedure were free of spurious ambiguity, this subset would contain just one member, i.e. there would be only one way of closing the axiom set *A* under the [L],[R] inferences so as to produce a proof tree with root *T*⇒*X*:*Term*. As the derivations for the Division type transition show, the original axiomatization of **L** does not have this pleasant property: the [L],[R] inferences can be applied in different orders which do not affect the associated lambda term.

The normal form proof procedure proposed in König (1989) abstracts away from the don't-care non-determinism in the order of rule applications by fixing this order in such a way that the [L] and [R] inferences directly reflect the structure of the lambda term associated with a proof[4]. To this effect, the normal form proof procedure

- performs [R] inferences before [L] inferences, and
- selects the principal type *A*/*B* in a [L] inference in such a way that the head of the principal type is equal to the succedent goal type *C*, and makes *A* the principal type of the right premise. (Define *head*(*A*)=*A* for atoms *A*, and *head*(*X*/*Y*)=*head*(*Y**X*)=*head*(*X*).)

$$\frac{T \Rightarrow B{:}t' \qquad U,A{:}tt',V \Rightarrow C}{U,A/B{:}t,T,V \Rightarrow C}[/L]$$

In the next section we show that the problem of irrelevant non-determinism in the order of rule applications can be avoided by transforming types into a clausal form representation.

3.2. Partial deduction

Consider the relation between Gentzen sequent calculus and Resolution theorem proving, in the categorial or in the logical setting. Gentzen sequent calculus deals with types (formulas) of arbitrary complexity built from a set of type-forming operators (connectives). The proof system consists of a *set* of [L],[R] inference rules, matching the set of type-forming operators (connectives), which decompose types (formulas) into the atomic subparts that make up the axiom sequents. Resolution theorem proving replaces this set of inference rules by a *single* one, the resolution rule. The price one has to pay for this simplification is that one cannot deal with types (formulas) of arbitrary complexity, but with a syntactically restricted set in conjunctive normal form. A type (formula) is in conjunctive normal form iff it is a conjunction C_1 & ... & C_n of clauses C_i, where a clause is a disjunction of literals $L_1 \vee ... \vee L_n$ a literal being an atom or its negation.

In this section we will eliminate the don't-care non-determinism of the sequent system for **L** by transforming the types into their clausal form equivalents[5]. The transformation is an application of the well-known partial deduction (or partial execution) compilation technique. The idea of partial deduction is extremely simple: we perform the basic inference rule of SLD resolution at *compile* time (i.e. on program clauses), rather than at *run* time (i.e. on goal clauses). As discussed in Section 2.3, resolution proceeds by selecting a literal in a goal and replacing it with the body of a matching program clause under the unifying substitution of the selected literal and the head of the clause, i.e. from the negative clause

$$\leftarrow B_1,...,B_n$$

and the program clause

$$A' \leftarrow B_1',...,B_m'$$

where B_i unifies with A' with mgu θ, we derive the resolvent

$$(\leftarrow B_1,...,B_{i-1},B_1',...,B_m',B_{i+1},...,B_n)\theta$$

From an implementation point of view, the partial deduction transformation can be seen as a compilation of the *lexicon*. The original axiomatization of **L** derives a goal sequent from *trivial* axioms and logical rules [L],[R]. Partial deduction of the lexicon adds *non-trivial* axioms — the unfolded clausal form representations of the lexical type assignments — and, as a result, can do *without* the original logical rules. The proof of a goal sequent in the clausal form axiomatization (which we will refer to as $\mathbf{L}_C$) is an SLD derivation from the unfolded lexical type assignments. In order to perform the partial deduction transformation, we will have to deal with two cases: unfolding with respect to [L] inferences, and with respect to [R] inferences.

Partial deduction with respect to [L] *inferences*

As an example, consider the unfolding of a type *(a\b)/c* with semantic recipe t^6. An antecedent (positive) occurrence of *(a\b)/c* matches the head of the [/L] inference which decomposes the type in its immediate subtypes *c* and *a\b*. The antecedent occurrence of *a\b*, in its turn, matches the head of the [\L] inference, leading to decomposition into the atoms *a* and *b*. There is no occasion for further unfolding here, as there are no connectives left. But the rightmost premise matches the Axiom scheme, instantiating *Z* as the head of the unfolded type (subtype *b*), and associating it with the application term *t(t')(t")* where *t'* and *t"* are the lambda terms computed for the argument subtypes *c* and *a* respectively. (We use *D,G,H,...* here as variables over non-empty sequences.)

$$[/L]\ \frac{H'\Rightarrow c{:}t' \qquad [\backslash L]\ \dfrac{D''\Rightarrow a{:}t'' \qquad \dfrac{\textbf{true}}{D',b{:}t(t')(t''),H''\Rightarrow Z}\{D'=H''=[],Z=b{:}t(t')(t'')\}}{D,a\backslash b{:}t(t'),H''\Rightarrow Z}\{D=(D',D'')\}}{D,(a\backslash b)/c{:}t,\ H\Rightarrow Z}\{H=(H',H'')\}$$

Partial deduction with respect to the two connectives in *(a\b)/c* thus yields the following derived rule of inference:

$$G,(a\backslash b)/c{:}t,H\Rightarrow b{:}t(t')(t'')\ \textbf{if}\ H\Rightarrow c{:}t'\ \textbf{and}\ G\Rightarrow a{:}t''$$

Partial deduction with respect to [R] *inferences*

The above case, where the atoms are reached by means of partial deduction with respect to just [L] inferences, covers *first order* types. In order to generalize the compilation technique to *higher order* types, we have to consider partial deduction with respect to the [R] rules. As an example, consider the unfolding of the third order type *(s/(np\s))\s:t*.

$$[\backslash L]\ \frac{[/R]\ \dfrac{D'',np\backslash s{:}v\Rightarrow s{:}t''}{D''\Rightarrow s/(np\backslash s){:}t'}\{t'=\lambda v.t''\} \qquad \dfrac{\textbf{true}}{D',s{:}t(t'),H\Rightarrow Z}\{D'=[],H=[],Z=s{:}b(t')\}}{D,(s/(np\backslash s))\backslash s{:}t,H\Rightarrow Z}\{D=(D',D'')\}$$

The first unfolding step generates the premise *D"⇒s/(np\s):t'*, with a complex goal type that can be further unfolded by partial execution of the relevant [R] inference. But now unfolding with respect to [R] introduces a new *positive* (antecedent) type, *np\s:v*, which in its own turn must be handed over to the unfolding transformation.

$$[\backslash L]\ \frac{G''\Rightarrow np{:}u \qquad \dfrac{\textbf{true}}{G',s{:}v(u),H\Rightarrow Z}\{Z=s{:}v(u),G'=[],H=[]\}}{G,\ np\backslash s{:}v,\ H\Rightarrow Z}\{G=(G',G'')\}$$

Note that one cannot perform the partial deduction of *np\s:v* within the premise *D",np\s:v⇒s:t"*, completing the unfolding of this premise as indicated below.

$$[\backslash L]\ \frac{D''\Rightarrow np{:}u \qquad s{:}v(u)\Rightarrow s{:}v(u)}{D'',np\backslash s{:}v\Rightarrow s{:}t''}\{t''=v(u)\}$$

In order to be sound, the unfolding transformation has to undo the binding of the D'' factor here. The premise $D'',np\backslash s{:}v \Rightarrow s{:}t''$ expresses the constraint that $s{:}t''$ has to be derived from the sequence $D'',np\backslash s{:}v$. It does *not* require $np\backslash s{:}v$ to be the head of this derivation — the head might occur in the D'' factor as well, as we will see in an example shortly.

Unfolding of higher order types, then, results in a *set* of derived inference rules: one of them yields the applicative term for the partial deduction of the immediate arguments of the type, the others represent the result of the partial deduction transformation for the types of bound variables corresponding to complex arguments[7]. In our example:

$$D,(s/(np\backslash s))\backslash s{:}t \Rightarrow s{:}t(\lambda v.t'') \textbf{ if } D,np\backslash s{:}v \Rightarrow s{:}t''.$$
$$G,\ np\backslash s{:}v \Rightarrow s{:}v(u) \textbf{ if } G \Rightarrow np{:}u.$$

SLD Resolution

We are in a position now to show how SLD resolution derives a goal sequent from the program database that results from the compilation by partial execution of the lexical atoms in this goal. As a first example, we look at an unambiguous sequent. Next we consider the case of an ambiguous sequent, and we show that the success branches of the SLD search tree are in a one-to-one correspondence with the readings.

The non-ambiguous example is the sequent

$$np{:}a,\ (s/(np\backslash s))\backslash s{:}b \Rightarrow s{:}Sem.$$

Partial deduction of the types of the antecedent expressions yields the following database of program clauses, where (1) is the unfolding of the atom $np{:}a$, and (2) and (3) of the third order type $(s/(np\backslash s))\backslash s{:}b$. (For ease of reference later on, we have numbered the positive (head) and negative (body) literals in the clauses separately. We refer to a clause with the number of its head.) Note that instead of the general axiom scheme $X{:}t \Rightarrow X{:}t$ we have the instance $np{:}a \Rightarrow np{:}a$. Instead of the [R] and [L] inference schemes, we have the clausal representation of the complex type built from (positive and negative occurrences of) the type-forming operators.

Database:

(1) $np{:}a \Rightarrow np{:}a.$

(2) $D,\ (s/(np\backslash s))\backslash s{:}b \Rightarrow s{:}b(\lambda v.t) \leftarrow$
(21) $\quad D,\ np\backslash s{:}v \Rightarrow s{:}t.$

(3) $H,\ np\backslash s{:}v \Rightarrow s{:}v(u) \leftarrow$
(31) $\quad H \Rightarrow np{:}u.$

From this database we want to derive the goal clause

(4) $\leftarrow np{:}a,\ (s/(np\backslash s))\backslash s{:}b \Rightarrow s{:}Sem.$

Below is the SLD search tree. (The unifying substitutions are indicated for the lambda term argument; we omit the matching for the string argument. The empty clause is written as '⊥')

	(4)	← np:a, (s/(np\s))\s:b⇒s:Sem.
Step 1	(2)	{Sem=b(λv.t)}
	(21)	← np:a, np\s:v⇒s:t.
Step 2	(3)	{t=v(u)}
	(31)	← np:a⇒np:u.
Step 3	(1)	{u=a}
	⊥	

Sem=b(λv.va)

From a *parsing as deduction* perspective, SLD resolution in $\mathbf{L}_C$ implements a *head driven* control regime. The head subtype of a complex type produces the head of the corresponding $\mathbf{L}_C$ clause $A \leftarrow B_1,...,B_n$; the argument subtypes yield the body $B_1,...,B_n$. Resolution then proceeds by parsing a phrase of type X on the basis of the unfolded clause $T \Rightarrow X \leftarrow B_1,...,B_n$ of the head of that phrase.

Let us turn now to the claim that in $\mathbf{L}_C$ we regain the one-to-one correspondence between proofs and readings: each SLD-derivation on the basis of the clausal form representation of the antecedent types produces a distinct reading. To establish the claim we inspect the two options for backtracking of the proof search algorithm:

- Find another match for the same program clause $A \leftarrow B_1,...,B_n$. This results in a different instantiation for the premise sequents $B_1,...,B_n$, hence a different lambda term for the derivation.
- Select a different program clause to resolve against. Since each program clause is the partial deduction for a different atom, this also results in a different term.

We illustrate these options with the ambiguous sequent

s/(np\s):a, (s/(np\s))\s:b⇒s:Sem.

The program database consists of the following four inference rules, where (1) and (2) represent the partial execution of *s/(np\s):a*, and (3) and (4) the partial execution of *(s/(np\s))\s:b*.

(1) s/(np\s):a, F⇒s:a(λv.t) ←
(11) np:v, F⇒s:t.

(2) np:v⇒np:v.

(3) D, (s/(np\s))\s:b⇒s:b(λw.t') ←
(31) D, np\s:w⇒s:t'.

(4) H, np\s:w⇒s:w(u) ←
(41) H⇒np:u.

We want to derive the following goal clause from the database of program clauses:

(5) ← s/(np\s):a, (s/(np\s))\s:b⇒s:Sem.

The SLD search tree contains two solutions. The branching occurs at the top level, because the goal sequent (5) resolves against both (1) and (3).

(5) ← s/(np\s):a, (s/(np\s))\s:b⇒s:Sem.

(1) {Sem=a(λv.t)}	(3) {Sem=b(λw.t')}
(11) ← np:v,(s/(np\s))\s:b⇒s:t	(31) ← s/(np\s):a,np\s:w⇒s:t'
(3) {t=b(λw.t')}	(1) {t'=a(λv.t)}
(31) ← np:v,np\s:w⇒s:t'	(11) ← np:v,np\s:w⇒s:t
(4) {t'=w(u)}	(4) {t=w(u)}
(41) ← np:v⇒np:u	(41) np:v⇒np:u
(2) {u=v}	(2) {u=v}
⊥	⊥
Sem = a(λv.b(λw.wv))	Sem = b(λw.a(λv.wv))

3.3. Complexity

Consider the effect of the unfolding transformation on the two dimensions of the search space.

Depth The maximal depth of a standard cut-free **L** derivation equals the complexity degree of the goal sequent, since each inference step removes an occurrence of a type-forming operator. $\mathbf{L}_C$ reduces the depth of the search tree, because the unfolding transformations replace subproofs of depth *n* consisting of primitive [L],[R] inferences by subproofs of depth 1 with *n* premises.

Width The horizontal expansion (*ramification*) of the search tree is determined by the number of resolvents that can be formed for a given goal sequent, i.e. the different ways of matching the goal sequent against the head of a program clause for the derivability relation '⇒'. Imposing more stringent constraints in the program clauses has a decreasing effect on the ramification — it reduces the number of possible matches in forming resolvents. Compare **L** and $\mathbf{L}_C$ in this respect.

Example. Consider the goal sequent below (which is underivable!)

((np\s)/pp)/np:put,np/n:a,n:book,pp/np:on,np/n:the,n:table⇒s:Sem

Each of the four functors in this sequent matches the head of the primitive [/L] inference below, with one or more unifying substitutions for the premise sequent $T \Rightarrow B{:}t'$. There is a total of 12 immediate descendants for this node in the search tree. Only in the *third* generation failure will be detected — when it turns out to be impossible to form a resolvent with respect to the *np\s* subtype which has no argument sequence to its left.

$$\frac{T \Rightarrow Y{:}t' \qquad U,A{:}tt',V \Rightarrow C}{U,A/B{:}t,T,V \Rightarrow C}[/L]$$

In the $\mathbf{L}_C$ version of the problem, the goal sequent immediately fails: it does not match the only eligible inference rule:

$$\frac{L \Rightarrow np{:}t_1 \qquad G \Rightarrow pp{:}t_2 \qquad D \Rightarrow np{:}t_3}{D,((np\backslash s)/pp)/np{:}put,L,G \Rightarrow s{:}put(t_1)(t_2)(t_3)}$$

Lexical ambiguity As we have remarked elsewhere (Moortgat 1988), lexical ambiguity causes a dramatic explosion of the ramification at the very root of the search space. If we want to find out under what lexical type assignment a sequence of morphemes $m_1,\ldots,m_n$ derives a goal type X, we have to consider the total $a_1 \times\ldots\times a_n$ of possible type combinations for the antecedent, where a_i is the number of initial lexical type assignments to m_i. For incompatible type combinations (i.e. combinations that do not derive the goal type X), one will have to exhaustively explore the relevant portion of the search space in order to detect failure. Pruning on the basis of count invariance partially alleviates this problem: one can discard type combinations that violate the count balance. But incompatible type combinations that pass the count test (but violate directionality requirements, for example) can only be rejected by a failing attempt to build a proof. A naive backtracking regime for trying out the possible type combinations will be highly inefficient.

$\mathbf{L}_C$ avoids this problem by providing a natural 'parallel' treatment of lexical ambiguity. The $\mathbf{L}_C$ database for a sequent with lexically ambiguous atoms simply contains the unfolded clausal form representations of the distinct type assignments to these atoms. The SLD derivation leaves the clauses of irrelevant type assignments unused.

4. L_C and proof nets for Linear Logic

There is a close connection between the unfolding transformation discussed above and the *proof nets* of Linear Logic (Girard 1987) adapted to the case of non-commutative **L** in Roorda (1990). In this section, we will briefly establish this correspondence. The data structures manipulated in the construction of proof nets and in the unfolding transformation of the preceding paragraph differ in one respect: the unfolded clauses carry a *string argument* (i.e. the factorization of the antecedent) which keeps track of the string covered by an atom. The string argument is absent from the proof nets. We will show that by adding the string argument to the proof net data structure, we can directly characterize the derivable lambda terms corresponding to well-formed proof nets, thus avoiding problems of overgeneration with the more parsimonious types *cum* lambda terms representation.

The proof nets of Linear Logic are designed to remove the redundancy from sequent derivations by providing an ambiguity-free representation of a proof. A proof net is defined in terms of a wider class of graphs, viz. *proof structures*. A proof structure is a graph constructed from two components: *axiom links* and *logical links*. The logical links (cf. the logical rules of sequent calculus) recursively decompose a type of arbitrary complexity into its atomic subtypes. As in the sequent case, we distinguish unfolding of *antecedent* occurrences (the [L] rules) and unfolding of *succedent* occurrences (the [R] rules). The axiom links (corresponding to axiom sequents *X:Term⇒X:Term*) cancel out a succedent occurrence of a literal against a matching antecedent occurrence, thus directly implementing the *count* balance. Let us refer to literals with opposite signature as *conjugates*. The term construction algorithm of Section 2 can be straightforwardly added to the unfolding rules, associating a proof net with the corresponding lambda term. We give a slightly adapted presentation of Roorda's formulation of **L** below. To highlight the correspondence with the preceding paragraph, the rules for antecedent occurrences are formulated so as to operate on unmarked types *X*, the rules for succedent occurrences — the negated goal types of our SLD refutations — on marked types *~X*. Note that the inert context sequences that have to be copied unchanged in the course of a regular sequent derivation are absent from the proof net construction rules.

Axiom links: $\overline{A{:}t \qquad {\sim}A{:}t}$ $\qquad$ $\overline{{\sim}A{:}t \qquad A{:}t}$

Logical links:

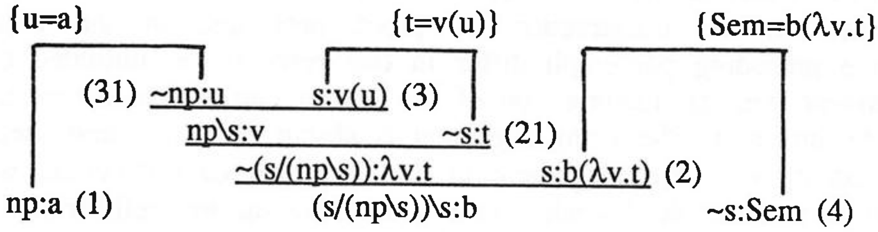
$$[/L]\ \frac{A{:}t(t') \qquad {\sim}B{:}t'}{A/B{:}t} \qquad\qquad \frac{B{:}v \qquad {\sim}A{:}t}{{\sim}(A/B){:}\lambda v.t}\ [/R]$$

$$[\backslash R]\ \frac{{\sim}A{:}t \qquad B{:}v}{{\sim}(B\backslash A){:}\lambda v.t} \qquad\qquad \frac{{\sim}B{:}t' \qquad A{:}t(t')}{B\backslash A{:}t}\ [\backslash L]$$

The attentive reader will observe the symmetry of the premise ordering in the display of the logical rules. The major result of Roorda (1990) is a strengthening of the count invariant which exploits this symmetry: in the case of non-commutative **L**, a well-formed proof net must be *planar*, i.e. the axiom links may not cross.

In order to establish the correspondence between $\mathbf{L}_C$ and Roorda's proof nets for **L**, we construct the proof net for the sequent derivation given in Section 3.2:

np:a, (s/(np\s))\s:b⇒s:Sem



Axiom links:

Step 1: (4)+(2) {Sem=b(λv.t)}
Step 2: (21)+(3) {t=v(u)}
Step 3: (31)+(1) {u=a}

One notes the one-to-one correspondence between nodes of the proof net and the literals in the clausal representation of the types, and the one-to-one correspondence between resolution steps and axiom links.

We remarked above that the SLD resolution proof search produces a *head driven* parsing algorithm. In the case of the proof net representation of sequent derivations, the head driven construction of axiom links is not enough to implement the planarity constraint. The planarity constraint suggests a straightforward resolution strategy based on a non-deterministic push-down automaton with the following transitions.

proof_net/2: proof_net(Atoms,Stack)

Shift-reduce machine to check the planarity constraint:

```
proof_net([],[]).
proof_net([X|T],[Y|R]):-          Reduce
    conjugates(X,Y),
    proof_net(T,R).
proof_net([X|T],Stack):-          Shift
    proof_net(T,[X|Stack]).
```

The shift-reduce automaton tries to reduce an ordered list of literals to the empty list (cf. derivation of the empty clause in the SLD resolution refutation). The automaton has to make a choice from two options. When the input literal is a conjugate of the top literal on the stack, the stack is popped and search continues under the unifying substitution used to match the conjugates. The second option is to push the input literal on the stack. The automaton is non-deterministic: the shift option can always be taken, also when the reduce option applies. The non-deterministic action makes it possible to associate a truly ambiguous sequent with its possible readings. The example below is a simple illustration. The displayed sequence of literals is the yield of the proof structure corresponding to the unfolding of the sequent *a/a:x,a:y,a\a:z⇒a:Term.*

a:x(t),~a:t,a:y,~a:t',a:z(t'),~a:Term

With a reduce preference, the automaton will match the literals *~a:t* and *a:y*, which ultimately produces the reading {*Term=z(x(y))*}. With a shift preference, this opportunity for a reduction can be skipped. But in order to accept the input with empty stack, *a:y* and *~a:t'* must then be reduced, producing the reading {*Term=x(z(y))*}.

If the planarity condition would completely determine the well-formed proof nets for **L**, the above shift-reduce automaton would provide an efficient parsing algorithm to compute the possible axiom links over the (precompiled) unfolding of the antecedent types, just like the SLD resolution procedure does for the clausal version of the problem. Unfortunately, the planarity condition, like the count invariant, is a necessary but not a sufficient condition in the characterization of proof nets. A well-formed proof net is inductively defined from the full set of the proof structures by the following clauses (Girard, Lafont & Taylor 1989) which have to be satisfied (in the case of **L**) in addition to the planarity constraint.

- Axiom links are proof nets.
- An [L] proof structure forms a proof net if its two premises are the conclusions of distinct proof nets N, N' (cf. the *two-premise* sequent rules).
- An [R] proof structure forms a proof net if its two premises are the conclusions of the same proof net N (cf. the *one-premise* sequent rules).

Although the proof nets for **L** are redundancy-free representations of sequent proofs, the insufficiency of the planarity constraint leaves open the computational problem of *checking* the constraints on the [L] and [R] structures. Roorda (1990) shows that if one omits this check, the following cases of overgeneration can occur:

- lambda terms with closed subterms (corresponding to deductions from an empty antecedent in the sequent derivations);
- terms with vacuous lambda abstraction;
- terms that leave antecedent terms unused.

We can avoid this overgeneration by making the correspondence between $\mathbf{L}_C$ and **L** proof nets complete, i.e. by adding a *string argument* to the proof net construction rules. The augmented data-structures are given below.

Proof structure rules with added string factorization argument

The nodes in the links are extended to *triples* Type:Term:String, corresponding to the *⇒(Type,Term,String)* representation of $\mathbf{L}_C$. The String parameters *F,G* stand for non-empty strings (lists [Head|Tail] in the Prolog notation); as in the previous section, the concatenation of strings *F* and *G* is written as *(F,G)*.

$$[/L]\frac{A{:}t(t'){:}(F,G) \qquad \sim B{:}t'{:}G}{A/B{:}t{:}F} \qquad\qquad \frac{B{:}v{:}[v] \qquad \sim A{:}t{:}(F,[v])}{\sim(A/B){:}\lambda v.t{:}F}[/R]$$

$$[\backslash R]\frac{\sim A{:}t{:}([v],F) \qquad B{:}v{:}[v]}{\sim(B\backslash A){:}\lambda v.t{:}F} \qquad\qquad \frac{\sim B{:}t'{:}G \qquad A{:}t(t'){:}(G,F)}{B\backslash A{:}t{:}F}[\backslash L]$$

Let us investigate how the string argument takes care of the conditions on the [L],[R] proof structure rules.

Vacuous abstraction

Consider the sequent

(pp/np)/(pp/np):vlak,pp/np:naast,np:jan⇒pp:Term

('just next-to John', where *vlak* is a *pp/np* modifier, rather than a *pp* modifier.) Without the string argument, a planar proof structure can be built resulting in a case of vacuous abstraction. (1) to (8) below is a vertical display of the yield of the proof structure prior to the construction of the axiom links. Linking (2) and (3) produces the unifying substitution {t'=v}, (4) and (5) match with substitution {t=naast(t")}, (6) and (7) with {t"=jan}, and finally (1) and (8) with {Term = vlak(λv.t)(t')}. Composition of the substitutions produces the reading {Term = vlak(λv.naast(jan))(v)}.

(1) pp:vlak(λv.t)(t')
(2) ~np:t'
(3) np:v
(4) ~pp:t
(5) pp:naast(t")
(6) ~np:t"
(7) np:jan
(8) ~pp:Term

Term = vlak(λv.naast(jan))(v)

Adding the string argument to the proof structure unfolding rules out this term. Matching of (2) and (3) produces the unifying substitution {t'=v, Q=[v]}. Matching of (4) and (5) gives {t=naast(t"), P=[naast], R=[v]}. The binding of R then blocks matching of (6) and (7). The **L** derivable reading {Term = vlak(λv.naast(v))(jan)} is obtainable by matching (2) and (7) under the unifying substitution {t'=jan, Q=[jan]}, (3) and (6) with {t"=v, R=[v]}, (4) and (5) with {t=naast(t"), P=[naast]}, and finally (1) and (8). Of course, this reading is also obtainable on the basis of the more parsimonious Type:Term representation.

(1) pp:vlak(λv.t)(t'):([vlak],P,Q)
(2) ~np:t':Q
(3) np:v:[v]
(4) ~pp:t:(P,[v])
(5) pp:naast(t"):([naast],R)
(6) ~np:t":R
(7) np:jan:[jan]
(8) ~pp:Term:[vlak,naast,jan]

Term = vlak(λv.naast(v))(jan)

Closed subterms

Consider the sequent

np/(n\((np/n)\np)):pegasus⇒np:Term

A reading with closed subterm is derivable in the proof net representation without the string parameter. Matching (2) and (3) gives the substitution {t=w(t')}, (4) matches (5) with {t'=v}, and (1) matches (6) with {Term=pegasus(λvλw.t)}.

(1) np:pegasus(λvλw.t)
(2) ~np:t
(3) np:w(t')
(4) ~n:t'
(5) n:v
(6) ~np:Term

Term=pegasus(λvλw.w(v))

There is no derivation under the enriched representation, as the reader can easily check. (2) matches (3) under the unifying substitution {t=w(t'), Q=([v],P)}. But now the identification of the string parameter blocks the matching of (4) and (5): Q=([v],P) does not unify with [v]. Similarly for other attempts to complete the axiom links.

(1) np:pegasus(λvλw.t):([pegasus],P)
(2) ~np:t:([w],[v],P)
(3) np:w(t'):([w],Q)
(4) ~n:t':Q
(5) n:v:[v]
(6) ~np:Term:[pegasus]

Unused antecedent terms The rejection of this case of overgeneration is immediate. For a sequent $A_1{:}t_1,\ldots,A_n{:}t_n \Rightarrow B{:}Term$, the goal type will be represented as $\sim B{:}Term{:}\ [t_1,\ldots,t_n]$, i.e. the antecedent terms $t_1,\ldots,t_n$ all have to be used in the derivation of the lambda term associated with *B*. We leave it to the reader to construct an example.

We can summarize our findings as follows. By adding a string factorization argument to the proof net data structure, we make the correspondence between the proof nets and the clausal form unfolding discussed in the previous section complete. The string argument plays the role of the factorized antecedent: it keeps track of the antecedent terms out of which the succedent lambda term has to be built. Metalogical checking of the lambda terms (to throw out **L**-underivable readings that pass the planarity test) can be avoided. The construction of the proof net axiom links is reduced to resolution on the yield of the proof structure for the unfolded types of the goal sequent.

Notes

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[1] The lambda term construction algorithm can be extended to product types by extending the semantic algebra with pairing and projection, as shown in Van Benthem (1989). We will ignore explicit product types, however.

$$\frac{P \Rightarrow A{:}t \qquad Q \Rightarrow B{:}t'}{P,Q \Rightarrow A \bullet B{:}\langle t,t' \rangle} \qquad \frac{U,A{:}\pi_1(t),B{:}\pi_2(t),V \Rightarrow C}{U,A \bullet B{:}t,V \Rightarrow C}$$

[2] For the directional systems under consideration the Curry-Howard correspondence turns up as an isomorphism *modulo* directionality, unless one distinguishes two directional forms of lambda abstraction as proposed e.g. in Buszkowski (1988).

[3] For a detailed account of the cut-elimination as normalization issue in terms of the internal language of a multicategory, the reader is referred to Lambek (1990). See also Hepple (1990) for normal form theorem proving in **L**. Wansing (1989) discusses the 'formulas-as-types' perspective for a hierarchy of categorial logics.

[4] Although König discusses the readings of **L** derivations without reference to their lambda terms, her auxiliary notion of a 'syntax tree' associated with a proof has essentially all the relevant structural properties of the lambda terms.

[5] For a closely related proposal, the reader should consult Heylen (1990). The proof tables presented in this paper offer a *constraint-propagation* parsing procedure for **L**, based on a conjunctive normal form decomposition of types.

Also related to our proposal is Pareschi's (1989) Definite Clause version of categorial systems in an extension of Prolog ('Hereditary Harrop Logic', *hhl*). An essential difference is that we directly axiomatize **L** derivability, whereas the *hhl* proof procedure has full access to the structural rules, and thus, for example, has to throw out cases of vacuous λ-abstraction (which are underivable in **L** due to the lack of structural rules) by means of metalogical tests.

[6] Strictly speaking, we should distinguish a syntactic atom (e.g. the string 'love') and the associated constant in the semantics, say **love'**. Here and below, we simplify the exposition by conflating these two.

[7] The λ bound variables introduced in the unfolding of a higher order type are taken to be expressions the occurrence of which is limited to the clauses resulting from the unfolding (technically, GENSYM generated expressions). The shared λ bound variables, in other words, tie together the set of clauses one obtains when unfolding a higher order type.

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EPISTEMIC SEMANTICS FOR COUNTERFACTUALS

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§ 1 INTRODUCTION

The semantic idea underlying most accounts of conditional sentences goes back to a suggestion by Frank Ramsey. As elaborated by Stalnaker in "A Theory of Conditionals", Ramsey's rule for evaluating conditionals is this:

...first, add the antecedent (hypothetically) to your stock of beliefs; second, make whatever adjustments are required to maintain consistency (without modifying the hypothetical belief in the antecedent); finally, consider whether or not the consequent is then true.

Stalnaker and Lewis fixed Ramsey's suggestion into possible world semantics in effect by turning it into truth conditions. More natural, perhaps, would be to interpret it epistemically, as a belief or acceptability condition: A conditional is to be accepted in a particular body of beliefs just incase adjusting those beliefs so as to accomodate its antecedent would result in belief in its consequent. Where the conditional is a counterfactual the adjustments to be made in accomodating its

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antecedent would involve *revision*, or suspending old beliefs so as to make way for new.

Relatively recently some philosophers have become interested in revision at a quite abstract level, seeking general methods for revising bodies of information and general constraints on the rationality of these methods. One such constraint is already implicit in Stalnaker's paraphrase of the Ramsey rule, where adjustments are to be made in order to maintain consistency. It would be irrational to start believing just anything on learning something new, unless of course the new belief is itself contradictory.

In order to study the formal properties of belief revision, Peter Gärdenfors has introduced the notion of belief revision models, which are systems of belief states together with revision functions saying how to revise these so as to accomodate new beliefs.The Ramsey rule provides an obvious interpretation for conditionals in belief revision models, and it might be hoped that they can be given a natural epistemic semantics in this way. Gärdenfors seemed however to definitively dash any such hopes in his "Belief Revisions and the Ramsey Test for Conditionals", which claims that the Ramsey rule is incompatible with any form of belief revision which is, in a certain very weak sense, rational.

After presenting a version of the Gärdenfors impossibility result which makes even weaker assumptions on belief revision, I will show in section 4 how belief revision models can be constructed which satisfy the Ramsey rule and all of the (weakened) rationality requirements, thus providing a natural epistemic semantics for unnested conditionals. In view of the theorem something has to go, and it will turn out that what these models do not satisfy is an additional and peripheral requirement which Gärdenfors places on belief revision models. To show the compatibility of some of a number of incompatible things is of course not to discredit the remainder, so the model construction does nothing to discredit this additional assumption. That is what I will try to do in section 5. Section 6 gives a syntactic characterisation of the conditional logic generated by these models, which is both weaker and stronger than David Lewis conditional logic VC. In the final section 7 the epistemic semantics contributes a solution to two problems for most earlier accounts of the semantics of counterfactuals: Tichy's man with the hat and Hansson's man with the hamburger.

§ 2 REQUIREMENTS ON BELIEF REVISION

For the meantime, beliefs are to be modelled as the sentences ϕ of a formal language L, namely propositional logic augmented with a conditional connective >. L_0 refers to the descriptive part of L, or that in which > does not appear. L_1 refers to that part of L where > is nested to a depth of at most 1. Bodies of beliefs are to be modelled as *belief sets* κ, these being sets of sentences closed under some monotonic background entailment notion $\Vdash$, which as in Gärdenfors is assumed at least as strong as classical entailment and to satisfy the deduction theorem.
The revision of beliefs is to be modelled by means of a revision function +, which maps belief sets and sentences onto sets of belief sets. Intuitively, the belief sets in $\kappa+\phi$ are different candidates for the result of revising κ with ϕ. Revision functions which do not admit different candidates, taking only singletons as values, are said to be *deterministic*. A belief revision model is a pair $\langle K,+\rangle$ of which K is a set of belief sets, and + a revision function defined on K.
Now here are three requirements on belief revision, which are non-deterministic restrictions to the descriptive language of the assumptions made by Gärdenfors in his original proof of the impossibility theorem. The first speaks for itself:

I SUCCESS For $\phi \in L_0$: $\phi \in \bigcap(\kappa+\phi)$.

The result of revising a body of beliefs so as to accomodate some new belief always includes that belief. This requirement must be satisfied by any belief revision model which conforms to the Ramsey rule, as adjustments made in maintaining consistency may not modify your belief in an antecedent you have hypothetically added to your stock of beliefs. The second requirement may be in need of more explication:

II EXPANSION For $\phi, \psi \in L_0$: $\psi \in \kappa$ and $\sim\phi \notin \kappa \Rightarrow \psi \in \bigcap(\kappa+\phi)$.

Expansion says that in accomodating a new descriptive belief which is compatible with all of the old ones, none of the old ones may be thrown away. It makes revisions minimal by preventing unneccessary loss of beliefs. One strong argument for expansion is that when it comes to accomodating compatible beliefs, revision should parallel Bayesian conditionalisation, which leaves the probability of a proposition unchanged once it has been assigned unit prior probability, that is to say accepted. Expansion, then, is a *rationality* requirement, constraining revision to behave like conditionalisation wherever possible.

The final requirement also concerns the minimality of revisions, but from the opposite direction. It makes revisions minimal by keeping out unneccessary new beliefs:

III PRESERVATION For $\phi \in L_0$: $\bot \notin \kappa$ and $\phi \nVdash \bot \Rightarrow \bot \notin \bigcap(\kappa + \phi)$.

One should not end up believing anything at all on accomodating a new belief unless the new belief is contradictory (in which case success makes this unavoidable). Like success this is required by the Ramsey rule, which asks that after adding an antecedent such minimal adjustments be made as are needed to maintain consistency. In fact in this non-deterministic formulation consistency preservation is weaker than my paraphrase would suggest, since all that is required is that not all belief sets which result from a revision be inconsistent.

These requirements are not absolute in the sense that all reasonable belief revision must satisfy them. Success might for example reasonably be suspended in a context where some older beliefs, perhaps beliefs about the laws of nature, are allowed to override incoming information which contradicts them. But the impossibility theorem of the next section does not require them to be absolute in this sense. All the theorem requires in order to do its damage is that the hypothetical belief revision presupposed by the Ramsey rule should conform to them, and this I think is incontestable.

§ 3 THE GÄRDENFORS IMPOSSIBILITY THEOREM

Interpreted epistemically as a belief condition for conditionals, the Ramsey rule places the following constraint on the interaction between conditionals and revision:

IV RAMSEY RULE For $\phi, \chi \in L_0$: $\phi > \chi \in \kappa \Leftrightarrow \chi \in \bigcap(\kappa + \phi)$.

There is one final requirement which Gärdenfors places on belief revision models. Defining the expansion κ/ϕ of κ by $\phi \in L_0$ to be the theory $\{\psi: \kappa \cup \{\phi\} \Vdash \psi\}$, models are assumed to satisfy an

V ADDITIONAL ASSUMPTION

If <K,+> is a belief revision model and $\kappa \in K$, then for all $\phi \in L_0$, $\kappa/\phi \in K$.

It is at this point worth noting that the additional assumption differs from all of the other assumptions made up until now in not obviously being required by the Ramsey rule, or for the rationality of belief revision.

1 EXAMPLE Let $K = \{L_1\}$, and let $+ = \{(L_1, \phi, \{L_1\}): \phi \in L_0\}$. Thus K contains just the inconsistent belief set, and + revises this belief set in the obvious way, by leaving it as it is. Then it may easily be verified that each of assumptions I - V is trivially satisfied by the belief revision model <K,+>.

The above model is of course uninteresting, and it is to be hoped that we can model less trivial systems of beliefs than this, with more realistic dynamics. The following notion of *nontriviality* provides a minimal requirement on belief revision models which excludes models like the above:

2 DEFINITION A belief revision model <K,+> is said to be *nontrivial* if there are sentences $\phi, \psi, \chi \in L_0$ and some $\kappa \in K$, such that while ϕ, ψ and χ are jointly inconsistent (that is to say $\phi, \psi, \chi \Vdash \bot$), any two of them are consistent with κ (thus $\sim(\phi \wedge \psi) \notin \kappa$, and so on).

To get a feeling for this definition, it is worth verifying that the model of example 1 is indeed trivial in this technical sense. More motivation for the notion of nontriviality is provided by the following two examples, the first of which shows that if we were not interested in conditionals and were to drop the Ramsey rule. then it would be very easy indeed to find nontrivial belief revision models:

3 EXAMPLE Let K be the set $\{T \subseteq L_0 : Cn(T) = T\}$ of classical L_0 theories. Take classical entailment as $\Vdash$, and define for each $\kappa \in K$ and $\phi \in L_0$:

$\kappa+\phi = \{\kappa\}$ if $\bot \in \kappa$.

$\kappa+\phi = \{\kappa/\phi\}$ if $\bot \notin \kappa$ and $\sim\phi \notin \kappa$,

$\kappa+\phi = \{Cn\{\phi\}\}$ if $\bot \notin \kappa$ and $\sim\phi \in \kappa$,

Now <K,+> satisfies assumptions I, II, III and V. Furthermore, assuming the language contains at least two atoms p and q it may easily be verified that the belief set Cn{ } and the descriptive sentences p, q and $\sim(p \wedge q)$ together establish the nontriviality of <K,+>.

The following is an example of a nontrivial model satisfying I, II, IV, and V:

4 EXAMPLE Let K be the set $\{T \subseteq L_0: Cn(T) = T, A \subseteq T\}$ of classical L_0 theories, and identify > with material implication. Take classical entailment as $\Vdash$, and define for each $\kappa \in K$ and $\phi \in L_0$: $\kappa+\phi = \{\kappa/\phi\}$.
It is obvious that I and II are satisfied, and IV is just the the deduction theorem for classical logic. V is satisfied since + is defined for all κ and ϕ. Nontriviality follows as in example 3.

From the above examples it is clear that given a language in which negation and conjunction may be expressed, nothing more is required for nontriviality than two logically independent contingent sentences and some belief set which does not contain anything contingent about them. One is regularly ignorant about any number of different matters, so nontriviality is clearly a minimal requirement if belief revision models are supposed to model what people actually do. Nontriviality constrains belief revision models to be realistic.

We are now in a position to state and prove Gärdenfors' impossibility result for belief revision models satisfying the Ramsey rule. This result shows that it is not accidental that the the model of example 1 is trivial, or that the model of example 3 does not satisfy IV, or that the model of example 4 does not satisfy III. In fact there are no nontrivial belief revision models at all satisfying I through to V. As such, Gärdenfors' result would seem to dash any hopes of giving a realistic epistemic semantics for conditionals. Either the belief revision is irrational or the models are trivial, so you take your pick. The proof given here is essentially that of Segerberg in "A Note on an Impossibility Theorem of Gärdenfors".

5 THEOREM
No nontrivial belief revision model satisfies I - V.

Proof:
Suppose for the contradiction that $\phi, \psi, \chi \in L_0$ and κ establish the nontriviality of a belief revision model.
As $\sim(\phi\wedge\chi) \notin \kappa$, by the deduction theorem $\sim\chi \notin \kappa/\phi$. Applying expansion to this set, which assumption V assures us to be in the model, we have
$\phi \in \bigcap(\kappa/\phi+\chi)$.
The Ramsey Rule applied to κ/ϕ then gives $\chi>\phi \in \kappa/\phi \subseteq \kappa/\phi\wedge\psi$. So
(i) $\chi>\phi \in \kappa/\phi\wedge\psi$
Swapping ϕ's and ψ's in this argument (our assumptions about these sentences are the same), we obtain:

(ii) $\chi{>}\psi \in \kappa/\phi\wedge\psi$

Now applying the Ramsey Rule to $\kappa/\phi\wedge\psi$, which assumption V assures us to be in the model, (i) and (ii) inform us that each belief set in the result of updating this set with χ contains both ϕ and ψ. By Success we also get χ, so

$$\{\phi, \psi, \chi\} \quad \subseteq \quad \bigcap((\kappa/\phi\wedge\psi) + \chi)$$

By the choice of ϕ, ψ, χ and the monotonicity[1] of $\Vdash$, we then have $\bigcap((\kappa/\phi\wedge\psi) + \chi) \Vdash \bot$, which by the closure of belief sets under $\Vdash$ gives us

$$\bot \in \bigcap((\kappa/\phi\wedge\psi) + \chi)$$

This is however not in accordance with the assumption that + satisfies consistency preservation, since by the pairwise compatibility of ϕ, ψ and χ with κ and by the monotonicity of $\Vdash$ we have:

$\kappa/\phi\wedge\psi \nVdash \bot$, so that $\bot \notin \kappa/\phi\wedge\psi$, and $\chi \nVdash \bot$.

We have thus obtained a contradiction, and proved Gärdenfors' result.

It is worth noting that although the letter of the above nontriviality notion is stricter than that of Gärdenfors, which does not require the sentences establishing nontriviality to be descriptive, in spirit his and my nontriviality requirements are the same. Indeed, if the nontriviality of a belief system were to show up only in conditional sentences then it would still be uninteresting as a model of beliefs and their dynamics, whether technically speaking trivial or not. The remaining assumptions in the above are weaker than Gärdenfors' original ones. For one thing, whereas Gärdenfors assumes I - V for the whole language, with unlimited nesting of >, in the above they are assumed only for descriptive ϕ, ψ and χ. Secondly, Gärdenfors assumes the revision function + to be deterministic. Essentially, then, the above version of the theorem is stronger than the original, and this in two instructive ways: it shows that triviality closes in on us even before we come to consider nested conditionals. And it shows that whatever is causing the triviality, it is not deterministic revision.[2]

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[1] It is interesting that very little of the monotonicity is in fact used here. All that is needed is that a set be inconsistent whenever one of its subsets is.

[2] Contrast this with Lewis' trivialisation result in "Conditional Probabilities and the Probabilities of Conditionals", which does make essential use of nested conditionals.

Also worthy of note is that an impossibility theorem can still be proved if assumption V is replaced by the weaker assumption that, for any κ and ϕ, there is among the belief sets containing κ and ϕ some belief set which is included in all the others. Assumption V of course equates this minimal belief set with κ/ϕ.

So then what is causing the trouble? In the period since the result was published various suggestions have been made. It has been argued that it is the Ramsey rule which is at fault, and that the result requires its modification so as to ban conditionals from belief sets. Belief in them ought not be modelled as elementhood in belief sets. Others have argued that the biconditional of the Ramsey rule is not appropriate, or have sought to place the responsibility with the monotonicity of the notion of logical consequence.
The impossibility theorem has however proved surprisingly robust. For example, in "Variations on the Ramsey Test: More triviality results" Gärdenfors shows how any number of variants of the Ramsey rule succumb to one or the other form of triviality, while in "The Gärdenfors Impossibility Theorem in Nonmonotonic Contexts" Makinson shows that allowing a nonmonotonic background entailment notion doesn't help either.
I claim that responsibility for the triviality is not to be placed with the Ramsey rule, or with any of the minimal requirements on rational revision. To support this, belief revision systems which satisfy all of assumptions I to IV will be constructed in the next section. The only assumption of the impossiblity theorem which they do not satisfy is the additional assumption, V, that belief revision models be closed under expansions. The case for giving up this assumption instead of any of the others is presented in section 5.

§ 4 TWO WAYS OF CONSTRUCTING BELIEF REVISION MODELS

Below I have given two ways of getting nontrivial belief revision models, which differ in the bodies of beliefs they contain. In the first the bodies of belief are just sets of sentences, or belief sets. I have not included these models because I think they contribute to an understanding of the dynamics of knowledge or belief. They do not. It is just that it is very easy to get them and to see that they have the right properties. They show there is nothing mysterious or tricky about nontrivial belief revision models satisfying the Ramsey rule. The second and more instructive construction borrows ideas familiar from the conditional logics of Stalnaker and Lewis, representing belief states as sets of possible worlds and interpreting conditionals by means of selection functions. Here there is more work to be done, imposing suitable restrictions on selection functions and making them in a sense context dependent so that the chosen revision method, imaging, gets the required formal properties.

4.1 THE EASY WAY

What follows is nothing more than an elaboration of example 3 above.

6 EXAMPLE Let M be the set $\{\mu \subseteq L_0: Cn(\mu) = \mu\}$ of classical L_0 theories. Take classical entailment as ⊫, and define for each $\mu \in M$ and $\phi \in L_0$:

$$\begin{array}{lll} \mu*\phi = \{\mu\} & \text{if} & \bot \in \mu \\ \mu*\phi = \{\mu/\phi\} & \text{if} & \bot \notin \mu \text{ and } {\sim}\phi \notin \mu \\ \mu*\phi = \{Cn\{\phi\}\} & \text{if} & \bot \notin \mu \text{ and } {\sim}\phi \in \mu. \end{array}$$

We have in <M,*> a belief revision model defined just for L_0 sentences which satisfies assumptions I, II and III for the same reasons as the almost identical model in example 3. But it doesn't satisfy IV, the Ramsey rule, as there are no conditional sentences describing the belief sets' revision behaviour. So we'll put some there. Let:

$$\begin{array}{lll} \kappa_\mu & = & Cn(\mu \cup \{\phi > \chi : \chi \in \mu*\phi\}), \text{ for each } \mu \in M. \\ K & = & \{\kappa_\mu : \mu \in M\}, \text{ and} \\ + & = & \{(\kappa_\mu, \phi, \kappa_{\mu*\phi}) : \mu \in M\}. \end{array}$$

Now <K,+> satisfies assumptions I, II and III just as <M,*> does, since adding a layer of conditionals does nothing to disturb this. In addition we have made it satisfy IV. Furthermore, assuming the language contains at least two atoms p and q it may easily be verified that the belief set Cn{ } and the descriptive sentences p, q and $\sim(p \wedge q)$ together establish the nontriviality of <K,+>. It is well worth verifiying that although <M,*> does satisfy assumption V, the same argument cannot be used to show that its successor <K,+> does too. As indeed it may not according to the trivialisation result.

4.2 AND ANOTHER WAY

Bodies of Belief

The bodies of belief or *belief states* in the following belief revision models are sets of L_1 interpretations. Letting At be the set of L's atoms, the set W of L_0 interpretations is defined as $\wp(At) \cup \{w_\perp\}$. This $w_\perp$ is the *absurd world* : $w_\perp \models \phi$ for every sentence $\phi \in L$. The interpretation of L_0 in the other, non-absurd worlds is standard: for $p \in At$ and $w \in W \setminus w_\perp$ we have $w \models p$ iff $p \in w$, the usual clauses for ~ and $\wedge$ extending $\models$ to the rest of L_0. In order to interpret the remainder of L_1, which involves >, we make use of Stalnaker's *selection functions*, *s*. These are functions selecting, for each $w \in W$ and each $\phi \in L_0$, some subset of W. By way of motivation, $s(w,\phi)$ may be thought of as the set of those worlds which are most like w, but where ϕ holds. In view of this motivation, it is reasonable to require selection functions to satisfy *centering:* $s(w,\phi)=\{w\}$ if $w \models \phi$. If ϕ is true at w then w is itself the unique world most like w where ϕ holds.

Following Stalnaker, conditionals are interpreted in possible worlds by means of selection functions.We define for $\phi, \chi \in L_0$:

$w,s \models \phi \quad \Leftrightarrow \quad w \models \phi$, and

$w, s \models \phi > \chi \quad \Leftrightarrow \quad s(w,\phi) \models \chi$.

This phrase $s(w,\phi) \models \chi$ just means that for all $v \in s(w,\phi)$, $v \models \chi$.The usual clauses for ~ and $\wedge$ then extend $\models$ to the rest of L_1. Having said how L_1 is to be interpreted relative to worlds and selection functions, we can now go on to describe our belief sets. But first we need some more restrictions on selection functions. They have been named so as to suggest the properties of revision functions to which they will later give rise:

7 DEFINITION A selection function is termed:

a) *Successful* iff for every $\phi \in L_0$, and every $w \in W$: $s(w,\phi) \models \phi$.

b) *Expansive with respect to* $P \subseteq W$ iff for every $\phi \in L_0$: If there is $v \in P$ with $v \models \phi$, then for all $w \in P$, $s(w,\phi) \subseteq P$. (Note that a selection function which is expansive with respect to one set of worlds need not be expansive with respect to another.)

c) *Preservative* iff for every $\phi \in L_0$ and $v \neq w_\perp$: If there is some $w \neq w_\perp$ such that $w \models \phi$, then $w_\perp \notin s(v, \phi)$.

It should be noted that in line with my simplistic modelling of beliefs as sentences, selection functions have been defined on sentences rather than on propositions, and this without ensuring that classically equivalent sentences are treated the same way. Nothing in the following depends on this permissiveness however, and suitable restrictions may be added which do ensure that equivalent sentences are treated the same way.

8 DEFINITION

A *belief state* P_s consists of a set $P \subseteq W$ and a selection function *s*, where

a) P is *coherent* (ie. $w_\perp \notin P \neq \{\}$) or *absurd* (ie. $P = \{w_\perp\}$).

b) s is successful, expansive with respect to P, and preservative.

The *belief set corresponding to the belief state P_s* is the set $\{\phi \in L_1$: for all $w \in P$, $w,s \models \phi\}$. Having distinguished between belief states and the corresponding belief sets it is however convenient to confuse them, using P_s to refer to whichever suits best.

These two definitions need some motivation. The idea is that a belief state P_s encodes two distinct kinds of beliefs, corresponding to its two components. The set P of worlds contains the epistemic possibilities with which the believer is still reckoning, containing a world w just incase the believer believes that w may yet turn out to be the real world. And the selection function s encodes his opinions about similarities between worlds. This much is standard, as is the first additional requirement of success placed on selection functions: the worlds most like w but where ϕ is true must in any case be worlds where ϕ is true.

The second additional requirement that the selection function s of a belief state P_s be expansive with respect to P is more novel, and introduces an important difference with earlier work on conditionals. Whereas selection functions have in the past been taken to be absolute in the sense that different belief states of a belief revision system all have the same one, by requiring selection functions to be expansive I have made them essentially context dependent. Roughly speaking, the intuitive content of the requirement is this: when it comes to judging which worlds are most like a given world which is still being reckoned with, other worlds which are still being reckoned with take precedence. In other words, an epistemic *im*possibility is never thought as much like a given epistemic possibility as another epistemic possibility. At the level of conditionals, this has the effect that states of affairs which are considered counter to the facts bear on the evaluation of a conditional only if it is a counterfactual. The third requirement placed on selection functions speaks for itself. It just says that the absurd world is never considered as much like a given consistent world as is another consistent world.

It is now the following lemma which does most of the work of getting us nontrivial belief revision models:

9 EXISTENCE LEMMA

For every coherent or absurd set $P \subseteq W$, there is a belief state P_s.

Proof: We have to show that for each coherent or absurd P there is a selection function s which is successful, expansive with respect to P, and preservative.

a) Suppose P is coherent. For each $\phi \in L_0$, choose some $w_\phi \vDash \phi$ subject to the following two constraints:

i) If there is some $v \in P$ with $v \vDash \phi$, choose $w_\phi \in P$.

ii) If not, in any case choose $w_\phi \neq w_\perp$ if there is $w \neq w_\perp$ such that $w \vDash \phi$.

Now for each $w \in W$, put $s(w,\phi) = \{w\}$ if $w \vDash \phi$, and $s(w,\phi) = \{w_\phi\}$ otherwise.

It is clear that s is successful.

That s is expansive with respect to P is also easily seen: let $\phi \in L_0$, and let there be $v \in P$ with $v \vDash \phi$. Then we can be sure that $w_\phi \in P$. Now taking any $w \in P$, by definition of s either $s(w, \phi)=\{w\}$ or $s(w, \phi)=\{w_\phi\}$. Either way $s(w,\phi) \subseteq P$, which is what is required.

That s is preservative is due to the coherence of P. Assume for the contradiction that s is not preservative. Then there is a $\phi \in L_0$ and a $v \neq w_\perp$ such that although there is $w \neq w_\perp$ with $w \vDash \phi$, still $w_\perp \in s(v,\phi)$. Given the definition of s, it follows from the latter that $s(v,\phi)=\{w_\perp\}$. As in this case $s(v,\phi) \neq \{v\}$, by definition of s it follows that (iii) $w_\phi = w_\perp$. In view of the coherence of P however, (iii) means that w_ϕ was not chosen from within P, in which case w_ϕ was chosen according to constraint (ii) above. By assumption there is $w \neq w_\perp$ such that $w \vDash \phi$, so by constraint (ii) $w_\phi \neq w_\perp$. But this is in direct contradiction with (iii) above.

b) Suppose $P=\{w_\perp\}$. Then as by centering $s(w_\perp,\phi)=\{w_\perp\}$ independently of s and ϕ, every selection function s is expansive with respect to P. So we only need to find one which is both successful and preservative. To this end choose, for each $\phi \in L_0$, some $w_\phi \vDash \phi$. Again, don't choose $w_\perp$ if this can be avoided, that is if there is $w \neq w_\perp$ with $w \vDash \phi$. And again put $s(w, \phi)=\{w\}$ if $w \vDash \phi$, and $s(w,\phi)=\{w_\phi\}$ otherwise.

It is then easy to verify that this s is both successful and preservative.

A Revision Method: Imaging

Selection functions have been introduced as revision functions for individual L_0 interpretations. But they generalize to revision functions for sets of L_0 interpretations in the following manner:

10 DEFINITION

a) The *image of* P *cast by* $\phi \in L_0$ *relative to* s, written s(P,ϕ), is defined as $\bigcup_{w \in P} s(w,\phi)$

Note that an image s(P,ϕ) of P, like P before it, is a set of L_0 interpretations.

b) A belief revision model is a pair <K,+> where K is a set of belief states, and $+: K \times L_0 \rightarrow \wp(K)$.

c) The revision function + of a belief revision model <K,+> is said to *agree with imaging* just incase for every $P_s \in K$, every $\phi \in L_0$, and every $\kappa \in P_s + \phi$, κ may be written $s(P,\phi)_t$ (for some selection function t). Intuitively, a revision function defined on belief states agrees with imaging just incase imaging determines its descriptive component.

If a selection function tells you which ϕ-worlds are most similar to some particular possible world you have in mind, then imaging tells you which ones are most similar to some set of alternatives between which you cannot yet choose. The notion has been named so as to acknowledge an heuristic debt to David Lewis' probabilistic notion of imaging, invented for very similar purposes in his "Probabilities of Conditionals and Conditional Probabilities". Indeed, if Lewis' imaging is thought of as the transfer of probability from the domain of a selection function to its image set, then my notion is just a qualitative version of his: here it is not measures of probability which are transferred along the selection functions, but epistemic possibility.

It is not difficult to check that for any $\phi \in L_0$ and any belief state P_s, s(P,ϕ) is coherent just incase P is coherent and ϕ classically consistent, and absurd otherwise. Using existence lemma 9 we can then prove:

11 EXISTENCE LEMMA For any belief state P_s there is a belief revision model <K,+> such that $P_s \in K$, and + agrees with imaging.

Proof: By existence lemma 9, let P_s be a belief state and let *f* be a function which assigns to each coherent or absurd set $Q \subseteq W$ some selection function which is successful, expansive with respect to Q and preservative. In addition, it is required that $f(P)=s$.
Now set:

$$K_0 = \{P_s\}, \text{ and}$$
$$K_{i+1} = \{t(Q,\phi)_{f(t(Q,\phi))}: Q_t \in K_i,\ \phi \in L_0\}$$
$$K = \bigcup K_i$$
$$+ = \{(Q_t,\phi,\{t(Q,\phi)_{f(t(Q,\phi))}\}) : Q_t \in K,\ \phi \in L_0\}$$

It may easily be verified that <K,+> is a belief revision model, and from the definition it is immediately obvious that $P_s \in K$, and that + agrees with imaging. Note that + is in fact deterministic.

A Background Entailment Notion

It remains to define an entailment notion ⊫ under which belief sets are to be closed, and relative to which rationality requirement III, preservation, is to be satisfied by our final belief revision models. Additional requirements are that ⊫ be monotonic, that it extend classical entailment, and that it satisfy the deduction theorem, since all of these assumptions were made by the impossibility theorem.

12 DEFINITION

a) Let S be the class of all belief revision models <K,+> such that + agrees with imaging. Existence lemmas 8 and 10 assure us that S is not empty.

b) For $\Gamma \subseteq L_1$, and for $\phi \in L_1$, let $\Gamma \Vdash \phi$ iff for all <K,+> ∈ S and all $P_s \in K$: if $\Gamma \subseteq P_s$, then $\phi \in P_s$.

It is easy to check that the relation ⊫ is a Tarskian entailment notion: it is reflexive, monotonic, and satisfies lemma generation. That ⊫ is stronger than classical entailment and that belief sets are closed under it are also readily demonstrated. In fact, an easy application of existence lemmas 9 and 11 shows ⊫ to be a conservative extension of ⊨: for all $\Gamma \subseteq L_0$ and $\phi \in L_0$, $\Gamma \Vdash \phi$ just incase

$\Gamma \models \phi$. That it satisfies the deduction theorem will be obvious once we get to section 6.

A Nontrivial Belief Revision Model

The suggestive restrictions on selection functions in definition 7 are now justified by the following lemma:

13 LEMMA Every structure <K,+> in S satisfies the requirements of success, expansion, and the Ramsey rule. Relative to $\Vdash$ they all satisfy consistency preservation.

Proof: Let $\langle K,+\rangle \in S$.
i) *success.* Let $P_s \in K$ and $\phi \in L_0$. We want to show that $\phi \in \bigcap(P_s+\phi)$. To this end, suppose for the contradiction that for some $\kappa \in P_s+\phi$, $\phi \notin \kappa$. As + agrees with imaging, for some selection function t we have $\kappa = s(P,\phi)_t$, so $\phi \notin s(P,\phi)_t$. By definition this just means that there is some $v \in s(P,\phi)$ such that $v,t \not\models \phi$ or, as selection functions don't contribute to the interpretations of descriptive sentences, $v \not\models \phi$. By definition of $s(P,\phi)$ there is some w with $v \in s(w,\phi)$. Now w and ϕ constitute a counterexample to the fact that s is successful.
ii) *expansion.* Let $P_s \in K$ and $\phi, \psi \in L_0$ such that $\psi \in P_s$ and $\sim\phi \notin P_s$. We want to show that $\psi \in \bigcap(P_s+\phi)$, which is to say that $\psi \in \kappa$ for any $\kappa \in P_s+\phi$. Note first that as $\sim\phi \notin P_s$ there is some $v \in P$ with $v,s \models \phi$. Since $\phi \in L_0$, $v \models \phi$. As s is expansive with respect to P this means that $s(w,\phi) \subseteq P$ for all $w \in P$. So

$$s(P,\phi) = \bigcup_{w \in P} s(w,\phi) \subseteq P.$$

So now consider any $\kappa \in P_s+\phi$. As + agrees with imaging, we know that κ may be written $s(P,\phi)_t$ for some suitable t. So to show that $\psi \in \kappa$ it is sufficient to show that $w,t \models \psi$ for each $w \in s(P,\phi)$. Or equivalently, as $\psi \in L_0$, that $w \models \psi$. To this end, let $w \in s(P,\phi)$. By the above inclusion $w \in P$. But then $w,s \models \psi$, since $\psi \in P_s$. So once again, as $\psi \in L_0$ we have $w \models \psi$.[1]

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[1]In section 5.4 of *Knowledge in Flux*, Gärdenfors proves that no nontrivial revision function based on imaging can satisfy expansion (he calls it *preservation*, and I apologise for interchanging the names). This is however quite compatible with the above. Gärdenfors is assuming that selection functions do not vary with belief states. It is precisely by making them context dependent that imaging can be made expansive. This trick is related to the technique of *preservative imaging*

iii) *preservation.* Let $P_s \in K$ such that $\bot \notin P_s$ and $\phi \in L_0$ such that $\phi \not\Vdash \bot$. We want to show that $\bot \notin \bigcap(P_s+\phi)$. As $\Vdash$ is an extension of classical logic, $\phi \not\models \bot$, so there is some $w \neq w_\bot$ such that $w \models \phi$. As s is preservative, this means that for all $v \neq w_\bot$, $w_\bot \notin s(v, \phi)$. As $\bot \notin P_s$ and P is either absurd or coherent, $w_\bot \notin P$. But then $w_\bot \notin \bigcup_{w \in P} s(w,\phi) = s(P,\phi)$.

So now consider any $s(P,\phi)_t \in P_s+\phi$, and any $w \in s(P,\phi)$. As $w \neq w_\bot$, $w,t \not\models \bot$, so $\bot \notin s(P,\phi)_t$. Thus $\bot \notin \bigcap(P_s+\phi)$.

iv) *the Ramsey rule.* Let $\phi, \chi \in L_0$, and $P_s \in K$. Then the following equivalences may be verified with much the same reasoning as in the above:

Then $\phi > \chi \in P_s \Leftrightarrow$ for all $w \in P$, $s(w,\phi) \models \chi \Leftrightarrow \bigcup_{w \in P} s(w,\phi) \models \chi$

$\Leftrightarrow s(P,\phi) \models \chi \Leftrightarrow$ for all $s(P,\phi)_t \in P_s+\phi$, $\chi \in s(P,\phi)_t$

$\Leftrightarrow \chi \in \bigcap(P_s+\phi)$.

This completes the proof that revision functions agreeing with imaging have the desired properties.

14 THEOREM There are nontrivial belief revision models satisfying I to IV.

Proof: By lemma 13 all S structures satisfy the requirements I - IV. It only remains to be seen that some of them are in addition nontrivial. To this end, assume that L has at least two atoms, p and q. Let P be the set {{p,q}, {p}, {q}}. P is clearly coherent, so by existence lemmas 9 and 11 there is a selection function s and some <K,+> ∈ S such that $P_s \in K$. Now together with the belief set P_s the sentences p, q and $\sim(p \wedge q)$ establish the nontriviality of <K,+>:

That p, q, $\sim(p \wedge q) \models \bot$, and thus p, q, $\sim(p \wedge q) \Vdash \bot$, is obvious.

That $\sim(p \wedge q) \notin P_s$,

$\sim(p \wedge \sim(p \wedge q)) \notin P_s$, and

$\sim(q \wedge \sim(p \wedge q)) \notin P_s$ is easily seen by inspection, since independently of s:

{p,q}, $s \not\models \sim(p \wedge q)$

{p}, $s \not\models \sim(p \wedge \sim(p \wedge q))$, and

{q}, $s \not\models \sim(q \wedge \sim(p \wedge q))$.

This completes the proof that there are nontrivial belief revision models.

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discussed by Gärdenfors in section 5.6 but is somewhat more general, since preservative imaging requires that one can express, as a sentence of the object language, everything one wants to preserve.

Theorem 14 and example 6 both provide nontrivial belief revision models satisfying assumptions I - IV, and the obvious question arises as to what relation there is between the two constructions, if any. Is the model of example 6 perhaps even to be found among the structures in the class S of definition 12? In fact it is. The model of example 6 may in effect be obtained with the construction of existence lemma 11, which was used to show S nonempty, in the following manner. First, the set P of the initial beliefstate P_s of that construction is chosen to be the universal set $W \backslash w_\perp$ of all nonabsurd possible worlds. And second, a new restriction is placed on all selection functions. Among selection functions satisfying the earlier requirements of success, expansion and preservation, the selection functions which figure in belief states must additionally be chosen maximal on the partial order ≤ defined as follows: $s \leq t$ just incase for all $w \in W$ and all $\phi \in L_0$, $s(w,\phi) \subseteq t(w,\phi)$. It is not difficult to verify that such maximal elements always exist, and that with these extra requirements on P and the selection functions the construction of existence lemma 11 does indeed embed the model of example 6 in S. Actually this extra restriction on selection functions is of interest quite apart from the embedding, corresponding at the macroscopic level of belief revision to the requirement that imaging add as little to belief states as is needed in order to ensure success.

§ 5 A CLOSER LOOK AT THE ADDITIONAL ASSUMPTION

So by dropping assumption V, which requires belief revision models to be closed under expansions, nontrivial belief revision models can be obtained which satisfy the Ramsey rule and the requirements I to III. But isn't the impossibility theorem then just being hidden from view instead of being solved, by thinning out the belief sets in which it would otherwise show up? Are the thinned out models, if not trivial, then in any case impoverished, no longer containing what on the face of it are perfectly reasonable sets of beliefs? Indeed, shouldn't any theory at all be a possible set of beliefs? As far as beliefs are concerned, surely just about anything goes.

Well, no. By imposing the Ramsey rule on belief revision models we have introduced an idealisation: the belief sets in our models stand for idealised believers who, however incompletely or wrongly informed they may be about the world around them, nevertheless have complete and correct conditional beliefs about their own belief revisions. Now it just might have been that ideal sets like this could be obtained simply by forming expansions, that is by taking other such sets, adding arbitrary sentences, and then closing under the background logic. And if that had been so it would have been perfectly reasonable to require belief revision models to satisfy V. As I see it, however, the impossibilty theorem is saying precisely that is not so. Expansions are not ideal in this way, so that closure under expansions is just incompatible with the idealisation.

The belief revision models are no less suited to the semantics of conditionals for not being closed under expansions, since dropping assumption V does not mean losing any intuitively acceptable belief sets which are compatible with the idealisation imposed by the Ramsey rule. As far as descriptive sentences are concerned, believers are still free to believe whatever they like. This is clear from example 6, a belief revision model in which every descriptive theory shows up. So why introduce assumption V in the first place? It originated, in Gärdenfors work, in a setting without conditional sentences, where the belief sets were not required to be idealised in the way that they are here. There it arose in a stronger form sanctioned by Bayesianism: If $\sim\phi \notin \kappa$, then $\kappa+\phi = \{\kappa/\phi\}$, where κ and ϕ are taken from L_0. Now one way of extending this — obviously desirable — assumption to the conditional setting would be to simply allow κ to range over the belief sets containing conditionals. In this form it is no longer sanctioned by Bayesianism, but much worse is that since + is a function it immediately implies assumption V and, as theorem 5 shows, trivialises the whole enterprise. A more cautious way to extend the assumption to the conditional setting would be to assume: If $\sim\phi \notin \kappa$, then $(\kappa+\phi) = \{\lambda\}$ for some λ such that $\lambda \cap L_0 = (\kappa_0/\phi) \cap L_0$. In this form it is still sanctioned by Bayesianism. And, better, the belief revision models can be made to satisfy it. Example 6 once again shows this most clearly, since there + is explicitly constructed in this way. And not surprisingly the models in S can also be made to satisfy it by means of the additional restriction on selection functions which at the end of section 4 was seen to embed the structure of example 6 in S.

The impossibility theorem, then, involves extending to conditional sentences an assumption which as far as I can see can only be defended for descriptive ones. Unlike assumptions I through to IV and the requirement of nontriviality, assumption V does not contribute to belief revision models in any way which is relevant to the semantics of conditionals. It neither makes the conditionals' interpretation more natural, nor makes the belief revision more rational, nor makes the models in any way more realistic. All this assumption does is to get fouled up with other assumptions which do contribute in these ways, and it should be dropped.

§ 6 THE CONDITIONAL LOGIC OF ⊫

We turn now to a syntactic characterisation of ⊫.

15 DEFINITION $\quad \Gamma \Vdash \phi$ iff ϕ follows with classical propositional logic from Γ together with the following axiom schemes, in which ϕ, ψ and χ range over L_0 formulas:

A1. $\phi > \phi$

A2. $\phi \wedge \psi \;\rightarrow\; \phi > \psi$

A3. $(\phi > \psi \wedge \phi > \chi) \;\rightarrow\; \phi > (\psi \wedge \chi)$

A4. $\phi > \psi \;\rightarrow\; \phi > \chi$ $\qquad$ for all ψ and χ such that $\psi \vDash \chi$.

A5. $\sim(\phi > \bot)$ $\qquad$ for all ϕ such that $\phi \nvDash \bot$.

These schemes are, with the exception of the last, all quite familiar from conditional logic. The following completeness theorem admits of a straight-forward Henkin-style proof:

16 THEOREM $\quad$ For all Γ and ϕ taken from L_1: $\Gamma \Vdash \phi \;\Leftrightarrow\; \Gamma ⊫ \phi$

Proof:

$\Rightarrow$ $\quad$ It is easy to check that each of the schemes holds in every world relative to every selection function, so that any belief state includes them. The closure of belief states under classical logic does the rest.

$\Leftarrow$ $\quad$ Before the proof we need two facts:

FACT A. $\quad$ For all ϕ and ψ, $\Vdash \; \phi > \psi \;\rightarrow\; \phi \rightarrow \psi$

Note first that the standard rules of natural deduction can be used for $\Vdash$. The reason for this is that $\Vdash$, having been characterised as classical logic applied to extra axioms but without any extra rules, inherits the (syntactic analog of) the deduction theorem from classical logic.

Now two cases are distinguished:

i) $\phi \models \bot$, in which case the natural deduction is trivial, and
ii) $\phi \not\models \bot$, in which case we have the following (summary of a) natural deduction:

1.	$\phi > \psi$	assumption.
2.	ϕ	assumption.
3.	$\sim\psi$	assumption.
4.	$\phi \wedge \sim\psi$	2 and 3.
5.	$\phi > \sim\psi$	4, A2, modus ponens.
6.	$\phi > (\psi \wedge \sim\psi)$	1, 5, A3, modus ponens.
7.	$\phi > \bot$	6, A4, modus ponens.
8.	$\sim(\phi > \bot)$	A5, $\phi \not\models \bot$.
9.	$\bot$	7 and 8.
10.	ψ	
11.	$\phi \rightarrow \psi$	
12.	$\phi > \psi \;\rightarrow\; \phi \rightarrow \psi$	

FACT B Any successful and preservative *partial* selection function — one which is defined for just some pairs of worlds and sentences, and which is successful and preservative to the extent that it is defined — may be extended to a successful and preservative total selection function.
A simple proof of this fact is implicit in the proof of existence lemma 9.

So now assume $\Gamma \not\Vdash \phi$. We need to find $<K,+> \in S$ and $P_s \in K$ such that $\Gamma \subseteq P_s$ but $\phi \notin P_s$. Sufficient for this is that P_s be consistent and $\sim\phi \in P_s$.
Let Γ^* be a maximal $\Vdash$—consistent extension of $\Gamma \cup \{\sim\phi\}$. Sufficient is now to find a possible world $w \neq w_\bot$ and some successful preservative partial t such that $w,t \models \Gamma^*$. For then fact B ensures that t may be assumed total, and since every selection function is expansive with respect to a singleton (as may easily be checked), with existence lemma 11 we obtain a $<K,+>$ with $\{w\}_t \in K$. $\{w\}_t$ is consistent since $w \neq w_\bot$, and $\Gamma^* \subseteq \{w\}_t$, so then $\{w\}_t$ is a suitable P_s.
So now for the $w \neq w_\bot$ and a successful preservative partial t such that $w,t \models \Gamma^*$. Defining *lit* $= \{\psi > \chi : \psi > \chi \in \Gamma^*\} \cup \{\sim(\psi > \chi) : \sim(\psi > \chi) \in \Gamma^*\}$, an easy induction on the complexity of $\gamma \in \Gamma^*$ shows it to be sufficient that $w,t \models lit \cup (\Gamma^* \cap L_0)$.
So choose $w \neq w_\bot$ such that $w \models \Gamma^* \cap L_0$, and define for each $\psi \in L_0$:

$\psi^+ = \{\chi : \psi > \chi \in \Gamma^*\}$, and

$\psi^- = \{\chi : \sim(\psi > \chi) \in \Gamma^*\}$.

Now, for each $\psi \in L_0$, choose $t(w,\psi)$ as follows:

a) Suppose $w \vDash \psi$, or equivalently $\psi \in \Gamma^*$. Then let $t(w,\psi) = \{w\}$ so as to satisfy centering. Now we only have to check that $w,t \vDash lit$, since t is obviously successful and preservative.

i) If $\psi > \chi \in \Gamma^*$ then since $\psi \in \Gamma^*$ and as by fact A $\psi > \chi \rightarrow \psi \rightarrow \chi \in \Gamma^*$, $\chi \in \Gamma^*$. But then $w \vDash \chi$, so that $w,t \vDash \psi > \chi$.

ii) If $\sim(\psi > \chi) \in \Gamma^*$ then $\sim\chi \in \Gamma^*$. (Otherwise, by the maximality of Γ^*, $\chi \in \Gamma^*$. As $\psi \in \Gamma^*$, with A2 we would then have $\psi > \chi \in \Gamma^*$, which would make Γ^* inconsistent.) Thus $w \vDash \sim\chi$, so as $w \neq w_\perp$, $w \nvDash \chi$.
Thus $w,t \vDash \sim(\psi > \chi)$.

b) Suppose $w \vDash \sim\psi$. Note first that for any $\chi \in \psi^-$, $\psi^+ \cup \{\sim\chi\} \nvDash \perp$. For if $\psi^+ \cup \{\sim\chi\} \vDash \perp$ there is a finite $T \subseteq \psi^+$ such that $T \cup \{\sim\chi\} \vDash \perp$, or $T \vDash \chi$. As $T \subseteq \psi^+$ a number of applications of A3 gives $\psi > \wedge T \in \Gamma^*$. As $T \vDash \chi$, this together with A4 gives $\psi > \chi \in \Gamma^*$. By choice of $\chi \in \psi^-$ we also have $\sim(\psi > \chi)$, which makes Γ^* inconsistent. It is consistent, so $\psi^+ \cup \{\sim\chi\} \nvDash \perp$.
Using this fact, choose for each $\chi \in \psi^-$ some $w_{\sim\chi} \neq w_\perp$ such that $w_{\sim\chi} \vDash \psi^+ \cup \{\sim\chi\}$. We can then be sure that $w_{\sim\chi} \vDash \psi^+$, but $w_{\sim\chi} \nvDash \chi$.
Now define $t(w,\psi) = \{w_\perp\}$ iff $\psi^- = \{\}$, and $t(w,\psi) = \{w_{\sim\chi}: \chi \in \psi^-\}$ otherwise.
We have to check that $w,t \vDash lit$, and that t is succesful and preservative.

i) If $\psi > \chi \in \Gamma^*$ then $\chi \in \psi^+$. By the construction $t(w,\psi) \vDash \psi^+$, so $t(w,\psi) \vDash \chi$, and $w,t \vDash \psi > \chi$.

ii) If $\sim(\psi > \chi) \in \Gamma^*$ then $\chi \in \psi^-$, so by the construction there is $w_{\sim\chi} \in t(w,\psi)$ with $w_{\sim\chi} \nvDash \chi$. So $w,t \nvDash \psi > \chi$ and thus $w,t \vDash \sim(\psi > \chi)$.

That t is successful is obvious, in that with A1 we have $\psi \in \psi^+$. The only way that a counterexample to preservation could have crept in is through the clause which puts $t(w,\psi) = \{w_\perp\}$ iff $\psi^- = \{\}$. We would have a counterexample if $t(w,\psi) = \{w_\perp\}$ while $\psi \nvDash \perp$. This can however never happen as $\perp \in \psi^-$ whenever $\psi \nvDash \perp$, in virtue of A5.
We now have a world $w \neq w_\perp$ and a successful preservative partial t such that $w,t \vDash lit \cup (\Gamma^* \cap L_0)$, and have completed the completeness proof.

One spinoff from the completeness theorem is to make it plain that $\Vdash$ satisfies the deduction theorem, as was promised after definition 12, since as we have already seen $\Vdash$ satisfies the deduction theorem. A comparison of A1 - A5 with the axioms of Lewis' **VC** shows only A5 to be novel. Apart from this axiom the logic is weaker than **VC**, in that for example classically equivalent antecedents may not be exchanged. As suitable axioms and corresponding restrictions on selection functions may be added to bring it up to strength, however, the logic generated by this epistemic semantics for conditionals does not differ greatly from that of Lewis.

§ 7 THE MEN WITH THE HAT AND THE HAMBURGER

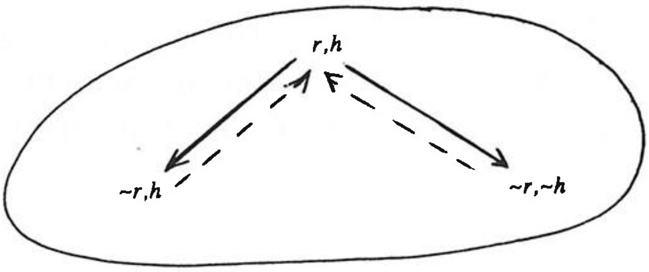
We have seen in the previous section that the logic is fairly familiar. Now the fact that such a logic should emerge from an epistemic semantics of the kind given above is in itself perhaps not very interesting, nor very surprising given that the semantics was constructed using selection functions and imaging, which are standard fare in conditional logic. So what is gained by the undertaking? Epistemic semantics would be a barren exercise indeed if all it showed was that familiar conditional logics may be provided with a semantics which stays closer to the letter of Ramsey's suggestion than the semantics of Stalnaker say, or that of Lewis.

But this is not all there is to be learned. For although the techniques and the resulting logic are familiar, the epistemic semantics given here is in some ways still quite novel, throwing new light on examples which remain puzzling for earlier accounts of conditionals based on the Ramsey rule. Two examples which illustrate this are Tichy's man with the hat, and Hansson's man with the hamburger.

In "A Counterexample to the Stalnaker-Lewis Analysis of Counterfactuals", Tichy asks us to imagine a man who always wears his hat when it is raining, but who sometimes does and sometimes does not wear it when it is not raining. Furthermore, we are to suppose that it is in fact raining today, so that he does in fact have his hat on. Finally, we are asked to evaluate by means of the Ramsey rule the counterfactual *if it hadn't been raining he would have had his hat on.* Adding the antecedent *it is not raining* hypothetically to ones stock of beliefs, one finds that the adjustments required to maintain consistency do not include changing ones belief that *he has his hat on.* Apparantly then, and counterintuitively, the counterfactual is to be believed.

The above depends on judging the minimality of adjustments by weighing up new beliefs accepted and old ones discarded. By identifying a body of belief with a set of possible worlds accompanied by a suitable selection function, however, the epistemic semantics given above suggests a different possibility: minimal adjustments to a body of beliefs are those which leave each of these components unchanged wherever possible.

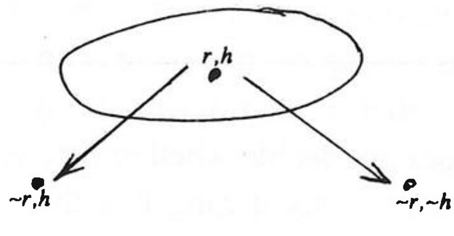
To see how this idea contributes a more satisfactory analysis of Tichy's example, watch how this example translates into a belief revision model. The initial information state you are in when the situation is explained, in which you know only that the man has his hat on when it is raining but that there is no telling if it is not, contains neither the information that it is raining today, *r*, nor the information that it is not. Similarly, it does not decide whether he has his hat on, *h*, or not. Pictorially, the initial belief state is something like this (the broken arrows are for *r* , the others for *~r*):[1]



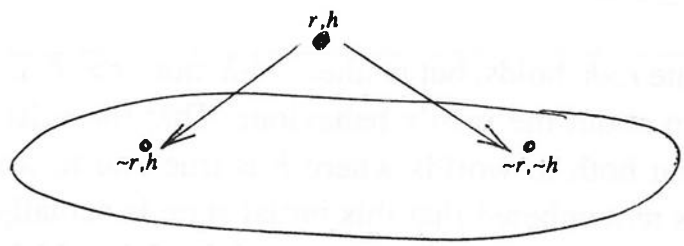
Note that in this initial belief state *r>h* holds, but neither *~r>h* nor *~r>~h* holds, which is true to what is known about the man's behaviour. That the selection function for *~r* must branch out both to worlds where *h* is true and to worlds where *~h* is true is clear if it is remembered that this initial state is actually the successor of your state of belief before the situation was explained, in which you believed nothing at all about the man's habits. It may therefore contain no more positive information about him than is required in order to accomodate the facts that he has his hat on when it's raining and that when it is not you cannot tell. Weakness of a belief state increases as its selection function becomes more branching, so the selection function must branch as much as consistent with these facts. Here we see the *dynamics* of belief revision models entering into things.
Now you are told that it is raining. Your new belief state contains only those worlds of the previous one where both *r* and *h* are true, and comes with some selection function which differs minimally from that of the previous one. The only reason for changing the selection function in this case is to keep it expansive, which pictorially means that wherever possible, arrows stay within the set of epistemically possible worlds. But then as far as the sentence *~r* is concerned, no change at all is required, since no epistemically possible worlds at all remain

[1]Here the set P of epistemically possible worlds has been encircled, and the selection function has been drawn in only to the extent that is needed for the problem. The points do not stand for individual worlds, but for sets of these which the selection function treats in the same way.

where *~r* holds. So your state of belief when asked to evaluate *if it hadn't been raining he would have had his hat on*, or *~r>h*, looks like this:



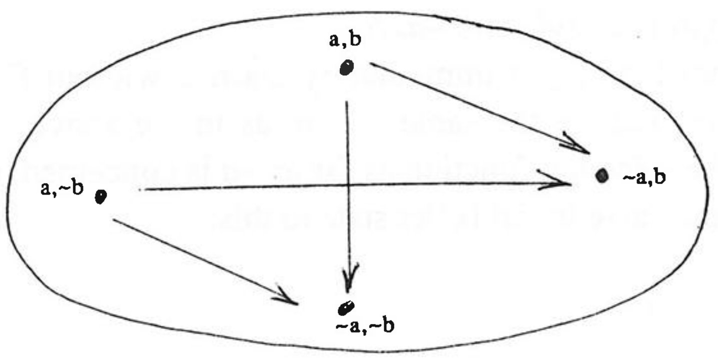
So now evaluate the counterfactual. It is clear that *~r>h* still doesn't hold (or *~r> -h* for that matter) which is as it should be. And in accordance with this, on adding the antecedent *~r* we arrive at the following belief state where neither *h* nor *~h* holds:



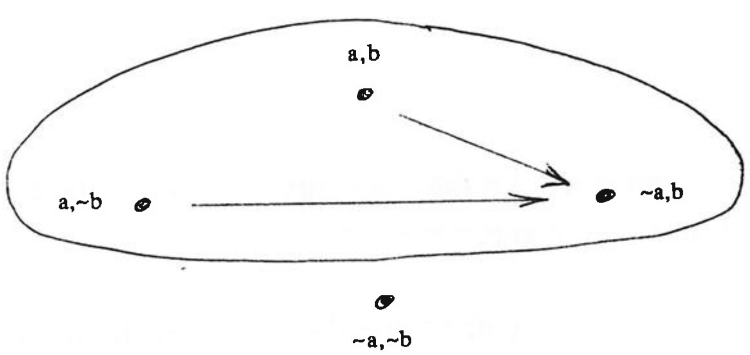
We have now seen how two aspects of this epistemic semantics contribute to a more satisfactory analysis of Tichy's example: the nature of its belief states, and the fact that it is dynamic.

The role played by the dynamics of belief is especially clear in the next intuitive counterexample to Stalnaker-Lewis semantics, recently described by S. Hansson in "New Operators for Theory Change". It consists of two stories: In the first, you enter a town which you believe to have just two snackbars, *A* and *B*. There you see a man walking along the street with a hamburger, and form the belief *a∨b* that at least one of the snackbars is open. At the same time you form the belief *~a>b* that if *A* is not open, then *B* is. Now as you approach one of the snackbars, it happens to be *A*, you see that the lights are on there. As a result you form the belief *a* that *A* is open, and you still believe that *a∨b* and that *~a>b* (since even if you are mistaken about *A* being open and they are just doing the cleaning, the hamburger must have come from somewhere). The second story has you entering the same town with the same belief about there being just two snackbars. This time however you do not run into the man with the hamburger, but just see that the lights are on at *A*. Again you form the belief that *a*, and with it (since you know logic) again the belief that *a∨b*. But this time you do not believe that $\sim a>b$. Together these stories create a problem for counterfactual semantics along the lines of Stalnaker and Lewis, where the interpretation of conditionals is supposed to be absolute, in the sense that it is independent of the beliefs one happens to have about the world. For there is no difference between the two stories which is visible to this kind of semantics, and which could account for the fact that the one belief state contains $\sim a>b$, while the other does not.[1]

The relevant difference which is visible to a dynamic semantics such as that given here lies in the different histories of the two belief states, which become encoded in their conditional components. Both stories start off with one having no positive beliefs at all about whether *A* and *B* are open or not, and this lack of positive belief corresponds once again to a selection function which branches out as much as possible. So the initial belief state is like this (all arrows are $\sim a$ arrows):

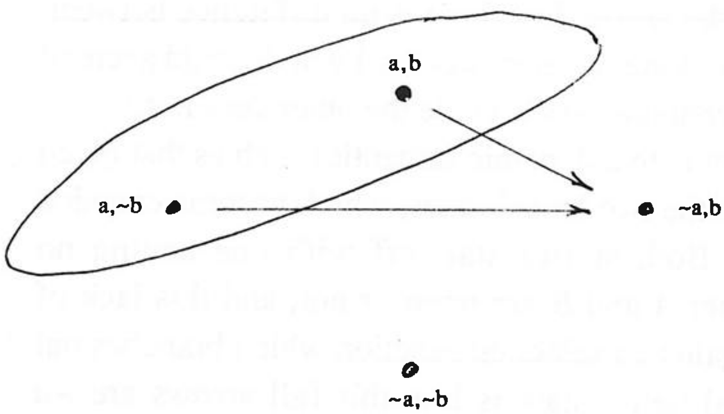


In the first story you then see the man with the hamburger and form the belief that $a \vee b$. Minimal changes made to the selection function so as to keep it expansive (namely removing offending arrows) then result in a belief state like the following one, which as is required supports $\sim a>b$:

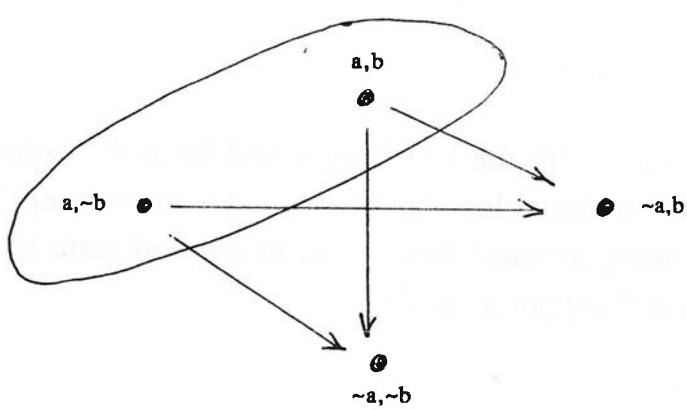


[1]An objection which has been made here is that there is indeed a visable difference: the one body of belief contains beliefs about a man with an hamburger while the other does not. The example can however easily be elaborated so as to eliminate man, hamburger, and objection.

As in the previous example, subsequently learning *a* requires no further changes in the selection function as far as *~a* is concerned, so the final belief state of the first story looks like this:



As required, this belief state supports *a*, *a∨b*, and *~a>b*.
In the second story on the other hand, you immediately learn *a* without first believing that *a∨b*. In this case, and for the same reason as in the above, no change at all need be made to the selection function as far as *~a* is concerned, so that you jump in one step from the naive initial belief state to this:



This final belief state supports *a* and *a∨b* like the previous one. But unlike the previous one, and as is required, it does not support *~a>b*.

In this analysis of Hansson's example it is clearly the dynamics of the models which does the work, since this semantics makes the different histories of the belief states involved visible in their conditional components. The future of a belief state is determined by its conditionals. That is the meaning of the Ramsey rule. But we see that the conditional component of a belief state is in turn determined by its past.

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Grammar and Logical Types*

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Abstract

This paper represents categorial grammar as an implicational type theory in the spirit of Girard's linear logic, and illustrates linguistic applications of a range of type-constructors over and above implication. The type theoretic perspective is concerned with a correspondence between the logic of types, and computational operations over the objects inhabiting types. In linguistic applications this correspondence is between rules of grammar which are theorems of type inference, and compositional operations in the various algebras in which linguistic objects, i.e. signs, are assumed to have dimensions: syntax, semantics, etc. Rule-to-rule description is familiar from Montague Grammar, but the idea here is to classify signs with structured types satisfying universal type laws determined by the semantics of the type connectives, in contrast to classification by categories satisfying stipulated rules. On this scheme an object language is to be specified by a type assignment to its finite vocabulary: a formal grammar is just a lexicon, plus perhaps some improper type axioms, and a grammar formalism is just a meta-language of types with its uniform logic and interpretation is each linguistic dimension. The aim is to develop a language of types which has sufficient transparency, sensitivity, and generality to implement interesting descriptions of natural language. The paper will illustrate sentence grammar, and also use of the semantic term algebra as a functional programming language for presentation of lexical semantics.

1 Introduction

The grammatical architecture exemplified in Montague Grammar is one which sees linguistic objects as having dimensions in syntactic and semantic algebraic domains, and in which rules of grammar correlate operations in these algebras. For this design in general, a linguistic object or sign is a vector across the elements in the linguistic domains under consideration, and a rule is a vector across the operations; a language is described by the componentwise closure of the rules of grammar over the lexical signs.[1]

This paper develops a tool for language description by extending the categorial type system of Lambek (1958).[2] Syntactic interpretation of Lambek categorial types is reviewed in e.g. Buszkowski (1988), and semantic interpretation in van Benthem (1983). On the present design these schemes are to be integrated in a compositional interpretation for type-constructors specifying the sign vectors in the composite types in terms of the sign

*The development of this material over a period of time has benefited from a wide range of influences; for extensive comments and suggestions I thank most of all Guy Barry, Mark Hepple, and Neil Leslie.

[1]For discussion of the rule-to-rule grammatical architecture see Oehrle (1988).

[2]See Moortgat (1989).

vectors in the operand types. The rules of grammar — theorems of type inference — are the validities according to the interpretation of types. The extension of Lambek categorial grammar given is related to linear logic (see e.g. Girard 1989) which drops structural rules from intuitionistic logic; in the present application this corresponds to the fact that grammar is concerned with e.g. presence, and number of occurrences, of linguistic objects.

Categorial grammar implements in a rather direct way a mode of linguistic analysis which is due in essence to Frege. Under such analysis certain not necessarily basic expressions are taken to be the primary bearers of meaning, and other expressions are attributed with meanings in terms of the meanings of the expressions in which they occur. Bidirectional categorial grammar provides for the classification of linguistic objects by starting with primitive types representing 'complete' (or: primarily meaningful) expressions, and building further classes by means of type-constructors / and \. Linguistic objects will be taken to have two dimensions, syntax and semantics. Assignment of a type X/Y ($Y\backslash X$) to an expression can be seen as a simultaneous classification according to form and meaning stating that the expression prefixes (postfixes) itself to expressions of type Y to form expressions of type X, and stating that the meaning of the resultant expression is given by the application of the meaning of the affix expression to that of the stem expression.

Consider for instance a language containing as complete expressions some proper names "John", "Mary", ...indexed by type NP and some sentences "John walks", "Mary walks", ..., "John likes John", ... indexed by type S. Then "walks" has type NP\S; so also do "likes John", "likes Mary", ..., and "likes" has type (NP\S)/NP. But other type assignments are valid under the intended meaning of the type-constructors. The expression "likes" also has type NP\(S/NP); "John" also has types S/(NP\S), ((NP\S)/NP)\(NP\S), and so on.

As a basis for grammar, the idea is to assign types to the vocabulary in such a way that the rest of the language, and the inhabitation of the sentence type in particular, is determined by rules of type inference. Assume for instance additional types PP and N for prepositional phrases and common noun phrases. Then types may be assigned to words as follows:

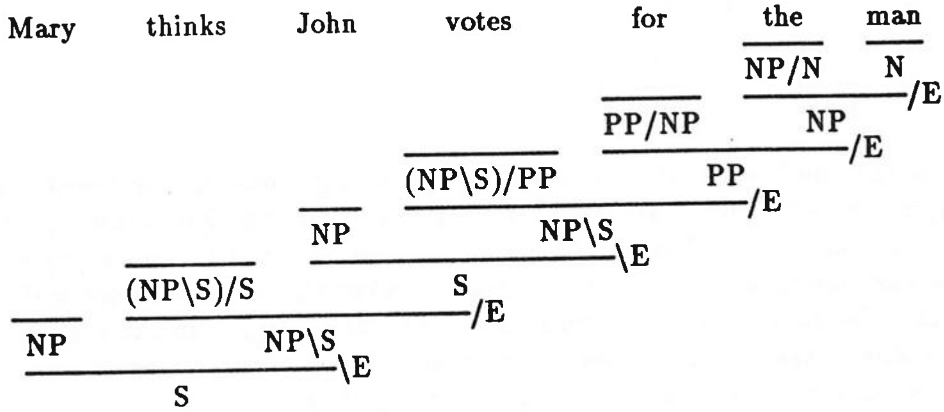
(1)

for	:= PP/NP
John, Mary	:= NP
likes	:= (NP\S)/NP
man	:= N
the	:= NP/N
thinks	:= (NP\S)/S
votes	:= (NP\S)/PP

The categorial calculus AB, essentially that of Ajdukiewicz (1935) and Bar-Hillel (1953), contains just the following two rules which state that (*functor*) expressions of types X/Y and $Y\backslash X$ combined with (*argument*) expressions of types Y form expressions of types X.

(2) a. $$\frac{\begin{array}{cc}\vdots & \vdots \\ X/Y & Y\end{array}}{X}/E$$ b. $$\frac{\begin{array}{cc}\vdots & \vdots \\ Y & Y\backslash X\end{array}}{X}\backslash E$$

A type under vertical ellipses signifies a derivation of that type, and a sole type constitutes a derivation of itself. By way of example, "Mary thinks John votes for the man" is derived as a sentence as follows:

(3) 

The 'ordered natural deduction' representation of categorial derivations used here is presented in Morrill, Leslie, Hepple, and Barry (1990). It has the convenient property that deductions are syntactically interpreted as the left-to-right concatenation of the premiss forms. Thus the rules of deduction in (2) are syntactically interpreted as concatenation of the subexpressions in left-to-right order, and semantically interpreted as *functional application* of the functor to the argument. The semantics of the composite expression in (3) is as follows, where the semantic operation of application is written as ' and elements of the semantic algebra are represented in boldface.

(4) **(thinks ' ((votes ' (for ' (the ' man))) ' John)) ' Mary**

Not all rules of inference are expressible in the inference figure format being used, and sequent rules will also be given. A sequent is of the form $\Gamma \Rightarrow X$ where Γ is a sequence of types, and asserts that there is a deduction of the *consequent* type X from the *antecedent* types Γ, i.e. that inhabitation of the antecedent types implies inhabitation of the consequent type. Sequents can be annotated with operators in the syntactic and semantic algebras indicating how the syntax and semantics of a consequent type inhabitant can be constructed out of the syntactic and semantic components of signs inhabiting the antecedent types. However, since the syntactic constructions here are always concatenation in left-to-right order, annotation will be limited to semantics.

(5) $\alpha : X \Rightarrow \alpha : X$ [Ax]

$\Delta(\Gamma) \Rightarrow \beta : Y$ [CUT]
$\Gamma \Rightarrow \alpha : X$
$\Delta(\alpha : X) \Rightarrow \beta : Y$

$\Gamma(\delta : X/Y\ \Delta) \Rightarrow \gamma : Z$ [/L]
$\Delta \Rightarrow \beta : Y$
$\Gamma(\delta'\beta : X) \Rightarrow \gamma : Z$

$\Gamma(\Delta\ \delta : Y\backslash X) \Rightarrow \gamma : Z$ [\L]
$\Delta \Rightarrow \beta : Y$
$\Gamma(\delta'\beta : X) \Rightarrow \gamma : Z$

The calculus AB does not capture all the type inferences that are valid according to the interpretation of types described above, e.g. it does not capture that inhabitation of type NP by "Mary" together with inhabitation of type (NP\S)/NP by "likes" implies inhabitation of type S/NP (by "Mary likes"). This completeness is achieved in the associative Lambek calculus L (Lambek 1958). L is obtained in the present 'natural deduction' format by adding the following conditionalisation rules to the directional modus ponens rules of AB. These state that from a derivation of Y from assumptions including an (appropriate) occurrence of X, a further derivation is obtained by withdrawing that assumption occurrence.

(6) a. $\dfrac{\begin{matrix}[X]^n\\ \vdots\\ Y\end{matrix}}{Y/X}/\mathrm{I}^n$ b. $\dfrac{\begin{matrix}[X]^n\\ \vdots\\ Y\end{matrix}}{X\backslash Y}\backslash \mathrm{I}^n$

There is the condition that X is respectively the rightmost and leftmost undischarged assumption above Y in /I and \I; there is also a condition that a sole assumption in a derivation cannot be withdrawn; thus no types are assigned to the empty string. That exactly one assumption must be withdrawn makes this an *occurrence* logic like linear logic; the directional system is non-commutative linear logic. In the introduction rules, the meaning of the result is given by the ***functional abstraction*** over the meaning of the discharged assumption. The sequent rules are as follows:

(7) $\dfrac{\Gamma \Rightarrow \lambda x\beta : Y/X}{\Gamma\ \ x : X \Rightarrow \beta : Y}$ [/R] $\qquad$ $\dfrac{\Gamma \Rightarrow \lambda x\beta : X\backslash Y}{x : X\ \ \Gamma \Rightarrow \beta : Y}$ [\R]

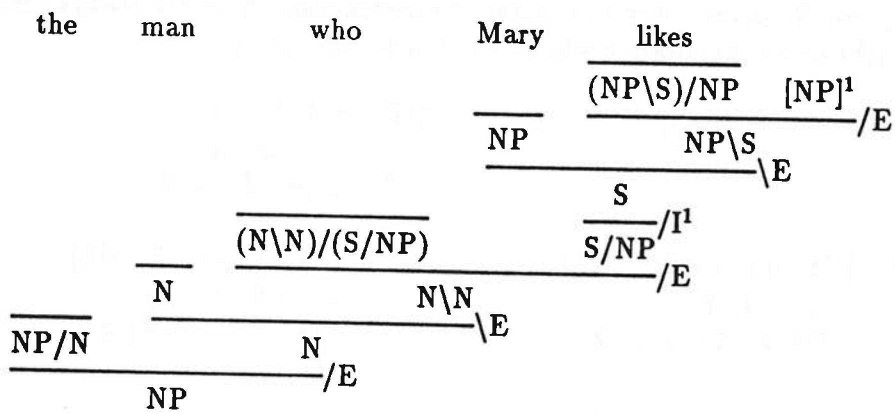
The rules in (6) are analogous to the usual natural deduction rule of →I, except exactly one assumption, in a particular place, must be withdrawn. Derivations are semantically interpreted in a single-bind lambda calculus satisfying the usual law of β-reduction so that the following equality holds (unary operators such as λx will be assumed to bind more tightly than binary operators such as '):

(8) $\lambda x\alpha\ '\ \beta = \alpha[\beta/x]$

This Curry-Howard 'formulas-as-types' interpretation of categorial deductions is introduced in van Benthem (1983) and implemented in a sequent context in Moortgat (1989). The simplification in (8) means that introducing and then eliminating implication constitutes a detour in derivation, producing a reducible, non-normal form, proof (cf. Prawitz 1965).

By way of illustration of conditionalisation, the non-canonical constituent "John likes" can be derived as of type S/NP, and assignment of type (N\N)/(S/NP) to an object relative pronoun produces relativisation: (cf. Ades and Steedman 1982).

(9)



The construction for the semantics is in (10).

(10) **the ' ((who ' λx((likes ' x) ' Mary)) ' man)**

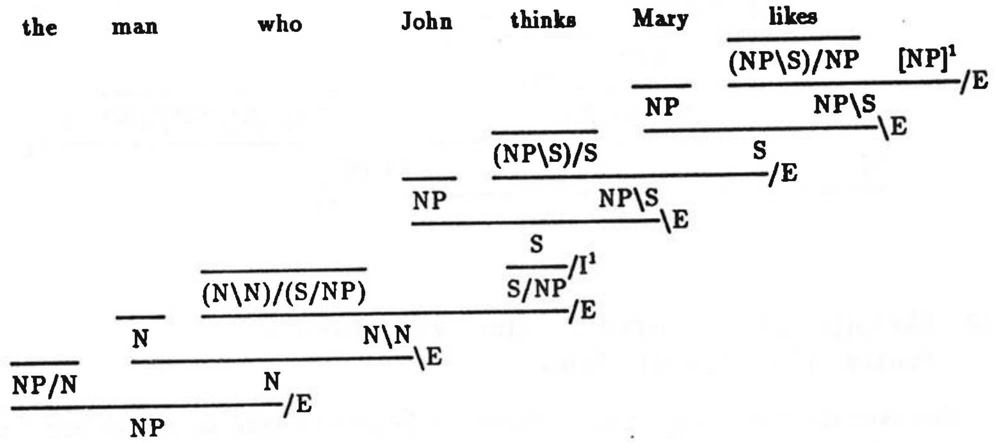
This specifies the meaning of the composite expression in terms of the meanings of the words. In order to give content to the semantics of words, these can themselves be expressed as terms in the semantic algebra. Thus the meaning of a relative pronoun might be described as follows:

(11) **who** = $\lambda x \lambda y \lambda z$((**AND** ' ($x$ ' z)) ' (y ' z))

Then (10) simplifies as shown in (12) (throughout variables will be renamed as is convenient).

(12) **the** ' (($\lambda x \lambda y \lambda z$((**AND** ' ($x$ ' z)) ' (y ' z)) ' λw((**likes** ' w) ' **Mary**)) ' **man**) =
the ' ($\lambda y \lambda z$((**AND** ' (λw((**likes** ' w) ' **Mary**) ' z)) ' (y ' z)) ' **man**) =
the ' λz((**AND** ' ((**likes** ' z) ' **Mary**)) ' (**man** ' z))

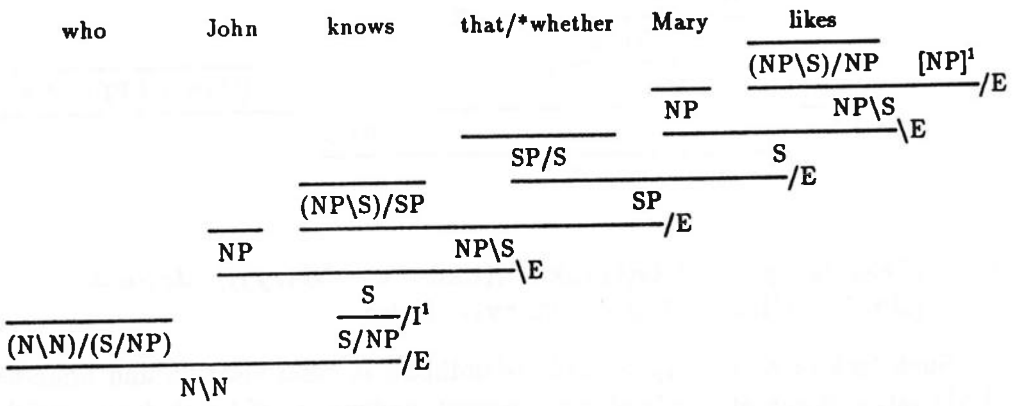
The operations ' and λx are like APPLY and LAMBDA (X) in the functional programming language Lisp, which was partly inspired by the lambda calculus. This treatment allows for long-distance relativisation:

(13)


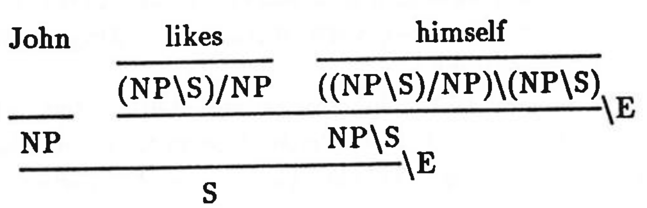
The meaning representation in (14) would reduce much as in (12).

(14) **the** ' ((**who** ' λx((**thinks** ' ((**likes** ' x) ' **Mary**)) ' **John**)) ' **man**)

However, a straightforward type assignment does not capture the following contrast (an instance of Chomsky's *wh*-island constraint):

(15)


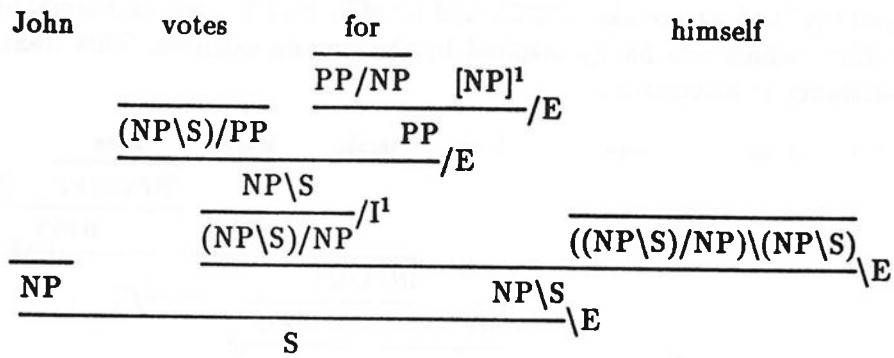
In relation to another kind of discontinuity in natural language, Szabolcsi (1987) observes that assignment of type ((NP\S)/NP)\(NP\S) to reflexives provides a characterisation of reflexivisation:

(16)


She notes that the reflexive should have the semantics of combinator **W** (lambda term $\lambda x \lambda y((x \text{ ' } y) \text{ ' } y)$). Desclés, Guentchéva and Shaumyan (1986) also make this proposal.

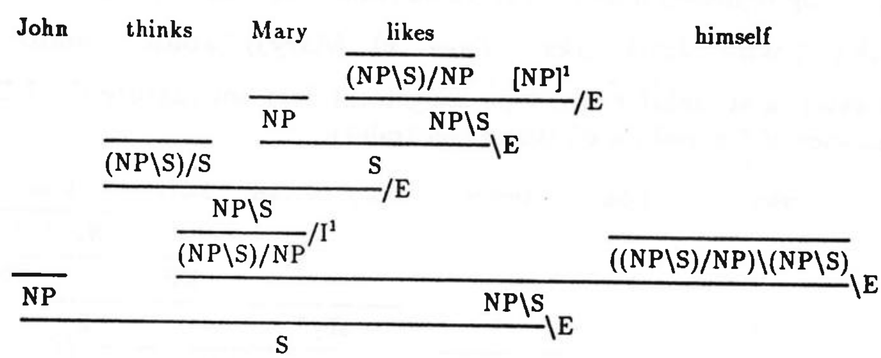
(17) $(\lambda x \lambda y((x \text{ ' } y) \text{ ' } y)$ ' **likes**) ' **John** =
$\lambda y(($**likes** ' $y)$ ' $y)$ ' **Mary** =
(**likes** ' **John**) ' **John**

Conditionalisation allows for less trivial cases:

(18)


(19) $(\lambda x \lambda y((x \text{ ' } y) \text{ ' } y)$ ' $\lambda z($**votes** ' (**for** ' $z)))$ ' **John** =
(**votes** ' (**for** ' **John**)) ' **John**

However the current grammar allows a reflexive to take an antecedent from a superordinate clause:

(20) *


(21) $(\lambda x \lambda y((x \text{ ' } y) \text{ ' } y)$ ' $\lambda z($**thinks** ' ((**likes** ' $z)$ ' **Mary**))) ' **John** =
(**thinks** ' ((**likes** ' **John**) ' **Mary**)) ' **John**

Such lack of sensitivity towards islandhood in relativisation and boundedness in reflexivisation is symptomatic of the apparent inadequacy of Lambek categorial type theory as a grammar formalism for natural language. There are many other shortcomings. For instance, natural language abounds with ambiguity: homonymy, and also ambiguity with respect to some but not all of the context of an element, e.g. transitive prepositions always take an object, but may then be either adnominal or adverbial, and many verbs can take different complements while uniformly forming verb phrases. Treating all such cases as lexical ambiguity misses the generalisations.

Further problems arise with respect to the kind of polymorphism that is treated with features in unification grammars: introduction of distinct primitive types for subclasses differing in minor features will lose those generalisations that can be made on the basis of major classes.

There are additionally well-known puzzles raised by an assumption of 'like-type' coordination: what type is shared by the conjuncts in "John is rich and an excellent cook"? And in "John or Mary walks" an attempt to match the gender of the conjuncts will fail.

Finally, the Lambek calculus is a sequence logic, and while this is appropriate for classification of linear linguistic forms, it of itself does not offer any control over the subtleties of occurrence and order in natural language.

The thesis of this paper is that with appropriate new type-constructors, categorial grammar can approach the kind of simultaneous sensitivity and generality required for description of natural language. Developing Morrill (1989b) more fully, the paper will introduce successive connectives together with their logic and informal interpretations, applying these to linguistic problems such as those mentioned above.

The calculus of Lambek (1958) actually included already the *product* type-constructor. The type $X \cdot Y$ is interpreted as the set of pairings of objects of types X and Y; there are the following rules of deduction:[3]

(22)
$$\frac{\begin{matrix}\vdots & \vdots\\ X & Y\end{matrix}}{X \cdot Y}\cdot\mathrm{I} \qquad \frac{\begin{matrix}\vdots\\ X \cdot Y\end{matrix}}{X \quad Y}\cdot\mathrm{E}$$

The annotated sequent rules are as follows.

(23)
$$\begin{array}{ll} \Gamma\ \Delta \Rightarrow \alpha,\beta : X \cdot Y \quad [\cdot\mathrm{R}] & \Gamma(\gamma : X \cdot Y) \Rightarrow \delta : W \quad [\cdot\mathrm{L}] \\ \quad \Gamma \Rightarrow \alpha : X & \quad \Gamma(\pi^1\gamma : X \quad \pi^2\gamma : Y) \Rightarrow \delta : W \\ \quad \Delta \Rightarrow \beta : Y & \end{array}$$

That pairing and projection are appropriate to the interpretation of product is noted in van Benthem (1987); they satisfy the following, being analogous to CONS, CAR and CDR in Lisp.

(24) $\pi_1(\alpha,\beta) = \alpha \qquad \pi_2(\alpha,\beta) = \beta$

These equalities again mean that feeding a product introduction into elimination results in a *redex*, i.e. a proof form which can be simplified. Complementarity in the semantic operations interpreting proofs will mean that the subsequent connectives have this introduction/elimination cancellation, which is a characteristic feature of type theory and its use in functional programming (see e.g. Girard, Taylor, and Lafont 1989). Hobbs and Rosenschein (1978) point out the close relations between Lisp and Montague's IL.

Linguistic applications of product are discussed in Wood (1988). In the following example it is used to give a Government-Binding style 'small-clause' analysis where the object and predicative following the verb form a constituent:

(25)
$$\dfrac{\dfrac{\text{Mary}}{\mathrm{NP}} \qquad \dfrac{\dfrac{\text{considers}}{(\mathrm{NP\backslash S})/(\mathrm{NP}\cdot(\mathrm{N/N}))} \qquad \dfrac{\dfrac{\text{John}}{\mathrm{NP}} \quad \dfrac{\text{tall}}{\mathrm{N/N}}}{\mathrm{NP}\cdot(\mathrm{N/N})}\cdot\mathrm{I}}{\mathrm{NP\backslash S}}/\mathrm{E}}{\mathrm{S}}\backslash\mathrm{E}$$

The semantic construction is given in (26).

[3] In fact unless product pairing is interpreted by a non-associative concatenation it is not clear that product elimination can be constructively interpreted in the syntax, since the appropriate partitioning cannot be recovered from a string

(26) **(considers ' (John , tall)) ' Mary**

The meaning of "considers" can be further specified:

(27) **considers** $= \lambda x \lambda y$((**CONSIDERS** ' y) ' ($\pi^2 x$ ' $\pi^1 x$))

Then (26) reduces thus:

(28) ($\lambda x \lambda y$((**CONSIDERS** ' y) ' ($\pi^2 x$ ' $\pi^1 x$)) ' (**John , tall**)) ' **Mary** =
λy((**CONSIDERS** ' y) ' (π^2(**John , tall**) ' π^1(**John , tall**))) ' **Mary** =
(**CONSIDERS** ' **Mary**) ' (**tall** ' **John**)

A product unit 1 would be as in (29).

(29) $\Rightarrow 1$ $\qquad$ $\dfrac{\Gamma(1) \Rightarrow X}{\Gamma() \Rightarrow X}$

2 Booleans

The conjunction type-constructor $\wedge$ is interpreted syntactically as intersection and semantically as pairing. Thus $X \wedge Y$ is the set of all sign vectors with an ordered pair semantics such that the syntax with the first meaning is a sign in X and the syntax with the second meaning is a sign in Y. There are the following rules of deduction.

(30) a. $\dfrac{\begin{array}{c}\vdots \\ X \wedge Y\end{array}}{X}\wedge E_a$ $\qquad$ b. $\dfrac{\begin{array}{c}\vdots \\ X \wedge Y\end{array}}{Y}\wedge E_b$

The $\wedge$I rule cannot be stated in the ordered natural deduction format but the sequent rules are as follows:

(31) $\dfrac{\Gamma(\gamma : X \wedge Y) \Rightarrow \delta : W}{\Gamma(\gamma_1 : X) \Rightarrow \delta : W}[\wedge L_a]$ $\qquad$ $\dfrac{\Gamma(\gamma : X \wedge Y) \Rightarrow \delta : W}{\Gamma(\gamma_2 : Y) \Rightarrow \delta : W}[\wedge L_b]$

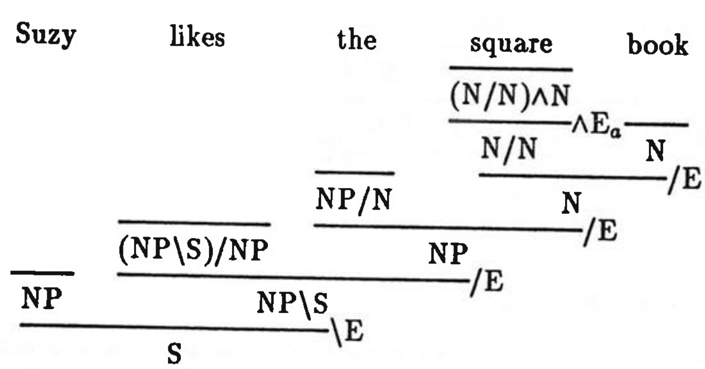
(32) $\dfrac{\Gamma \Rightarrow \alpha * \beta : X \wedge Y}{\Gamma \Rightarrow \alpha : X \qquad \Gamma \Rightarrow \beta : Y}[\wedge R]$

Intuitively the operator $*$ represents a point of nondeterminism and the operators $_1$ and $_2$ decisions to take the first and second branches. They obey the usual laws of pairing and projection:

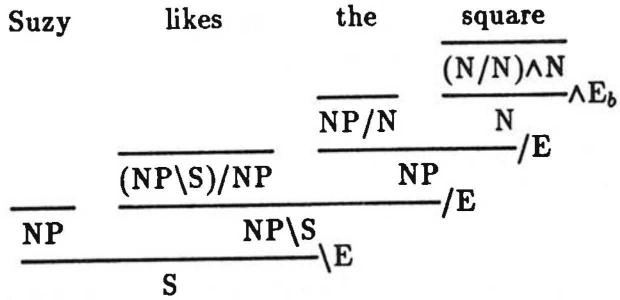
(33) $(\alpha * \beta)_1 = \alpha \qquad (\alpha * \beta)_2 = \beta$

Then e.g. the homonymy of "square" may now be captured by assignment of the type (N/N)$\wedge$N:

(34)

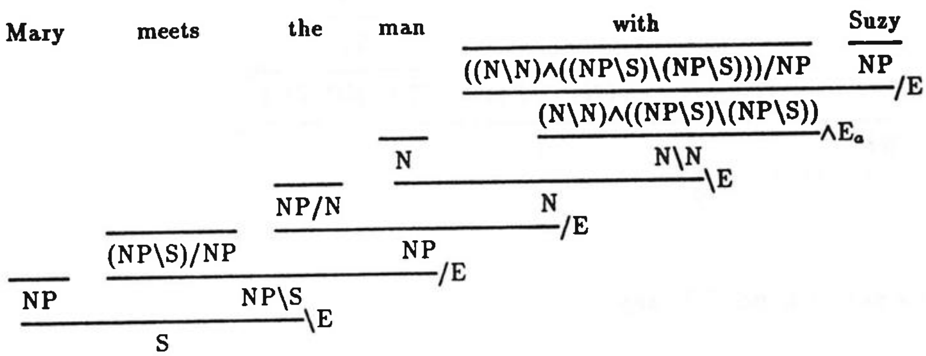


(35) **(likes ' (the ' (square$_1$ ' book))) ' Suzy**

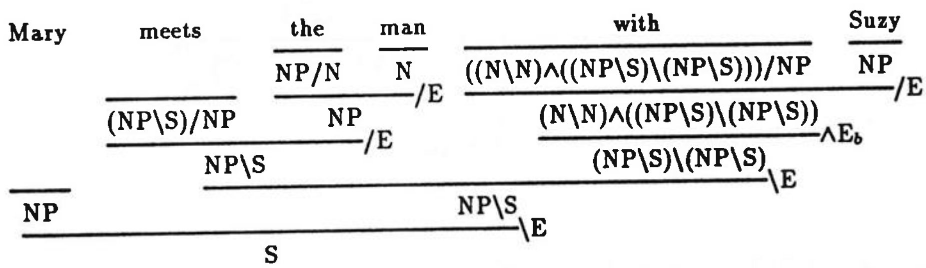
(36)


(37) **(likes ' (the ' square$_2$)) ' Suzy**

The conjunction connective also allows for the description of 'value ambiguity', such as that of a preposition which having once combined with its object may play the role of either an adnominal or adverbial. The two readings of "Mary meets the man with Suzy" are obtained from a single type assignment:

(38)


(39) **(meets ' (the ' ((with ' Suzy)$_1$ ' man))) ' Mary**

(40)


(41) **(((with ' Suzy)$_2$) ' (meets ' (the ' man))) ' John**

The disjunction type-constructor $\vee$ is interpreted syntactically as union and semantically as disjoint union. Thus $X \vee Y$ is the type of all sign vectors consisting of a syntax, and semantics flagged according as the syntax and unflagged semantics is in X or Y. There are the following rules of deduction:

(42) a.
$$\frac{\begin{matrix}\vdots \\ X\end{matrix}}{X \vee Y}\vee I_a$$

b.
$$\frac{\begin{matrix}\vdots \\ Y\end{matrix}}{X \vee Y}\vee I_b$$

This time the $\vee$E rule is hard to state as an ordered natural deduction, but the sequent rules are thus:

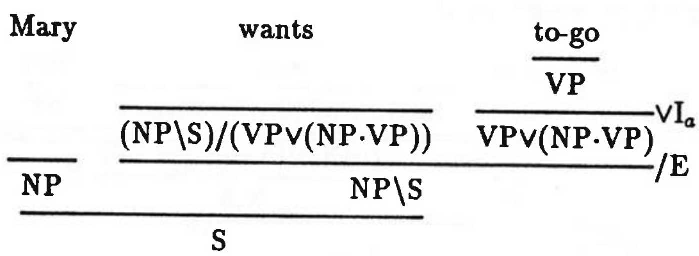
(43) $$\frac{\Gamma \Rightarrow i\alpha : X \vee Y \quad [\vee R_a]}{\Gamma \Rightarrow \alpha : X} \qquad \frac{\Gamma \Rightarrow j\beta : X \vee Y \quad [\vee R_b]}{\Gamma \Rightarrow \beta : Y}$$

(44) $$\Gamma(\delta : X \vee Y) \Rightarrow \delta \rightarrow x.\gamma^i; y.\gamma^j : Z \quad [\vee L]$$
$$\Gamma(x : X) \Rightarrow \gamma^i : Z$$
$$\Gamma(y : Y) \Rightarrow \gamma^j : Z$$

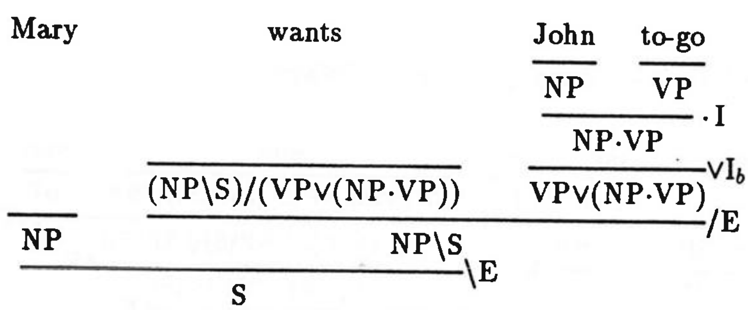
Intuitively the operator in (44) is a case operator keyed on i and j. Depending on the labelling, the first or second branch is taken:

(45) $i\alpha \rightarrow x.\gamma^i; y.\gamma^j = \gamma^i[\alpha/x]$
$j\alpha \rightarrow x.\gamma^i; y.\gamma^j = \gamma^j[\alpha/y]$

The disjunction connective finds application in e.g. the ambiguity of "wants" with respect to its complementation:

(46)


(47) **(wants ' *i*to-go) ' Mary**

(48)


(49) **(wants ' *j*(John , to-go)) ' Mary**

The semantics of wants can be spelled out in terms of a more primitive **WANTS** as follows:

(50) **wants** $= \lambda x \lambda y((\textbf{WANTS}\ '\ y)\ '\ x(\rightarrow z.(z\ '\ y); w.(\pi^2 w\ '\ \pi^1 w)))$

Then (47) simplifies as in (51).

(51) $(\lambda x \lambda y((\textbf{WANTS}\ '\ y)\ '\ x(\rightarrow z.(z\ '\ yx); w.(\pi^2 w\ '\ \pi^1 w)))\ '\ i\textbf{to-go})\ '\ \textbf{Mary} =$
$(\textbf{WANTS}\ '\ \textbf{Mary})\ '\ (i\textbf{to-go} \rightarrow z; (z\ '\ \textbf{Mary}); w.(\pi^2 w\ '\ \pi^1 w)) =$
$(\textbf{WANTS}\ '\ \textbf{Mary})\ '\ (\textbf{to-go}\ '\ \textbf{Mary})$

Similarly, (49) reduces as in (52).

(52) $$(\lambda x \lambda y((\mathbf{WANTS}\ '\ y)\ '\ (x \to z.(z\ '\ yx); w.(\pi^2 w\ '\ \pi^1 w)))\ '\ j(\mathbf{John}\ , \mathbf{to\text{-}go}))\ '\ \mathbf{Mary} =$$
$$(\mathbf{WANTS}\ '\ \mathbf{Mary})\ '\ (j(\mathbf{John}\ , \mathbf{to\text{-}go}) \to z.(z\ '\ \mathbf{Mary}); w.(\pi^2 w\ '\ \pi^1 w)) =$$
$$(\mathbf{WANTS}\ '\ \mathbf{Mary})\ '\ (\pi^2(\mathbf{John}\ , \mathbf{to\text{-}go})\ '\ \pi^1(\mathbf{John}\ , \mathbf{to\text{-}go})) =$$
$$(\mathbf{WANTS}\ '\ \mathbf{Mary})\ '\ (\mathbf{to\text{-}go}\ '\ \mathbf{John})$$

Disjunction suggests a means by which coordination of unlike types can be licensed if there is a suitable functor over the disjunction of their domains:

(53) a. John is rich.

b. John is an excellent cook.

c. John is rich and an excellent cook.

(54) $$\frac{\text{is}}{(\mathrm{NP}\backslash \mathrm{S})/(\mathrm{NP}\vee(\mathrm{N}/\mathrm{N}))} \qquad \cfrac{\cfrac{\text{rich}}{\mathrm{N}/\mathrm{N}}}{\mathrm{NP}\vee(\mathrm{N}/\mathrm{N})}\vee I_b \qquad \cfrac{\cfrac{\text{an excellent cook}}{\mathrm{NP}}}{\mathrm{NP}\vee(\mathrm{N}/\mathrm{N})}\vee I_a$$

The conjunction type actually appears in Lambek (1961), and use of booleans is implied in van Benthem (1989a). Keenan and Timberlake (1988) use an 'n-tuple' type constructor such that if $X_1, Y_1, \ldots, X_n, Y_n$ are types then so is $< X_1, \ldots, X_n > / < Y_1, \ldots, Y_n >$. This would be defined in terms of the proposal above as $(X_1/Y_1) \wedge \ldots \wedge (X_n/Y_n)$; the n-tuple types for the earlier "with" and "wants" would be <N\N, (NP\S)\(NP\S)> / <NP, NP> and <NP\S, NP\S> / <VP, NP·VP>, i.e. they do not capture the generalisations that domains or ranges are the same.

A negation type-constructor could be syntactically interpreted as set complement. A universal type t and null type $\bot$ have logic as follows:

(55) $\Gamma \Rightarrow \mathrm{t} \qquad\qquad \bot \Rightarrow X$

3 Quantification

The proposal of this section is to achieve increased sensitivity by moving from a *propositional* system of types, to a *predicational* one. Unification will be represented as being a way of implementing part of such a proposal.

Instead of just the primitive types as propositional non-logical constants, there will now be feature, feature-function, and predicate constants.[4] Thus the type of a feminine noun phrase might be NP(f), or aiming for more information the type of a nominative third person feminine noun phrase might be NP(third(f), nom). In order to ensure coherent occupency of argument positions, the system should be *sorted*, i.e. distinguishing gender from case, etc.

A facility of variables and quantification over features enables description of polymorphisms. A universally quantified type signifies feature-dependent elements which for any feature of the quantified sort can adapt to a member of the class represented by the body of the quantified type under a valuation where the quantified variable is assigned that feature.

[4]The term 'feature' is used here to mean just a characteristic; the more usual unification grammar terminology would be 'feature-value'

The rules of deduction for universal quantification are as follows:

(56)
$$\dfrac{\begin{array}{c}\vdots\\ X\end{array}}{\forall v X}\forall I \qquad \dfrac{\begin{array}{c}\vdots\\ \forall v X\end{array}}{X[F/v]}\forall E$$

There is the condition on ∀I that v is not free in any undischarged assumption above X, and in ∀E F must be a feature with the sort of v. The introduction rule is semantically interpreted as abstraction over the quantified sort, and the elimination as application to features of the quantified sort. The annotated sequent rules are as follows:

(57)
$$\dfrac{\Gamma \Rightarrow \alpha : X}{\Gamma \Rightarrow Lv\alpha : \forall v X}[\forall R] \qquad \dfrac{\Gamma(\gamma'F : X[F/v]) \Rightarrow \beta : Y}{\Gamma(\gamma : \forall v X) \Rightarrow \beta : Y}[\forall L]$$

The feature application and abstraction operators satisfy the usual lambda conversion; feeding ∀I into ∀E produces a proof redex.

(58) $Lv\alpha\ '\ F = \alpha[F/v]$

Existentially quantified types are semantically interpreted as pairs consisting of a feature of the quantified sort, and a member of the body domain under a valuation where the quantified variable is assigned that feature. The elimination deduction cannot be represented by an ordered natural deduction inference figure, but the introduction rule is thus:

(59)
$$\dfrac{\begin{array}{c}\vdots\\ X[F/v]\end{array}}{\exists v X}\exists I$$

The sequent rules are as follows:

(60)
$$\dfrac{\Gamma \Rightarrow \alpha : X[F/v]}{\Gamma \Rightarrow F`\alpha : \exists v X}[\exists R] \qquad \dfrac{\Gamma(2\gamma : X[1\gamma/v]) \Rightarrow \beta : Y}{\Gamma(\gamma : \exists v X) \Rightarrow \beta : Y}[\exists L]$$

(61) $1(\alpha\ `\ \beta) = \alpha \qquad 2(\alpha\ `\ \beta) = \beta$

Type assignments and derivations may now be as illustrated in (62).

(62) Mary likes the tall man

$$\dfrac{\dfrac{\overline{NP(f)}}{\exists g NP(g)}\exists I \quad \dfrac{\overline{(\exists g NP(g)\backslash S)/\exists g NP(g)} \quad \dfrac{\dfrac{\dfrac{\overline{\forall g[NP(g)/N(g)]}}{NP(m)/N(m)}\forall E \quad \dfrac{\dfrac{\overline{\forall g[N(g)/N(g)]}}{N(m)/N(m)}\forall E \quad \overline{N(m)}}{N(m)}/E}{NP(m)}/E}{\exists g NP(g)}\exists I}{\exists g NP(g)\backslash S}/E}{S}\backslash E$$

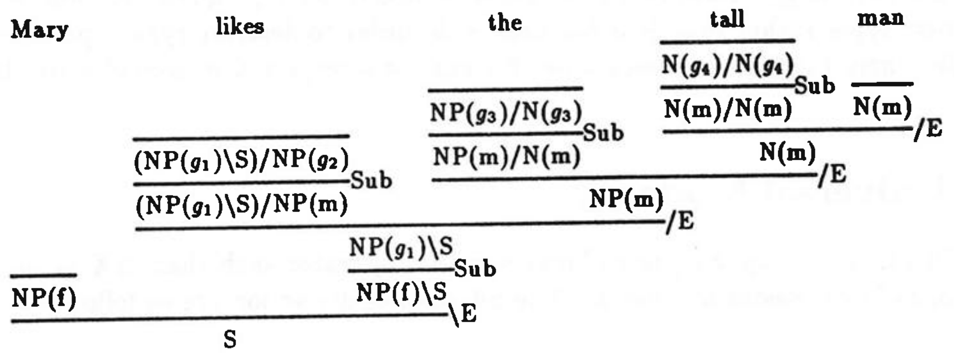
(63) **(likes ' (m ' ((the ' m) ' ((tall ' m) ' man)))) ' (f ' Mary)**

In the case that there is a quantifier free logic with all variables implicitly quantified at the outermost level, this scheme looks like a term unification formalism such as that used in

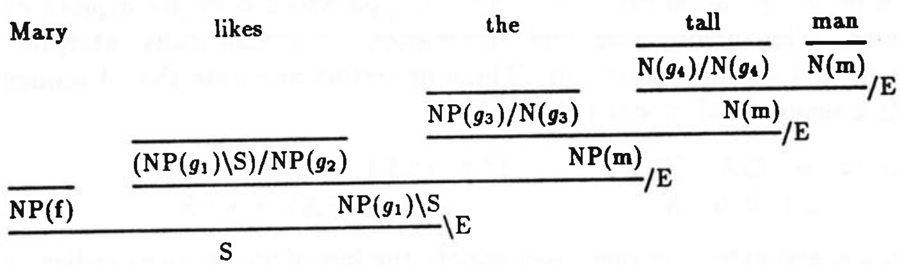
Unification Categorial Grammar (Zeevat, Klein, and Calder 1987); the universal quantifier rules are replaced by a rule of substitution:

(64)
$$\frac{\begin{array}{c}\vdots\\ X\end{array}}{X[a/v]}\text{Sub}$$

Then the previous derivation may be as follows:

(65)


Alternatively, as noted in van Benthem (1989b), a system of deduction based on resolution (such as that implemented in Prolog) could be used. For AB deductions, a most general unifier of a functor's argument type and an actual argument type is to be applied to the functor's value to obtain the actual value:

(66)


Finally, type structures could be notated as sets of paths, e.g. NP(third(f), nom) could be encoded [[Maj NP] [Min [Agr [[Per third] [Gen f]]] [Case nom]]], resulting in a graph unification formalism such as that used in Head-Driven Phrase Structure Grammar (Pollard and Sag 1987).

In term and graph unification formalisms it is possible to use the implicit universal quantification implemented by unification to assemble semantic (and syntactic) representations within the type structures. With explicit quantification however, the lambda semantics for universally quantified types is seen to be just like that for any other type-constructor, with the usual type theoretic relation between proofs over types, and functional terms. Limitation to universal quantification retains much expressive power, but it may be noted that existential quantification will provide a shared type for the conjuncts in such cases as (67) which universal quantification could not provide without the disjunction of the previous section.

(67) John or Mary walks.

(68) $\dfrac{\dfrac{\overline{\text{John}}}{\text{NP(m)}}}{\exists g\text{NP}(g)}\exists\text{I}$ $\qquad$ $\dfrac{\dfrac{\overline{\text{Mary}}}{\text{NP(f)}}}{\exists g\text{NP}(g)}\exists\text{I}$

The current proposals do not include boolean operations on features as in e.g. NP(f&m), though there seems no reason why such logical feature-functions should not be included. Another natural generalisation is to allow second order type quantification — quantification over types rather than just features — in order to describe type dependent elements; van Benthem (1989b) discusses how this can be interpreted in second order lambda calculus.

4 Universal Modality

Morrill (1989a, to appear) introduces a modal operator such that $\Box X$ is the type of intensions of expressions of type X. The S4 rules of deduction are as follows:

(69) $\dfrac{\begin{matrix}\vdots\\ X\end{matrix}}{\Box X}\Box\text{I} \qquad \dfrac{\begin{matrix}\vdots\\ \Box X\end{matrix}}{X}\Box\text{E}$

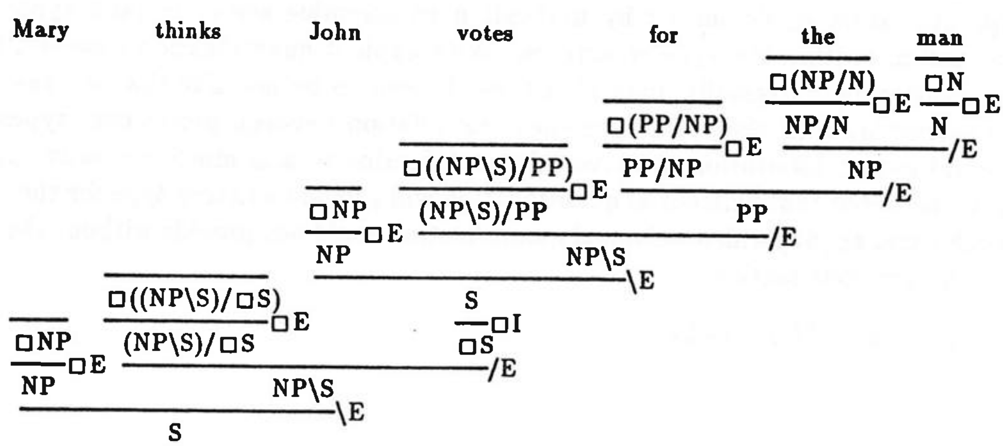
There is the condition on $\Box$I that every path from the root to an undischarged assumption contains a licensing modal type, i.e. a modal type which does not depend on discharged assumptions. The introduction and elimination are semantically interpreted as intensionalisation and extensionalisation. These operators annotate the S4 sequent rules ($\Box\Gamma$ represents a sequence of modal types).

(70) $\Box\Gamma \Rightarrow {}^{\wedge}\alpha : \Box X \quad [\Box\text{R}] \qquad \Gamma(\beta : \Box X) \Rightarrow \gamma : Z \quad [\Box\text{L}]$
$\qquad \Box\Gamma \Rightarrow \alpha : X \qquad\qquad\qquad \Gamma({}^{\vee}\beta : X) \Rightarrow \gamma : Z$

The intension and extension operators satisfy the law of 'down-up cancellation', and feeding $\Box$I into $\Box$E produces a proof redex; these operators have analogues in the QUOTE and EVAL of Lisp.

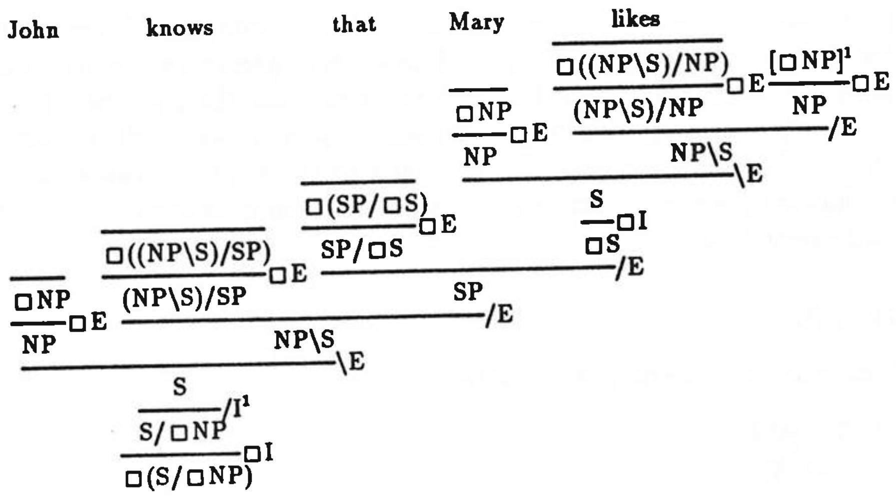
(71) ${}^{\vee\wedge}\alpha = \alpha$

An intensional analysis of "Mary thinks John votes for the man" is given in (72).

(72)


(73) (ˇ**thinks** ' ˆ(ˇ**votes** ' (ˇ**for** ' (ˇ**the** ' ˇ**man**))) ' **John**) ' **Mary**

Consider the effect of assigning a relative pronoun type □(N\N)/□(S/□NP). Then the body of "who John knows that Mary likes" is obtained as follows:

(74) 

The meaning of the embedded clause is thus:

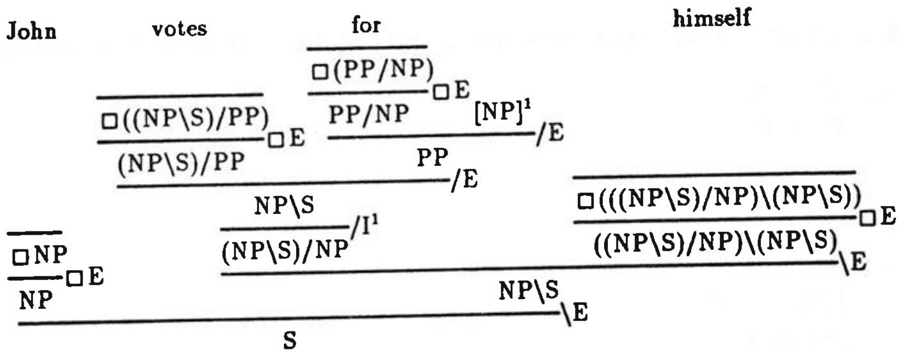
(75) **who** ' ˆλx((ˇ**knows** ' (ˇ**that** ' ˆ((ˇ**likes** ' ˇx) ' **Mary**)) ' **John**))

However, if "whether" is of type □SP/□S, the earlier *wh*-island violation is not obtained: the sequent in (76) is not a theorem.

(76)

John	knows	whether	Mary	likes	
□NP	□((NP\S)/SP)	□SP/□S	□NP	□((NP\S)/NP)	⇒ □(S/□NP)

Reflexivisation might be treated as follows.

(77) 

(78) (**himself** ' λx(ˇ**votes** ' (ˇ**for** ' x))) ' **John**

Then the long distance reflexivisation is no longer obtained; the sequent in (79) is not a theorem.

(79)

Bill	thinks	Mary	likes	himself	
□NPm	□((NPm\S)/□S)	□NPf	□((NPf\S)/NPm)	□(((NPm\S)/NPm)\(NPm\S))	⇒ S

5 Structural Operations

In an *occurrence* logic such as linear logic, the structural rules of weakening and contraction are not valid: the presence and number of occurrences of a premiss are significant.

However, Girard reintroduces these operations via *exponentials* or *structural modalities* which are unary type connectives that essentially license the structural rules on formulas bearing the appropriate connective.

Taking this strategy as inspiration, Morrill, Leslie, Hepple, and Barry (1990) present structural modalities for categorial calculi. Linear logic preserves the structural rule of permutation; since this is also dropped in sequence logics, like the Lambek calculus, we will again adopt Girard's strategy and invoke exchange exponentials. Although the proposals are in the spirit of linear logic, they will be tuned to the sequence calculi and linguistic applications that are the current concern, and similarities in notation do not imply identity with standard linear logic.

5.1 Optionality

The structural rule of weakening is as follows:

(80) $\dfrac{\Gamma\, Y \Rightarrow X}{\Gamma \Rightarrow X}$ [W]

The assertion is that when some premisses yield a conclusion, the conclusion is still yielded when a further premiss is added. The optionality of the presence of a premiss has a linguistic analogue in the optionality of certain elements. Assume a unary connective ? such that $?X$ is the type of those expressions which are either an X, or else are the empty expression. A sequence $\Gamma(?X)$ can then be regarded as abbreviating the disjunction $\Gamma(X)$ or $\Gamma()$. The following rules of deduction are valid.

(81) a. $\dfrac{\begin{matrix}\vdots \\ X\end{matrix}}{?X}\,?I_1$ b. $\dfrac{}{?X}\,?I_2$

Expressed as sequent proof rules these are as in (82); the rule in (83) is also valid.

(82) a. $\dfrac{\Gamma \Rightarrow ?X}{\Gamma \Rightarrow X}$ [?R₁]

b. $\Rightarrow ?X$ [?R₂]

(83) $\dfrac{\Gamma(?X) \Rightarrow Y}{\Gamma(X) \Rightarrow Y \quad \Gamma() \Rightarrow Y}$ [?L]

Then for example, the optionality of the sentential complement of *belief* is characterised by assignment to N/?SP:

(84) the belief that John lies

$$\dfrac{\dfrac{}{NP/N} \quad \dfrac{\dfrac{}{N/?SP} \quad \dfrac{\dfrac{}{SP}}{?SP}\,?I_1}{N}\,/E}{NP}\,/E$$

(85)
$$\dfrac{\text{the} \qquad \dfrac{\dfrac{\text{belief}}{N/?SP} \quad \dfrac{}{?SP}\,?I}{N}\,/E}{NP}\,/E$$

5.2 Iterability

The structural rules of contraction and expansion are given in (86) (note that expansion is subsumed by weakening).

(86) a. $\dfrac{\Gamma\ Y \Rightarrow X}{\Gamma\ Y\ Y\ \Rightarrow X}\ [C]$

b. $\dfrac{\Gamma\ Y\ Y \Rightarrow X}{\Gamma\ Y \Rightarrow X}\ [E]$

The rules describe invariance of validities under numbers of occurrences of premisses. Such flexibility has linguistic analogy in e.g. the variable number of conjuncts that can occur left of a coordinator, and the variable number of gaps that can be filled by a fronted element in a language permitting parasitic extraction. Two possibilities are as follows: there may be a unary type constructor $^+$ indicating *some* degree of iteration, or one ! indicating *any* degree of iteration. The sequence $\Gamma(X^+)$ abbreviates the infinite disjunction $\Gamma(X)$ or $\Gamma(X\ X)$, or $\Gamma(X\ X\ X)$, etc. The sequence $\Gamma(!X)$ abbreviates the infinite conjunction $\Gamma(X)$ and $\Gamma(X\ X)$, and $\Gamma(X\ X\ X)$, etc. The following rules of deduction suggest themselves[5].

(87) a. $\dfrac{\begin{matrix}\vdots\\ X\end{matrix}}{X^+}\,{}^+I$ b. $\dfrac{\begin{matrix}\vdots\\ X^+\end{matrix} \quad \begin{matrix}\vdots\\ X^+\end{matrix}}{X^+}\,{}^+E$

(88) a. $\dfrac{\begin{matrix}\vdots\\ !X\end{matrix}}{X}\,!E$ b. $\dfrac{\begin{matrix}\vdots\\ !X\end{matrix}}{!X\ \ !X}\,!I$

Correspondingly there are the following sequent rules:

(89) $\dfrac{\Gamma \Rightarrow X^+}{\Gamma \Rightarrow X}\ [^+R]$ $\dfrac{\Gamma(X^+\ X^+) \Rightarrow Y}{\Gamma(X^+) \Rightarrow Y}\ [^+Con]$

(90) $\dfrac{\Gamma(!X) \Rightarrow Y}{\Gamma(X) \Rightarrow Y}\ [!L]$ $\dfrac{\Gamma(!X) \Rightarrow Y}{\Gamma(!X\ !X) \Rightarrow Y}\ [!Exp]$

[5] Interactions of iteration and optionality are not considered here.

Iterated coordination may be treated by assignment of coordinators to $(X^+\backslash X)/X$:

(91) John Bill Mary and Suzy

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{NP}}{\text{NP}^+}{+}\text{I} \quad \dfrac{\text{NP}}{\text{NP}^+}{+}\text{I}}{\text{NP}^+}{+}\text{E} \quad \dfrac{\text{NP}}{\text{NP}^+}{+}\text{I}}{\text{NP}^+}{+}\text{E} \quad \dfrac{(\text{NP}^+\backslash\text{NP})/\text{NP} \quad \text{NP}}{\text{NP}^+\backslash\text{NP}}/\text{E}}{\text{NP}}\backslash\text{E}$$

5.3 Word Order Variation

The structural rule of exchange (or: permutation) is as follows:

(92) $\dfrac{\Gamma(X\ Y) \Rightarrow Z}{\Gamma(Y\ X) \Rightarrow Z}\ [\text{P}]$

Just as linear logic introduces weaken and contract structural modalities to replace the structural rules, we may propose to supply exchange or permute connectives to sequence logic. Let $\Gamma \rhd X\ Y_1 \ldots Y_n$ abbreviate the conjunction $\Gamma\ X\ Y_1 \ldots Y_n$ and $\Gamma\ Y_1\ X \ldots Y_n$ …and $\Gamma\ Y_1 \ldots X\ Y_n$ and $\Gamma\ Y_1 \ldots Y_n\ X$. Likewise, let $Y_1 \ldots Y_n\ X \lhd \Gamma$ abbreviate $Y_1 \ldots Y_n\ X\ \Gamma$ and $Y_1 \ldots X\ Y_n\ \Gamma$ …and $Y_1\ X \ldots Y_n\ \Gamma$ and $X\ Y_1 \ldots Y_n\ \Gamma$. Then the following rules are valid:

(93) a. $\dfrac{\begin{matrix}\vdots\\ X\end{matrix}}{\rhd X}\rhd\text{I}$ b. $\dfrac{\begin{matrix}\vdots\\ X\end{matrix}}{X\lhd}\lhd\text{I}$

(94) a. $\dfrac{\begin{matrix}\vdots\\ \rhd X\end{matrix} \quad \begin{matrix}\vdots\\ Y\end{matrix}}{Y \quad \rhd X}$ b. $\dfrac{\begin{matrix}\vdots\\ Y\end{matrix} \quad \begin{matrix}\vdots\\ X\lhd\end{matrix}}{X\lhd \quad Y}$

(95) a. $\dfrac{\begin{matrix}\vdots\\ \rhd X\end{matrix}}{X}\rhd\text{E}$ b. $\dfrac{\begin{matrix}\vdots\\ X\lhd\end{matrix}}{X}\lhd\text{E}$

The rules of $\rhd$I and $\lhd$I are subject to the condition that every path from root to undischarged assumption contain a licensing modal type (right permute and left permute types respectively which do not depend on discharged assumptions). There are the following sequent rules, where $\rhd\Gamma$ and $\Gamma\lhd$ indicate sequences of the appropriate modal types. Note that the Left and Right rules are those for S4.

(96)

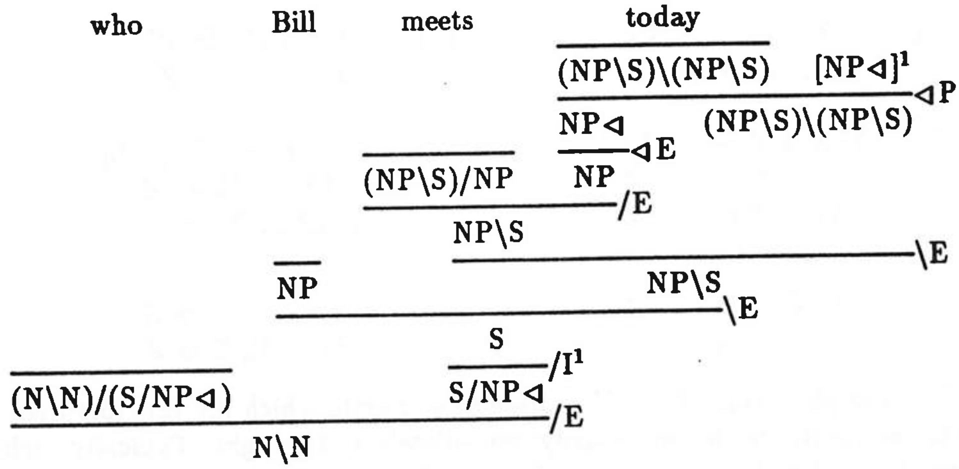
$\dfrac{\rhd\Gamma \Rightarrow \rhd X}{\rhd\Gamma \Rightarrow X}\ [\rhd\text{R}]$ $\qquad \dfrac{\Gamma\lhd \Rightarrow X\lhd}{\Gamma\lhd \Rightarrow X}\ [\lhd\text{R}]$

$\dfrac{\Gamma(\rhd X\ Y) \Rightarrow Z}{\Gamma(Y\ \rhd X) \Rightarrow Z}\ [\rhd\text{P}]$ $\qquad \dfrac{\Gamma(Y\ X\lhd) \Rightarrow Z}{\Gamma(X\lhd\ Y) \Rightarrow Z}\ [\lhd\text{P}]$

$\dfrac{\Gamma(\rhd X) \Rightarrow Y}{\Gamma(X) \Rightarrow Y}\ [\rhd\text{L}]$ $\qquad \dfrac{\Gamma(X\lhd) \Rightarrow Y}{\Gamma(X) \Rightarrow Y}\ [\lhd\text{L}]$

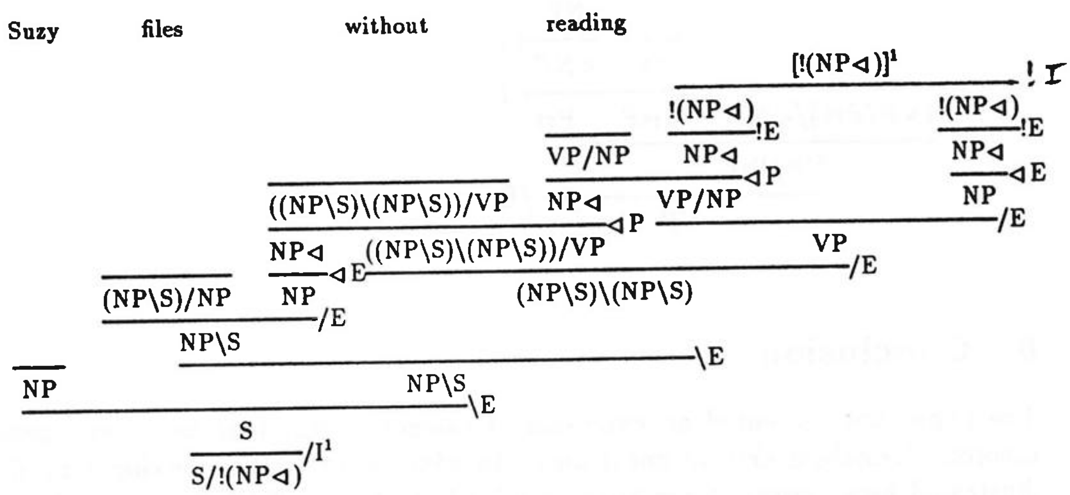
Assignment of type (N\N)/(S/NP◁) to a relative pronoun will now allow it to fill a gap in *any* position, not just a clause-peripheral one:

(97)



In this way it is possible to *define* the kind of discontinuous type-constructors introduced in Moortgat (1989). Furthermore, assignment of a type (N\N)/(S/!(NP◁)) will allow it to fill *any number of* gaps in any positions. Thus "the paper which Suzy files without reading" can be derived as follows:

(98)



The permute modalities that have been given signify that a type belongs *anywhere* to one side. Dual permute modalities can signify that a type belongs *somewhere* to one side. The logic may be as follows:

(99) a. $\dfrac{\begin{matrix}\vdots\\ X\end{matrix}}{X>}>I$ b. $\dfrac{\begin{matrix}\vdots\\ X\end{matrix}}{<X}<I$

(100) a. $\dfrac{\begin{matrix}\vdots\\ X>\end{matrix} \quad \begin{matrix}\vdots\\ Y\end{matrix}}{Y \quad X>}$ b. $\dfrac{\begin{matrix}\vdots\\ Y\end{matrix} \quad \begin{matrix}\vdots\\ <X\end{matrix}}{<X \quad Y}$

(101)

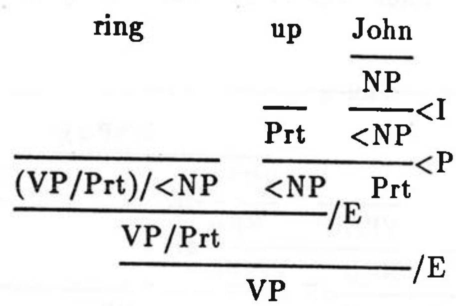
$$\frac{\Gamma \Rightarrow X>}{\Gamma \Rightarrow X}\ [>R] \qquad \frac{\Gamma \Rightarrow <X}{\Gamma \Rightarrow X}\ [<R]$$

$$\frac{\Gamma(X> \ Y) \Rightarrow Z}{\Gamma(Y \ X>) \Rightarrow Z}\ [>P] \qquad \frac{\Gamma(Y \ <X) \Rightarrow Z}{\Gamma(<X \ Y) \Rightarrow Z}\ [<P]$$

$$\begin{array}{l} Y_1 \ldots Y_n X > \Gamma \Rightarrow Z \quad [>L] \\ Y_1 \ldots Y_n X \Gamma \Rightarrow Z \\ Y_1 \ldots X Y_n \Gamma \Rightarrow Z \\ \vdots \\ Y_1 X \ldots Y_n \Gamma \Rightarrow Z \\ X Y_1 \ldots Y_n \Gamma \Rightarrow Z \end{array} \qquad \begin{array}{l} \Gamma < X Y_1 \ldots Y_n \Rightarrow Z \quad [<L] \\ \Gamma X Y_1 \ldots Y_n \Rightarrow Z \\ \Gamma Y_1 X \ldots Y_n \Rightarrow Z \\ \vdots \\ \Gamma Y_1 \ldots X Y_n \Rightarrow Z \\ \Gamma Y_1 \ldots Y_n X \Rightarrow Z \end{array}$$

Then for example a type $X/<Y$ represents elements which are functors over Xs *somewhere* to the right, but not necessarily immediately to the right. Typically such specifications would need to be made with reference also to the domains within which the word order variation is allowed, e.g. by using the modal device illustrated in section 4, but the essential idea is as illustrated in (102).

(102)



6 Conclusion

The paper has presented an extension of categorial grammar to a more general type theoretic formalism akin to linear logic. In addition to further development of the kind illustrated here, several tasks arise: precise formulation of the logic and determination of its properties, specification of a formal semantics, and the development of effective automated type inference. No doubt much of relevance is to be found in the Computer Science literature on type theory and linear logic. This paper at least aims to have provided a survey of the possibilities for practical description of natural language in the categorial tradition.

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A Discourse Perspective on Verb Phrase Anaphora *

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Introduction

Existing treatments of VP anaphora (VP ellipsis) view this phenomenon in a very local fashion, as only involving an anaphoric VP and an antecedent VP (see section 1). In this paper, however, we shall show that this approach only works for simple cases, and that it cannot account for the interaction between VP anaphora, quantifier scope and pronominal reference. We shall demonstrate that the process of resolving a VP anaphor must involve the entire structure of the clause in which the VP anaphor occurs, as well as the entire structure of the clause in which the antecedent VP occurs: a VP anaphor induces syntactic/semantic parallelism between the 'anaphoric clause' and the 'antecedent clause'. This means that the traditional notion of 'VP identity' needs to be broadened to a notion of 'clausal parallelism' (section 2.1). Furthermore we show that the resolution of VP anaphora is also constrained by the structure of discourse (2.2).

Our treatment of ellipsis is embedded in a general framework for the description of discourse structure and discourse semantics (section 3). A crucial mechanism in the integration of a new sentence within the previous discourse is a mechanism for establishing the common ground of syntactic/semantic structures of adjacent clauses, or rather of adjacent discourse constituent units. Extracting common ground, and with that clausal parallelism, is important not only for a correct interpretation of ellipsis but also for the interpretation of pronomina, the articulation of discourse coherence and discourse structure. This means that establishing common ground is an independently motivated component of the discourse grammar.

If we assume a discourse grammar of this sort, (VP) anaphora resolution is in fact a side-effect of the general discourse parsing process. This implies that there is no need to stipulate an autonomous mechanism for resolution because it is integrated in a formal account of discourse structure and discourse semantics. Calculation of common ground is obtained by matching the syntactic/semantic structures of the relevant clauses (section 4). The clausal parallelism constraints on VP anaphora that were found in section 2 are formulated in terms of conditions on the matching process that effectuates common ground.

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1 Setting

Verb Phrase anaphora, mostly embodied by an auxiliary refering to a VP, is an intriguing natural language phenomenon. Several linguistic theories have provided a treatment of VP anaphora and still many aspects of the phenomenon have not been uncovered. Example (1-1)/(1-2) shows an occurrence of a simple Verb Phrase ('does') anaphor.[1] It has been shown convincingly that VP anaphora is for the most part a semantic problem ([Sag 77] and [Williams 77]). Nevertheless it has also become clear that some strong syntactic constraints are to be respected. Example (1-3) illustrates the fact that the auxiliaries must match. The influence of syntactic parallelism is illustrated by (1-4) and (1-5) (an example from [Partee 81] page 460]). In contrast with (1-4), the possible antecedent of the VP anaphor in (1-5) is not a syntactic VP, which blocks the anaphoric reference. It is clear that VP anaphora may occur within or across sentences. We abstract from this fact for the moment, and come back to it in section 2.2 and further on.

(1-1) John loves Mary.
(1-2) Peter does too.

(1-3) Bill was sitting in the garden. *Jerry did too.
(1-4) Almost every student submitted a paper. But Bill didn't.
(1-5) A paper was submitted by almost every student. *But Bill didn't.

Existing approaches to VP anaphora (whether formulated in a transformational theory ([Sag 77]), a Montagovian framework ([Partee 81]) or Discourse Representation Theory ([vanEijck 85] and [Klein 87])) all treat VP anaphora in terms of 'identity of predication'. This central notion leads to formulating some (copying- or deletion-) mechanism to interpret semantically identical VP's. Simple cases like (1-2) can be explained by such a mechanism based on the notion of identity of predication, interpreting the 'does' VP as identical to the VP 'love Mary'.

Complicated cases of VP anaphora show that important constraints on these anaphora cannot be formulated in terms of the notion of identity of predication: parallelism constraints essentially 'go beyond the scope of verb phrase identity' and need to be formulated in sentence level i.e. clausal constraints. Furthermore, the discourse structure in which clauses are embedded plays an important role.

Before we demonstrate this, we first highlight some main points of existing approaches. In [Sag 77], Ivan Sag explains VP anaphora in terms of deletion of 'identical VP's' which are represented by means of λ-expressions. A sentence like 'Peter does too' in (1-2) is derived in a transformational framework from (1-2') 'Peter loves Mary (too)' by deletion of the identical predicate which is represented as a λ-expression at the level of logical form[2]:

(1-1) John loves Mary.	(John, λx(x loves Mary))
(1-2') Peter loves Mary (too).	(Peter, λy(y loves Mary))

After VP deletion a syntactic rule of Do-support is applied. The predicate deletion is obtained by means of the transformation VPD; its application is subject to the following constraints ([Sag 77], page 74):

"With respect to a sentence S, VPD can delete any VP in S whose representation at the level of logical form is a λ-expression that is an alphabetic variant of another λ-expression present in the logical form of S or in the logical form of some other sentence S' which precedes S in discourse."

Sag convincingly shows that some notion of semantic VP identity is needed. His notion is formulated in terms of alphabetic variance[3] and incorporates a parallelism requirement: "Quantifier scope must be parallel if VPD is to apply" ([Sag 77], page 40). This, for instance, accounts for the observation that (1-6) is ambiguous with regard to quantifier scope whereas continuation with (1-7), according to Sag, implies that (1-6) can only have a single interpretation in that context (namely the interpretation where the existential quantifier has wide scope because only that interpretation shows an alphabetic variant of the VP in (1-7)).

(1-6) Someone hit everyone,
(1-6)a. $(\exists x)[x, \lambda y((\forall x_1)[y \text{ hit } x_1])]$
(1-6)b. $(\forall x)(\exists y)[y, \lambda x_1(x_1 \text{ hit } x)]$
(1-7) and (then) Bill did.
(1-7)' Bill, $\lambda x((\forall y)[x \text{ hit } y])$

In Sag's treatment, the VP anaphor in (1-7) only receives an interpretation if (1-6) has interpretation (1-6)a. (because $\lambda y((\forall x_1)[y \text{ hit } x_1])$ and $\lambda x((\forall y)[x \text{ hit } y])$ are alphabetic variants), whereas no deletion is possible if interpretation (1-6)b. is assigned (this does not incorporate any alphabetic variant of $\lambda x((\forall y)[x \text{ hit } y])$).

Although it is clear that some notion of semantic parallelism is needed to treat VP anaphora, the actual implementation of such a notion in Sag's theory is inadequate. Consider for instance (1-8), which shows the same quantifier ambiguity as (1-6). In example (1-8)/(1-9) both interpretations of (1-8) give rise to a correct interpretation of (1-9). There seems no reason why only (1-8)a. would give rise to a correct interpretation of (1-9) and not (1-8)b. A correct interpretation of (1-9) can only be obtained by assigning a quantifier structure which parallels the quantifier structure in (1-8). Each of the two possible interpretations of (1-8) leads to a parallel interpretation of (1-9). However, Sag's mechanism only permits deletion on the basis of (1-8)a..

(1-8) Every man hit someone,
(1-8)a. $(\forall x_{MAN})(\exists y)[y, \lambda x_1(x_{MAN} \text{ hit } x_1)]$
(1-8)b. $(\exists y)[y, \lambda x_1((\forall x_{MAN})[x_{MAN} \text{ hit } x_1])]$
(1-9) and (then) every boy did.
(1-9)a. $(\forall x_{BOY})(\exists y)[y, \lambda x_1(x_{BOY} \text{ hit } x_1)]$
(1-9)b. $(\exists y)[y, \lambda x_1((\forall x_{BOY})[x_{BOY} \text{ hit } x_1])]$

This example shows that the way in which Sag interweaves the syntactic subject/predicate distinction into the semantics leads to incorrect results with regard to parallel quantifier scope. We believe that the same argument holds for (1-6)/(1-7), i.e. both readings are possible in (1-7) but they have to be parallel to the interpretation of (1-6). It's just that the wide scope reading is somewhat harder to get.

In general it seems that the preferred quantifier reading of a sentence is the one that corresponds to the syntactic surface structure of the sentence. Of course this is not always the case. Consider for instance a (perfectly acceptable) wide scope reading of 'at least two Dutch soccer players' in (1-10).

(1-10) Every Italian knows at least two Dutch soccer players.
(1-11) Every Dutchman does (too).

Even though this quantifier reading is does not directly correspond to the syntactic surface structure, the VP anaphor in (1-11) seems to induce parallelism, i.e. the VP anaphor enforces an interpretation of clause (1-11) that has a quantifier structure parallel to the quantifier structure in the interpretation of (1-10). We come back to this in section 2.

[Williams 77] employs an approach to VP anaphora in the transformational framework that involves a VP rule for the interpretation of a phonologically null VP in the syntactic surface structure (instead of deletion). He proposes a grammar that is composed of two distinct subgrammars: a Sentence Grammar and a Discourse Grammar. Discourse rules only apply to the logical forms provided by the Sentence grammar. Williams sees the VP rule as a rule of the Discourse grammar. However, the notion of Discourse Grammar is not worked out; Williams states that "What is urgently needed is a careful articulation of the discourse component of the grammar, ..." ([Williams 77], page 138). Both [Williams 77] and [Sag 77] argue that a correct interpretation of ellipsis in the transformational framework must take place after quantifier scoping. In the next section we show how we can deal with this by using Flexible Montague Grammar [Hendriks 88] to attach quantifier distribution to the verb.

Another interpretative approach to ellipsis is found in [Partee 81]. Partee & Bach build on the work of [Sag 77] and [Williams 77], based on the notion of identity of predication, but place their treatment in a Montagovian framework. They encounter a fundamental problem which leads to the conclusion "that there is no semantic value that can be assigned to VP's such that VP deletion can be characterized in terms of identity". Consider an instance of the problem in (1-12)/(1-13) ([Partee 81], page 466]).

(1-12) Bill believes that Sally will marry him,
(1-13) but everyone knows that she won't.

In (1-13) the VP anaphor must be interpreted as 'marry him', with 'him' refering to Bill ('him' can not be bound by 'everyone'). So the referential pronoun 'him' should not be considered as a 'free variable within the VP' but as a context-dependent element which must be resolved (refer to Bill) before the interpretation of the VP anaphor can be obtained. Partee & Bach argue that, for cases such as (1-12)/(1-13), it is necessary to formulate a restriction which involves global properties of the logical representation, which is unwanted.[4] We shall argue, however, that the 'global properties' are restricted to well definable constraints on discourse structure and on the syntactic/semantic structures of clauses (or rather of discourse constituent units). For the resolution of VP anaphora this leads to the definition of a matching process which may involve insertion of bound variables only in contexts where this doesn't change their binding, whereas referential pronouns can only be part of the interpretation of a VP anaphor if they are resolved. Section 3 and 4 will show that matching of clausal structures provides a means to formulate restrictions on resolution of VP anaphors in terms of their clausal context. This provides a solution to problem of variables bound outside the verb phrase.

All theories mentioned up till now try to account for VP anaphora at the sentence level ([Sag 77] and [Williams 77] only indicate that a discourse level is necessary). Both [Klein 87] and [vanEijck 85] give an account of ellipsis in Discourse Representation Theory. They treat ellipsis as an anaphoric relation between a predicate DRS (Discourse Representation Structure) and a discourse referent of the predicate type. The influence of clausal parallelism on the resolution of VP anaphora, which we'll discuss in section 2, is not taken into account in either treatment. The notion of discourse structure in standard DRT is too limited to account for the constraints on the resolution of VP anaphora which result from discourse structure.

2 Neglected Constraints on VP anaphora

2.1 Clausal Parallelism

In this section we shall argue that VP anaphors induce syntactic/semantic parallelism between the 'anaphoric clause' (i.e. the clause that incorporates a VP anaphor) and the 'antecedent clause'. The syntactic aspects of clausal parallelism have been shown clearly in the literature, the semantic aspects, however, have not. Semantic parallelism exhibits two components at least: structural and indexical parallelism. Examples (2-1)/(2-2) and (2-3)/(2-4) illustrate the contribution of structural parallelism to the interpretation of the VP anaphors in question. Sentence (2-1) may be given a so-called 'collective' interpretation (one single pizza ordered by three boys) or a 'distributive' reading (each of the three boys ordered his own different pizza).

(2-1) Three boys ordered a pizza (and)
(2-2) two girls did (too).

The interpretation of (2-2) is partly determined by the interpretation of (2-1), in particular a collective (respectively distributive) interpretation of (2-2) is strongly preferred if (2-1) is assigned a collective (respectively distributive) interpretation. Another instance of structural parallelism is shown in (2-3), a sentence well-known for its semantic ambiguity.

(2-3) Every man loves a woman.
(2-3)a. $\forall x_1$ [MAN(x_1) --> $\exists y_1$ [WOMAN(y_1) & LOVE(y_1)(x_1)]]
(2-3)b. $\exists y_1$ [WOMAN(y_1) & $\forall x_1$ [MAN(x_1) --> LOVE(y_1)(x_1)]]

If sentence (2-3) is part of a discourse, the context will probably provide reasons to decide between (2-3)a. and (2-3)b. (and it will probably also 'narrow down' the interpretation of 'every', 'man' and 'woman'). However, lacking context, (2-3) receives two different interpretations. Consider a continuation of (2-3) by (2-4).

(2-3) Every man loves a woman (and)
(2-4) every boy does (too).

Whatever the interpretation of (2-3) may be, the interpretation of (2-4) will have to correspond with that interpretation of (2-3): if the interpretation of (2-3) is to be (2-3)a., the interpretation of (2-4) will have to be (2-4)a. Interpretation (2-4)b. can only be assigned to (2-4) if sentence (2-3) has semantics (2-3)b.

(2-3)a. $\forall x_1$ [MAN(x_1) --> $\exists y_1$ [WOMAN(y_1) & LOVE(y_1)(x_1)]]
(2-4)a. $\forall x_2$ [BOY(x_2) --> $\exists y_2$ [WOMAN(y_2) & LOVE(y_2)(x_2)]]

(2-3)b. $\exists y_1$ [WOMAN(y_1) & $\forall x_1$ [MAN(x_1) --> LOVE(y_1)(x_1)]]
(2-4)b. $\exists y_2$ [WOMAN(y_2) & $\forall x_2$ [BOY(x_2) --> LOVE(y_2)(x_2)]]

The two combinations of interpretations of these sentences where the quantifier scopes differ are ruled out.

(2-3)a. $\forall x_1$ [MAN(x_1) --> $\exists y_1$ [WOMAN(y_1) & LOVE(y_1)(x_1)]]
(2-4)b. $\exists y_2$ [WOMAN(y_2) & $\forall x_2$ [BOY(x_2) --> LOVE(y_2)(x_2)]] *

(2-3)b. $\exists y_1$ [WOMAN(y_1) & $\forall x_1$ [MAN(x_1) --> LOVE(y_1)(x_1)]]
(2-4)a. $\forall x_2$ [BOY(x_2) --> $\exists y_2$ [WOMAN(y_2) & LOVE(y_2)(x_2)]] *

One might get the idea that structural parallelism points at the necessity of a notion that is broader than the notion of 'VP identity' to treat ellipsis adequately. This is not necessary, however, because there is a way to attach quantifier distribution to the main verb of a clause if we adopt the theory of 'Flexible Montague Grammar' ([Hendriks 88]). FMG deals with scope ambiguities as in (2-3) by flexible typing of the verb 'love' so that the landingplaces of quantifiers are attached to the verb. In FMG, every expression is assigned a lexical translation of the minimal type for this expression. By means of general rules of raising and lowering, translations of higher types are derived. For instance sentence (2-3) needs the following basic translations:

every man	$\lambda P.\forall x$[MAN(x) --> P(x)]
a woman	$\lambda P.\exists y$[WOMAN(y) & P(y)]
love	$\lambda y \lambda x$.LOVE(y)(x)

Two nonequivalent ways of raising the lexical translation of 'love' give rise to the following derived translations:

(a) $\lambda T_1 \lambda T_2.T_2(\lambda y.T_1(\lambda x.\text{LOVE}(y)(x)))$
(b) $\lambda T_1 \lambda T_2.T_1(\lambda x.T_2(\lambda y.\text{LOVE}(y)(x)))$

Combining (a) with the basic translations of 'every man' and 'a woman' results in translation (2-3)a., translation (b) leads to (2-3)b. (T_1 and T_2 are variables of term/NP-type.) We shall not go into technical details of FMG here. The important point is that the two interpretations of (2-3) are obtained by raising the lexical translation of the verb 'love'. So quantifier distribution can be considered a property of the (derived) translation of the verb.

When combined with a proper name 'Mary' as in (2-5), both derived translations of the verb 'love' result in the same verb phrase interpretation. This also holds for the VP anaphor in (2-6).

(2-5) Every man loves Mary (and)
(2-6) every boy does (too).

(2-7) John loves a woman (and)
(2-8) every boy does (too).

In (2-7) the two different interpretations of the VP 'love a woman' result in one and the same interpretation of the whole sentence: sentence (2-7) has only one reading (despite of two different VP interpretations of 'love a woman'). The second (anaphoric) sentence, however, has two readings, a wide scope and a narrow scope reading for 'a woman'. These can be obtained by preserving both VP interpretations of the first clause despite the fact that they give rise to the same sentence interpretation.

So it seems that as far as structural parallelism is concerned, we can still build on the notion of 'identity of predication' for resolution of VP anaphora if we use Flexible Montague Grammar for semantic representations because in FMG VP's incorporate quantifier structure. However, indexical parallelism goes beyond this extended notion of 'identity of flexible predication'. If the antecedent of a VP anaphor contains a pronoun, the pronoun including its interpretation must be incorporated in the interpretation of the VP anaphor.

In (2-9) the pronoun 'him' refers to some discourse referent not mentioned in this fragment. The preferred interpretation for (2-10) is an interpretation that parallels (2-9), i.e. where both pronouns refer to the same entity. However, there is a possiblity to escape this parallelism: when deictically interpreted, the pronouns can in principle have different referents (for instance, when accompanied by different pointing gestures). In (2-11)/(2-12), however, it is impossible to escape parallelism: Whatever interpretation the pronoun 'him' receives in (2-11), the interpretation of the VP anaphor in (2-12) will have to incorporate the same pronoun interpretation.

(2-9) Mary likes him (and)
(2-10) Fred likes him.

(2-11) Mary likes him (and)
(2-12) Fred does (too).

Sentences (2-13)/(2-14) give another clear example of indexical parallelism. The interpreted pronoun of (2-13) is in the antecedent of the VP anaphor. Interpreting the anaphor just as 'likes him' with some free variable for the pronoun, is obviously wrong. (In that case the pronoun might even become bound by the universal term.)

(2-13) Mary likes him (and)
(2-14) every man thinks that Georgette does.

To obtain the correct interpretation of the elliptic sentence (2-16), the binding structure of (2-15) needs to be considered. If 'him' in (2-15) is bound by the existential term 'a man', this must also be so in the interpretation of (2-16). However, if 'him' in (2-15) is bound by the universal term 'everyone', the pronoun in the VP interpretation of (2-16) is bound parallel, namely by 'John'. (If 'him' in (2-15) refers to some other discourse entity, this interpretation is just 'copied' in the interpretation of the VP anaphor.)

(2-15) Everyone told a man that Mary likes him (and)
(2-16) John did (too).

Crucial examples of clausal parallelism are found when both aspects of semantic parallelism are present at the same time. A paradigmatic example is given below. In this example the VP anaphor induces both structural and indexical parallelism i.e. the interpretation of sentence (2-18) requires strict parallelism with the interpretation of (2-17) regarding both quantifier structure and pronoun interpretation.

(2-17) Everyone told a man that Mary likes him
(2-18) (and) everyone told a boy that Suzy does.

Without any context to interpret it in, sentence (2-17) has six different interpretations, which are listed below in two groups. The difference between the two groups is in the quantifier structure of the formulas. In the compositional construction of the semantic representation in FMG, the quantifier distribution is attached to the verb 'tell'.

(2-17) Everyone told a man that Mary likes him
a. ∀x[HUMAN(x) --> ∃y[MAN(y) & TELL(like(x)(MARY)) (y)(x)]]
b. ∀x[HUMAN(x) --> ∃y[MAN(y) & TELL(like(y)(MARY)) (y)(x)]]
c. ∀x[HUMAN(x) --> ∃y[MAN(y) & TELL(like(<u>he</u>)(MARY))(y)(x)]]

d. ∃y[MAN(y) & ∀x[HUMAN(x) --> TELL(like(x)(MARY)) (y)(x)]]
e. ∃y[MAN(y) & ∀x[HUMAN(x) --> TELL(like(y)(MARY)) (y)(x)]]
f. ∃y[MAN(y) & ∀x[HUMAN(x) --> TELL(like(<u>he</u>)(MARY))(y)(x)]]

Each group is subdivided with regard to the interpretation of the pronoun 'him', which can be interpreted in three different ways, respectively related to 'everyone', to 'a man' or to some other discourse referent not mentioned in the sentence. (In the latter case the pronoun is represented by a special variable '<u>he</u>'.) The interpretation of (2-18) strongly depends on the interpretation of (2-17). A first incomplete description of the semantic content of (2-18) will be twofold, depending on the order of quantifiers.

(2-18) everyone told a boy that Suzy does.
A. $\forall x_1$[HUMAN(x_1) --> $\exists y_1$[BOY(y_1) & TELL(<u>does</u>(SUZY))(y_1)(x_1)]]

B. $\exists y_1$[BOY(y_1) & $\forall x_1$[HUMAN(x_1) --> TELL(<u>does</u>(SUZY))(y_1)(x_1)]]

Now it turns out that the interpretation of (2-18) must be constructed on the basis of its parallelism with (2-17).

First of all, as in the foregoing examples, the VP anaphor induces structural parallelism: the quantifier structure in the interpretations of the two sentences must be the same. The first interpretation for (2-18), (2-18)A., is only possible if (2-17) has been assigned an interpretation from the first group; interpretation (2-18)B. is possible only if (2-17) has received an interpretation from the second group. Note that parallel quantifier structures cannot be obtained by means of some notion of copying: both clauses have their own quantifier structure which will have to be 'checked' for the required parallelism. In our use of FMG, quantifier distribution is attached to the main verb so the semantics of the main verbs of the clauses involved will have to match to obtain parallel quantifier distribution. This example shows that notions of copying or deletion of verb phrases are too coarse to account for VP anaphora because clausal contexts need to be checked for certain properties.

Secondly it is important to notice that the interpretation of the pronoun 'him' in (2-17) forces a parallel interpretation in (2-18). For instance if 'him' refers to some discourse entity 'John' (interpretation c. or f.), the pronoun in the resolution of the VP anaphor must refer to the same entity 'John'. More importantly, if in (2-17) the pronoun 'him' is interpreted as being bound by 'a man' (interpretation b. or e.), the interpretation of (2-18) will have to be parallel, binding the pronoun in the interpretation of the VP anaphor to 'a boy'. In order to account for this indexical parallelism, the binding structures (particularly which quantifier binds the pronoun) of the clauses involved have to be parallel. It is not possible to treat the binding structure of a clause as a property of the verb, i.e. these parallelisms cannot be 'attached to the verb' in an FMG way. Semantic, in particular indexical, parallelism shows that the notion of 'VP-identity' is too narrow as a notion to account for VP anaphora and must be replaced by some notion of 'clausal parallelism'.

The paradigmatic example (2-17)/(2-18) clearly shows that semantic constraints on ellipsis cannot be limited to VP identity but must be stated in terms of clausal parallelism (i.e. parallelism between clausal syntactic/semantic structures). Complicated, embedded, VP anaphora in an ambiguous quantifier structure induce clausal parallelism concerning quantifier- and binding-structures. In order to account for this parallelism, we propose to match the syntactic/semantic structures of the clauses involved. Before section 4 introduces the matching mechanism, section 2.2 discusses some influence of discourse structure on VP anaphora and section 3 describes the discourse grammar that is used, and in which the notion of semantic parallelism is formalized.

2.2 Scope Restrictions

In the foregoing section we have clearly shown that syntax and semantics at the sentence level impose restrictions on VP anaphora. Here we give some examples of the influence of discourse syntax/semantics on the interpretation of VP anaphora.

A sentence containing a VP anaphor, is always assumed to be preceded by a sentence (or conjunct) wich incorporates the antecedent. The next examples clearly show that this assumption is not generally correct.

(2-19) John went to the library.
(2-20) He borrowed a book on computer science.
(2-21) Bill did too.

Clause (2-21) is ambiguous because the VP anaphor can be interpreted in two different ways. The anaphor might refer to the VP in (2-20), or to a conjunction of the VP's in respectively (2-19) and (2-20). It can hardly refer to the VP in (2-19) only.[5]

The influence of discourse structure on the interpretation of ellipsis is even more clear in (2-22)/(2-26)

(2-22) John prepared his exam thouroughly.
(2-23) He decided not to go on holiday.
(2-24) He spent the whole week studying.
(2-25) Every day he went to bed early.
(2-26) Bill did too.

Here the VP anaphor may refer to the whole preceding text, or to the elaboration (2-23) through (2-25), or maybe to VP in the directly preceding sentence (2-25). Because of the structure of this small discourse ((2-23) through (2-25) being an elaboration), the VP anaphor is limited in its reference possibilities. The anaphor cannot refer to (2-23) or (2-24) only or only to both. So it turns out that the reference possibilities of VP anaphora are constrained by the structure of discourse. We shall give an account of these observations in section 4.

3 A Formal Discourse Grammar

In [Scha 88] a unification grammar is presented as a formal tool for discourse parsing, i.e., for building a structural and semantical representation of an incoming discourse. Discourses have a hierarchical structure which is accounted for by assigning them tree structures. The grammar consists of a set of explicit rules which assign structural analyses and meaning representations to discourses.

The constituent segments of a discourse are so-called 'discourse constituent units' (DCU's). Clauses (sentences) are taken to form elementary DCU's, 'sentence-DCU's'. Every clause is initially interpreted in a context-independent way. This results in a meaning representation which will usually contain special variables, standing for the discourse dependent elements in the utterance meaning such as pronouns and VP anaphors. These variables are interpreted when the sentence is integrated in the ongoing discourse. In placing a DCU in its structural context, a matching process plays a crucial role.

The grammar comprises different rules for different structural types. The two main structural types (DCU's) are: subordination and coordination. Subordinations are binary structures in which all or most of the relevant features (reference time, spatial index, modal index etc.) are inherited from the subordinating constituent. Among the subordinate structures covered are rhetorical subordinations, topic-dominant chainings and interruptions. Coordinations are n-ary ($n \geq 2$) structures in which all elements have equal status. Rules are formulated for coordinate structures such as lists, narratives and question/answer pairs. In the context of this paper we only highlight the aspects of the grammar that are relevant to the treatment of VP Anaphora.

3.1 Grammar Rules

To describe the hierarchical structure of discourse, a context free grammar is used, augmented with attribute/value pairs (such as a set of discourse referents, an attribute representing the semantics and attributes for reference time, modal index and spatial index). The context free rules describe how compound DCU's are built up out of their constituating DCU's. The rules which are relevant for the purpose of the present paper are the rules for building 'list' structures. The rule which is crucially relevant for VP anaphora is the following one (somewhat modified from the notation of [Scha 88], $S_1 \sqcap S_2$ is the 'unification' of S_1 and S_2, and $S_1 \sqcup S_2$ the 'common ground'):

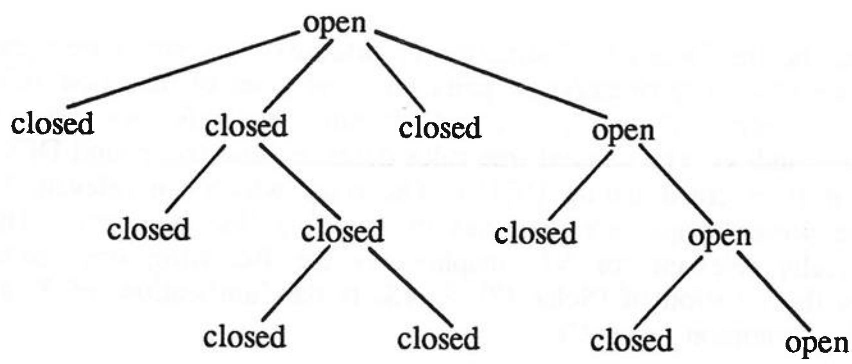
list [**drs**:$D_1 \cup D_2$, **schema**:$S_1 \sqcup S_2$), **sem**:S_1 & $(S_2 \sqcap (S_1 \sqcup S_2))$] (I)
==> DCU1 [**drs**:D_1, **sem**:S_1]
+ DCU2 [**drs**:D_2, **sem**:S_2]

This rule formalizes the idea that a 'list' structure may consist of two DCU's of arbitrary types that have some common semantics. Capital letters in the rule stand for variables (therefore 'DCU1' and 'DCU2' may be of any type). A list DCU comprises several attributes: the Semantics attribute (**sem**) records the syntactic/semantic representation of the DCU, the Discourse Referents Set (**drs**) records the entities introduced in the discourse unit, the **schema** attribute records local semantic coherence. Other attributes like reference time, spatial index, modal index etc. are not considered here (but see [Scha 88]).

A central notion in the construction of list structures is the notion of 'most specific common ground' (mscg) of two syntactic/semantic structures. The intuitive idea behind it is, that the mscg of two syntactic/semantic structures S_1 and S_2, $S_1 \sqcup S_2$, indicates what these two structures have in common. In general, the integration of a sentence(DCU) in its structural context by means of the grammar rules is based on calculating the most specific common ground of the clauses involved. The mscg should provide a flexible and yet precise way to characterize local semantic coherence. As in [Scha 88] the notion of mscg is not used, we shall define it in more detail in the next section.

The unification grammar is used for discourse parsing. As mentioned above, every sentence is initially interpreted in a context-independent way. This results in a meaning representation which possibly contains special variables for overt context-dependent elements. The interpretation of a sentence in its structured context is based on the syntactic/semantic aspects it shares with the DCU it is attached to. The contribution of the newly integrated structured sentence (S_2) to the semantics of the list structure is therefore obtained by its unification with the common ground: $S_2 \sqcap (S_1 \sqcup S_2)$. (See next section.)

The parsing process is assumed to be incremental. An incoming sentence is placed in its (structured) discourse context, thus forming a new (structured) discourse context for a next sentence. The incrementality of the parsing process brings with it that at any point in the process, only the right edges in the discourse tree are 'open' to form new (sub)structures whereas the rest of the edges are 'closed' as, for instance, in the schematic example below. The parsing process only uses information on the right edge of the existing discourse tree. Expansion of the open discourse constituent units is carried out in accordance with the grammar rules.



3.2 Establishing Common Ground

We have demonstrated in section 2.1 that an important factor in the interpretation of ellipsis is the common ground of the anaphoric and the antecedent clause: syntactic/semantic parallelism is a crucial factor. Syntactic/semantic parallelism, i.e. clauses sharing syntactic/semantic structure, is a well-known notion in the discourse literature. It has been invoked to account for pronoun resolution (see for instance [Grober 78], [Cowan 80], [Kameyama 86] and [Brennan 87]), for topic/focus articulation and coherence ([Polanyi 85]). In [Scha 88] it is assumed that parallelism plays a significant role in the structuring of discourse. Despite the importance of this notion, no one, as far as we know, has come up with a precise formalization of this concept. (A first attempt can be found in [Scha 88], where semantic parallelism is embodied in the grammar rules. However, this is not worked out in much detail.)

The formalization that we propose now is based on the notion of 'generalization'. Generalization is a computational notion that is the dual of 'unification' (cf. [Robinson 65]). Given two structures S_1 and S_2, a generalization of those structures is a third structure S of which both S_1 and S_2 are instances. In other words, structure S is at least as general as both S_1 and S_2. We want to employ the notion of generalization to compute a formula which indicates what two structures have in common. We shall do this by means of the standard notion of 'most specific generalization' (msg).

Formally, generalization is based on 'antisubstitution' (see [Knight 89]). An antisubstitution γ is a mapping of terms into variables. Two terms **a** and **b** are generalizable if there exists an antisubstitution γ such that $\gamma(\mathbf{a})=\gamma(\mathbf{b})$. In that case $\gamma(\mathbf{a})$ is called a generalization of **a** and **b**. A generalizer γ of terms **a** and **b** is called the most specific generalizer of **a** and **b** if for any other generalizer η, there is an antisubstitution ζ such that $\zeta(\gamma(\mathbf{a}))=\eta(\mathbf{a})$. In the latter case, $\gamma(\mathbf{a})$ is the most specific generalization of **a** and **b**, sometimes represented as $\mathbf{a} \sqcup \mathbf{b}$.

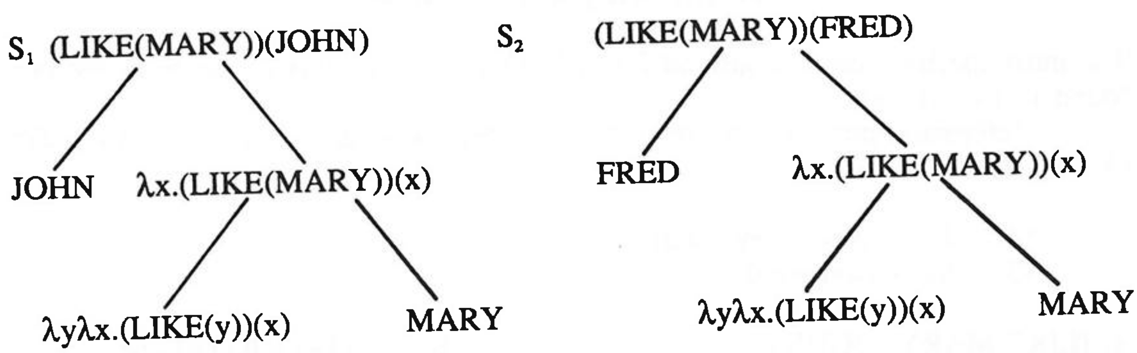
The maximalization of common information of structures that we pursue in our concept of generalization differs somewhat from the standard concept of 'most specific generalization'. Therefore we introduce the notion of 'most specific common ground' (mscg). The mscg is the generalization over structures that extracts the shared structure of clauses as specifically as possible. This especially effects underspecified elements such as anaphora.

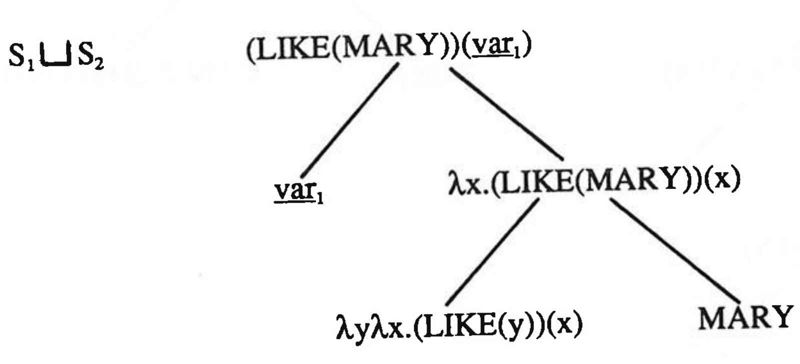
In the semantic representations, anaphoric terms that are discourse referent searchers (such as pronouns and VP anaphors) are represented by special variables, sometimes called 'context variables'.[6] A variable is a placeholder for future instantiation. Another common way to look at a variable is to regard it as the set of all its possible instances. From that point of view, a clause that contains an underspecified element, such as a VP anaphor, that is represented by a context variable, is considered a multiple ambiguous clause: Each possible instance of the variable gives an interpretation. This entails that when anaphors are involved, the search for common information between a pair of clauses results in a set of msg's. The most specific common ground of two structures S_1 and S_2, written $S_1 \sqcup S_2$, selects that msg, $(S_1 \sqcup S_2)$, that incorporates the maximum of common information. In particular, the mscg of an anaphoric clause, being a multiple ambiguous clause, and its antecedent clause is the most specific of their msg's.

Formally the set of msg's of a constant **c** and a context variable $\underline{\mathbf{v}}$ consists of the msg's of all pairs from $\{\mathbf{c}\} \times \mathbf{V}$ (if **V** is the set of all possible instances of $\underline{v}$). The most specific common ground is defined as the most specific element in this set, i.e. the constant **c**.

We illustrate the application of the notion of most specific common ground with some examples. Consider the syntactic/semantic structures of clauses (3-1) and (3-2), respectively S_1 and S_2. What the two subject terms ('John' and 'Fred') share is that they are both quantifying terms. So the subjects generalize to a variable for a quantifying term ('$\underline{var}_1$').[7] The rest of the structure S_1 is the same as S_2 (alphabetic variance of semantic variables considered). The generalization of these substructures is therefore that very substructure.

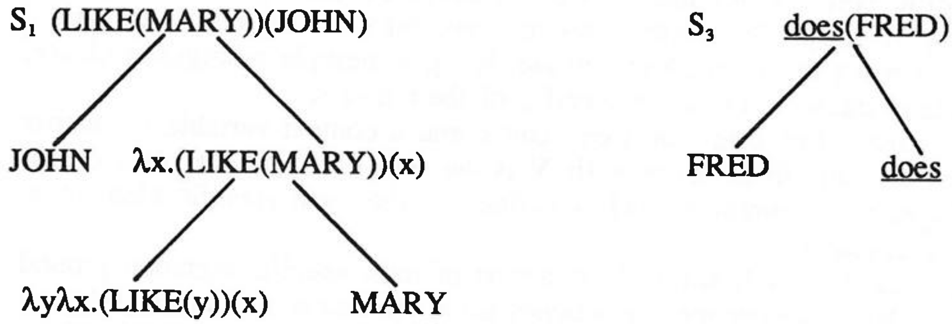
(3-1) John likes Mary (and)
(3-2) Fred likes Mary.

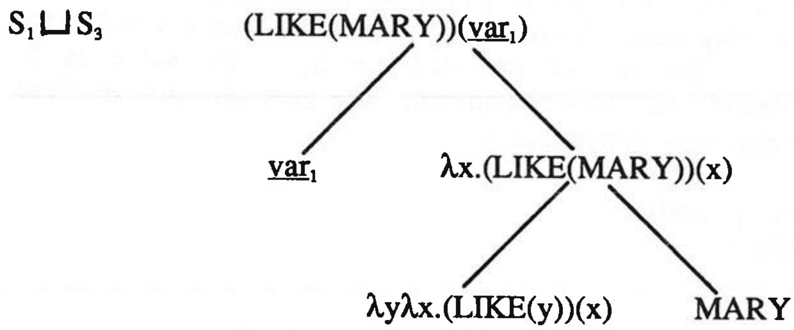




In example (3-1)/(3-2) no anaphors (and therefore no context variables) are involved. Compare, however, the following simple example, which contains a VP anaphor. The most specific common ground of the substructures of the VP's in (3-1) ('λx.(LIKE(MARY))(x)') and (3-3) (the context variable '<u>does</u>') is the substructure labelled 'λx.(LIKE(MARY))(x)'.

(3-1) John likes Mary (and)
(3-3) Fred does (too).

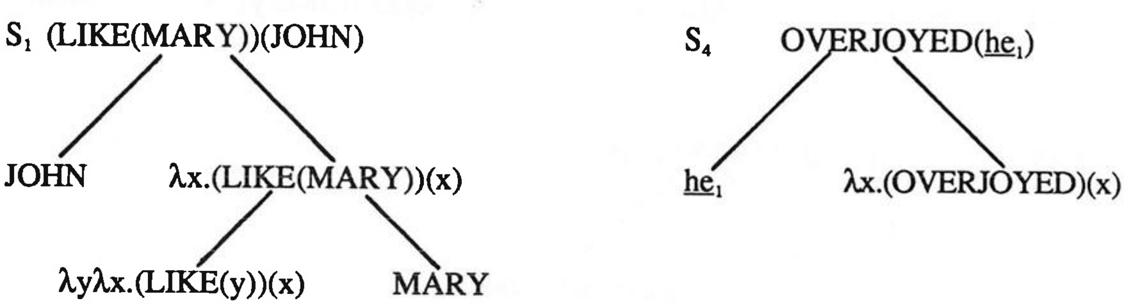


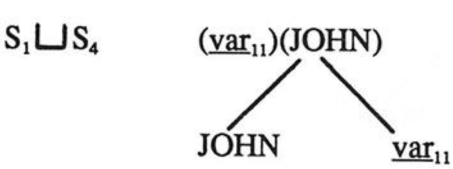


The most specific common ground found in (3-1)/(3-3) is thus the same as the one found in (3-1)/(3-2).

Referential pronouns are treated in the same way as VP anaphors.[8] Consider (3-1)/(3-4).

(3-1) John likes Mary (and)
(3-4) he is overjoyed.





If we abstract from the syntactic composition of the semantic structures, we can articulate the semantic common ground that is obtained by means of generalization explicitly in λ-calculus. The free variables that occur in the mscg are equivalent to variables that are (λ-)bound directly outside the expression. So using λ-abstraction, the top-level semantics of the mscg's can be represented as follows:

S_1	'John likes Mary'
S_2	'Fred likes Mary (too)'
$S_1 \sqcup S_2$	λx.LIKE(MARY)(x)
S_3	'Fred does (too)'
$S_1 \sqcup S_3$	λx.LIKE(MARY)(x)
S_4	'He is overjoyed'
$S_1 \sqcup S_4$	λP.P(JOHN)

Calculating the mscg provides the value for the **schema** attribute in rules of the discourse grammar. This attribute records local semantic coherence. Building a DCU out of two adjacent sentences thus involves computing the mscg of the syntactic/semantic structures.

The common ground of two DCU's is also essential in the articulation of the semantics of the resulting list structure. The semantics of the list structure is formed by the conjunction of the 'old' semantics (S_1) and the contribution of the newly integrated sentence (S_2). The latter contribution is not, of course, the initial context-independent interpretation of the sentence, but its interpretation with regard to the preceding discourse. Intuitively, the contextual influence on the interpretation is based both on the aspects the sentence shares with the preceding discourse and on the new information it contributes to the discourse. The interpretation of the new sentence in the discourse is therefore obtained by means of unification of the initial interpretation (S_2) with its common ground ($S_1 \sqcup S_2$): $S_2 \sqcap (S_1 \sqcup S_2)$. 'Unification' of two structures **S** and **T**, $\mathbf{S} \sqcap \mathbf{T}$, is a substitution of terms for variables that makes two structures identical (cf. [Robinson 65]). Thus the influence of the local context is incorporated in the initial context-independent interpretation.

As observed above, parallelism is an important linguistic device, therefore the computation of common ground provides an adequate means for the treatment of several phenomena. In the next section we show how the constraints on ellipsis are implemented in the discourse grammar via the notion of most specific common ground.

4 Implementation of the Constraints on Resolution

4.1 Clausal Parallelism

In order to guarantee the required parallelism between anaphoric clause and antecedent clause, the VP anaphor resolution mechanism must carry out a matching process between the syntactic/semantic structures of the clauses involved. If we would only think about sentence-level syntax and semantics, this might seem a rather drastic extension of the required machinery. But of course we must realize that VP anaphor resolution is in fact a discourse-level process, and that the mechanism for accomplishing it should be embedded in a mechanism which is capable of assigning a structure and an interpretation to a discourse, such as the grammar presented in section 3. From the discourse perspective, the situation looks rather different, and more attractive: it turns out that one of the structural relations which have been postulated to play an important role at the discourse level is exactly the relation of syntactic/semantic parallelism, and that the process that is necessary for VP anaphor resolution is parasitic upon the matching process that is needed for establishing discourse structure and discourse coherence.

It has become clear that an algorithm for resolution of VP anaphora cannot be based on the concepts of 'copying' and 'VP identity', but must consider common ground of clausal structures. The computational notion of matching offers a solution to implement the calculation of mscg's and the clausal parallelism requirements on the resolution of VP anaphora. Based on the discourse grammar, the general parsing mechanism matches the syntactic/semantic structures of two adjacent clauses and computes the most specific common ground. The mscg is stored in the **schema** attribute to record local semantic coherence. In case of VP anaphora, the general mechanism provides the mscg of the anaphoric clause and the antecedent clause. The mscg incorporates the antecedent VP and all relevant information (on quantifier structure and binding structure) to verify the constraints on resolution.

We have argued in section 2 that clausal parallelism comprises two aspects: structural parallelism - the quantifier structure of the anaphoric clause parallels the quantifier structure of the antecedent clause - and
indexical parallelism - variables can only be 'copied' if a structurally parallel binding quantifier is present (copying of free variables in VP anaphor resolution gives wrong results). These parallelism constraints on the relation between the structured clauses can now be formulated in terms of conditions on the required matching. As we have seen the matching process takes syntactically composed semantic structures of the relevant clauses as input. The fact that only the syntactically composed semantic structures are considered, accounts for the observation (in section 1) that VP anaphora cannot refer to non-constituents. In placing a clause in its structural context, a match between the structures of the relevant clauses is established. In case of an anaphoric clause, the structure of the anaphoric clause must be subsumed by the structure of the antecedent clause, i.e. there must be a match. For this match the following conditions hold:

Syntactic Agreement
- The auxiliaries must match.

Parallelism
- The quantifier structure of the anaphoric clause parallels the quantifier structure of antecedent clause.
- The binding structures are parallel. (Semantic variables can only be "copied" (unified) if a structural parallel binding quantifier is present.)

The parallelism constraint concerning quantifier structure is implemented as a condition on the verbs that take care of quantifier distribution: The (FMG) raising and lowering operations that are applied to the basic translation of the main verb in order to attach quantifier distribution must be identical. The constraint on the binding structures is a structural condition on the matching.

The interpretation of a new sentence in its context takes shape when it is integrated in the preceding (structured) discourse. The contribution of a new sentence to the total discourse semantics consists of the result of unification of the sentence structure (possibly containing context variables for context-dependent elements) with the common ground. See rule (I) for forming a list, which is repeated below. The interpretation of a VP anaphor results as a side-effect of the integration of the anaphoric clause in the discourse. In unifying the anaphoric clause with the common ground, as described in section 3.2, the computational variable that represents the VP anaphor is instantiated. The conditions on the matching garantee that the instantiation only yields the antecedent under the relevant constraints for VP anaphora resolution.

list $[\textbf{drs}{:}D_1 \cup D_2,\ \textbf{schema}{:}S_1 \sqcup S_2),\ \textbf{sem}{:}S_1\ \&\ (S_2 \sqcap (S_1 \sqcup S_2))]$ (I)
==> DCU1 $[\textbf{drs}{:}D_1,\ \textbf{sem}{:}S_1]$
+ DCU2 $[\textbf{drs}{:}D_2,\ \textbf{sem}{:}S_2]$

Using this mechanism, VP anaphors are resolved in the required parallel way. We illustrate this with some examples. As we have shown in section 2, the quantifier distribution in the ambiguous sentence (2-3) is incorporated in the semantics of the verb 'love' by raising its translation. This also determines the quantifier structure in the interpretation of the following clause (2-4). Consider the syntactic/semantic structures (i.e. the **sem** attributes of the DCU's) of (2-3)a. and (2-4).

(2-3) Every man loves a woman.
(2-3)a. $\forall x[MAN(x) \dashrightarrow \exists y[WOMAN(y)\ \&\ LOVE(y)(x)]]$

$\lambda Q \forall x[MAN(x) \dashrightarrow Q(x)]$ $\quad \lambda T_2.T_2(\lambda x.\exists y[WOMAN(y)\ \&\ LOVE(y)(x)])$

$\lambda T_1 \lambda T_2.T_2(\lambda y.T_1(\lambda x.LOVE(y)(x)))$ $\quad \lambda Q.\exists y[WOMAN(y)\ \&\ Q(y)]$

(2-4) Every boy does (too).
(2-4)' <u>does</u>$(\lambda P \forall x_1[BOY(x_1) \dashrightarrow P(x_1)])$

$\lambda P \forall x_1[BOY(x_1) \dashrightarrow P(x_1)]$ $\quad$ <u>does</u>

The two sentences are parsed into a list structure according to rule (I). When the syntactic/semantic structures of (2-3) and (2-4) undergo matching, the resulting mscg incorporates the relevant interpretation of the verb phrase. In constructing the semantics of the list, the '<u>does</u>' variable is instantiated with its antecedent VP. Thus the relevant readings of (2-4) are obtained: parallel quantifier distribution is simply incorporated because it is attached to the antecedent VP (by means of the raised translation of the verb). The condition on binding structures is irrelevant in this example. The theory makes essential use of the compositionality of the FMG semantics in that the complete semantics of the anaphoric clause depends on the compositional construction of both clauses.

Consider example (2-17)/(2-18).

(2-17) Everyone told a man that Mary likes him
a. $\forall x[HUMAN(x) \dashrightarrow \exists y[MAN(y) \ \& \ TELL(like(x)(MARY)) \ (y)(x)]]$
b. $\forall x[HUMAN(x) \dashrightarrow \exists y[MAN(y) \ \& \ TELL(like(y)(MARY)) \ (y)(x)]]$
c. $\forall x[HUMAN(x) \dashrightarrow \exists y[MAN(y) \ \& \ TELL(like(\underline{he})(MARY))(y)(x)]]$

d. $\exists y[MAN(y) \ \& \ \forall x[HUMAN(x) \dashrightarrow TELL(like(x)(MARY)) \ (y)(x)]]$
e. $\exists y[MAN(y) \ \& \ \forall x[HUMAN(x) \dashrightarrow TELL(like(y)(MARY)) \ (y)(x)]]$
f. $\exists y[MAN(y) \ \& \ \forall x[HUMAN(x) \dashrightarrow TELL(like(\underline{he})(MARY))(y)(x)]]$

(2-18) everyone told a boy that Suzy does.
A. $\forall x_1[HUMAN(x_1) \dashrightarrow \exists y_1[BOY(y_1) \ \& \ TELL(\underline{does}(SUZY))(y_1)(x_1)]]$

B. $\exists y_1[BOY(y_1) \ \& \ \forall x_1[HUMAN(x_1) \dashrightarrow TELL(\underline{does}(SUZY))(y_1)(x_1)]]$

The VP anaphor can be resolved only if the syntactic constraints are fulfilled (which is clearly the case) and, if, moreover, quantifier and binding structure are parallel. As before, quantifier structure is attached to the verb. Contrary to the foregoing simple example, the VP anaphor is situated in an embedded structure. The requirement of parallel quantifier structure functions as a condition on the matching of the main verbs of the clauses. The condition on binding structures in this case results in the following. All Noun Phrases are treated as quantifiers and are incorporated as such (as being a quantifier) in the mscg. The interpretation of the pronoun in (2-17) (represented in the syntactic/semantic structure as alternatives x, y and he) is 'passed on' to the anaphoric clause. The 'logical variable' interpretations (represented by x and y) are only passed on to the interpretation of the VP anaphor if a structurally parallel binding quantifier is present, whereas the context variable (he) is only included in the VP anaphor interpretation if it is resolved. Thus non-parallel interpretations are 'filtered' on basis of the formulated conditions.

4.2 Scope Restrictions

We have shown in section 2.2 that VP anaphora may refer to more than one verb phrase. Formulating the restrictions on the reference possibilities of VP anaphora that we observed in terms of the parsing process, it turns out that VP anaphora can only refer to open DCU's. If a VP anaphor does not refer to the preceding (sentence-)DCU_n, the parsing algorithm recursively tries to attach the anaphoric clause to the previous DCU_{n-1} that is open for expansion.[9] This implies that the matching process that possibly results in resolution of the VP anaphor involves the semantic structure stored in the **sem** attribute of DCU_{n-1}. The discourse structure constraints on VP anaphora resolution thus simply follow from the stipulation of incrementality of the parsing process that results in open versus closed discourse constituent units.

In the foregoing section, it has become clear that the interpretation of a sentence depends on its integration in the preceding (structured) discourse on the basis of the grammar. This is formalized in terms of unification of the initially context-independent interpretation of a sentence and its common ground with the DCU it is attached to. The reference possibilities of VP anaphora indicate that the common ground not only effects the interpretation of the newly integrated sentence but also urges a re-formulation of the already processed semantic part of the DCU that is expanded.

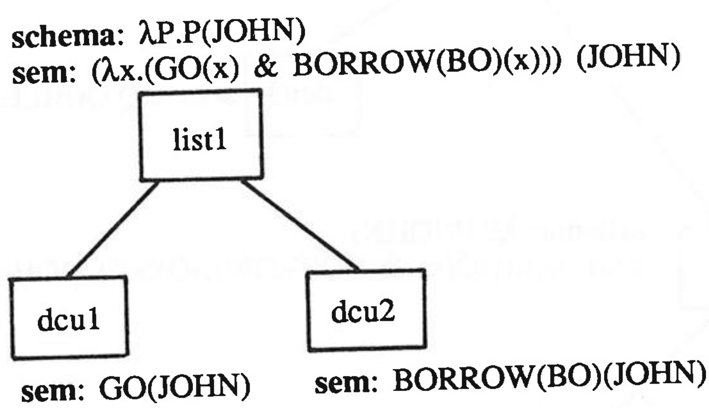
In order to account for the full impact of the common ground of DCU's on the semantics of the resulting structure, we propose to adjust the grammar rules in the following way. As before, the common semantics of DCU's is computed as their mscg. The mscg now becomes a self-contained part of the semantics of the compound DCU (see rule (II)). To achieve this, the mscg ($S_1 \sqcup S_2$) is lifted from the semantics of both constituting DCU's and added to the semantics of the whole. The formal details of this remains to be investigated, as is the associated semantics. The result seems to bear a more than accidental resemblance to structured meaning. However tentative, we do think the idea is interesting enough to describe here briefly.

list [**drs**:$D_1 \cup D_2$, **schema**:$S_1 \sqcup S_2$), **sem**:((S_1 & S_2)/($S_1 \sqcup S_2$))($S_1 \sqcup S_2$)] (II)
==> DCU1 [drs:D_1, **sem**:S_1]
+ DCU2 [drs:D_2, **sem**:S_2]

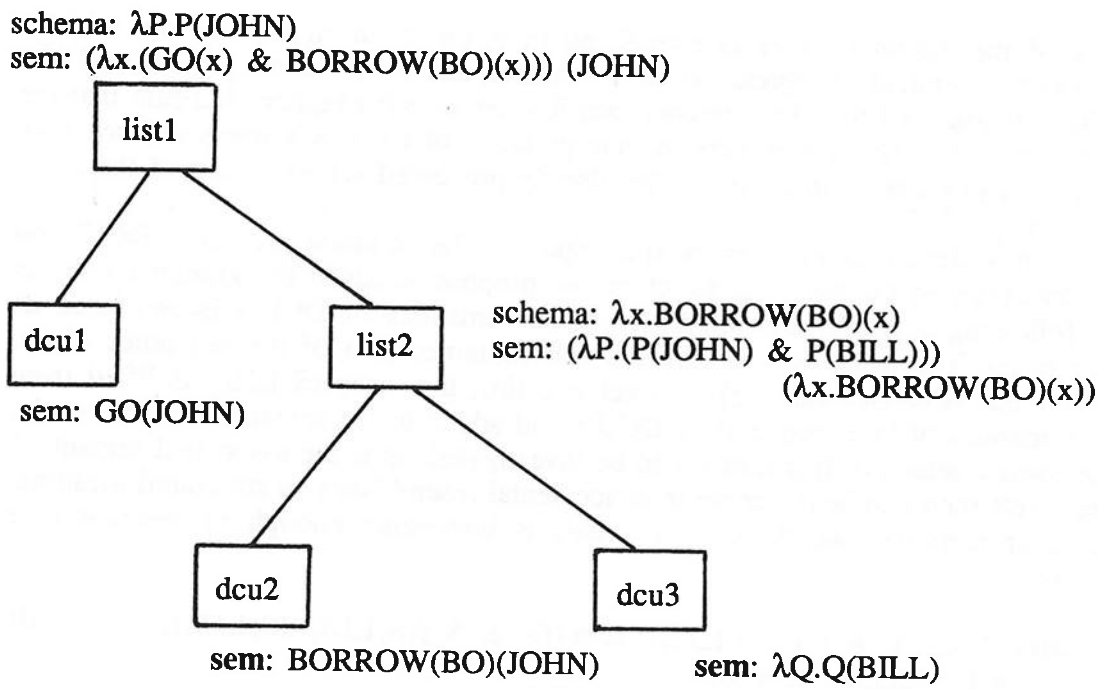
We illustrate the effect of this rule with example (2-19)/(2-21).

(2-19) John went to the library.
(2-20) He borrowed a book on computer science.
(2-21) Bill did too.

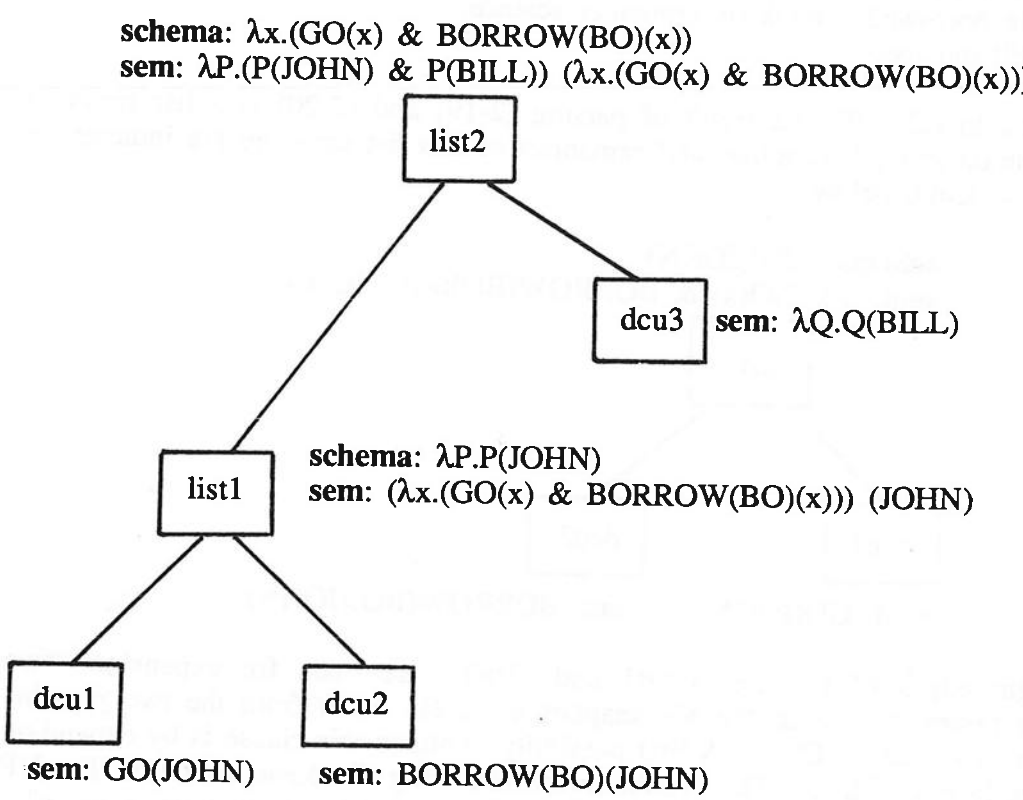
According to rule (II), the result of parsing (2-19) and (2-20) is a list structure. The common ground (**schema**) and semantics of this list structure are indicated in the tree structure below.



The right edges of this tree ('list1 and 'dcu2') are open for expansion. Two possible interpretations of the VP anaphor in (2-21) result from the two possible ways to attach clause (2-21). A first possiblity to attach this clause is by expanding the edge labelled 'dcu2'. This results in the following discourse structure. The VP anaphor is interpreted here as refering to "borrow a book on computer science".



The other possible way to build a discourse structure according to the grammar rule is by expanding the edge labelled 'list1'. The result of this (below) has as a side-effect that the elliptic clause is interpreted as refering to both VP's in (2-19) and (2-20).



What remains to be investigated is how exactly the matching process in the second case proceeds.

5 Conclusions & Further Research

In this paper we have argued that a formal account of VP anaphora (ellipsis) must be embedded in a theory of discourse semantics and discourse structure. There are at least two reasons why the problem of ellipsis needs a discourse perspective to provide an adequate solution. In the resolution of VP anaphora, the clausal structures of both antecedent and anaphoric clause have to be considered to account for the parallelism constraints that were formulated. So some linguistic framework is needed to handle whole clauses and to place structured clauses in their context. Furthermore, a discourse perspective is indispensable to cover the reference possibilities of VP anaphora.

The discourse theory that has been developed in [Scha 88] turns out to be a fruitful framework for the treatment of ellipsis. An essential aspect of the theory is the integration and interpretation each clause in its (structured) context on the basis of computing the most specific common ground of the relevant discourse constituent units. The notion of mscg is an intuitively appealing notion that also plays an important role in an account of discourse structure and coherence.

The parallelism constraints were formulated as conditions on the mscg. VP anaphora resolution comes out as a side-effect of the general discourse parsing process for constructing discourse structure and discourse semantics. This means that it is not necessary to stipulate autonomous mechanisms for resolution. An account of gapping along the same lines seems plausible. In the near future we hope to generalize this approach to a treatment of anaphora which also covers pronouns, where parallelism plays an important role as well.

Footnotes

1. Although the role of the word 'too' is important, we shall not deal with it in this paper. One of the effects of the word 'too' is that it 'balances' parallelism of subject and verb phrase: The semantics of the subject of the anaphoric clause can not subsume the semantics of the subject in the antecedent clause whereas the semantics of the verb phrase of the second clause must subsume the semantics of the first clause verb phrase. Compare for instance the next two examples (in contrast with (1), example (2) implies that John is no man):

(1) John loves a woman. Every man does.
(2) John loves a woman. Every man does too.

2. Sag uses this unconventional notation of logical formulas because they 'reflect surface word order extremely closely'([Sag 77], page 72). The notion of VPD depends heavily on the unconventional syntax of the semantic representations, which we therefore adopt for the moment.

3. The notion of identical predicates hinges on what Sag calls 'the standard notion of "alphabetic variance" ' of λ-expressions. Alphabetic variance of two λ-expressions in Sag's view depends on binders (i.e. quantifiers or λ-operators) outside the two expressions: "Crucially, if $\lambda x(A)$ contains a variable bound outside of $\lambda x(A)$ and $\lambda y(B)$ contains a corresponding variable bound outside of $\lambda y(B)$ the two λ-expressions are not alphabetic variants." ([Sag 77], page 73]). For instance in: $(\forall x)[John,\lambda y(y \text{ likes } x)]$ and
$(\forall x_1)[John,\lambda y_1(y_1 \text{ likes } x_1)]$
the two λ-expressions are not alphabetic variants (whereas the whole quantified expressions are). Sag states that his definition of alphabetic variance entails that in cases like (3) deletion is (rightly) impossible.

(3) John likes someone and Bill likes everyone.
$(\exists y)[John,\lambda x(x \text{ likes } y)]$ & $(\forall y_1)[Bill,\lambda x_1(x_1 \text{ likes } y_1)]$

His (non-standard) definition of aphabetic variance is not unproblematic, however. For instance, it incorrectly implies that in the following interpretation of (4) deletion is impossible.

(4) John likes someone and Bill likes someone.
$(\exists y)[John,\lambda x(x \text{ likes } y)]$ & $(\exists y_1)[Bill,\lambda x_1(x_1 \text{ likes } y_1)]$

4. Bach & Partee propose the following tentative restriction on the logical representation in terms of an example ([Partee 81], page 467):
"Two occurrences of "LOVE(x)" are semantically identical iff either (i) both occurrences of x are free, or (ii) both occurrences are bound by the same (token) variable-binder."
In our approach no free variables in the sense of [Partee 81] occur: referential pronouns are represented by context variables (cf. [Janssen 80]). The condition on binding is formulated in terms of clausal matching.

5. A continuation using the contrastive word 'but', might allow the VP anaphor in (7) to refer to the VP in (5) only:

(5) John went to the library.
(6) He borrowed a book on computer science.
(7) Bill did too,
(8) but he borrowed two books on French.

In this case the parallelism between (5)/(6) and (7)/(8) will have to be taken into account.

6. Context variables are also called 'context-dependent constants' (see [Janssen 80]). A context variable is semantically a constant, but dependent on the context for its interpretation. We use '<u>does</u>', '<u>he</u>', '$\underline{he}_1$' etc. in order not to clutter the semantic structures with types.

7. Of course the subject terms of (3-1) and (3-2) also share the information that they both refer to a male person. The standard notion of generalization is based on a rather poor hierarchy of terms as it always introduces variables when some information conflicts, thereby loosing further common information. For instance the terms 'John' and 'Fred' generalize to a variable of the appropriate type, thereby loosing the information that both terms refer to a male person. We will assume a more sophisticated notion of generalization that incorporates this kind of lexical

information. A more sophisticated notion of generalization also provides a means to formalize the notion of coherence in the sense of [Polanyi 85]. This notion covers, for example, the fact that sentences (9) and (10) both describe 'physical attributes' of John ([Polanyi 85], page 316).

(9) John is a blond.

(10) He weighs about 215.

(11) He's got a very nice disposition.

The contribution of (11), in this respect, results in the establishment that (9) through (11) all describe 'generally known attributes' of John. Such a notion of coherence can be moulded into concrete form by computing the common ground of DCU's on the basis of a lexical thesaurus.

8. In principle, a pronoun receives a twofold translation: it is translated into a special context variable ($\underline{he, he_1, he_2, ..}$) and into a standard (logical) variable ($x, y, x_1, y_1, ...$). The latter, of course, only when it is bound by a quantifier. This twofold translation reflects the dichotomy of 'bound variable' versus 'referential' pronouns. The overhead due to this twofold translation of pronouns can be reduced substantially on the basis of syntactic constraints on the distribution of bound variable versus referential pronouns (cf. [Reinhart 83]).

9. This backtracking to the previous open DCU might need refinement in order to incorporate more aspects of discourse structure and discourse semantics as criteria for attachment of a new DCU. Although the simple backtracking seems to work fine for the interpretation of VP anaphora, generalization of this approach to pronominal anaphora might need a more refined backtracking mechanism.

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Against Groups

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My purpose here is to compare two approaches to plurals. The first will be called the sums approach and the second will be called the groups approach. The sums approach, has some affinities to Link (1983) and Massey (1976). It presupposes a domain of discourse having individual entities as well as plural entities, or sums, which correspond to sets of individuals. The groups approach has affinities to work by, among others, Bennet, Bartsch, Hoeksema, Link (1984), Landman and Lasersohn. This approach presupposes a richer domain of discourse with entities corresponding not only to individuals and sets of individuals but also entities corresponding to higher order sets. In the first half I outline the two approaches and present evidence favoring the groups approach. I will temporarily adopt it. Then I will argue for the introduction of two shifting operations which can apply to predicates to allow them to apply to entities of different types. Next, I will show that once these operations are in place, the motivation for the groups approach is eroded. I end this section by returning to the simpler sums approach. In subsequent discussion, I outline a role for the pragmatics in explaining the intuitions associated with the original examples used to motivate the groups approach.

Part I. The two approaches.

We begin with a set of individuals "IN". Our domain of discourse D will be generated from this set IN. On the sums approach, D is just the power set of IN, on the groups approach it is gotten by iterative set-formation applied to IN resulting in something more than the power set. Singular common nouns denote subsets of IN, a plural common noun denotes the power-set of its singular counterpart minus the empty set. The definite article, "the" denotes the supremum operator, it takes a set of sets and returns the largest one of them. Here is an example. The common noun "cow" will denote the set of cows. The plural noun "cows" will denote the power set of the set of cows minus the empty set. The noun phrase "the cows" will denote the largest member of the aforementioned power set, which is just the set of all the cows. This way of analyzing definite plurals is essentially the set-theoretic counterpart of the analysis in Link(1983).

Now we come to noun phrase conjunction and this is where the two approaches differ. On the sums approach, the conjunction "and" is interpreted as set-union. On the groups approach it is interpreted as set-formation. Another example will show the effects of this difference.

Let us assume for the remainder of our discussion here that there are some cows and some pigs and that the cows and the pigs comprise all the animals there are. As we said above, the noun phrase, "the cows" will denote the set of all the cows. The noun phrase "the pigs" likewise will denote the set of all the pigs and the noun phrase "the animals" will denote the set of all the animals. So far we have three distinct noun phrases, and three distinct entities. Now we come to the noun phrase "the cows and the pigs". On the sums approach we get the union of the cows and the pigs which is the set of all the cows and pigs which is just the set of all the animals. So the cows and the pigs are just the animals on this approach and we still have only three entities. On the groups approach, the noun phrase "the cows and the pigs" denotes a set of two sets, or a group of two groups, a cow group and a pig group. This is different from the noun phrase "the animals" which denotes a group of individuals, not a group of groups. So now, on the groups approach, we have four entities, a cow group, a pig group, an animal group and now a group of two groups. Of course, these are not the only new animal entities that the groups approach has but that the sums approach lacks. For there is also the young animals and the old animals, another group of two groups and there is the male animals and the female animals yet another distinct entity. And so on. All we want now is some linguistic evidence to show that we need these extra entities.

1. Summary of two approaches.

IN is a set of (non-plural) individuals.
D is the domain of discourse generated from IN.

Sums approach:	D is the power set of IN.
Groups approach:	D is formed by iterative set formation applied to IN.

If cn is a singular common noun, then $\|cn'\| \subseteq$ IN.

If pl-cn is the plural of cn, then $\|pl\text{-}cn'\| = Power(\|cn'\|) - \phi$

"the" denotes the supremum operator.

$\|cow'\|$ is the set of all cows in IN.

$\|cows'\|$ is $Power(\|cow'\|) - \phi$

$\|$the-cows'$\|$ is the set of all the cows.

Sums approach: $\|$A and B$\|$ = $\|$A$\|$ U $\|$B$\|$

Groups approach: $\|$A and B$\|$ = { $\|$A$\|$, $\|$B$\|$ }

Assume: $\|$animal'$\|$ = $\|$cow'$\|$ U $\|$pig'$\|$

Sums approach: $\|$the-cows-and-the-pigs'$\|$ =
$\|$the-cows'$\|$ U $\|$the-pigs'$\|$ =
$\|$the-animals'$\|$

Groups approach: $\|$the-cows-and-the-pigs'$\|$ =
{ $\|$the-cows'$\|$, $\|$the-pigs'$\|$} ≠
$\|$the-animals'$\|$

The list in (2)-(4) below, inspired by examples in work of Hoeksema, Link and Landman, contains the evidence we want to show the need for the extra higher order entities of the groups approach. Let me note that I will be ignoring distributive readings throughout the discussion here. The context for these examples is one in which there are young and old cows and young and old pigs and there are no other animals.

2. a. The cows and the pigs were separated.
   b. The young animals and the old animals were separated.

3. a. The cows and the pigs talked to each other.
   b. The young animals and the old animals talked to each other.

4. a. The cows and the pigs were given different foods.
   b. The young animals and the old animals were given different foods.

Each example consists of an a. and b. pair which seem to be independent in the sense that one could be true while the other is false. Consider the pair in (2). It has been claimed that we do not want it to follow necessarily from the fact that the cows and the pigs were separated that the young animals and the old animals were separated, even with our assumption that the animals are just the cows and the pigs. On the sums approach the noun phrase subjects of the a. and b. sentences would have the same denotation, namely something corresponding to the set of all the animals. This would mean that if any of the a. sentences was true the corresponding b. sentence would also have to be true. In order to avoid this undesirable consequence, we adopt the groups approach, under which the noun phrase "the cows and the pigs" and the noun phrase "the young animals and the old animals" have different denotations, and hence the a. and b. sentences remain independent.

Before going on let me alert you to a terminological quirk about my exposition. I use the term "group" for something corresponding to a set of any order, while the term "sum" is reserved for something corresponding to a set of individuals.

Now, having adopted the groups approach, the first problem we run into is that unlike those in (2)-(4), many predicates are insensitive to the differences among higher order groups formed out of the same individuals. Two examples of this appear in (5) and (6):

5. a. The animals filled the barn to capacity.
   b. The cows and the pigs filled the barn to capacity.
   c. The young animals and the old animals filled the barn to capacity.

6. a. The animals were sleeping in the barn.
   b. The cows and the pigs were sleeping in the barn.
   c. The young animals and the old animals were sleeping in the barn.

Notice, that if (5)a. is true then in the context we are assuming, (5)b. and c. will follow. On the groups approach which we have just adopted, nothing guarantees this since the subject noun phrases in (5)a.-c. are not coreferent. Clearly, all that matters for a predicate like "fill to capacity" is what individuals are involved. The inference from (5)a. to (5)b. and c. is an example of what I call the Upward Closure phenomenon, whereby

7. Upward Closure Phenomenon

An English predicate that is true of a first order group G, (sum, set of individuals) is true as well of all higher order groups formed using all the members of G.

Something needs to be added to our system to take care of the Upward Closure Phenomenon. Following ideas in Landman(1989), I suggest the operation "LIFT" defined in (8).[1] LIFT(P) denotes the set containing any set Y in D

[1] Actually, we might have to add the following to the definition of LIFT:

if $\|P\| \cap \text{Power(IN)} = \phi$ then $\|\text{LIFT(P)}\| = \|P\|$

such that the set of individuals from which Y is constructed is in the extension of P.

8. $\|LIFT(P)\| = \{ Y \in D \mid \{x \in IN \mid x \in^{*} Y\} \in \|P\| \}$

[$\in^{*}$ is the transitive closure of $\in$]

if P is true of some group of individuals A, then LIFT(P) will be true of a group of any order just in case the individuals involved are just the members of A.

e.g. $\{a,b,c,d\} \in \|P\|$ --> $\{\{a,b\},\{c,d\}\} \in \|LIFT(P)\|$

9. if α is a predicate of English and α translates as α' then it also translates as LIFT(α')

In (5)a. the verb phrase "fill the barn to capacity" will translate as a predicate P true of groups of individuals. In (5)b. and (5)c. the verb phrase is translated as LIFT(P). Given the way LIFT is defined, if (5)a. is true, b. and c. will also be true, since the subject noun phrases in (5) have in common that they are all built from the same set of individuals.

In (9), I allowed for this operation to apply to any predicate. In this respect, I follow the tactic employed in Landman(1987), though I may note that he has a more elaborate shifting mechanism. Let me add that I am not in principle opposed to the approach of Hoeksema (1987) according to which the effect of the LIFT operation would be achieved through meaning postulates.

Reviewing briefly, because of the examples in (2)-(4) we adopted the groups approach. In order to prevent the inference from the truth of an a. sentence to the corresponding b. sentence, we interpret conjunction as set-formation and this insures that the noun phrases "the cows and the pigs" and "the young animals and the old animals" will have different denotations. Next, we saw that certain predicates which apply to sums or first order groups can apply as well to higher order groups formed from these sums, thus blurring the distinctions introduced with the adoption of the groups approach. To handle this upward closure phenomenon, we include an operation that allows predicates of English to be true of a higher order group just in case they are true of the set of individuals from which that group is constructed.

This is to prevent a sentence with a higher groups predicate from having a reading on which it is false, just because LIFT has applied.

Next I would like to bring to your attention a significant distributional fact about the predicates of English which I have chosen in (10) to call "the Mereological Generalization".

10. Mereological Generalization

1° There are no predicates of English that have higher order groups in their extension but that cannot also have first order groups (i.e. groups of individuals)

2° If a predicate of English is true of a group G of any order, it will also be true of that first order group G' which is composed of the individuals used to generate G.

The first part says that there are no predicates of English that have higher order groups in their extension but that cannot also have first order groups (i.e. groups of individuals). In other words, there are no predicates that are strictly typed for higher order groups. Support for this claim comes from the fact that all of the predicates used in (2)-(4) to argue for adopting the groups approach can be applied to noun phrases denoting first order groups. Examples of this appear in (11)-(13).

11. The boys were separated.
12. The boys talked to each other.
13. The boys were given different foods.

This means that although our universe is populated with various types of entities, curiously predicates of English are not sorted along these lines in any fine grained way. In fact the only distinction that is respected in the sorting of predicates is that between groups and individuals. A predicate such as "eat together" is true of groups but cannot truthfully apply to individuals. The sentence "!John ate together" is illformed. In other words, if all we were interested in was the sorting of predicates we would be happy sticking to the simpler sums approach.[2]

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[2]At the conference J. Hoeksema wondered about the predicate "be equally numerous" suggesting that it may apply only to groups of groups. However, I think the following piece of discourse is well-formed:

i. After years of separation a very puzzling thing happened. Whereas the women from the Western communities far outnumbered the women from the Eastern communities, the men were just about equally numerous.

The second part of the Mereological Generalization says that if a predicate of English is true of a group G of any order, it will also be true of that first order group G' which is composed of the individuals used to generate G. This is, in a sense, a stronger version of the first part. It speaks not about the kinds of things that can be in a predicate's extension, but about specific entities that we find there. Evidence for this claim follows in examples (14)-(16). Let me caution you again that I am ignoring distributive readings here and that we are assuming that the animals are just the cows and the pigs. I claim that in each example the b. sentence follows from the a. sentence. An appropriate context for these sentences might be one where a speaker says a. and his hearer replies with b. adding that he is not interested in how it was done, just that it was done.

14. a. The cows and the pigs were separated.
    b. The animals were separated.

15. a. The cows and the pigs talked to each other.
    b. The animals talked to each other.

16. a. The cows and the pigs were given different foods.
    b. The animals were given different foods.

Another context might be one in which the a. sentence is true, but the speaker didn't have enough information to say that, for example she did not realize that there were only cows and pigs or she could not distinguish a cow from a pig.

So far, there is nothing in the system we are working with that will guarantee the types of inferences exemplified in (14) - (16) since the noun phrase "the animals" has a different denotation from the noun phrase "the cows and the pigs". As a remedy, I propose the shifting operation, LOWER, defined in (17) to account for the Mereological Generalization.

LOWER(P) denotes the set containing every set of individuals Y, such that there is some group K and P is true of K and K is constructed from the members of Y.

17.
$\|LOWER(P)\| =$

In i. the predicate "be equally numerous" applies to "the men" which on the semantics I am assuming would refer to a set of individual men.

$\{Y \subseteq IN \mid \bigvee K \, [\, K \in \|P\| \;\&\; \{x \in IN \mid x \in^{*} K\} = Y \,]\, \}$

If P is true of a group G of any order, then LOWER(P) will be true of that first order group G' which is composed of the individuals used to generate G.

e.g. $\{\{a,b\},\{c,d\}\} \in \|(P)\| \dashrightarrow \{a,b,c,d\} \in \|LOWER(P)\|$

18. if α is a predicate of English and α translates as α' then it also translates as LOWER(α')

In (14)a. the verb "separate" will translate as a predicate P true of a group of two groups. In (14)b. the same verb translates as LOWER(P) and hence applies truthfully to the first order group denoted by the NP "the animals".

Reviewing again, because of the examples in (2)-(4) we introduce groups into our domain and we interpret conjunction as set-formation. This means that the noun phrases "the animals" and "the cows and the pigs" cannot be not coreferent. But then we come to find out that English does not want to cooperate. Predicates of English are just not fine-grained enough. So the next thing we need to do is introduce two shifting operations that will blur the distinctions. But now the possibility arises that these two operations could feed one another. In particular, LOWER produces predicates that are true of sums and LIFT seems to apply quite generally to predicates that are true of sums so we might be able to have LIFT(LOWER(P)) for some predicate P. I take the piece of reasoning given in (19) as evidence that the operations do feed each other.

19. [Note, I have shown in curly brackets how the main predicate is translated.]

Given:

a. The young animals and the old animals are just the cows and the pigs.

Assume:

b. The young animals and the old animals were separated. {were-separated'}

Then:

c. The animals were separated. {LOWER(were-separated')}

d. The animals were separated by age. {LOWER(were-separated')}

e. The cows and the pigs were separated by age. {LIFT(LOWER(were-separated')) }

I assume that "by age" is a standard modifier, that is, if somethings are separated by age, then they are separated.

So, from (19)e. I conclude:

f. The cows and the pigs were separated.
{LIFT(LOWER(were-separated'))}

I do not deny that (19)f. is a misleading thing to **say**, if you know that (19)b. is true. It is misleading, but not false. (19)f. follows from (19)b. simply because English does not respect the distinctions that the groups approach makes. Here is another example of reasoning in which LOWER feeds LIFT:

20. Scene: four lawyers: Mary (defense), John (defense), Bill (prosecution), Mike (prosecution).

Given:

a. The defense lawyers and the prosecution lawyers used to fight each other in court every day. {fight-e.o.'}

Then:

b. The lawyers used to fight each other in court every day. {LOWER(fight-e.o.')}

c. That woman and those three men used to fight each other in court every day. {LIFT(LOWER(fight-e.o.'))}

But now if LOWER can feed LIFT then our original motivation for moving from the sparse universe of the sums approach to the more complicated world of groups is undercut.

Recall, our original motivation was to achieve a logic that would allow a sentence like (21) to be true, without it following, in the relevant context, that (22) was true.

21. The cows and the pigs were separated.

22. The young animals and the old animals were separated.

Having accommodated the Upward Closure Phenomenon and the Mereological Generalization, we now have a logic that precisely allows us to go from (21) to (22), using the definitions of LIFT and LOWER and the fact that, taken together, (9) and (18) above imply (23):

23. if α is a predicate of English and α translates as α' then it also translates as LIFT(LOWER(α'))

Let me put this another way. In order to argue for a groups approach we need to have predicates that distinguish plural entities which differ only by the way they are grouped. We thought we had this. However, English has two properties that conspire against us. The first, the Mereological Generalization, says that a predicate of English that is true of a particular grouping of a sum will be true of the sum itself. The second is that the Upward Closure Phenomenon is rampant allowing predicates of English that are true of a sum to be true of that sum on any grouping. Putting these two together, it turns out that a predicate of English that is true of a sum on one grouping will have to be true of the sum itself, and hence true of that sum on any grouping, so groupings cannot matter. But this is exactly what the sums approach predicts.

At this point various embellishments may come to mind for salvaging the groups approach. For example, thinking of our shifting operations along the lines of Partee & Rooth's (1983) type shifting, we might suggest some sort of constraint, pragmatic or otherwise, barring the multiple application of these shifting operations. This might work, however I prefer a more direct route as follows. Let's revert to the sums approach in which case the Mereological Generalization and the Upward Closure Phenomenon fall out automatically and then give some type of explanation, pragmatic or otherwise, of the difference between the truth conditionally equivalent pair (21) and (22). In part II of this paper I will begin to pursue this program.

All of the examples discussed so far involve subject noun phrases consisting of a conjunction of two plural noun phrases. Before turning to a sketch of a 'sums only' program, I would like to briefly discuss some examples that do not fit in this pattern.

Consider the following example:

24. The pigs from the two communities hated each other.

This example has a 'reading' or understanding of the kind that was initially appealed to in examples (2)-(4) as an argument for a groups approach. On this reading there is only inter-communal hatred. The hating goes on between two groups of pigs. The same reading arises in the following example:

25. The pigs from community A and the pigs from community B hated each other.

In the case of (25), on the groups approach, the subject noun phrase would denote a group of two (hating) groups and the predicate would have this higher order entity in its extension. We have shown how a higher order entity is arrived at in a compositional manner for the case of (25). The problem is how to get the subject of (24) to denote a group of two groups. Without actually proposing a solution let me suggest that at some point the meaning of the definite article will have to be revised in order to accommodate this example on a groups approach. Perhaps we can lower the cost of this revision by assuming that something like our LIFT operation would apply to the meaning given above for "the". Up to now I have made it seem like the only difference between the two approaches lies in the interpretation of conjunction. I mention this example to show that more is at stake.

The next type of example has been discussed in the groups literature. It involves multiple conjunction of noun phrases.

26. [[Blücher and Wellington] and Napoleon] fought against each other near Waterloo. ≠ [Blücher and [Wellington and Napoleon]] fought against each other near Waterloo. (Hoeksema 1983)

Hoeksema argues on the basis of the example in (26) that the interpretation of "and" must not be associative otherwise the bracketing on the conjunction would not affect the denotation of the noun phrase subject. The problem with this argument is that, as with the examples used above, it relies on having predicates that are sensitive to these groupings. In this particular example, the extension of the predicate "fighting each other" would have to encode information not only about who was fighting but also about who was allied with whom in the battle. However as I have argued up to now, English predicates do not seem to be as fine grained as the groups approach requires. Compare (26) to the following example:

27. Despite their current membership in a common market, only 45 years ago, Germany, England, France and Italy were battling each other in one of the worst wars in history.

I take (27) to be true. If the groups approach is correct and the alignment of the forces is encoded in the extension of the predicate "were battling each other" then (27) should only be true if the subject noun phrase denotes a group of two groups, one containing Germany and Italy and the other France and England. But again I see no compositional way of assigning this denotation to the subject of (27).

(28) below is yet another example of this type:

28. John and Mary and Bill and Sue played tennis with each other. In the first match, the men played the women, and in the second match John and Mary played Bill and Sue.

A non-associative "and" might seem attractive on the basis of the second match, but must be abandoned on the basis of the first match.

Returning to the main theme, I have argued up to now against capturing the differences between (21) and (22) in the truth conditions. I have suggested instead that there is a role for the pragmatics in this story. At this point I will elaborate on that role.

Part II. Pragmatics.

Under a pure sums approach all the plural noun phrases we have seen up to now denote sums, or sets of individuals. Predicate extensions include sums and nothing more complicated than that. But there is more we can say. The predicates we have discussed so far carry subentailments that apply to sub-parts of the sums. Thus if a sum is separated then some sub-part of that sum is separated from some other subpart. If there is a reciprocal hating predicated of a sum, this entails that hating is going on between certain subparts of this sum, perhaps even on the level of individuals. This term "subentailment" was introduced by Dowty to refer to entailments of collective predicates that apply to some individual members of the group to which the collective predicate applies. For example, if the predicate "voted for the proposal" applies to a group of individuals then the predicate "voted for the proposal" will apply to some (a majority) of the individual members of the group. Dowty uses this notion to explain the use of "all". Briefly, "all" requires the subentailments to apply to everyone of the individual members of the group, hence the differences in the a. and b. pairs below:

29. a. The students voted for the proposal.
 b. All the students voted for the proposal.

30. a. The students gathered in the cafeteria.
 b. All the students gathered in the cafeteria.

31. a. The students destroyed the house.
 b. All the students destroyed the house.

For (29) it would have been simpler to interpret "all" as a universal quantifier over individuals, however this will not work for the examples in (30) and (31).

Dowty's subentailments apply only to individuals while mine apply to sums as well. Now the question of which particular subsums are involved is not, I claim, to be answered by studying the denotation of the noun phrase subject or the extension of the main verb phrase predicate. The details concerning what part of the sum is separated from what part, is indicated instead with an adverb as in (32).

32. The animals were separated by age.

or through contextual information. Thus I would claim that in a sentence such as (33)a.:

33. a. The pictures that came from Bill's parents and the pictures that had come from Sheila's parents were separated.

b. The books were separated that way too.

the way that the pictures were separated is implicated by the choice of the noun phrase used to refer to the sum of all the pictures. Note the continuation of (33)a. in (33)b. The phrase, "that way" refers back to the pragmatic information coming from the noun phrase that is used in (33)a. to tell you how the pictures were separated. This is essentially like the situation in (34).

34. John fed the cow and killed the cat in that order.

The order of the actions is understood from pragmatic information coming from the order of the words in the sentence. This pragmatic information is referred to explicitly by the phrase "in that order".

Let us call the particular parts of the sum to which these subentailments apply the operative subsums allowing individuals to count in some instances as operative subsums. What I claim then is that in certain cases, such as (33), the identity of the operative subsums is indicated by the choice of the subject noun phrase.

Perhaps a little more can be said at this point about the relation between examples like (32) and (33)a. It seems to me that the adverb in (32) specifies a partition, the result of the separation, by naming a set of properties to which the cells of the partition correspond. Thus the animals were separated by age, means something like the animals were divided into separate groups and that set of groups forms a partition induced by the family of properties denoted by "age". Notice that the object of "by" in these cases must be a bare noun denoting a set of properties (or perhaps more correctly a property of properties):

35. The pigs were separated by gender.

36. # The pigs were separated by female/sow.

37. The cows were divided by age.
38. # The cows were divided by 10 year old.

So the idea then is that the operative subsums are identified as the cells of a partition of the subject's referent. This partition is induced by a family of properties. In (32), this family comes in through the adverbial by-phrase. In (33)a., the family of properties is explicitly mentioned in the subject NP. These are the properties of 'coming from Bill's parents' and 'coming from Sheila's parents'. (33)a. can be understood to mean that the pictures were divided into separate groups and that set of groups forms a partition the cells of which correspond to these properties. Using this idea, we return to an earlier type of example:

39. The pigs from the two communities were separated.

which can be understood to mean:

40. The pigs were separated by community.

In (39) two communities are mentioned. In this case it the properties of membership in these communities is what induces the partition.

In the examples in (33)a. and (39) the set of properties that were used to play the role of the by-phrase of (32) were somehow included in the noun phrase that was used to refer to the thing that was separated. I don't think this is necessary. Consider the following example:

41. Farmer Smith and Farmer Jones said that although their cows could stay together, <u>the pigs had to be separated</u>.

Here the pigs can be understood to be separated in such a way that the pigs that belong to Smith are separated from those belonging to Jones. The properties of "belonging to Smith" and "belonging to Jones" come in through prior discourse. Unfortunately, I do not have a formal theory of how these properties become 'live' in the discourse. This effect is reminiscent of discussions of generics in which the mention of a common noun is taken to introduce the name of kind which can be refered to later in the discourse. (cf. van Eijck, Wilkinson). More also needs to be said about how these properties get picked up in the interpretation of predicates such as "separate".

In all the cases we have seen so far, the contextual information that was employed in the determination of the operative subsums was linguistic. Now I would like to look at an example in which the non-linguistic context determines the operative subsums.

The example has to do with reciprocals and before discussing it, I want to mention a fact or two about reciprocals. In their work on this topic, Fiengo and Lasnik claim that if a stative reciprocal sentence is true of a set S of individuals and its main verb denotes the relation R then the following holds:

42. $\bigwedge x,y_{\in S}\ [x \neq y \rightarrow xRy]$

To show this they adduce, among others, the example in (43):

43. The numbers in the list below are congruent mod 3 to each other.

2,3,4,5,6,7,8,9,10

(43) is taken to be false, because it is not the case that any two numbers in the list are congruent mod 3.

Link(1984) assumes roughly the same thing in analyzing the sentence:

44. The Leitches and the Latches like each other.

on the reading where the liking is between families only. He achieves this by allowing the subject noun phrase to denote a sum of only two entities: a Leitch group and a Latch group and allowing the verb phrase to be true of this sum of two groups. In this case (42) says that each group likes the other group. Concerning this example I would argue that the verb phrase applies to the sum of all the individual Leitches and Latches. Fiengo and Lasnik's condition in (42) is a subentailment that applies not to members of S, but to operative parts of S. In the case of (44), reference to the two families suggests a particular choice of subsums as operative namely the sum of the Leitches and the sum of the Latches. (42) applies to these subsums with R representing the liking relation. With this in mind, let us turn to the example in (45) from a syllabus for a literature course:

45. The books in the chart below complement each other.

Fiction	Non-fiction
Richard III(Shakespeare)	The Prince (Machiavelli)
Oedipus Rex (Sophocles) Elektra (Euripides)	Totem and Taboo (Freud)
Fantastic Voyage(Asimov)	Grey's Anatomy
David Copperfield, Hard Times(Dickens)	Das Kapital (Marx), The Wealth of Nations(Smith)
Alice in Wonderland (Lewis Carroll)	Aspects of the Theory of Syntax. (Chomsky) Language(Bloomfield)

Here the chart determines the operative subsums. The operative subsums of books are those that are opposite each other in the chart. The relation of "complementing" is understood to apply reciprocally between these. Notice that if the chart was rearranged, the meaning of the utterance in (45) would change as well, even if the same books were on the rearranged chart. Now a theory that is bent on keeping the operative subsums distinct in the extensions of predicates would have the predicate in this case apply to some special type of group whose structure was determined via a mapping from the chart. The noun phrase "the books in this chart" in (45) would then have to denote this complex object and that would explain why a change in the chart would effect the meaning of the utterance. The problem is that there does not seem to be any compositional way to get the noun phrase in this case to denote anything but a simple sum.

Reviewing so far, I am advocating here a semantics that includes only sums in the universe of discourse. Certain predicates that are true of sums carry subentailments concerning subparts of sums. The question of which specific subparts these entailments apply to or in other words the question of what the operative subsums are, is addressed either through adverbial modification or through contextual information. In some of these cases the contextual information is linguistic. These examples have been used to argue for groups, in effect building the operative subsums into the meanings of noun phrases and verb phrases. However, this will not always work since the operative subsums are sometimes determined non-linguistically.

Finally, I would like to briefly discuss a kind of example where the operative subsums are determined by the linguistic context, but in a rather surprising way. The examples I have in mind, depend on predicates that Fiengo and Lasnik call the 'linear configurationals'. "Linear configurationals", they write, "define a set of objects which are in a common linear configuration". The examples I have in mind appear in (46) and (47) below.

46. The red plates and the blue plates were stacked on top of each other.

47. The bells and the lights followed each other in rapid succession.

I have found that speakers often interpret (46) to be about an alternating red/blue stack. Similarly, (47) has an interpretation whereby the lights and the bells alternate. Again, I contend that the predicates in these sentences have sums in their extensions and the subject noun phrases refer to sums. Furthermore, these sentences have subentailments to the effect that certain parts of these sums are stacked on other parts and that certain subsums of bells and lights follow other subsums. Clearly, it is the particular phrasing of the subject noun phrases that is producing these alternating readings. But this case is unlike (44) for example in that we do not just allow each conjunct of the subject NP to be an operative subsum and then apply the transitive verb "stack" to pairs of subsums. The subentailments in this case are quite unique and show that the actual form of the subentailment is dependent upon the meaning of the main predicate and does not follow from a general theory of reciprocals as pursued by Fiengo and Lasnik and others.[1]

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[1] I wish at this juncture to express my gratitude to Max Cresswell, Angelika Kratzer, Barbara Partee, Paul Portner, Karina Wilkinson and Ede Zimmermann for their help in this project. Also thanks to Robyn Ratt for linguistic intuitions and everything bagels.

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Notes on Aspect in Bulgarian and English

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1. General

Anyone who is after a quick introduction into the problems of aspectology is in for a shock as soon as their second book on the subject has been opened. Few fields of human knowledge can boast of greater terminological chaos and instability, and almost none - of such geographical variation.

Two major "schools" of aspect stand out in Europe: an "Eastern" school with concepts and methods reflecting data from Slavonic languages - where the major objects of research are Action Modes and the aspectual opposition Perfective/Imperfective and a "Western" school where aspectual distinctions are analyzed within "Aspectual classes".

The aim of the present paper is, on the basis of a brief outline of the aspectual systems of two languages: Bulgarian and English, to propose a unified frame of analysis of aspectual distinctions that could serve as a foundation for cross-language investigations. The choice of languages has some attractions beyond the fact that this author would find it hard to make another: among Slavonic languages, Bulgarian stands out with the amount and regularity of aspectual markers; among Germanic languages, English is distinguished with the scarcity of these. And it is a reflection of the oddities of life that this empoverished system has attracted so much more classification-makers than its better-endowed cousin. Hence another purpose we have here is to make a step along the lines of remedying this unfairness.

2. Bulgarian

2.1. Bulgarian may not at first blush appear to be a very typical representative of Slavonic languages. It has a highly specific typological contour deriving from a unique and liberal interplay of "genes" and "social contacts". While belonging to the Slavonic branch of Indoeuropean languages, it has proved not averse to non-Slavonic Balkan influence. The outcome can, in a nutshell, be presented thus:

- systematic analyticity
- systematic grammatical expression of definiteness
- a full-blown system of inflection in the sphere of finite verbal forms (one of the pioneers of Bulgarian grammar, A. Teodorov-Balan, referred to the verb as "the elephant of Bulgarian grammar").

The aspectual system is usually described in terms of the Trinity: Action Modes (AM) - a calque I have adopted as a substitute for the tonguetwister Aktionsarten, Aspect and Aspect-cum-Tense - an opposition within the Preterit consisting of two members: the Aorist and the Imperfect.

2.2. Aspect and Action Mode

While in languages like English the difficulties of arriving at rigid classifications of aspectual distinctions are due to the lack of regular formal markers, in Slavonic languages problems arise due to the close interplay of separate aspectual phenomena. One of the hardest nuts to crack in this respect is the distinction between Aspect and Action Mode.

Slavonic languages make use of a system of affixes (mainly prefixes) which modify in specific ways the characteristics of the situation as denoted by the lexical meaning of the verb. The most detailed study of Action modes in Bulgarian, Ivanova 1974, lists no less than 53 AMs derived with the help of more than 20 prefixes. AM types listed in her monograph are: General Resultative, about 20 more subtypes of Resultative such as Resultative-Incursive, Resultative-Distributive, Resultative-Saturative etc., Attenuative, Augmentative, Effective, Iterative etc. Non/prefixed verbs are as a rule Imperfective while the prefixation of these Imperfectives yields Perfective forms:

peja - IZpeja
sing-Imp sing-Perf

stroja - POstroja
build-Imp build-Perf etc.

This interplay of Aspect and Action Mode has often given rise to attempts to equate them - both in older work on the subject and in more recent papers (Cf. e.g. Walinska's 1989 analysis in this volume where aspectual derivation and inflection are viewed as a unified process). Indeed, in Slavonic languages Perfectivization is impossible without affixation - although the opposite need not be true. On the one hand, however, the semantic information contributed by prefixation is not restricted to the transition from Perfect to Imperfect. On the other hand, even more noteworthy is the fact that in Bulgarian, for instance, Aspect as a truly grammatical opposition (Perfective - Secondary Imperfective) is realized within one Action Mode, with the help of Imperfectivizing inflectional suffixes:
Postroja - Postrojavam
build(up)-Perf build(up)- Imp

where the second form can approximately be translated as "be building up".

Hence Aspect and Action Mode, although simultaneously realized, are two well-distinct categories. Aspect is best studied as a SEMANTIC oppposition that in some languages can find realization as a grammatical category. Grammatical categories are realized as oppositions of linguistic meanings within one lexeme with the help of fixed explicit markers.The opposition Primary Imperfective/Perfective is realized within different lexemes.

Another question that can be asked relates to the content of the term Action Mode and whether it comprises ALL kinds of information contributed by the prefix. This approach is adopted by Ivanova (op. cit.) who considers the prefix to be the bearer of the semantics of Action Mode. She separates three general semantic components of this category: space coordination, directionality and intensity.

However, I fully agree with Lindstedt's objection (Lindstedt 1984) that the staggering number of Action Modes Ivanova sets out does not speak in favour of her classification. While I expect that a more careful analysis of Preposition + Primary Imperfective combinations may well reveal deeper regularities of semantic compositionality , here I will concentrate on the topologically describable parameters of Action Modes, as only these are aspectually relevant. In the model I propose, Action Mode is a feature of every verb, perfective or imperfective, defining the inner structure of the situation described. Action modes can be: Statal]_____[, Inchoative []_____, Momentaneous [], Culminative [______[]. A pertinent question to be asked here is whether Statal does not in fact define Imperfective aspect rather than a certain Action Mode and similarly, whether the other configurations are not manifestations of Perfective. In order to answer this, I will briefly turn to the definition of Aspect.

In Stambolieva 1989 (forthcoming) I have argued in favour of an analysis of the opposition Perfective/Imperfective in terms of the notion of Change. This is not, of course, a novel approach - Cf. Russell 1903, von Wright 1965, Lindstedt op.cit., Gruber 1976, Chafe 1970, Vet 1979, among others. My analysis comes close to Russell's definition of Change as

"the difference, in respect of truth or falsehood, between a proposition concerning an entity and a time T and a proposition concerning the same entity and a time T', provided the two propositions differ only by the fact that T occurs in the one where T' occurs in the other "
(op. cit., 469).

A more compact formulation is provided by von Wright (op.cit.) who speaks of a transition between two propositions, T(p,q).

Topologically, this transition can be presented in terms of "state" and "change of state": a sequence of open and closed intervals along a time axis:

] state 1 [change] state 2[

————————————————→

p T q

Fig. 1

It cannot be overstressed that p and q in the above figure should not be viewed as standing for objective states: they denote conceptualizations of states of affairs of varying nature and varying degrees of generality. Thus, for instance, to make use of our AM types above, p can stand for the conceptualization of a situation as Resultative; T will then mark a change of the "inner state of affairs" leading to a different conceptualization q of this same situation as, say, Statal. This analysis is needed in order to account for "negative" or "remaining" sentences where no change of state of the objective situation is denoted, yet the verbs are marked as perfective:

Petar ne sčupi vazata
Perf
(Peter did not break the vase)

Vse pak me razbra i ostanaxme prijateli
Perf
(But he understood me and we remained friends).

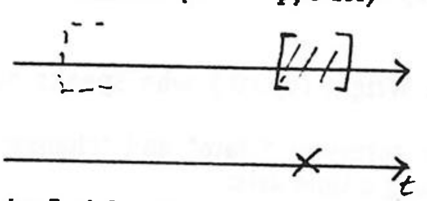
Fig 1 is misleading in one non-trivial respect: it presents /CHANGE OF STATE/ as a situation separate from the two adjoining states. For some situations this is indeed the case but not for all: in the case of Inchoatives and Culminatives, the event forms part of the situation, as an initial or final subinterval:

[]_____ [_____[] fig.2
Inchoative Culminative

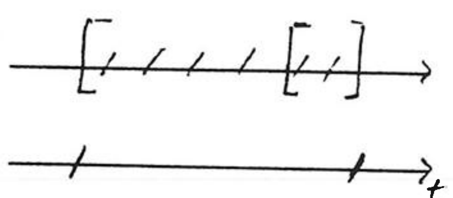
Aspect in the above-defined sense has not received the attention I believe it deserves within interval-based aspectual theories. This is largely due to the fact that the "aspects" defined in IS are "situation types". Within our framework, Aspect is accounted for in terms of its interaction with AM and the understanding of change in terms of conceptualization.

Action Modes reflect the "inner structure" of an interval as a maximal realization of a given situation concept. Aspect can be viewed as a function from situations to situations, i.e. as defining regions within the region defined by the AM interval. Perfective covers regions either entirely defined by or including a closed (sub)interval while the sphere of Imperfective are open intervals. It is thus that Perfective and Imperfective can create different situations within one Action Mode:

POSTROJA (build up, Perf)



(at 5 o'clock)



(from January to May)

POSTROJAVAM (be building up)

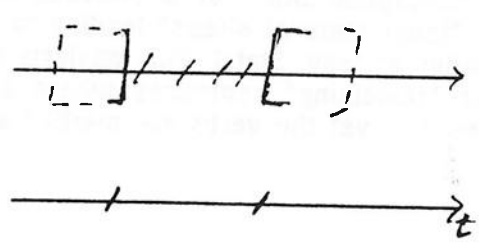


Fig. 3

These configurations can be defined in terms of the following IS interpretations:
For Perfectives obtaining at minimal intervals:

If $\Phi \in ME_{<et>}$ Mom.Perf and $<t,h> \in I$, then:

If $[[\Phi]]^{T,t,h,g} \neq \emptyset$, then t is an instant ot time

For Perfectives obtaining at non-minimal intervals:

If $\Phi \in ME_{<et>}$ NonMinPerf and $<t,h>$, then

If $e \in [[\Phi]]_{T,t,h,g}$ then $\forall e'$ [If e' is a temporal part of e then $e' \notin [[\Phi]]_{T, time(e'), h, g}$].

Imperfectives can be defined thus:

If $\Phi \in ME_{<et>,IMP}$ and <t,h> I, then if $e \in [[\Phi]]_{T,t,h,g}$ and e' is a temporal part of e,

then there is a time t' such that $e' \in [[\Phi]]$ and t'=time (e').
(ME is the set of meaningful expressions of type x. The analysis is based on Saurer 1984).

Here, it may be wise to assume a certain gappiness. A somewhat informal definition of this can be found in Vlach 1981:
"The sentence Φ is true for the interval I if and only if Φ is true at a set of subintervals of I which are more or less scattered all over I in such a way that there are no large gaps between the subintervals of I at which Φ is true".
This account presents the first stages only of the "process" that I call "Vertical compositionality" in Bulgarian - a term coined to stress the contrast with the "horizontal" compositionality of English. Another stage of analysis integrates the two forms of the Preterit.

2.3. The Aorist and the Imperfect

Bulgarian has fully preserved the Old Church Slavonic (Old Bulgarian) opposition Aorist/Imperfect - largely by virtue of being a Balkan language. Balkan languages, the temporal systems of which are of an archaic Indoeuropean type display some striking similarities in their modal and temporal systems, especially clear parallelisms existing between Bulgarian, Greek and Albanian (Cf. Asenova 1989: 198ff). What is specific for Bulgarian is not simply the preservation of these forms but the fact that the opposition Aorist/Imperfect is not in the least contaminated by the opposition Perfective/Imperfective (Cf. Maslov 1984: 91).
Thus, four distinct forms appear in the preterit:
1. Perfective Aorist
2. Imperfective Aorist
3. Perfective Imperfect
4. Imperfective Imperfect.
These, of course, will grow to six upon taking into account the Primary/Secondary Imperfective distinction.

One major problem in Bulgarian linguistics that can here only briefly be mentioned derives from the fact that the nature of the Imperfect/Aorist opposition is not yet conclusively and unanimously settled. Some authors deny the presence of aspectual distinctions and insist on the temporal character of the opposition, defining the Aorist as Past and the Imperfect - as Praesens in the Past (Cf. Stankov 1969).
I do not find this view very convincing: were the distinction only temporal, it would not affect the aspectual nature of the oppositions. But the forms listed above

are realized within different configurations. Before proceeding to their definition, this may be an appropriate place to draw attention to a couple of methodologically relevant points.

First, the "time axis" presentations I have made use of so far do not present an axis of deictic time, following a distinction that a number of authors are already making (Cf. e.g. Descles 1989, Oversteegen 1989 etc.)An illustration in this respect is fig. 3. Others do not even speak of a non-deictic axis but rather present situations as "hovering" over the axis of deictic time:

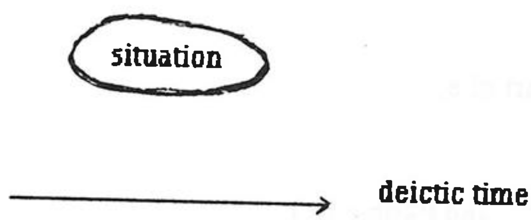


Fig. 4

(Cf. de Vuyst 1983). Inasmuch as AMs and Aspect involve in our model presentations within specific interval configurations where relations of precedence and following are important, I have here opted for the "non-deictic time " approach.

Second, although one would expect to find Imperfect and Aorist mapped directly on the deictic time axis and hence forming there open or closed interval configurations, I have chosen to "decompose" them into an "aspectual" and a "temporal" element. Following Lindstedt op. cit., I will consider combinations like Imperfective Aorist and Perfective Imperfect as instances of aspectual nesting. Deictic time limits have very little to do with aspectual distinctions - an open situation may obtain at a point on the deictic time axis while a closed situation may be evaluated for a period.

Returning now to our aspectual configurations: tentatively assuming the equipollence of the two aspectual oppositions, the following configurations are obtained:

1. Imp(Imp)]]______[[

2. Imp(Perf)] [] [

3. Aor(Imp) []______[]

4. Aor(Perf) [[]]

1 and 4 are straightforward and high frequency cases: there is no conflict in the aspectual marking and "pure" imperfective obtains for 1, "pure" perfective - for 4. No distinction is deemed necessary for the iterative and durative realizations of the Imperfective: both can be equally well accounted for in terms of a statal AM with a homogeneous subinterval structure.In the case of 3, the net result of the "nesting" is neutralization - so that Aor(Imp) is aspectually ambiguous (Cf. Stankov 1980, Maslov op.cit.). It would in fact have been possible to formulate a "neutralization of aspects" principle, if only the nesting in 2 led to the same result. But Imp(Perf) forms are realized as iteratives with a marked element of sporadicity - a possibility well predicted in fact by our topological scheme:

] [] [] [] [] [] [] [

Fig 5

(The best English equivalent I can think of are WOULD + INFINITIVE forms where the separate occurrences of an iterative situation are put into profile:
Cf. On such evenings he would come and sit by the fire...
Bulg. V takiva veceri toku DOJDEŠE i SEDNESE...)
It therefore seems that Maslov may have been right in defining the Imperfect/Aorist and Perfective/Imperfective oppositions are privative ones, where the marked member of the former is the Imperfect and the marked member of the latter - the Perfective. Such an analysis would account for the tenacity of the perfective configuration as opposed to the neutralization of the imperfective one, as well as for the clearly greater strength of the imperfective.

Accordingly modifying the presentation we introduce a forbidden "unmarked" interval (/ /). Of course, situations can only be defined in terms of open and closed intervals but the fuzzy notation will only be needed at this preliminary stage of aspectual composition. Our revised calculus now gives the following results:

```
//+][      -][
IMP  IMP        IMP(IMP)

//+//      -//
IMP  AOR        AOR(IMP)

[]+][      -][] [][] [
PERF IMP        IMP(PERF)

[]+//      -[]
PERF AOR        AOR(PERF)
```

Fig.6

The IS interpretations of IMP(IMP) and AOR(PERF) are identical to the ones given on p. for the AMs. In the case of IMP(PERF) some stronger requirements for gappiness will have to be made. As to the AOR(IMP) nesting, it will just have to be provided with two interpretations, with the ambiguity resolved at a higher level of compositionality.

At still another stage, these configurations are projected onto the deictic time axis as open or closed intervals of deictic time. As only aspectual distinctions are the subject matter of this paper, however, we will not consider this problem.

2.4. TO SUM UP:

We saw in this chapter that Bulgarian displays two formally marked aspectual categories: Aspect and Action mode. Action mode is a feature of both primary and projected verbal forms, defining that aspect of the semantics of a verb that relates to the "inner structure" of the situation denoted. Action Modes do not form a grammatical category. While grammatical categories are inflexionally marked and act within a single lexeme contributing thus to its paradigm of word-forms, the process of verbal prefixation is purely derivational and gives rise to new lexical units.

Aspect as a semantical category is non-grammatically realized within "primary" verbal forms : these being INHERENTLY perfective or imperfective(in the vast majority of cases, the latter). Aspect is realized as a grammatical oppposition within derived verbal forms. There, the unmarked form is the Perfective while the inflexionally marked one is the Imperfective(Imperfective 2). Perfective and Imperfective in such cases were described as defining different "regions" within a single Action mode. In general, the sphere of "Imperfective" are open intervals while that of Perfective - closed intervals. This presentation ties up well with the analysis proposed for the semantic category of aspect involving the semantic opposition [± Change of state].

3. ENGLISH

Opposed to the abundance of formal aspectual markers in Bulgarian, English (as noted earlier) has very few of these - if any. This does not mean that it cannot express the semantics of aspect and action mode; but the means of expression in the two languages are widely asymmetrical.

As it is not possible here to make a survey of the staggeringly abundant literature on aspectuality in English, I will direct my efforts towards a few undoubted classics. The major problem posed in this section is: Which are the aspectual classes of English, what is their linguistic status and how do they relate to the categories of Action Mode and Aspect as defined above.

The classification of aspectual classes which is I think most widespread, if not most popular, is the one proposed in Vendler 1967. This classification involves four classes: States, Activities, Accomplishments and Achievements, which are defined as follows:

STATE: A LOVED SOMEBODY FROM t1 TO t2 means that any instant between T1 and T2 A loved that person.

ACTIVITY: A WAS RUNNING AT TIME t means that time instant t is on a time stretch throughout which A was running,

ACCOMPLISHMENT: A WAS DRAWING A CIRCLE AT t means that t is on the time stretch in which A drew that circle.

ACHIEVEMENT: A WON A RACE BETWEEN t1 AND t2 means that the time instant at which A won the race is between t1 and t2.

Vendler's classification raises a number of questions. One relates to some methodological considerations: one of the basic criteria for setting up the four classes is their behaviour w.r.t. the so-called "Progressive test" A serious objection to be made here is that classes claimed to present more or less universally valid semantic distinctions are set out and defined in terms of intralinguistic regularities. The test, besides being inapplicable to other languages, is not even a satisfactory criterion for English:

YOU ARE LOOKING WELL, SHE IS WINNING THE GAME (examples quoted in Verkuyl 1989). Of course, this latter objection is only valid if we assume that: 1/ the classification applies to verb classes and 2/ the class-membership of English verbs is constant, i.e. they cannot appear in more than one class. And here we come to a second problem: it is not quite clear what kind of classification Vendler has offered. Throughout his paper, he speaks of VERB classes while in fact exemplifying these with larger syntactic units.

Most authors taking Vendler's classes as the basis for further investigation accept the classification as one relating to sentences or situations(Cf.Vlach 1981, Saurer 1984etc.) Inasmuch as the main factor presented as contributing to the class membership besides the verb is the [NP,VP], the point made is obviously that the presence or absence of a noun phrase within the VP matters for aspectual class determination. This thesis has been successfully developed for English in Verkuyl 1972 and 1989. It is for Activity/Accomplishment distinctions that the issue is relevant and it is hence on this opposition that I will concentrate.

The contrast between Activities and Accomplishments stands out very clearly in works presenting attempts to formalize the distinction. Thus, Dowty 1979 and Bennett 1981 use a heterogeneous approach within an interval-based semantics to provide a semantic description of Vendler's classes. In Dowty's framework, the truth value of an interval I of a sentence denoting an Accomplishment is determined at the endpoints of I which is not the case for Activities. Bennett 1981 presents the distinction between sentences denoting Accomplishments and those denoting Activities as one between open and closed intervals.

The terminology of Moens - Processes for Activities and Culminated processes for Accomplishments - might also be important in defining the contrast.

A very typical way of treating the issue where oppositions between Western and Eastern schools of thought are, besides, defined is given in Dahl 1981:

" Whereas the "Westerners" recognize one distinction only, the adherents of the "Eastern" view claim that two distinctions must be made, one of which would distinguish (7) on the one hand from (8) and (9) on the other, whereas the other would distinguish (7) and (8) from (9).

(7) I was writing.

(8) I was writing a letter.

(9) I wrote a letter. (taken to imply "I finished it")

(Dahl op.cit. p.81),

Now, in this short introductory statement already, there are at least three inaccuracies. First, the Slavonic equivalents of (7) and (8) are not aspectually opposed in, I think, any Slavonic language:

Cf. Bulgarian: Pišex (7') IMP(IMP)

Pišex pismo (8') IMP(IMP)

Russian: Ja pisal (7") IMP

Ja pisal pismo (8") IMP

Second, (9) will not necessarily be translated in Slavonic languages as Perfective. It is a grammar book fact of English that the simple past tense is aspectually ambiguous. Bulgarian corpus-based investigations of the simple past tense show that translation equivalents are evenly distributed in Bulgarian IMP(IMP) and AOR(IMP) groups.(Cf. Danchev and Alexieva 1977).Third, even if (9) IS perfective in definite contexts, the sentence does not simply imply "I finished it". Perfectives of Processes can be evaluated both at minimal closed intervals and at non-minimal, closed again, situation intervals. And finally, just an open question to justify the qualification of the inaccuracies as "at least three": Does I WAS WRITING A LETTER YESTERDAY imply that the letter was not finished?

Dahl defines the (7)/(8) & (9) opposition as expressing the absence or presence of a "T-property" (with "T" standing for Telic, I guess):

"A situation, process, action etc. or the verb, verb phrase, sentence etc. expressing this situation etc. has the T property iff

(DEFINITION 1, S.G. Andersson 1972) it is directed toward attaining a goal or limit at which the action exhausts itself and passes into something else.

(DEFINITION 2, Comrie 1976) it leads up to a well-defined point behind which the process cannot continue" (op. cit. p. 81).

the "P-Property" ((7) & (8)/(9)) is then described as follows:

"A situation , process, action etc. has the P-property iff it has the T property and the goal, limit or terminal point in question is or is claimed to be actually reached."

Thus defined, the T-property corresponds to what was in the preceding section defined as "Culminative AM", and the P-property - as being a Culminative AM in the Perfective Aspect. From this perspective, definite NPs can indeed be viewed as providing the final, bounding portion of the situation, i.e. as introducing a final, closed subinterval within a non-minimal interval that would otherwise have been left open.

There is one problem, however: WRITE-A-LETTER phrases need not necessarily define Culminative AMs - they can also denote Statals, corresponding to processes where "write a letter" stands for a more specific or "narrowed down" type of Activity concept than just "write". In this meaning, English WRITE A LETTER would correspond to Bulgarian PIŠA PISMO (IMP) which is a Statal AM. But then, if V + NP do not solve the aspectual ambiguity of the VP, why not just define VERB classes and then compositionally form AMs and Aspect with the help of sufficient contextual indicators or lexical additives. Since English verbs and larger units show regular cross-class fluctuation between Activities and Achievements (He ran in an hour/for an hour; He ran a race in an hour/for an hour), there is little point in setting these two groups apart.

A more realistic classification, I think, would involve three classes only: Events (minimal situations), Processes (culminated or not) and States. (For more on this proposal Cf. Stambolieva forthcoming)

These three lexical classes of verbs define the following AM types, respectively:

STATES: (know, believe)
STATAL] [
INCHOATIVE (this in Bulgarian is derivative, expressed with prefixed forms)
[]_____
EVENTS (win, reach)
MOMENTARY []

(Bulgarian derivatives can be formed out of Events but little semantic difference is felt as the non-derivatives, even if primary imperfectives, are nearly equivalent to perfectives. A similar pattern can be found in English: I win = I have won)

CULMINATIVE [_____](in cases where the "winning" situation is viewed as an "archisituation" of a Processual type - Cf. also Stambolieva 1989 b).

PROCESSES (run, run a race)
STATAL] [
CULMINATIVE [_____]
INCHOATIVE []_____

Lexical types can be realized as more than one AM; the Culminative AM can itself be realized in different configurations. The final AM configurations are a function of the interplay of verbal semantics and aspect - Imperfective or Perfective.

The "inner" aspect of English verbs can be tested with lexical additives - the FIT test, for instance (FOR and IN Adv Test). As to the hierarchy of contextual markers of English aspect, the list proposed in Danchev 1989 goes as follows:

- adverbs
- lexical meaning of the verb
- semantic configuration of the subject (± Animate)
- definiteness of the object NP
- larger context.

With the last factor, the "compositionality" of aspect goes beyond the bounds of the sentence. This "textual" marker of aspect is termed by Danchev "aspectual texteme" and is schematically described as follows:

X # V

aspectual texteme

Fig. 7

As to "outer" aspectuality which is usually defined in terms of the opposition Non/Progressive/Progressive, the most that can be said of it is that Non-Progressive forms for the Past and Future TEND to express "closed" situations while Progressive forms TEND to express "open" ones. But, as was noted in connection with Dahl's T and P properties, here too fuzziness looms large.

TO SUM UP:

In English aspectual distinctions are expressed by verbal semantics in combination with contextual indicators (above all, adverbs). Due to the absence of formal markers, reliance on context is much greater than it is in Slavonic languages. This is why in V + def. NP combinations, for instance, the NP is often felt as contributing to aspectual determination. It was demonstrated, however, that these constructions are aspectually ambiguous. As to formal markers of "outer" aspect, they are no less unreliable than V + NP structures.

The "implicit" nature of semantic distinctions in English is not restricted to aspectual phenomena. In a series of contrastive English-Bulgarian investigations, it has been shown to be a typological feature of English (Cf. Molhova 1977a, 1977b, 1979, Stambolieva 1987, 1988, 1989a, Stamenov 1977 etc.). This feature casts some doubt on the adequacy of compositional theories of English aspect based on aspectual definitions over tenseless VPs. Here, too, more flexible accounts will probably be needed.

4. Conclusions

A theory of aspect was presented making use of the central notions of Verb class, Action mode and Aspect.

According to their lexical meaning, verbs fall into three major classes: EVENTS, STATES and PROCESSES. Each of these classes can be realized within fixed Action modes. AMs were defined as patterns of "inner structure" of the situations denoted by verbs and were described in terms of interval structures (configurations).

Aspect is a semantic opposition with two members: perfective and imperfective. These were defined as specific conceptualizations of states of affairs of varying nature involving or not a transition from one conceptualization to another, i.e. a change of state.

Although in Bulgarian some Action Modes are derivatively obtained, i,e, they are realized within derivative verbs which form a large portion of Bulgarian verbs, the AM types of English and Bulgarian were shown to be identical. This result is due to the novel delimitation of AMs presented here obtained as a result of "extracting" aspectually relevant information from semantic matrices of both prefixed and non-prefixed verbs.

Unlike other Slavonic languages where Aspect is a semantic category "inherent" to lexemes (derivative or primary), Bulgarian has evolved an additional grammatical category. This is realized, within one and the same AM, as a Perfective/Imperfective opposition where the Imperfective is the inflectionally marked member. Due to the interplay of two distinct aspectual oppositions - Imperfective/Perfective and Aorist/Imperfect, Bulgarian demonstrates as a typological feature a specific kind of aspectual nesting.

The two contrasted languages can well be viewed as two extremes along a scale defining types of expression of aspectual distinctions and ranging from formal regularity to systematic fuzziness.

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NON-QUANTIFICATIONAL READINGS OF ADVERBS

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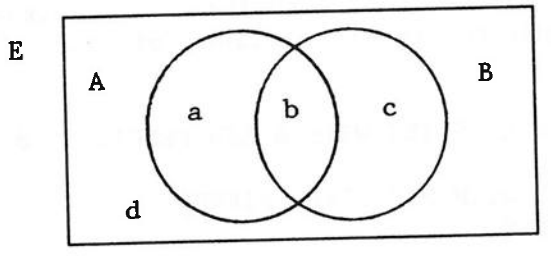
1. INTRODUCTION

The syntax and semantics of quantificational structures which we assume nowadays has its roots in Montague grammar. Barwise and Cooper (1981) show that we can use Generalized Quantifier theory to give a unified account of both standard and non-standard quantified NPs. In the relational perspective on Generalized Quantifiers adopted in the work of Zwarts (1983) and Van Benthem (1986), a determiner is defined as a function Q which assigns to each universe E a binary relation Q_E between subsets A and B of E. The relational interpretation of quantified NPs can be pictured in a Venn diagram:



$a = |A - B|$

$b = |A \cap B|$

$c = |B - A|$

$d = |E - (A \cup B)|$

Not every such binary relation between sets counts as a possible determiner denotation, though. A number of principles have been formulated in order to restrict the range of admissible interpretations of natural language determiners. The most important of these are Conservativity, Extension and Quantity. The combination of these three properties makes the general interpretation of the determiner only dependent on the cardinality of A - B and A ∩ B, that is **a** and **b** in the diagram above.

In the last few years the theory of quantification has been generalized from a theory of determiners to an analysis of all sorts of quantificational expressions, including conditionals, modals and adverbial quantifiers. In frameworks like Discourse Representation Theory (Kamp, 1981) and File Change Semantics (Heim, 1982) the general interpretation underlying different kinds of quantification is a tripartite structure where the quantifier denotes a binary relation between arguments of arbitrary complexity. In this paper we will focus on the interpretation of quantifying adverbs such as **often, always**, etc. and leave other quantificational expressions outside the discussion.

If we interpret adverbs like **sometimes, always**, etc. as Generalized Quantifiers over events, situations or 'cases', we can give a straightforward analysis of the sentences under (1) and (2):

(1) (a) When he gets up late, Marc sometimes has a headache
(b) On Sunday evening, Mary always goes to the movies

(2) (a) Anne never writes with a red pencil
(b) Paul mostly plays tennis with his girl-friend

In (1a) the **when**-clause provides the first argument of the quantifier: it indicates the type of situation **sometimes** quantifies over. In (2a) the restriction on the quantifier is given by the sentence adverbial. In both cases the main clause as a whole functions as the second argument of the adverb. This results in the (rather informal) interpretations under (1'):

(1') (a) MARC GETS UP LATE $\cap$ MARC HAS A HEAD ACHE $\neq \phi$

(b) SUNDAY EVENING $\subseteq$ MARY GOES TO THE MOVIES

The upper case is used to refer to the set-theoretic denotation of the linguistic expressions. For instance MARC GETS UP LATE is the set of situations denoted by **Marc gets up late**. The representations given here do not take into consideration the role of tense.

In examples such as (2) there is no explicit **when**-clause or sentence adverbial which provides the restriction on the quantifier. In most cases, though, the restriction can be quite easily recovered, sometimes with the help of topic-focus structures (as argued by Rooth, 1985). The most natural interpretation of (2) (again neglecting tense) is as in (2'):

(2') (a) ANNE WRITES $\cap$ ANNE WRITES WITH A RED PENCIL $= \phi$

(b) |PAUL PLAYS TENNIS WITH HIS GIRL-FRIEND| > ½ |PAUL PLAYS TENNIS|

The sentences under (1) and (2) give standard cases of what have been called 'relational readings' in De Swart (1989). The relational readings of quantifying adverbs provide a strong argument in favor of an analysis of quantifying adverbs as Generalized Quantifiers. In this perspective we can work out a semantic characterization of adverbs, based on well-known properties such as Conservativity, Monotonicity, etc. (cf. for instance Chierchia, 1988; Schwarzchild, 1989).

But we can go further than that. The introduction of tripartite structures underlying different kinds of quantification narrows down the set of 'real' quantifiers in a significant way. In DRT terms, real quantifiers lead to 'splitting of boxes': one box embodies the restriction, the other one describes the second argument of the quantifier. Expressions which do not require box splitting operations are considered to be non-quantificational in nature. In studies such as Kamp (1981) and Heim (1982) this approach turns all definite and indefinite NPs into non-quantificational expressions. Kadmon (1987) and Partee (1988) show that this distinction also affects the interpretation of NPs with bare numeral determiners (**two chairs**) or other weak determiners (**many children**). In certain contexts these NPs induce tripartite structures, in others they have non-quantificational readings and function as predicates of a group. In section 2 we will examine some examples of these NPs.

The distinction between quantificational and non-quantificational (readings of) NPs is useful, because it explains certain differences in anaphoric behavior (discourse anaphora as well as bound anaphora) and inferential patterns, related to the properties of strength, persistence and intersectivity. One of the interesting questions to ask at this point is whether this distinction is only relevant to NPs or if it plays a role for other quantificational expressions as well. And in the present context I mean in particular adverbs of quantification like **sometimes, often, always**, ... As noted above, the interpretation of these adverbs as binary operators has provided many valuable insights in the semantics of these expressions. But we might expect to find some adverbs which, at least in certain contexts, do not require box splitting operations. In this paper I claim that there are indeed non-quantificational readings of adverbs in addition to the well-known quantificational ones.

In a number of cases we observe that quantifying adverbs can be used without an explicit restriction. In some examples, such as (2), the restriction can be easily determined and made explicit in the representation of the sentence. In other cases, such as (3), it can be recovered from the context:

(3) Flo always lands on her feet

In their discussion of this kind of examples Schubert and Pelletier (1987) argue that **always** here does not mean literally 'all the time'. We have to specify a set of occasions for the quantifier to operate on, say the cases in which Flo, who is a cat, drops to the ground. We can use a contextual variable **C** to get this information in the representation of (3):

(3') C $\underline{c}$ FLO LANDS ON HER FEET

Although the introduction of a contextual variable seems to be the right move to make in view of examples like (3) I will argue in this paper that it is not always necessary, nor even appropriate. As we will see below, certain sentences can do without the kind of accomodation of the restriction on the quantifier we need in the representation of (3). These interpretations can be characterized as non-quantificational in the sense defined above. Of course this does not mean that they invalidate the overall interpretation of quantifying adverbs as Generalized Quantifiers. On the contrary, it will be shown that the class of adverbs which allow these interpretations can be defined in terms of notions familiar from Generalized Quantifier theory.

I will discuss three contexts in which the interpretation of the adverb does not fit the typical tripartite structure of a relation between sets of events. They concern iterative adverbs (section 3), weak frequency adverbs (section 4) and a special use of existential/ universal adverbs (section 5). I will show how to develop alternative analyses in the more procedural frameworks of semantic automata theory (cf. Van Benthem, 1987) and 'phase' quantification (Löbner, 1987).

2. QUANTIFICATIONAL AND NON-QUANTIFICATIONAL READINGS OF NPS

Kadmon (1987: 59 sqq) argues that NPs with bare numeral determiners give rise to splitting of boxes when they have the 'exactly n' reading, while they are treated just like indefinite NPs when they have the 'at least n' reading, cf:

(4) Three cats are pink
'at least' reading

X
three cats are pink
cats (X)
pink (X)
$|X| = 3$

(5) A hundred Hondas are defective. The others are getting sold fast
'exactly' reading

X
A hundred Hondas are defective
Hondas (X)
Defective(X)

Z Hondas (Z) Defective (Z)	=>	$Z \subseteq X$

$|X| = 100$

The capitals X and Z in these representations introduce sets of individuals as discourse referents in the discourse representation. N and VP are interpreted as predicates over that set. In (4) the numeral just gives the cardinality of the set X and does not induce splitting of boxes. The added conditional in (5) specifies that there are no more defective Hondas outside of the set X.

I do not want to go into the details of the construction rules of these boxes, but I think it is easy to see that (4) is intended to represent the non-quantificational, predicative interpretation, whereas (5) gives the strong, quantificational reading of the numeral. As a matter of fact, in Kadmon's view this is not to be considered a semantic ambiguity, but a pragmatic one. The 'exactly' reading is pragmatically derived from the 'at least' reading, by means of principles of scalar implicature (cf. Kadmon, 1987: p. 71 sqq).

A comparable distinction has been made in the interpretation of the weak quantifiers **many** and **few**. However, the ambiguity is here claimed to be of a semantic, rather than a pragmatic nature. Partee (1988) discusses the semantic properties associated with the cardinal and proportional readings of these quantifiers, and their treatment in the literature. The cardinal readings have truth conditions which concern only the intersection of A and B:

(6) Cardinal readings

MANY: $|A \cap B| \geq m$

FEW: $|A \cap B| \leq p$

That is, **many** and **few** as vague cardinal quantifiers have a meaning of the type 'at least m', respectively 'at most p'. The vagueness is located in the unsepcified choice of **m** and **p**. It is part of the meaning of **many** and **few** that the value of **m** counts as large in the given context, whereas **p** is generally understood to be small.

The proportional reading of **many**, on the other hand, requires a relatively large proportion of the N to satisfy the property denoted by the VP. **Many** then means 'at least m/n', where we may think of **m/n** as a fraction (say 1/3) or a percentage (say 40%). For **few** this has to be a relatively low proportion of course (say 1/10 or 10%).

(7) Proportional readings

MANY: $$\frac{|A \cap B|}{|A|} \geq m/n$$

FEW: $$\frac{|A \cap B|}{|A|} \leq p/q$$

A nice context in which we can demonstrate the different meaning effects of cardinal and proportional readings is given in existence asserting sentences like (8):

(8) Few egg-laying mammals turned up in our survey, perhaps because there are few

This example, which Partee cites from Huettner (1984), shows that in such constructions **few N** can amount to all the egg-laying mammals there are. One can argue that **few** can never be 'all' on the proportional reading. In that case the context-dependent value of **few** cannot be understood to be either 0% or 100%, but must be somewhere in between. Moreover, it is preferably low, so it will be closer to 0% than to 100%. The cardinal reading, on the other hand, is quite compatible with **few** being 'all', since it asserts that the number of N's that satisfy the predicate is small without saying anything about what proportion of the set of all N's that is. For instance, if the cardinality of A itself is already smaller than **p**, and all the A's are B's, the cardinal truth condition of **few** is clearly satisfied, but the proportional one is not.

Following Milsark (1977) and others, Partee points out that the cardinal reading is basically adjectival and non-quantificational, that is, it does not induce splitting of boxes. It thus patterns with the numerals (in their non-quantificational reading) and the indefinite NPs in a Kamp/Heim approach. The proportional reading on the other hand has properties which also characterize strong quantificational determiners, such as **each** and **most**. Consequently, it requires a genuine Generalized Quantifier analysis.

We can now turn to the discussion of the adverbs. Although they basically use the same mechanisms, they give rise to quite different meaning effects. Most of these are understandable, however, if we take into account the internal structure of the temporal domain.

3. ITERATIVE ADVERBS

The first example we will consider here concerns iterative adverbs such as **twice, three times, several times**, for instance:

(9) (a) I met him twice
(b) I tried to call him several times, but he never answered

Iterative adverbs specify the cardinality of a set of situations. Their semantic structure is close to simple event sentences, and they are not normally thought of as tripartite structures. Note the similarity in discourse function between (9b) and a sentence like (10):

(10) I invited several linguists, but none of them came to the party

In (10) we first introduce a set of individuals and then we quantify over that set, adding that none of them has a certain property. Similarly, (9b) introduces a set of situations and says that none of them extends to a specific larger situation. This introducing function of iteratives is typical for non-quantificational expressions, as observed by Kamp (1981), Heim (1982).

Iterative adverbs express first-order quantification. In earlier papers I argued that quantifying adverbs require a more procedural interpretation. For that purpose I used the semantic automata theory developed by Van Benthem (1987). In that framework iterative adverbs can be interpreted by means of finite state automata (cf. De Swart, 1988, 1989).

4. WEAK FREQUENCY ADVERBS

Obviously there are many sentences where adverbs like **sometimes, often, never**, ... involve tripartite structures, and where they alternate with strong quantifiers such as **always, mostly**. But in other sentences, weak quantifiers denote a pure frequency, rather than a relation between two sets of situations. An example inspired by Stump (1986) is:

(11) (a) Jane often complains
(b) Jane often complains when figuring her taxes

In (11b) we clearly dispose of a tripartite structure: we have a set of situations in which Jane figures her taxes and many of these extend to a situation in which she complains. In (11a) we can obtain the same kind of interpretation if we assume that the restriction on the quantifier is implicit, and given by context and/or situation (cf. 3 above). In that case the sentence says that many situations of this implicitly given set extend to a situation in which Jane complains. But (11a) also has a simpler reading in which we do not get a relational interpretation, but where we describe the frequency with which Jane complains as high in our domain of discourse.

Under that reading the two sentences (11a) and (11b) do not have the same truth conditions. Suppose there are few occasions where Jane has to figure her taxes, and those are the only situations where she complains, then (11b) might be true, while (11a) will be false. When we replace **often** by **seldom** in the same context, we obtain the opposite result, as shown in (12):

(12) (a) Jane seldom complains
(b) Jane seldom complains when figuring her taxes

The frequency of Jane complaining may be very low, maybe because there simply are very few occasions where she could complain about anything. Still she might do it often, or even always, when figuring her taxes. This means that **seldom** in (12a) can amount to all the complaining situations there are. This is very similar to Partee's remarks on **few** being 'all', which could only be true on the cardinal reading of **few**. In the same way, **seldom** in (12b) cannot amount to all the situations there are if the adverb has a proportional interpretation. It can only refer to the totality of Jane's complaining situations if it denotes a pure frequency, which is low without saying anything about what proportion of all the possible occasions that corresponds to.

So the meaning of the Q-adverb in sentences like (11a and 12a) is not construed in the same way as in (11b and 12b): there is a real tripartite structure involved in the interpretation of the b-sentences of (11)/(12), but the a-sentences of (11)/(12) have a non-quantificational reading. A natural paraphrase of simple frequency readings is by means of expressions like **twice a week, monthly, every year**, etc.

Since **often** and **seldom** are normative adverbs, they are vague in both their proportional and their frequency readings, and this vagueness tends to blur the picture. Still, their vagueness and context dependency are located in different areas. The proportional reading of **often** is true if a relatively large proportion of situations of type A can be extended to a situation B. This is similar to the proportional truth conditions Partee (1988) proposed for **many** (cf. 7 above). In the non-quantificational reading the vagueness of **often** resides in the number of situations per time unit, and the choice of the appropriate time unit (days, weeks, ...). We can take the general interpretation to be 'at least **m** times per time unit', where **m** is relatively high and/ or the time intervals are relatively short. Both the exact value of **m** and the selection of the time unit depend on context and situation. Similar truth conditions can be defined for **seldom**.

Note that the frequency reading is not limited to normative adverbs like **often, seldom**, but actually arises with other weak adverbs as well. In the case of **never**, the (a) sentence implies the (b) sentence, for **sometimes** the direction of the implication is inversed:

(13) (a) Jane never complains
(b) Jane never complains when figuring her taxes

(14) (a) Jane sometimes complains
(b) Jane sometimes complains when figuring her taxes

When Jane never complains at all, she obviously does not complain when figuring her taxes. When Jane sometimes complains when figuring her taxes, this implies that she does indeed sometimes complain.

On the other hand, strong adverbs such as **always, mostly** never give rise to frequency readings, as we can see in (15):

(15) (a) Jane always complains
(b) Jane always complains when figuring her taxes

In sentences such as (15a) strong adverbs are infelicitous unless the context provides a set of occasions to quantify over (cf. 3 above). But that means that we end up with a relational interpretation just as in (15b), only the restriction on the quantifier is not explicitly given, but inferred from the context. In other words, strong adverbs always create tripartite structures, and we do not get frequency readings.

Independent evidence for this use of weak frequency adverbs is provided by a temporal analogue to the existential **there** sentences, as in (16):

(16) (a) It happened { twice / several times } that we lost our way

(b) It { sometimes / seldom / never / frequently / ... / ??always / ??mostly / ??usually } happens that we order a pizza

Existential sentences in general have an introducing or presentative function. Both iterative and weak frequency adverbs occur naturally in existential sentences. But the strong quantifiers **always** and **mostly** are excluded or very hard to imagine in this construction. Also it is clear that the weak frequency adverbs here cannot have a proportional interpretation, but denote a pure frequency. So the 'it happens Q' construction is a context where only non-quantificational (readings of) adverbs can occur.

I assume that the weak Q-adverbials in pure frequency readings do not denote proportional relations, but are in some sense cardinal in nature. Still, this cardinality is not used to simply count a number of events in a bounded domain, as in the case of the iterative adverbs. The Q-adverbs in the (a) sentences of (11) - (14) are sensitive to the recurrence of a type of event after a certain time. The cyclic meaning aspect of frequency adverbs is inherent to the possibility of quantification over open, unbounded domains. The combination of cardinality and recurrence leads to the interpretation of pure frequency readings as iteratives in a cyclic perspective. In this way they get the same kind of interpretation as expressions like **twice a week, daily, every year**, ... The main difference is that adverbials of the latter category specify frequencies whose periods are of a fixed length, whereas **sometimes, often, seldom**, etc. are vague.

The semantics of expressions of cyclic iteration cannot be captured by ordinary cardinal truth conditions, because these would underdetermine the meaning of the Q-adverb: they would give weak adverbs in pure frequency readings the same interpretation as iterative adverbs. We need a more procedural theory to account for the crucial differences between iteration and frequency. The whole idea is that we verify a frequency gradually, as we go along in time. Technically this can be achieved when we use a procedural interpretation of Generalized Quantifiers based on semantic automata, as proposed by Van Benthem (1987). Gradual verification is here formulated as a constant evaluation procedure, and that can be realized by defining a push down automaton with a limited stack (cf. De Swart 1988, 1989).

Now that we know how to interpret the frequency readings of Q-adverbs two final questions remain to be answered. Why can a pure frequency only be expressed by weak frequency adverbs? And are there any non-quantificational readings for strong adverbs?

There are various (equivalent) ways to characterize the set of weak quantifiers. One of these is to say that weak determiners satisfy the intersection condition:

(17) A (conservative) quantifier is intersective iff for all A, A', B, B' $\subseteq$ E:

$$\text{if } A \cap B = A' \cap B' \text{ then } Q\,AB \Leftrightarrow QA'B'$$

The condition in (17) formalizes the intuition that weak quantifiers only take into consideration the number of elements in the intersection of A and B. This characterization leads to the property which Keenan (1989) calls 'sortal reducibility'. He notes that intersective determiners are not inherently sortal in that the common noun does not really delimit the domain of quantification. We can just as well quantify over all the objects in universe, as long as we compensate this by taking a boolean function of the noun and the predicate property. In the case of intersective functions the appropriate operation is conjunction. Sortal reducibility then makes (18a and b) logically equivalent:

(18) (a) Some student is a vegetarian
(b) Some individual is both a student and a vegetarian

This means that intersective quantifiers satisfy the equivalence under (19):

(19) $Q\,AB \Leftrightarrow Q\,E\,(A \cap B)$

E represents the property which every individual in our domain of discourse has. We can read this universal property for instance as "exist". Keenan (1987) uses this universal property in his analysis of existential sentences, which partly relies on the same equivalence

(19).

The property of sortal reducibility shows that, in fact, intersective quantifiers reduce to one-place operators. They do not denote a relation between two sets, but they quantify over all the individuals in our universe of discourse. This idea is captured by a function * from intersective determiner denotations to the set of Generalized Quantifier denotations. Keenan (1989) defines this function as follows:

(20) $Q^* A = Q E A$

Transposing this analysis to the adverbial domain we note that the pure frequency interpretations of weak adverbs are intersective, whereas their proportional readings obviously are not. If intersective quantifiers allow argument reduction from two-place relations to one-place operators, this explains why, intuitively, Q-adverbs in many instances of pure frequency readings take only one argument, not two. There is a direct quantification over our temporal domain of discourse. As a result, we no longer describe a relation between two sets, but we give the pure frequency with which a certain type of event occurs. In terms of Keenan's functions: weak adverbs in pure frequency readings do not denote a binary relation Q, but its one-place correspondent Q*.

As Keenan (1989) points out, intersective quantifiers are not the only sortally reducible relations. Among the conservative relations both intersective and co-intersective quantifiers are sortally reducible. Co-intersective quantifiers also take into account the cardinality of just one set, namely the difference of A and B. They satisfy the condition under (21):

(21) A (conservative) quantifier Q is co-intersective iff for all $A, A', B, B' \subseteq E$:

if $A - B = A' - B'$ then $Q\,AB \Leftrightarrow Q\,A'B'$

Co-intersective determiners are for instance **all**, **not all** and exception determiners (**all but at most n**). Their truth conditions only concern the number of elements in A - B.

This observation leads us to expect non-quantificational readings for co-intersective Q-adverbs as well. If they allow argument reduction from two-place relations to unary operators, their semantics does not necessarily impose a relational interpretation. A nice example of this kind of non-quantificational reading is given by **not always** in contexts like (22):

(22) If you have any problems, come and see me in my office. I am not always around, but when I am there I am willing to help you

As an informal interpretation rule for this kind of sentences we can say that **not always S** means 'sometimes not-S'. That is to say, if I am not always around, I am sometimes not around. In other words, the adverb gets a pure frequency reading, but in the complement set (A-B), not in the intersection ($A \cap B$). In view of the characterization just given, this is exactly what we would expect from a co-intersective quantifier.

A more precise and general procedure for calculating the meaning of co-intersective Q-adverbs in pure frequency readings as one-place operators can be defined if we use the following observation, which relates co-intersectivity to intersectivity:

(23) Fact:
For all functions Q denoting a (conservative) co-intersective relation:

$Q\ AB \iff Q^{-}\ (A)(-B)$

and Q^{-} denotes an intersective relation

Since Q^{-} is intersective, it can describe the pure frequency of the set A - B:

(24) $Q^{-}\ (A)(-B) \iff Q^{-}\ E\ (A - B) \iff (Q^{-})*\ (A - B)$

This boils down to the conclusion that the non-quantificational reading of **not always S** is exactly the pure frequency interpretation of **not always-not not-S**, that is, **sometimes not-S**. And this is indeed the interpretation we proposed for (22).

The hypothesis that intersectivity and co-intersectivity are responsible for the non-quantificational readings of frequency adverbs is confirmed by the absence of non-quantificational interpretations for an adverb like **mostly**. Interestingly, this adverb is neither intersective, nor co-intersective, and it can only get a proportional interpretation. This is as it should be in the light of the theory developed here.

5. A SPECIAL USE OF EXISTENTIAL AND UNIVERSAL QUANTIFIERS.

In some sentences universal/existential adverbs give rise to special meaning effects which cannot be interpreted as quantification over occasions/cases/situations, but where there is no cyclic iteration either. Examples are the following:

(25) (a) This is the first time in my life that I have been really alone at night. I have always lived in hives, surrounded by human presences behind walls.
(b) You can rub shoulders with the great, bald eagle, or with friendly mountain people whose doors are open and whose coffeepots are always perking
(c) Will you ever leave me?
- Never, I will always be at your side

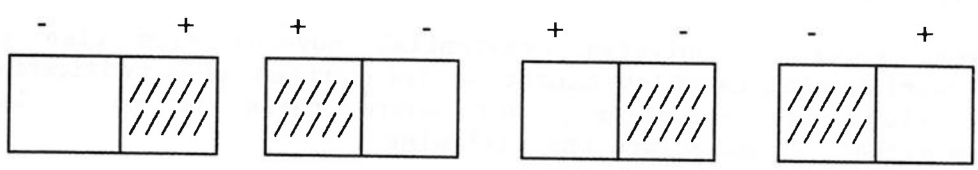
Examples are given in the Present Perfect, Present and Future in order to show how the temporal point of view bears on the meaning of the sentence as a whole. **Always** in the first sentence means that the situation described by the proposition P has been the case always up to now. Sentence (25b) tells us that some situation is true all the time, or equivalently that the negative case never occurs. **Always** in (25c) means that there will never be a moment where a certain situation is no longer the case. And so on ...

These informal characterizations show that there is no recurrence of situations in this use of the Q-adverbs, so an analysis in terms of cyclic iteration or quantification over cases does not fit this kind of examples. Rather, the fact that these interpretations arise only with universal/existential adverbs suggests a first order analysis. As pointed out in section 3, we can use finite state automata to interpret iterative adverbs, which also express simple first order quantification. But the crucial notion here is not iteration or even frequency, but continuation of a situation with at most one change in state. This meaning aspect is related to mass-like quantification. It cannot be captured by semantic automata, because they are based on quantification over discrete objects, which can be counted. This forces us to develop an alternative analysis.

A closely related theory, but which can account for continuous structures, is the 'phase quantification' approach proposed by Löbner (1987). Löbner uses this model to give an analysis of adverbs like **already/ still/ not yet** and **enough/ too**. He also treats scaling adjectives like **big/ small** and aspectual verbs like **begin/ continue/ stop** in this way. I will just give the general definition of phase quantification, and then show how we can apply it to the universal/ existential adverbs.

The phase operator semantically has two operands. It takes the predicate that we quantify over to define a range of values on a scale. There are two phases, a positive one, and a negative contrast phase. It does not matter if there is a zone of indetermination between the positive and the negative phase. The second operand is a parameter point, which is indicated with an arrow in the diagrams we will see further on. In the schemas in (26) the parameter point is set in the phase marked with a plus. The combination of the two operands defines four possible quantifiers:

(26) type 1 type 2 type 3 type 4



We can give a procedural description of these types. You start from within the leftmost phase and you run along the scale until you reach the parameter point. Just check on the way whether you enter any second phase. If the parameter falls in the first phase, the quantifier is of type 2 or 3. Quantifiers of type 1 and 4 require a change in phase before the parameter point is reached.

The four quantifiers form a square of opposition related by negation. Inner negation results in exchanging the positive and negative phases, while outer negation concerns the decision whether the parameter point falls into the first or the second phase. As a result, type 1 quantifiers are existential: there are positive cases of the predicate quantified. Type 2 quantifiers are universal, because all points within the domain of quantification fall into the positive phase. Type 3 and 4 are the negated existential and the negated universal quantifier respectively. A number of important characteristics of Generalized Quantifiers can be reformulated in this framework. I will not repeat Löbner's discussion here, but refer to his paper for more details on the properties associated with the square of opposition.

In the case of the universal/existential adverbs the scale on which the positive and negative phases are defined is the time scale. Because of the influence of tense on the meaning of the sentence as a whole we have to combine Löbner's phase quantification with a theory of tense and aspect. Since the basic idea is continuation of a situation with at most one change in state, we expect to find extended tenses like the Present Perfect and the Progressive. Also, if we need a continuous positive phase, we expect the quantifier to combine with states and processes, because these give rise to "durative" propositions in the sense of Verkuyl (1989). This is particularly important in the interpretation of **always** and **not always**. **Never** and **ever** do not have this constraint, because they require a continuous negative phrase, and probably negations of both states and events create durative situations (cf. Verkuyl, 1987 for discussion of the interaction between negation and Aktionsart).

In this paper I will use a very simple temporal semantics, much like the classical system of Reichenbach (1947) but extended to intervals. It is based on the three notions of speech time (S), reference time (R) and event time (E). In the simplest case the time of speech is identical with the reference time (R = S), which allows us to define three basic tenses: for the Present tense R $\subseteq$ E, that is, we look at the phase we are actually in. For the Present Perfect E < R, so we look to the past from R along the scale. For the Future R < E, so we look forward along the scale. This characterization of the tenses predicts the behavior of the universal/ existential adverbs in the Present Perfect and the Future to be mirror images, and this is indeed what we will see below.

In the following I will discuss the meaning of the different adverbs in the three basic categories of Present Perfect, Future and Present tense. In all categories I will give examples, propose an informal characterization of the four adverbs, and represent them in a diagram. Let us start with the Present Perfect.

I Present Perfect

Examples:

(27) (a) This is the first time that I have been really alone at night. I have always lived in hives, surrounded by human presences behind walls.
(b) He has not always been rich. There was a time at which he was poor and lonely
(c) I have never been really alone at night
(d) I wonder if I have ever been really alone at night

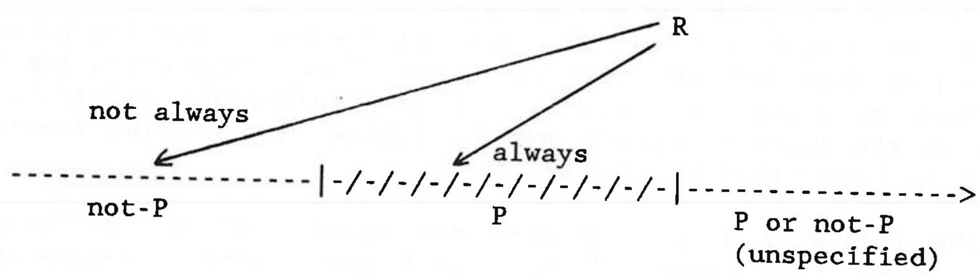
Informal characterization:

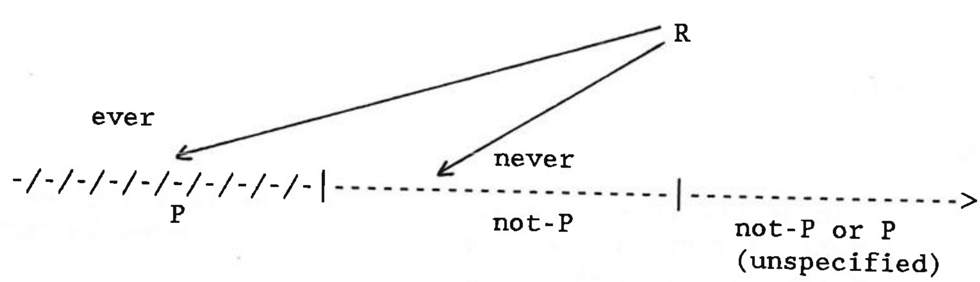
P is the untensed proposition expressed by the sentence;
R is the reference point;
in all basic tenses R is identified with the time of speech (R = S).

always: P up to or including R[1]
not always: There has been a time before R at which not-P
Presupposition: since then and up to now P
never: not-P up to or including R
ever: There has been a time before R at which P
Presupposition: since then and up to now not-P

These characterizations are more complex than the cases Löbner discusses insofar as the reference time of the sentence is different from the parameter point which is involved in the evaluation of the quantifier. R, the reference time, is a specific point in time (in the examples discussed here R = S). But as far as the parameter point is concerned, we only know that it has to be set "somewhere" in the relevant phase. Probably it is not in the immediate neighbourhood of R, because we have the impression that we go back quite far in time, but its location is not exactly determined. This makes it impossible to fix the parameter as a specific point in the diagrams. Instead, we use two arrows, starting from R, which symbolize the two possibilities that the parameter point either falls in the positive phase P (where the proposition P is true) or in the negative phase not-P (where the situation P describes is not the case).

Diagram:





1 Nothing can have been literally always the case. We might suppose then that there is an implicit or explicit starting point for the situation in the sentence or the context. This restriction does not seem to be part of the semantics of **always**, though, but is rather of a pragmatic nature. As such there is no need to limit the definition of the adverb to a bounded interval.

The definitions of the adverbs in the Present Perfect give a description of situations in the past and up till R, so in order to verify the sentence we start at the reference point R and then go backwards along the time scale. We accept a sentence containing **always** (respectively **never**) if the parameter point falls somewhere in the positive (respectively the negative) phase. That is, until the parameter point is reached, we do not want to encounter a change in phase. In the definitions of **always** and **never** there is no presupposition concerning a change in phase, so in fact there is just one phase we need to take into consideration. In this sense the diagrams picturing a double phase structure may be somewhat misleading.[2] For a sentence with **not always** (respectively **ever**) to come out true, we require a change in phase on our way from the starting point R to the parameter point from positive to negative (respectively negative to positive). This makes **always/ not always** and **never/ ever** external negations of each other.

Note that, in the diagrams the time scale runs from left ("past") to right ("future") as usual, but the phases must be read from right to left. At first sight this may seem counterintuitive, but it is not when we realize that R gives the point of view from which we start looking along the scale. As far as the Present Perfect is concerned, the definitions of the adverb require us to look backwards to the past.

This proposal correctly classifies **ever** as a type 1 quantifier according to the schema in (26): the parameter point is set in the positive phase which we enter after having considered a negative phase. Similarly **always** is a type 2 quantifier: the first (and in fact only) phase we take into consideration is a positive one, and the parameter point is set somewhere in that phase. Finally, **never** and **not always** are appropriately characterized as type 3 and type 4 quantifiers respectively.

II Future

Basically, the interpretation of the universal/ existential adverbs in the Future tense is a mirror image of what happens in the Past. The point of view is again the reference point R, which is identified with the moment of speech, but instead of looking back to the past we look forward along the scale. This implies that the two phases have to be read from left to right, that is, in the order of the time scale. Accordingly, the arrows in the diagrams, indicating the phase in which the parameter falls, now point to the right of R.

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2 In Löbner (1987) all the phase quantifiers which are discussed presuppose a double phase structure. The fact that **always** and **never** do not carry such a presupposition does not invalidate their analysis as phase quantifiers. A simpler analysis, based on just one phase, would seem possible, because the truth conditions of these adverbs only bear on the first phase, starting from R. However, the relations between the four quantifiers can only be described in a double phase model. This is the main argument for giving the complete diagrams.

Examples:

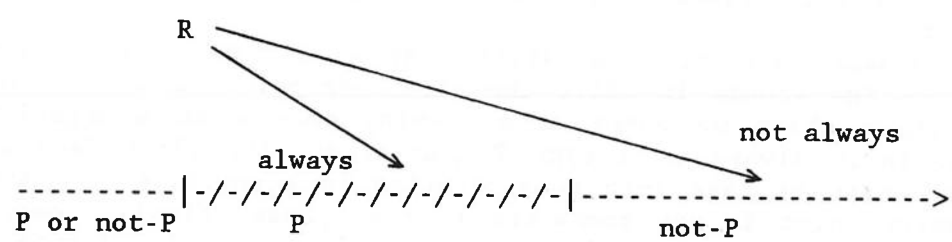
(28) (a) I will always be at your side
(b) We will not always sit and talk. One day we will stand up and fight
(c) I will never leave you alone
(d) I wonder if he will ever get a tenure track position

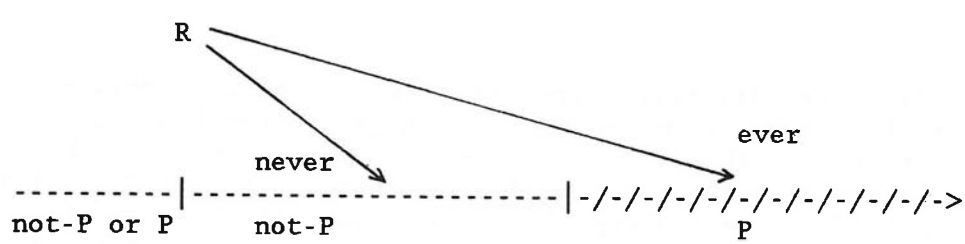
Informal characterization:

always: P from R onwards (including or not R)
not always: there will be a time after R at which not-P
presupposition: from now on and until then P
never: not-P from R onwards (including or not R)
ever: there will be a time after R at which P
presupposition: from now on and until then not-P

The change in the definitions only concerns the location of the phases with respect to the reference point R and the speech moment R. This is reflected in the diagrams.

Diagram:





III Present

The Present is somewhat different in that it only combines naturally with the phase quantifiers **always** and **never**. This is because the parameter point has to be set in the phase we are actually looking at, so we cannot use adverbs which require a change in phase, hereby excluding **ever** and **not always**.

Note that this concerns only the phase interpretation of **not always** and **ever**. A quantificational reading, introducing a tripartite structure remains possible, as is shown by (29):

(29) When she is tired Anne does not always go to bed early

Also, there is no reason to exclude the possibility of a non-quantificational reading in which the adverb expresses a pure frequency, either in the intersection of A and B (**ever** in 30a) or in the complement set A - B (**not always** in 22, repeated here as 30b):

(30) (a) - This shop is always closed when I pass by. I wonder if it is ever open.
- As far as I know it is only open on Saturday afternoon

(b) If you have any problems, come and see me in my office. I am not always around, but when I am there I am willing to help you

However, the observation that **ever** and **not always** are certainly not ungrammatical in combination with the Present tense does not invalidate the fundamental intuition that they can never be used as phase quantifiers in these sentences. Consequently, in this section we can restrict ourselves to the study of **always** and **never**, as the only phase quantifiers which occur in the Present tense.

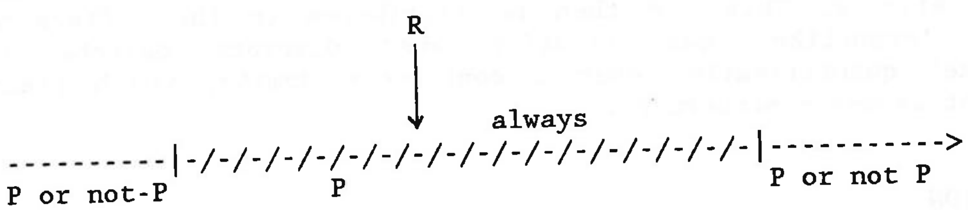
Examples:

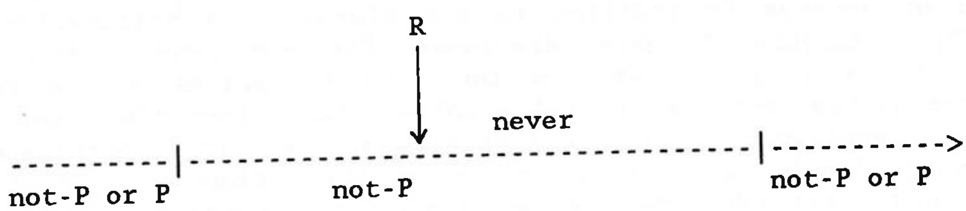
(31) (a) You can rub shoulders with friendly mountain people whose coffeepots are always perking
(b) The door is always open
(c) The monster is never asleep
(d) The door is never locked

Informal characterization:

<u>always</u>: P is the case at R and indefinitely long before and afterwards
<u>never</u>: not-P is the case at R and indefinitely long before and afterwards

Diagram:





If we take into account other points of view than R = S (in particular R < S), we can create more possible diagrams. Examples can be multiplied at will:

(32) (a) He talked to me as if he had always known me
(b) Carry did all the shopping, telling Louise she knew how hard it was to go into a supermarket when you were hungry. And Louise was always hungry
(c) I started to see the city in the way I would always remember it: an old centre of criminals and disappointments

(32a) combines **always** with the Past Perfect, and it looks back to the past from a given reference point which is itself already past. Similarly, (32b) and (32c) give examples of respectively the Simple Past (describing something like a present in the past) and the Future in the Past (looking forward to the future from a given moment in the past). These examples show that we were right in formulating the characterizations of the adverbs in terms of R rather than S: the reference time R is in all tenses the starting point for the evaluation of the phase quantifier.

There is no need to discuss examples of all the adverbs here, nor give the diagrams, because everything is very similar to the cases discussed above. Although some combinations are prohibited, because of incompatibilities between the character of the quantifier and the tense (as in the Present), the combinatory approach explains why we find so many different meaning effects.

We may conclude that the notion of phase quantifier gives us a good way of looking at these examples, especially if we combine it with a theory of tense and aspect in order to explain the influence of tense and the Aktionsart of the sentence. At the moment I do not know how to integrate phase quantification and semantic automata theory into one more general framework. On the other hand it might be good to have a separate phase quantification model. The reason is that iterative adverbs (**twice**, **several times**, etc.) and universal/existential adverbs in continuous temporal structures both express first order quantification, but clearly do not give rise to the same meaning effects. This can then be attributed to the differences between 'countlike' quantification over discrete objects and 'masslike' quantification over a continuous domain, which create different semantic structures.

CONCLUSION

I think that the examples I have discussed in this paper provide convincing evidence for the existence of non-quantificational readings of adverbs in addition to the classical quantificational ones. The analyses I have developed for non-quantificational interpretations rely in part on the same mechanisms as in the determiner system. But the special meaning effects they give rise to can only be explained if we take into account specific properties of the temporal domain. We need in particular the notions of (partial) temporal order and unboundedness, which make it necessary to come up with procedural analyses.

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SEMANTIC PROPERTIES OF COMPOSED FUNCTIONS AND THE DISTRIBUTION OF WH-PHRASES

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Introduction*

The following sentences exemplify WH-extraction:

(1)a. How do you believe that John behaved -?
b.*How do you wonder whether John behaved -?

The proper characterization of the intricate conditions on extraction has been one of the major research topics in generative grammar over the past decades. It has been agreed that the phenomenon is essentially syntactic in nature. In this paper we will suggest that it may in all probability be approached in semantic terms.

In Section 1 we will briefly review the "traditional" approach to extraction, pursued by Chomsky, Huang and Lasnik&Saito, which states the constraints on extractees as well as extraction domains primarily in terms of function-argument structure. We will then point out that the empirical generalizations underlying this approach have proven to be inadequate. In Section 2 we will turn to a recent alternative, pursued by Cinque, Obenauer and Rizzi, which suggests that constraints on extractees be in terms of referentiality, and constraints on extraction domains, in terms of intervening potential binders. We will observe that this account is halfway between syntax and semantics in that it appeals to semantic-minded notions but accomodates them in syntactic terms.

The question that immediately arises here is whether a formal semantic account is possible. In particular, we will explore the possibility of a semantic account compatible with the assumptions of combinatory categorial grammar.

In Section 3 we will point out that there is a precedent for an account of intervention effects within that theory, namely, Zwarts's proposal concerning the distribution of negative polarity items. This makes crucial use of the inheritance patterns for entailment properties under functional composition. The proposal will be briefly reviewed to introduce the methodology as well as a number of useful notions. In Sections 4 and 5 we will examine a larger set of extraction data, and point out that they suggest at least two different lines for research. The working hypothesis of the first is that islands are functions that are not monotone increasing, and island-sensitive extractees are those for which the inclusion relation is relevant. Or, if we draw the rather delicate demarcation line between grammatical and ungrammatical examples slightly differently, we may hypothesize that islands are contexts that have no dynamic effect, and island-sensitive extractees are the ones that, not being linked to an old discourse marker, should introduce a new one. We hope to be able to explicate both possibilities with sufficient clarity to promote further research.

Section 1

It is well-known that WH-extraction is subject to island constraints but not all WH-phrases are equally sensitive. A representative sample of the contrasts and descriptive generalizations that have been considered most important is given below. Extraction sites will be indicated with a dash for the sake of transparency. In each case a minimally different good example is provided for comparison. Judgments are contrastive. Unmoved WH-phrases (WH-in-situ) are supposed to move in Logical Form, hence their behavior is subsumed under extraction.

Constraints on Extraction Domains:

Subjects are islands:

(2) *Who are [pictures of -] on sale?
<u>vs</u>. Who did you see [pictures of -]?

Adjuncts are islands:

(3) ?Who did you laugh [because Mary liked -]?
<u>vs</u>. Who did you think [that Mary liked -]?

WH-complements are islands:

(4) ?Who did you wonder [whether Mary liked -]?
<u>vs</u>. Who did you think [that Mary liked -]?

Constraints on Extraction Sites:

THAT-t effect for subjects:

(5) *Who did you think [that - liked Mary]?
<u>vs</u>. Who did you think [that Mary liked -]?
How did you think [that Mary played the sonata -]?

WH-in-situ; problems with subjects and adjuncts:

(6)	*What did who buy?	LF: *who what did - buy
	*What did you buy why?	LF: *why what did you buy -
	<u>vs</u>. Who bought what?	LF: what who bought -
	Why did you buy what?	LF: what why did you buy -

Extraction from an adjunct-island; problems with adjuncts:

(7) *How did you laugh [because Mary got the job -]?
<u>vs</u>. ?Who did you laugh [because Mary said - was cool]?
?Who did you laugh [because Mary liked -]?

Extraction from a WH-island; problems with subjects and adjuncts:

(8) *Who do you wonder [what - saw]?
*How do you wonder [who played the sonata -]?
vs. ?What do you wonder [who saw -]?

(9) *Why do you wonder [whether Mary liked Bill -]?
vs. ?What do you wonder [whether Mary liked -]?
?Who do you wonder [whether Mary knew - was here]?

What this picture suggests is that the contrasts are basically in terms of function-argument structure: the crucial distinctions are those between arguments and adjuncts on one hand, and between subjects and objects on the other. (Here "object" is used as the prototypical VP-internal argument.) Objects themselves extract easily, and object complements make good extraction domains. Adjuncts hardly extract and make bad extraction domains. Subjects are somewhere halfway between. According to one influential line of research, summarized in Chomsky (1986), these are the core generalizations that the twin constraints of the Empty Category Principle and Subjacency need to account for.

Switching perspective for a minute, let us consider what another theory of WH-constructions, namely, combinatory categorial grammar, has to say about these contrasts. In view of the fact that the contrasts are claimed to reflect function-argument structure, the outsider might expect categorial grammar to yield spectacular results. Those more familiar with the working of the theory will know, however, that it is quite helpless with these data. While more or less ad hoc restrictions on categorial connectives, type-lifting, and higher-order quantifiers may help exclude a number of bad examples, the theory as it stands has nothing insightful to offer about these contrasts. See Hepple (1989) and Morrill (1989) for a discussion of some of these points.

Bearing this fact in mind, let us return to the Government-Binding Theory. Lasnik&Saito (1989) pursue a similar line of research as Chomsky (1986), but suggest a certain revision of the relevant data. Following to a great extent Huang (1982), they argue that the real distinction that the Empty Category Principle and Subjacency should account for is the argument-adjunct asymmetry. Of the constraints on subjects, it is only the complementizer-trace effect that ECP should cover (and they develop a rather roundabout suggestion how). All other problems concerning subjects need to be explained by independent mechanisms. To account for the subject condition, cf. (2), they invoke Kuno's Non-Final Incomplete Constituent Constraint, and to account for the interaction of subject WH with other WH-phrases, cf. (5)-(6), they invoke Superiority (the hierarchically superior WH-phrase must move first). They note that Superiority must be independent of ECP anyway, cf.:

(10) *What did you tell who to read -?
  vs. Who did you tell - what to read?

Kuno's Constraint and Superiority are taken to be primitive. A number of instances of subject-object asymmetry noted in the literature remain unaccounted for at present.

If Lasnik&Saito are correct in carving up the data differently than Chomsky did, then their claim causes an embarrassment for any other theory that managed to derive the subject-object-adjunct asymmetries in a uniform principled manner. At the same time, while combinatory grammar is still faulty of overgeneration, it is at least spared the embarrassment of deriving a generalization that is not there. It may also be noted that many cases falling under Kuno's Constraint and Superiority are theorems of (principled versions of) combinatory grammar, e.g., Steedman (1987).

Whether Lasnik&Saito are indeed correct, their proposal certainly seems to have brought out the best and the worst in the line of research they pursue. Let us therefore examine the empirical adequacy of their proposal in somewhat more detail.

The first observation we may make here is that the islandhood of WH-complements does not receive a natural explanation on this theory. The way fronted WH-phrases matter for ECP and Subjacency is kind of independent of the argument-adjunct issue. It is assumed that WH-movement proceeds in small steps through escape hatches. A WH-phrase, just like the complmenetizer THAT, will block such an escape hatch; that's why they create islands. With a little exaggeration we may say that in this theory WH-phrases end up creating islands with their bulk, not with their specific semantic content. One may wonder whether this is satisfactory.

Further, let us ask whether the argument-adjunct contrast is descriptively speaking as robust as Lasnik and Saito's theory makes us expect. As is clear from the above, adjuncts do not make as bad extraction domains as Huang assumed, and they are also not unique in constituting islands. So let us ask whether the demarcation line between arguments and adjuncts coincides with that between good and bad extraction sites. We will first review data that were noted well in the 'Eighties.

First, consider Belletti's observation (quoted in Chomsky 1982) that only NP arguments can possibly be extracted from an adjunct-island. This means that argument PPs are subject to the same restriction as adjuncts. Compare (7):

(11)a. ?What did you go home [without talking about -]?
  b. *About what did you go home [without talking -]?
  c. *How loudly did you go home [without talking -]?

Second, consider a more extended set of WH-in-situ data, mostly based on Pesetsky (1987). WH-in-situ phrases are underscored for transparency. The (a) examples come from (6) above:

(12)a. Who bought what?
b. ?Who bought how much milk?
c. *Who bought what the hell?

(13)a. *What did who buy?
b. Which book did which man buy?

(14)a. *What did you buy why/how?
b. Which book did you buy for which reason/ at which store?
c. Which problem did you solve which way?
d. *Who was healed by moonlight in what sense?

(12b,c) contradict our expectations in that HOW MUCH MILK and WHAT THE HELL are direct objects (canonical arguments) and should therefore be able to stay in situ; but they cannot. (14b) is unexpected because FOR WHICH REASON / AT WHICH STORE are adjuncts but may nevertheless stay in situ. Following Huang, Lasnik&Saito assume that this is explained by preposition stranding at LF. That explanation is difficult to extend to (14c), however. On the other hand, it should extend to (14d) but it does not, as Obenauer (1984:201) pointed out.

Prior to going further, let us note that even in view of rather well-known data, only extraction from WH-islands, cf. (8)-(9), is left as drawing a line between just arguments and adjuncts. The conclusion for combinatory grammar is, again, that its present form is surely faulty of overgeneration, but one thing it should not be blamed for is that it does not derive a prevalent argument-adjunct asymmetry.

The insight that the "definiteness" of the WH-phrase may make a difference is due to Pesetsky (1987). He develops a theory of Discourse-linked WH-phrases that we will not review here. Let it suffice that D-linked WH-phrases do not move at LF and do not therefore fall under strictly movement-related constraints. So, if only the WHICH-phrases are D-linked, the contrasts in (13) and (14) are accounted for. Note, though, that this account predicts only that (subjects and) adjuncts that are D-linked will be able to stay in situ; it does not by any means predict that objects that are not D-linked may not stay. Thus the marginality of (12b) and the ungrammaticality of (12c) do not follow. Pesetsky notes that Kitagawa (1984) suggests that WH THE HELL phrases may have to "be the WH-phrase that moves first"; that is, (12c) is out for obscure but independent reasons. In this connection, Lasnik&Saito suggest that WH THE HELL must be Focused, and derive the ungrammaticality of (12c) from that.

Let us at this point introduce a novel set of suggestive data from Hungarian. (For a background on Hungarian syntax, see É. Kiss 1987). Hungarian has a surface syntactic preverbal Focus position. Multiple interrogation requires that one WH-phrase be moved to Focus and the other(s) be adjoined to it. Thus, we suggest, the S-structures of (15) and (17) are much like the LF representations standardly assigned to (16) and (18), respectively.

(15) Mit miért vettél meg?
what-acc why bought-you
'*What did you buy why'
(16) Why did you buy what?

COMP
/\
what COMP
why

(17) *Miért mit vettél meg?
why what-acc bought-you
'Why did you buy what?'
(18) *What did you buy why?

COMP
/\
why COMP
what

The correspondence is with LF-structures and not with translations. The reason is that the organization of the answer mirrors the surface order of the question words in both languages, but the surface order is reversed by LF adjunction in English. Given that GB-theoretical claims are made about the LF-structures, we take it that the relevant parallelism holds. Now consider the following contrasts:

(19)a. Melyik fiú hány könyvet olvasott?
which boy how many book-acc read
b.* Hány könyvet melyik fiú olvasott?
how many book-acc which boy read
(20)a. Melyik könyvet hány fiú olvasta?
which book-acc how many boy read
b.* Hány fiú melyik könyvet olvasta?
how many boy which book-acc read

(21)a. Mit hány módszerrel tudsz bizonyítani?
what-acc how many method-with can-you prove
b.* Hány módszerrel mit tudsz bizonyítani?
how many method-with what-acc can-you prove
(22)a. Melyik módszerrel hány tételt bizonyítottál?
which method-with how many theorem-acc proved-you
b.* Hány tételt melyik módszerrel bizonyítottál?
how many theorem-acc which method-with proved-you

The contrasts in (19)-(22) can be straightforwardly summarized as follows. The grammaticality of a <WH1, WH2> sequence is independent of which WH is subject, object, or adjunct. What matters is that WH1 (the adjoined term) must not be "indefinite" in some sense.

If we now look back at English (12)-(14) we find that the same descriptive generalization appears to cover all the contrasts. What shall we make of this?

One way to interpret this finding is that it provides support for Lasnik&Saito's proposal that (12c) is out because WH THE HELL must be Focussed. Note that WH2 in Hungarian is in the canonical Focus position, and that the constraint might as well have been phrased as: an "indefinite" WH must be in WH2, i.e., in Focus. In other words, the WH THE HELL suggestion may even extend to the cases otherwise problematic for Lasnik&Saito.

There is a problem, however. Consider WHY, the prototypical adjunct for theories like Huang's. We are now trying to say that the ungrammaticality of (14a) follows from the alleged universal requirement that WHY, an "indefinite", must be Focused. However, from languages that make Focus overt it is quite clear that precisely this item can, but need not, be Focused. In Spanish POR QUÉ does not trigger inversion; in Hungarian MIÉRT is the only WH-phrase that may even adjoin to a non-WH Focus. (Otherwise both behave exactly as bad as expected, e.g., they they do not extract from their own clause at all.) This means that the desired extension is not straightforward.

Note, though, that even if the suggestion concerning Focus were easily generalizable, its very merits would undermine the theory it is intended to save. The reason is that the empirical scope of the argument-adjunct asymmetry is now extremely reduced; it picks out a small and almost random subset of the data. There is no denying that something having to do with function-argument structure is relevant for the distribution of the data; but it seems like a strategic mistake to assign such a central role to it. Quite simply, the moral of the story seems to be that the specific content -- primarily, semantics -- of the items involved is at least as important as the function-argument structure they enter into. The main problem with the Huang-Chomsky-Lasnik&Saito line of research just reviewed is not that it does not contain such semantic considerations. The problem is that it does not have a natural place for them. Therefore, as soon as such considerations are added, they end up blurring, rather than sharpening, the insight that the basic theory offered.

This state of affairs suggests to us that it is not just a matter of taste, but one of necessity, to look for an alternative.

Section 2

Let us now turn to another line of research within the GB paradigm, represented by the convergent results by Cinque (1989), Obenauer (1984), and Rizzi (1989). They invoke data that have been unknown, or ignored, by most workers in the field.

First, Obenauer (1984) argues that the VP-adverb BEAUCOUP 'a lot' will also give rise to a quantification-at-a-distance interpretation when (i) the event itself is countable and (ii) the object in the VP has an empty specifier:

(23)a. J'ai beaucoup conduit ø.
b. ce camion.
c. de camions.
'I drove a lot'
this truck a lot'
many trucks'

Second, he observes that while the WH-specifier (COMBIEN) of the object is generally extractable, such extraction is blocked by an intervening BEAUCOUP, even though the latter might just mean 'often' in this context:

(24)a. Combien as-tu conduit - de camions?
'How many trucks did you drive'
b. *Combien as-tu beaucoup conduit - de camions?
'How many trucks did you drive a lot'
c. Combien de camions as-tu beaucoup conduits -?
'How many trucks did you drive a lot'

Obenauer argues that these data are incompatible with the derivational view of the identification of empty categories, according to which a moved element and its trace are automatically linked. Rather, the empty variable will be blindly linked to the nearest operator that is a potential, even if not "intended", binder for it. I.e. BEAUCOUP will actually bind [- DE CAMIONS] in (24b), and the sentence is out because COMBIEN is left with nothing to bind.

Independently, Rizzi (1986) proposes a parallel account of *GIANNI SI È STATO AFFIDATO (since reflexive SI bears the same index as GIANNI, SI ends up binding both its own trace and the trace of GIANNI). Using these results as well as Obenauer's, Rizzi (1989) develops the notion of Relativized Minimality. This generalizes the above suggestion in essentially the following way:

(25) For each local relation μ, X μ-s Y if and only if X is the closest typical potential μ-er for Y. I.e., the minimal μ-er X becomes the μ-er, creating a μ-island, though not an absolute island, for Y.

The claim that islands are not absolute is important in connection with the THAT-trace effect but will not concern us here. We will be concerned with the consequence of (25) that extraction islands are created by the intervention of a potential antecedent between certain WH-phrases and their traces. Schematically:

    * ------------------------
(26) WH   POT.ANTEC.   TRACE(WH)

The crucial questions are, (i) When is it necessary for there to be a local relation between WH and its trace, i.e., when is Relativized Minimality relevant, and (ii) What are the potential antecedents for those traces?

As regards (i), Rizzi proposes a slight modification of the argument-adjunct distinction. He observes that the extraction of subcategorized (=argumental) manner adverbs is as sensitive to island constraints as that of adjunct manner adverbs:

(27) *How do you wonder whether John behaved -?
    <u>cf</u>. *How do you wonder whether John talked -?
    <u>vs</u>. ?What do you wonder whether John noticed -?

He proposes to distinguish two kinds of thematic roles. Agent, patient, etc. are "referential roles"; measure, manner, etc. are "non-referential roles". A phrase is now called referential iff it has a referential role. Further, he construes binding as a non-local relation that holds only between referential phrases. A referential WH-phrase can thus bind its trace, hence their linking is not subject to Relativized Minimality, i.e., to island constraints. A manner or measure WH-phrase cannot bind its trace, hence they need to be in some local relation. Thus their linking is subject to Relativized Minimality: it is ruined by the intervention of any potential antecedent between a non-referential WH-phrase and its trace.

As regards (ii), Rizzi has to define potential antecedents for non-referential WH-traces in a way that covers intervening WH-phrases and VP-adverb BEAUCOUP. What else? Ross (1984) made the important observation that negation creates "inner islands" for non-referentials:

(28)a. What did no imitation pearls touch -?
    b.*What did no imitation pearls cost -?

Rizzi defines the class of potential antecedents in terms of syntactic position. The position of WH-phrases is an "A-bar specifier" position, hence any intervening A-bar specifier must create an island. He argues that BEAUCOUP and negatives are indeed A-bar specifiers (at least in Logical Form). What exactly an A-bar specifier position is need not worry the non-syntactician reader. Suffice it to point out that not all operators are A-bar specifiers. Rizzi notes that quantifiers like EVERY GIRL acquire their scope by adjunction, cf. May (1985), not by movement into a specifier position. Hence they are predicted not to create an island:

(29) How did every girl think that John behaved -?
    <u>vs</u>. *How did no girl think that John behaved -?

Rizzi's proposal can be said to have the following properties. (i) The well-formedness conditions on extraction structures are formulated in terms of "filters" applied to arbitrary syntactic representations; (ii) The definitions of both island-sensitive extractees and island-creators are in purely syntactic terms; however, (iii) The classes of objects singled out as relevant, and some of the auxiliary notions employed have a distinct semantic flavor. In this last regard it squares with our conclusion in Section 1.

One refinement of Rizzi's theory in Cinque (1989) points even more in the direction of semantics. Given the definition of "referentiality" in terms of thematic roles, Rizzi predicts that role-wise identical phrases behave identically. We have already seen a set of counterexamples to this in connection with WH-in-situ. In (12)-(13) and (19)-(20) the extractees are all "referential" in Rizzi's sense but exhibit differential behavior depending on their "definiteness". Cinque demonstrates that the same holds for extraction from a WH-island, an inner (negative) island, and -- new facts -- from a factive island and an extraposition island. The examples below are modelled after Cinque's:

WH-island:
(30)a. *How much milk do you wonder whether he drank -?
b. Which muffin do you wonder whether he ate -?

Inner (negative) island:
(31)a. *How much milk don't you think that he drank -?
b. Which muffin don't you think that he ate -?

Factive island:
(32)a. *How much milk do you regret that he drank -?
b. Which muffin do you regret that he ate -?

Extraposition island:
(33)a. *How much milk is it time to drink -?
b. Which muffin is it time to eat -?

Cinque supplements Rizzi's theory with the claim that phrases that enter into a non-local binding relation and are therefore insensitive to such islands need not only be role-wise "referential", but also Discourse-linked in the sense of Pesetsky (1987).

As regards Wh-phrases with no "referential role", Cinque agrees with Rizzi that they cannot escape from islands. This is at varience with our (14) and (21)-(22). We return to the issue in Section 4, where a set of data will be discussed more carefully.

Let us summarize the results so far as follows. Constraints on extraction can (to an important extent) be stated in terms of "referentiality / definiteness" and "intervention". The need to substantiate these notions in semantic terms seems imperative. It is also obvious that they need to be substantiated in a way that explains why exactly the observed set of interveners create an island for exactly the observed set of extractees.

One area of semantics to look might be some version of Discourse Representation Theory. This is not only because of Cinque's reference to Pesetsky, who in turn refers to Heim (1982) in connection with Discourse-linking. Even more challenging is Obenauer's and Rizzi's notion of intervention. The description that a variety of operators whose presence in the sentence is originally unrelated to the presence of a certain variable (WH-trace) may however end up binding that variable is highly reminiscent of unselective binding. More precisely, we should say we are dealing with a kind of "forced unselective binding".

This track may be attractive to many but we will not follow it here. We propose to experiment first with developing a more restrictive theory. In earlier work by Steedman and Szabolcsi it was argued that combinatory grammar is (i) strictly compositional, and (ii) capable of handling phenomena like extraction and anaphora without appeal to variable binding in syntax. Phonetically empty bound variables (traces) are avoidable by using functional composition, and overt bound variables are avoidable by assigning a duplicator interpretation to anaphors. The above suggestion that island phenomena be explained in terms of the intervener unselectively binding the WH-trace is incompatible with this theory. It would require the introduction of traces, and binding, and an amount of non-compositionality. Since there appears to be no a priori reason why the conditions on extraction cannot belong to the combinatory portion of grammar, we find it more interesting to see first if the requisite kind of intervention can be captured within its limits.

Section 3

There has in fact been an explicit proposal for the treatment of semantic intervention effects in combinatory grammar: Zwarts' (1990) on the distribution of negative polarity items. In this section we review some aspects of it with two purposes in mind. One is to provide a methodological model of how to approach intervention in this theory. Another is to introduce a set of semantic notions that will be rather directly relevant in the discussion of extraction.

According to Ladusaw (1980), negative polarity items (NPIs) are licensed in the scope of a downward entailing expression (trigger):

(34) An expression d is downward-entailing iff

$$\forall x\, \forall y\, \Box\, [x \subseteq y \rightarrow [d'(y)\ \{ {\rightarrow \atop \subseteq} \}\ d'(x)]]$$

Zwarts (1990) discusses two observations that lead to suggestions that will be relevant in the context of the present paper. The first has to do with the non-uniformity of NPIs and triggers. Dutch (and German) examples reveal that not all NPIs are licensed by simply downward entailing (monotone decreasing) expressions. The requisite distinctions can be made in terms of how many of the de Morgan laws hold for each potential trigger. This indicates that finer patterns of entailment have empirical linguistic relevance. The second observation is that the intervention of certain items between the NPI and the trigger it is in the scope of blocks licensing. The suggestion is to abandon reference to scope and to require, instead, that the NPI be the argument of a function with appropriate licensing properties. That function may be atomic or obtained by composition. Blocking can then be accounted for by showing that the properties of the intervener block the inheritance of the licensing properties of the trigger under composition.

For non-uniformity, consider just one contrast: the Dutch polarity item HOEVEN 'need' can be licensed either by HOOGSTENS ÉÉN (KIND) 'at most one (child)' or by GEEN (KIND) 'no (child)'. On the other hand, OOK MAAR IETS 'anything' is licensed only by the latter. Similar contrasts hold between German BRAUCHEN and AUCH NUR IRGENDWAS:

(35) HOOGSTENS ÉÉN KIND zal zich HOEVEN te verantwoorden
at most one child will self need to justify
'At most one child need justify himself'
GEEN KIND zal zulk een beproeving HOEVEN te
no child will such an ordeal need to
doorstaan
go through
'No child need go through such an ordeal'

(36)*HOOGSTENS ÉÉN KIND heeft OOK MAAR IETS bemerkt
at most one child has anything noticed
'At most one child noticed anything'
GEEN KIND heeft van de regenbui OOK MAAR IETS
no child has of the rain anything
bemerkt
noticed
'No child noticed anything of the rain'

The differences between triggers can be demonstrated in terms of the de Morgan laws. For typograhic reasons we will use a mixture of notations, and f(x v y) -> fx & fy stands for all the following:

(37) John doesn't walk or talk ->
John doesn't walk and doesn't talk
No man walks or talks ->
No man walks and no man talks
At most two men walk or talk ->
At most two men walk and at most two men talk

(38)

	f=NOT	f=NO (N)	f=AT MOST n (N)
f(x v y) -> fx & fy	yes	yes	yes
f(x v y) <- fx & fy	yes	yes	no
f(x & y) -> fx v fy	yes	no	no
f(x & y) <- fx v fy	yes	yes	yes

The two yes's that all three items share correspond to the downward entailing (monotone decreasing) property. The yes that only NOT and NO N share corresponds to closure under (finite) unions. Apparently, OOK MAAR IETS requires its trigger to have this property, too.

For blocking, consider the following:

(39) GEEN KIND kent een jongen die hard HOEFT te werken
no child knows a boy who hard need to work
'No child knows a boy who need work hard'

(40)*GEEN KIND kent <u>de</u> jongen die hard HOEFT te werken
no child knows the boy who hard need to work
'No child knows the boy who need work hard'

The blocking effect of DE would be amenable to a scope treatment, as Ladusaw (1980) points out in connection with similar English examples. In (40) HOEVEN is contained in a relative clause headed by a definite. If definites always have wide scope, then HOEVEN is not in the scope of GEEN KIND. Take (41), however:

(41)*GEEN KIND kent <u>hoogstens één</u> jongen die
no child knows at most one boy who
OOK MAAR IETS gezien heeft
anything seen has
'No child knows at most one boy who saw anything'

Here HOOGSTENS ÉÉN does not force wide scope, hence OOK MAAR IETS can certainly be in the scope of GEEN KIND, which, as we saw above, is a good trigger for it. Nevertheless, the sentence is out. This can only be explained if we can show that while HOOGSTENS ÉÉN (which is merely monotone decreasing in both arguments) is not good enough to license OOK MAAR IETS, its intervention between GEEN KIND and OOK MAAR IETS blocks licensing.

The structure of this problem is in many respects similar to the island problem discussed above, where it was concluded that the intervention of certain items between the WH-phrase and its "trace" blocks their linking. It is therefore interesting to see whether the NPI-problem has a kind of solution that a solution to the WH-problem can in principle be modelled after.

As was mentioned above, the solution Zwarts proposes is to require that NPI be the argument of a function with the appropriate licensing properties. If the atomic function that NPI is the argument of is not suitable, a trigger can be assembled in arbitrarily many steps of function composition. If there is in fact a potential trigger f in the sentence such that f• ... • g can be assembled and NPI is the argument of f• ... • g, the question is whether the critical properties of f are preserved under composition.

We will use the following terminology and notation:

(42)	f is >additive	iff	f(x v y) -> fx v fy	[v>v]
	f is <additive	iff	f(x v y) <- fx v fy	[v<v]
	f is >multiplicative	iff	f(x & y) -> fx & fy	[&>&]
	f is <multiplicative	iff	f(x & y) <- fx & fy	[&<&]
	f is >anti-additive	iff	f(x v y) -> fx & fy	[v>&]
	f is <anti-additive	iff	f(x v y) <- fx & fy	[v<&]
	f is >anti-multipl.	iff	f(x & y) -> fx v fy	[&>v]
	f is <anti-multipl.	iff	f(x & y) <- fx v fy	[&<v]

(43) f is monotone increasing iff
f is >multiplicative or <additive

f is monotone decreasing iff
f is >anti-additive or <anti-multiplicative

f is a quasi-ideal iff
f is monotone decreasing and <anti-additive

To see how these properties are inherited under composition, consider (44), in which we check the preservation of the <anti-additive [v<&] property of a main function under composition:

(44)a.	f•g(x) & f•g(y)	=	f(gx) & f(gy)	by def. of comp.
b.	f(gx) & f(gy)	->	f(gx v gy)	f is [v<&] by ass.
c.	f(gx v gy)	->	f(g(x v y))	if g is [v<v]
	f(gx v gy)	->	f(g(x & y))	if g is [&<v]
	f(gx v gy)	-/->		if g is neither
d.	f(g(x v y))	=	f•g(x v y)	by def. of comp.
	f(g(x & y))	=	f•g(x & y)	by def. of comp.

Given that we are working in the < direction, the property of main function f becomes relevant first. Step (44b) can be carried out because f has a property defined for the connective & we have in the antecedent and for direction <. In our case, it was [v<&], hence it changed & into its dual v. For step (44c) the property of minor functor g needs to be defined for v and <. If g is <additive [v<v], then f• g is [v<&]. If g is <anti-multiplicative [&<v], then f•g is [v<v]. If g is neither, then nothing follows. That is, the <anti-additivity of the main function is preserved under composition iff the minor function is <additive. Otherwise it is reversed or simply lost.

The following calculus summarizes all the combinations that lead to a result along the same lines:

(45)	f	g	=>	f • g
a.	<anti-additive	<additive		<anti-additive
b.	>anti-multipl.	>multipl.		>anti-multipl.

c.	<anti-additive	<anti-multipl.	<multiplicative
d.	>anti-additive	>anti-multipl.	>multiplicative
e.	<multiplicative	<anti-additive	<anti-additive
f.	>additive	>anti-multipl.	>anti-multipl.
g.	<multiplicative	<multiplicative	<multiplicative
h.	>multiplicative	>multiplicative	>multiplicative
i.	<anti-multipl.	<multiplicative	<anti-multipl.
j.	>anti-additive	>additive	>anti-additive
k.	<anti-multipl.	<anti-additive	<additive
l.	>anti-multipl.	>anti-additive	>additive
m.	<additive	<anti-multipl.	<anti-multipl.
n.	>multiplicative	>anti-additive	<anti-additive
o.	<additive	<additive	<additive
p.	>additive	>additive	>additive

Let us now check what this entails for (39)-(41), recalling the theorems in (43). For (39), we need to show that the monotone decreasing properties of GEEN KIND are preserved under composition with EEN. They are, because EEN is monotone increasing wrto its N-argument, and a determiner of a minimal amount, whence it has the properties that let through the requisite properties of GEEN KIND. We are also assuming that the other items that intervene between GEEN KIND and HOEVEN (KENT, etc.) are harmless:

(46)

GEEN KIND			EEN
mon.de. =>	>anti-add.	>add.	<= min.amount
mon.de. =>	<anti-mult.	<mult.	<= min.amount
q-ideal =>	<anti-add.	<add.	<= mon.in.

As to the ungrammatical (40) and (41), DE is not monotone wrto its N-argument, and HOOGSTENS n is monotone decreasing wrto its N-argument. Hence they do not have the properties that would guarantee the preservation of the requisite properties of GEEN KIND under composition. Again, we assume that the other items that become part of the composed function all pass on the requisite properties.

Let us summarize this account of licensing and intervention as follows. The traditional approach defines the licensing of NPIs with reference to particular phrases with some trigger property. This approach, on the other hand, requires that there be at least one analysis of the sentence in which the NPI is the argument of a function with the trigger property. So to say, it imposes a requirement on the environment itself. In this way the blocking effect of certain "interveners" does not require any special treatment: in the ungrammatical cases the environment simply fails to have the trigger property. Putting it more sharply, even the notion of "intervention" is rendered superfluous. All the lexical items in the environment contribute their own semantic properties. The fact that some of those properties block the inheritance of some others should be recognized but has no distinguished status.

With this in mind, we return to extraction islands.

<u>Section 4</u>

Prior to going into any details, let us note that the above reviewed proposal clearly has the potential to extend to the extraction island problem. The reason is that extraction in combinatory grammar is handled by function composition. Therefore the lexical items connecting the WH-phrase and the smallest function that is looking for it will be composed into one big function in any case. Consider, for instance, the following somewhat simplified derivations. Index B indicates composition and index T, lifting. This latter is used to facilitate adjunct extraction. Adv is short for (S\NP)\(S\NP):

```
(47) how          you          think        John         behaved
     S/(S/Adv)    S/(S\NP)     (S\NP)/S     S/(S\NP)     (S\NP)/Adv
                                            --------------------B
                                            S/Adv
                               ---------------------------------B
                               (S\NP)/Adv
                  ----------------------------------------------B
                  S/Adv
     -------------------------------------------------------------
     S

(48) how          ...          John         played
     S/(S/Adv)                 S/(S\NP)     S\NP
                                            ---------------------T
                                            (S\NP)/((S\NP)\(S\NP))
                               ----------------------------------B
                               S/Adv
                  -----------------------------------------------B
                  S/Adv
     -------------------------------------------------------------
     S
```

The effect of function-argument structure on extraction is necessarily captured by the conditions on purely categorial composition. What we have to do, in addition, is to look at the semantic properties of the composed function. Notice that by doing so we do not introduce anything new into the theory. Those semantic properties were always there; we just ignored them. From now on, we won't.

Now, the kind of semantic properties that WH-phrases are sensitive to might in fact be quite different from the kind of properties that matter for NPIs. The methodological parallelism would hold even in that case. However, we will suggest that those properties, even if not identical, are drawn from the same inventory. To the extent this is correct, we will be able to make use of the specific apparatus reviewed in the preceding section.

Our task is now to find some coherence in the set of "interveners" (island creators) on the one hand, and to devise a compatible definition for "referentiality/definiteness" on the other. We will proceed by looking at some data, forming a hypothesis, looking at more data, and so on.

Let us begin by trying to get a picture of what island creators have in common. To this end, we take an uncontroversially island-sensitive extractee, HOW, and see what items create an island for it. (49a-f) are from Rizzi (1989), and (49g-h) are supplemented by Cinque (1989).

(49)a. How do you think that John behaved?
b. *How didn't you think that John behaved?
c. *How did no one think that John behaved?
d. *How did you deny that John behaved?
e. *How did you wonder who behaved?
f. How did every girl think that John behaved?
g. *How did you regret that John behaved?
h. *How is it time to behave?

We already know that Rizzi unifies his interveners (b-e) under the heading of an "A-bar specifier". (NO ONE and DENY become ones at Logical Form, and EVERY GIRL never does.) As to (g-h), Cinque actually claims that they contain no interveners, but the position of the embedded clauses prevents extraction.

We may note here that Rizzi also provides a semantic characterization for A-bar specifiers. He assumes they are all "affective operators, in Klima's (1964) sense, i.e., operators licensing negative polarity items." Unfortunately, this remark contains an overgeneralization in connection with WH.

In (50) we check these items for the properties defined in (42) and (43). Capital YES's mark the defining properties of monotone increasingness and decreasingness.

(50)	v<&	v>&	&>v	&<v	&<&	&>&	v<v	v>v
YOU	yes	no	yes	no	yes	YES	YES	yes
EVERY MAN	yes	no	yes	no	yes	YES	YES	no
NOT	yes	YES	yes	YES	yes	no	no	yes
NO ONE	yes	YES	no	YES	yes	no	no	yes
DENY THAT	yes	YES	no	YES	yes	no	no	yes
WHO	yes	no	--	--	yes	no	--	--

The properties of WHO were determined with the help of the following context:

(51)a. I know who walks and who talks v<& : yes
  -> I know who (walks or talks)
 b. I know who (walks or talks) v>& : no
  -/-> I know who walks and who talks
 c. I know who walks and who talks &<& : yes
  -> I know who (walks and talks)
 d. I know who (walks and talks) &>& : no
  -/-> I know who walks and who talks

The disjunction of WH-questions seems ungrammatical to us, so all cases involving such disjunction are left without a value. However, even the clear values show that WHO lacks the critical >anti-additive feature of monotone decreasing expressions. Hence it is a misfit for Rizzi. (The NPI-trigger properties of WH are also dubious, as is discussed in Ladusaw (1980).)

Since WHO also clearly lacks the defining properties of monotone increasing functions, we must conclude it is not monotone. But this observation suggests a new perspective that has to our knowledge never been explored in the literature: we should check whether other non-monotone expressions create islands.

Some of the data we have already seen can in fact be interpreted in such terms. REGRET, for instance, is surely not monotone decreasing. Now, it is not monotone increasing, either. The same suspicion arises in connection with IT IS TIME TO. Compare:

(52)a. I regret that John fell asleep in the bathtub
  -/-> I regret that John fell asleep
 b. It is time to fall asleep in the bathtub
  -?-> It is time to fall asleep

Then, take Obenauer's BEAUCOUP. It has been noted by Westerståhl that items like OFTEN are not monotone because frequency standards are variable. VERY MUCH may be similar:

(53)a. John often falls asleep in the bathtub
  -/-> John often falls asleep
 b. John very much likes to read in the bathtub
  -/-> John very much likes to read

OFTEN, in turn, seems to block the lower construal of WHY:

(54)a. Why do you say that Bill was fired?
  'I say: Bill was fired because he was sick'
 b. *Why do you often say that Bill was fired?
  'I often say: Bill was fired because he was sick'

If islands created by WH-phrases, REGRET, IT'S TIME, BEAUCOUP, and OFTEN appear to lend themselves to this treatment, let us examine the paragon of non-monotonicity, an EXACTLY-phrase. Many, though not all, speakers have the following judgment when we consider 'why Bill was fired':

(55)a. Why does every girl say that Bill was fired?
b. ?Why do more than five girls say that Bill was fired?
c. *Why do exactly five girls say that Bill was fired?

Incidentally, the ungrammaticality of (55c), as opposed to the grammaticality of (55a,b) would square with Rizzi's A-bar specifier generalization, since exhaustive listing is a property of Focus, and Focus is an A-bar specifier.

If we accept these judgments, and draw the line between (55a,b) and (55c), we arrive at the conclusion that non-monotone functions are among islands. In other words:

(56) Hypothesis 1A: <u>Monotone Increasingness</u>
Islands are functions which are not monotone increasing, i.e., which are monotone decreasing or not monotone.

This immediately leads to a hypothesis concerning the island-sensitivity of extractees:

(57) Hypothesis 1B: <u>Inclusion</u>
What monotone increasing functions do is to preserve the inclusion relation.
Inclusion is relevant only for WHs whose domain has a (partial) ordering. These WHs can be island-sensitive; others cannot be.

Not unetymologically, we will call the elements of any domain without a (partial) ordering individuals. We note that not only 'john,' 'mary,' 'bill,' and 'ninety' are individuals. Even properties etc. can be individuated by abstracting from their ordering (e.g., Szabolcsi 1983). Expressions ranging over individuals will be called atomic. A simple(-minded) test to determine whether a WH-phrase is atomic is to see if the question that contains it can be answered by listing individuals in the following fashion (the possibility of phrasing the same answer in a different way does not matter):

(58) A: Who do(n't) you invite?
B: I (don't) invite John, and
I (don't) invite Mary, etc.

Some WH-phrases are invariably atomic or non-atomic in this sense. WHICH-questions can always be answered in the manner of (58), and WHY-question can never be. They represent the two extremes of island-sensitivity, indeed:

(59)a. Which man did no one think that I invited -?
Which man did you wonder who invited -?

b. *Why did no one think that Bill was fired -?
   *Why did you wonder who was fired -?

Most of the other WH-phrases have both atomic and non-atomic readings. In those cases discourse context matters a lot. The following examples are modelled after Kroch (1989) and É.Kiss (1990), whose observations converge with ours, even if they evaluate them in slightly different terms.

(60) How many votes do you think this candidate received? ... Ninety.
(61) How many votes do you wonder which candidate received? ... Ninety.

In (60), the response NINETY can be understood as a pure cardinality predicate in the 'at least' sense. In (61), it cannot: it can only refer to the individual 'ninety.' This latter is also possible, but by no means necessary, in (60). HOW MANY VOTERS has an additional atomic use, roughly the same as 'which voters':

(62) How many voters do you wonder which candidate received? ... Three, namely, Mr. Smith, Mrs. Jones, and Ms. Black.

In pragmatic terms we can say the question-answer pair (61) is felicitous only if figures are salient in the discourse. However, salience does not constitute a minimal difference. The two uses of HOW MANY (VOTES) and NINETY differ in semantic type (cardinality predicate vs. individual). According to Hypotheses 1A-B, this latter is crucial, and salience is ancillary. Similarly, (62) is felicitous only if individual voters are salient. Again, this only facilitates the use of HOW MANY VOTERS in an atomic sense.

There are even cross-linguistic differences in the ambiguity potential of WH-phrases. Hungarian MIKOR 'when' easily allows atomic readings English WHEN does not:

(63) *When do you wonder who you have to visit -?
(64) Mikor nem tudod, hogy kit kell meglátogatnod -?
when not know-you that whom should visit-you
'At what time of the day / on which occasions etc. do you wonder who you have to visit -'

This example brings us to adjuncts. Rizzi, Cinque and Kroch (but not É.Kiss) maintain that true adjuncts can never be extracted. That claim, to which (64) is certainly a counterexample, is crucially based on this contrast:

(65) *For what reason don't you know if we can say [that Gianni was fired -]?
(66) What reason don't you know if we can give - for Gianni's firing?

Rizzi writes, "Since the entities involved are reasons in both cases, the distinction does not seem to be directly expressible in ontological terms" (1989, Ch. 3.5). We do not believe we are dealing with the same "reasons" in the two cases. FOR WHAT REASON (just as Hungarian MI OKBOL) is primarily synonymous with WHY, i.e., asks for a propositional answer in terms of BECAUSE ... In (66) on the other hand we are looking for objects that we directly classify as reasons (cf. also ACCOUNT, EXPLANATION, etc.). Plural FOR WHAT REASONS and, especially, FOR WHICH OF THESE REASONS fall together with WHAT REASON in (66), and extraction improves, too:

(67) For which of these reasons do you wonder [if they can fire you -]?

All in all, it appears that the data are consistent with Hypotheses 1A-B concerning monotonicity and inclusion. Note, however, that those hypotheses are not as strong as one would want them to be. They explain why only non-atomic expressions can be sensitive to islands defined as non-increasing functions. They do not explain why non-atomic expressions actually are sensitive. Unfortunately, we do not have a good answer to this question for the time being. It seems intuitively clear to us that something must go wrong with the evaluation of non-atomic WH-phrases if the function they combine with does not preserve inclusion. But we are unable to formally demonstrate what it is that goes wrong.

While the above question can be regarded as merely open for the time being, the monotone increasing proposal also encounters a genuine problem: that of double negation.

Note the following difference in the predictions made by intervention theories like Rizzi's and ours. According to Rizzi, the intervention of one negative item between the WH-phrase and its trace blocks their linking once for all. According to our theory, two negatives in the composed function may cancel each other out. Now, to the extent the following examples are processable at all, they appear to support Rizzi:

(68) a. *How did <u>no one</u> <u>deny</u> that Bill behaved?
b. *How did<u>n't</u> you <u>deny</u> that Bill behaved?
c. *How did<u>n't</u> you think that <u>not everyone</u> behaved?
d. *How did<u>n't</u> you think that <u>at most two men</u> said that Bill behaved?
e. *How did <u>no one</u> think that I did<u>n't</u> say that Bill behaved?
f. *How did<u>n't</u> you think that I did<u>n't</u> say that Bill behaved?

To be more precise, our proposal as it stands accounts for (68d) but not the rest. The reason is as follows.

In (43) we said that a function is monotone increasing iff it is either <additive or >multiplicative. For f•g to be <additive [v<v], either both f and g must be <additive (which no decreasing functions are), or f <anti-multiplicative [&<v] and g <anti-additive [v<&]. For f•g to be >multiplicative [&>&], either both f and g must be >multiplicative (which no decreasing functions are), or f >anti-additive [v>&] and g >anti-multiplicative [&>v]. Now consider the properties of a few decreasing functions:

(69)

	v<&	v>&	&>v	&<v	&<&	&>&	v<v	v>v
NOT	yes	YES	yes	YES	yes	no	no	yes
NO ONE	yes	YES	no	YES	yes	no	no	yes
DENY THAT	yes	YES	no	YES	yes	no	no	yes
AT MOST 2 N	no	YES	no	YES	yes	no	no	yes
NOT EVERY N	no	YES	yes	YES	yes	no	no	yes
YOU	yes	no	yes	no	yes	YES	YES	yes

We see that all decreasing functions have the properties required above for main function f and, with the exception of AT MOST 2 N, they have one or both of the properties required for g. In other words, they offer at least one way to prove the monotone increasingness of f•g. Only when AT MOST 2 N is the minor function do we fail to get that result. This is of course not what we want since all the combinations in (68) are ungrammatical.

There are two natural ways to go in trying to solve this problem. The first is to choose somewhat different algebraic structures as models, so that the above quoted monotonicity results do not obtain. The second is to show that island-sensitive extractees actually require the preservation of more than just the inclusion relation. In that case mere monotone increasingness will not be enough to exempt a function from islandhood. (In this regard, note that multiplicative functions preserve (finite) intersections, additive functions (finite) unions, consistent and complete functions complements etc.) If we can identify such a further property, we may well be able to show that the problematic increasing functions in (68) do not have it.

In this paper we will not explore this avenue in any further detail. We believe that at this preliminary stage of research, when the range of relevant data has not yet been very safely established, it is more useful to sketch, however briefly, at least one alternative approach.

Section 5

In this section we try to attack the double negation problem in a way that places the whole island issue in a different light.

One theory in which double negation is not generally eliminable is Dynamic Predicate Logic (DPL). In Groenendijk and Stokhof (1989) the meaning of a sentence specifies how it changes the interpreter's information. $\exists x\phi$ is associated with a pair of assignments g and h:

(70) $[\![\exists x\phi]\!] = \{\langle g,h\rangle \mid \exists k{:}k[x]g \;\&\; \langle k,h\rangle \in [\![\phi]\!]\}$

In common DRT parlance (which is literally not applicable here) we might say that the dynamic effect of $\exists x\phi$ consists in introducing a new discourse marker. Negation is defined so as to eliminate this possibility:

(71) $[\![{\sim}\phi]\!] = \{\langle g,h\rangle \mid h=g \;\&\; {\sim}\exists k{:}\langle h,k\rangle \in [\![\phi]\!]\}$

Hence, $\sim\sim\phi$ is equivalent to ϕ in its truth conditions but not in its dynamic effects. One negation freezes the formula and another cannot defrost it.

This may be relevant to us if we assume, roughly:

(72) Hypothesis 2A:
A WH-phrase is either linked to an old discourse-marker or introduces a new one.

(73) Hypothesis 2B:
A WH-phrase that is not linked to an old discourse-marker must introduce a new one; hence it only occurs in contexts where a new discourse marker can emerge. Other contexts constitute islands for it.

Adopting 2A-B we predict at least that contexts without a standard existential entailment or without a dynamic effect constitute islands. Besides (single or double) negative islands, this may cover WH-islands:

(74)a. (I know) who has a unicorn -/-> There is a unicorn
b. *What do you wonder who has?
[if WHAT is not linked to an old discourse marker]

There are more islands, of course. In those cases we certainly have an existential entailment but, interestingly, not necessarily a dynamic effect. For instance, REGRET seems to freeze the embedded proposition:

(75) I regret that you broke a plate. *It was my mother's.

Unfortunately, we do not know of systematic results concerning the dynamic properties of monotone increasing and non-monotone contexts in general, or of the specific contexts reviewed in the preceding sections in particular. It may well turn out that dynamic properties and monotonicity properties do not quite correlate and therefore a certain revision of our data is required. Given that the pertinent judgments are often subtle, such evidence would be extremely valueable. On the other hand, extraction data would provide a good testing ground for the empirical consequences of dynamic theories. In this way, the requisite investigations might be mutually rewarding.

Let us now ask what may motivate the adoption of Hypotheses 2A-B, beyond noticing that they may help solve the double negation problem. Just as in the case of 1A-B, we do not have a definitive answer. We may note, however, that motivation may come from the proposals this line of research converges with, namely, Pesetsky's, Cinque's and Kroch's on the one hand, or Berman's (1990) on the other. As to the former, Kroch assumes that questions carry an existential presupposition that introduces a discourse referent. As to the latter, Berman examines an entirely different set of data than we did, and arrives at the conclusion that WH-phrases (in interrogatives embedded under factives) are variables and the clause that contains them serves as their restrictor. Either approach may eventually motivate 2A. Care should be taken, however, not to tie the motivation to assumptions about one type of interrogative that do not apply to another type. Island phenomena do not, to our knowledge, depend on whether the whole WH-construction happens to be a direct question or is embedded under a factive matrix verb, for instance.

Finally, the question arises whether the approaches in Sections 4 and 5 are necessarily rivals, or can be combined. While we have by no means given up the hypothesis sketched in Section 4, it is with an eye on a conceivable combination that we have taken the compositional theory DPL as a point of departure in this section.

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Partial propositional and modal logic: the overall theory

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1 Introduction and program

During the last decade several partial logics have emerged in the literature. Our initial goal just was to give an overview of these proposals, possibly extend them to modal logic and incorporate them in an overall system. To facilitate comparison, we focussed on the purely propositional part of these theories. Then it turned out that choices concerning such key notions as *rule* and *validity* were sometimes seemingly arbitrary or *ad hoc*. In other words, the attempted regimentation forced us to consider alternatives and to formulate a general theory which covers both existing proposals and alternatives.

The point is that in classical logics some distinctions are suppressed that prove to be rather important for (partial) logics, even from a strictly deductive point of view. For instance, what do we mean when we state that $\varphi \Rightarrow \psi$ is a valid rule? One possible interpretation is that if φ is a theorem, so is ψ; another that given an (arbitrary) formula φ we can derive ψ. The celebrated rule of *modus ponens* is used both ways, and in classical propositional logic the difference between these interpretations is immaterial. But already in normal modal logic the difference is important: the rule of *necessitation* operates on theorems, not on arbitrary formulae. This distinction is especially relevant for partial semantics, even in the propositional case. Moreover, the distinction presented does not exhaust the possibilities: one might also want to *reduce* the validity of rules to that of formulas. This can be achieved by invoking the deduction theorem as a heuristic principle; then $\varphi \Rightarrow \psi$ is valid iff $\varphi \rightarrow \psi$ is a valid formula.

The choice of the type of rule interacts with the way validity is defined in the semantics. We can choose (among other things) between a 'true everywhere' (*verification*) and 'false nowhere' (*non-falsification* or *falsifiability*) concept of validity. E.g., the law of excluded middle $\varphi \vee \neg\varphi$ is valid for coherent situations under the falsification perspective (it is never false), but it is not valid under the verification ('always true') perspective, for the valuation may be undefined with respect to φ. For the modal language there is yet another option: validity can be obtained by evaluation on a designated subset of the worlds involved in a model; one type of this nonstandard approach is treated in subsection 3.4.1.

The effect of such distinctions also depends on the sort of indices and valuations used: are they *partial* or *total*, *coherent* or *incoherent*?

Finally, one may opt for different truth conditions. It is well-known from multi-valued logic that different truth-conditions compatible with classical logic are possible e.g. for $\rightarrow$. But in our general setting even in propositional logic truth conditions with a genuine 'modal' flavour qualify, in the sense that they make reference to extensions of the situation of interpretation. These 'eventual' conditions will be illustrated for the case of intuitionistic logic (section 2.3). Yet in what we consider the *standard* case, truth conditions will not be eventual, but 'actual'. Sections 2.1 and 2.2 deal with the standard propositional case.

This modal perspective is generalized in section 3, where modality is interpreted by means of an arbitrary accessibility relation, which may be different from plain extension. In all, this approach is very much in the spirit of ordinary Kripke models, with partiality and/or incoherence imposed on valuation. We have explored this new area and found completeness theorems for the standard types of partial modal semantics. Another deviation from what we consider the main trail is the case where the accessibility relation itself is partialized (see subsection 3.4.2).

The syntax of the logical languages treated in this paper is quite traditional; we have refrained from incorporation of new propositional and modal connectives.

We end this introduction with a list of parameters controlling partial semantics:

- situations & valuations: partiality and coherence
- truth-conditions: standard ('actual') vs. non-standard 'eventual'
- validity:(unrestricted) verification (VERIF) or falsifiability (FALSIF) or some restricted set of situations
- type of rule: absolute, relative, or mixed

2 Propositional logics

2.1 Coherent situation semantics

Presumably the most intuitive form of partial semantics is displayed in the following system, which originates essentially from [Barwise81]. Assume the propositional language to be construed in the obvious way with a set of propositional atoms called *Prop*. Since our prime goal here is to model modal rather than mere propositional logic, we have to index the valuation function. Given a model $\langle S, V\rangle$, S being a set of coherent situations (or: partial worlds) and V a partial function from $Prop \times S$ to $\{0,1\}$ (truth values), the usual truth conditions relative to $\langle S, V\rangle$ and $s \in S$ are:[1]

$$\begin{array}{ll} s \models p \Leftrightarrow V(p,s) = 1 \; (\forall p \in Prop) & s \mathrel{=\!\!|} p \Leftrightarrow V(p,s) = 0 \; (\forall p \in Prop) \\ s \models \neg\alpha \Leftrightarrow s \mathrel{=\!\!|} \alpha & s \mathrel{=\!\!|} \neg\alpha \Leftrightarrow s \models \alpha \\ s \models \alpha \wedge \beta \Leftrightarrow s \models \alpha \text{ and } s \models \beta & s \mathrel{=\!\!|} \alpha \wedge \beta \Leftrightarrow s \mathrel{=\!\!|} \alpha \text{ or } s \mathrel{=\!\!|} \beta \end{array}$$

One says that s ***verifies*** (or ***satisfies, supports***) φ whenever $s \models \varphi$ and that s ***falsifies*** (or ***rejects***) φ whenever $s \mathrel{=\!\!|} \varphi$.

The truth conditions for the other propositional connectives then follow directly from the given conventions and the usual recursive definitions:

$$\begin{array}{ll} s \models \alpha \vee \beta \Leftrightarrow s \models \alpha \text{ or } s \models \beta & s \mathrel{=\!\!|} \alpha \vee \beta \Leftrightarrow s \mathrel{=\!\!|} \alpha \text{ and } s \mathrel{=\!\!|} \beta \\ s \models \alpha \rightarrow \beta \Leftrightarrow s \mathrel{=\!\!|} \alpha \text{ or } s \models \beta & s \mathrel{=\!\!|} \alpha \rightarrow \beta \Leftrightarrow s \models \alpha \text{ and } s \mathrel{=\!\!|} \beta \\ s \models \alpha \leftrightarrow \beta \Leftrightarrow s \mathrel{=\!\!|} \alpha, \beta \text{ or } s \models \alpha, \beta & s \mathrel{=\!\!|} \alpha \leftrightarrow \beta \Leftrightarrow s \mathrel{=\!\!|} \alpha, s \models \beta \text{ or } s \models \alpha, s \mathrel{=\!\!|} \beta \end{array}$$

In fact it will be convenient for several results to make the presentation entirely symmetrical by adding $\vee$ as a basic symbol. Due to the above clauses a situation need not verify classical tautologies, such as the 'law of excluded middle'. So, possibly $s \not\models \varphi \vee \neg\varphi$ (which is intended when employing a truth value gap) and $s \not\models \varphi \rightarrow \varphi$ (which is counterintuitive)[2].

But the force of this semantics is not solely determined by truth conditions. One other factor is the notion of validity. In the classical propositional case (i.e. when V remains total) a formula is valid if it is verified everywhere, or, equivalently, falsified nowhere. Interestingly, in the partial case the notions of verification and falsifiability diverge widely, akin to what comes to mind from the perspective of philosophy of science. We shall discuss both options below.

Propositional verification

Presumably the first possibility which suggests itself is to call a formula valid if it is supported by every situation in each model. So we define

Definition 1 (VERIF)
φ is verifiably valid iff for **every** *model $M = \langle S, V\rangle$ and* **every** *$s \in S : M, s \models \varphi$*
(Notation: $\models \varphi$)

[1] $\langle S, V\rangle, s \models \varphi$ is abbreviated to $s \models \varphi$ when no confusion arises.

[2] Lukasiewics' proposal 'to fill the gap' by making $\alpha \rightarrow \beta$ true when both antecedent and consequent are undefined solves this problem but produces many other counterintuitive results, e.g. when α and β are independent unknown propositions.

This notion turns out to have weird logical consequences, given the fact that a logic is usually described to a large extent by the set of valid formulas. For the model that leaves every atom undefined in some situation does not verify a complex formula either, as can easily be shown by induction.

Theorem 1 *The set of verifiably valid formulas is empty.*

Since there are no valid formulas according to the verification perspective, there is no point in invoking a deduction theorem which reduces valid consequence to valid implication. So, for verification, the absolute and the reductive approaches to rules are uninformative: these notions would yield the total set of rules (**R**) or the empty set ($\emptyset$) respectively. But in the relative approach, an evaluation of arguments is required since the logic is fully specified by its *rules* rather than its *axioms*. What we need is a definition of relative verifiable validity, called strong consequence in [Barwise81]:

Definition 2 (VERIF$_{rel}$)
$\alpha_1,\ldots,\alpha_m/\beta_1,\ldots,\beta_n$ *is relatively verifiably valid iff for every* M,s *such that* $M,s \models \alpha_i$ *for all* i, $M,s \models \beta_j$ *for some* j *(Notation:* $\alpha_1,\ldots,\alpha_m \models \beta_1,\ldots,\beta_n$*)*

Notice that *Modus Ponens* is valid for relative verification on coherent models, but does not characterize the set of valid rules on its own. [Kamp83] contains such a complete set of inference rules. Though elegant, his characterization invokes a special normal form and corresponding proof technique which does not generalize easily to the modal case. The system we propose here is in the spirit of natural deduction. The usual starting rule, viz. $\alpha_1,\ldots,\alpha_m \vdash \beta_1,\ldots,\beta_n$ if some α_i equals some β_j, follows from this system. Other important properties such as distributivity and associativity of $\wedge$ and $\vee$, as well as the cut–rule, are derivable. [3]

(R1) $\neg\neg\varphi \dashv\vdash \varphi$ ('the law of double negation')

(R2) $\neg(\varphi\wedge\psi) \dashv\vdash \neg\varphi\vee\neg\psi$ (first 'de Morgan's law')

(R3) $\neg(\varphi\vee\psi) \dashv\vdash \neg\varphi\wedge\neg\psi$ (second 'de Morgan's law')

(R4) $\varphi\wedge\psi \vdash \varphi$ $\quad \varphi\wedge\psi \vdash \psi$

(R5) $\varphi \vdash \varphi\vee\psi$ $\quad \psi \vdash \varphi\vee\psi$

(R6) if $\varphi,\varrho \vdash \chi$ and $\psi,\varrho \vdash \chi$ then $\varphi\vee\psi,\varrho \vdash \chi$

(R7) if $\chi \vdash \varphi,\varrho$ and $\chi \vdash \psi,\varrho$ then $\chi \vdash \varphi\wedge\psi,\varrho$

(R8) $\varphi\wedge\neg\varphi \vdash \psi$ (*ex falso [sequitur quodlibet]*)

(R9) if $\varphi \vdash \psi$ and $\psi \vdash \chi$ then $\varphi \vdash \chi$

(R10) $A \vdash B$ iff there are nonempty sets $\{\alpha_1,\ldots,\alpha_m\} \subseteq A$ and $\{\beta_1,\ldots,\beta_n\} \subseteq B$ such that $\alpha_1\wedge\cdots\wedge\alpha_m \vdash \beta_1\vee\cdots\vee\beta_n$ (notation: $\alpha_1,\ldots,\alpha_m \vdash \beta_1,\ldots,\beta_n$)

[3] $\varphi \dashv\vdash \psi$ abbreviates $\varphi\vdash\psi$ & $\psi\vdash\varphi$. The alternative symbols $\Rightarrow$ and $\Leftrightarrow$ are quite common in deductive rules, but are here avoided since we use them for meta-level implication and equivalence, respectively.

Now let rL^+ be the set of deductive rules generated by R1–10. We obtain the following important

Theorem 2 $A \models \varphi \Leftrightarrow A \vdash_{rL^+} \varphi$,

which can be proved in a Henkin-style fashion (cf. theorem 9 in section 3.2).

Yet, the verification approach may still seem strange since it produces no tautologies. So let us pay attention to the other option suggested above.

Propositional falsifiability

[van Benthem84] suggests that already Beth's semantic tableaus, originating from the thirties, contain implicit partiality. In a Beth tableau one tests the validity of an inference by trying to construct a counterexample; the (in)validity only depends on propositional variables occurring in the premises and conclusion and often not even all of them! So the attempted falsification naturally leads to partiality.

Definition 3 (FALSIF)
φ is falsifiably valid ($\not\dashv \varphi$) iff for **no** *model $M = \langle S, V\rangle$ and* **no** *$s \in S$: $M, s \dashv \varphi$*

For example, $\varphi \vee \neg\varphi$ is valid under this definition simply because it is never rejected. Because there are valid formulas now, we do not really need a separate definition of *consequence*: the notion *falsifiably valid rule* can be reduced to that for *formulas* by stipulating $\alpha, \beta \not\dashv \gamma \Leftrightarrow\ \not\dashv (\alpha \wedge \beta) \to \gamma$, i.e. one can employ the familiar deduction theorem not as a derived theorem but as a guiding principle. Since $s \not\dashv (\alpha \wedge \beta) \to \gamma \Leftrightarrow$ if $s \models \alpha$ and $s \models \beta$ then $s \not\dashv \gamma$, the effect is that falsifiability of the conclusion 'mixes' with the truth of the premises. We shall therefore call this type of validity *mixed falsifiability*.

Definition 4 ($\mathbf{FALSIF}_{mix}$)
$\varphi_1, \ldots, \varphi_n/\psi$ is mixed falsifiably valid iff for every M and s: if $M, s \models \varphi_1, \ldots, M, s \models \varphi_n$ then $M, s \not\dashv \psi$.

This notion gives rise to a remarkable 'unpartial' result.

Theorem 3 (van Benthem)
FALSIF_{mix} on coherent models is completely describable by some system of classical propositional logic pL.[4]

We can obtain a similar result for *absolute* falsifiable validity.

Definition 5 ($\mathbf{FALSIF}_{abs}$)
$\varphi_1, \ldots, \varphi_n/\psi$ is absolutely falsifiably valid iff $\not\dashv \varphi_1$ and ... and $\not\dashv \varphi_n$ jointly imply $\not\dashv \psi$.

---

[4] [van Benthem84] uses Beth tableaus (or Gentzen sequents) and the notions of strong and weak consequence, where we (would) use ordinary models, and relative verifiable and mixed falsifiable validity, respectively.

Absolute rules such as $\vdash \varphi \Rightarrow \vdash \psi$ suffer from a complication not yet dealt with. The problem is that in addition to relatively or mixedly valid propositional rules there is a class of absolute rules that qualify for the simple reason that the premise is a contingent formula: e.g. $\vdash p \Rightarrow \vdash q$ whereas of course $p \not\vdash q$. [5] Without claiming elegance, we can give a very simple solution: let the rules of pL be combined with scheme

$$\vdash \varphi \Rightarrow \vdash \psi \text{ if } \vdash \chi \not\Rightarrow \vdash \varphi$$

together forming the absolute propositional logic pL*. But are not we overdoing things? For notice that the above scheme can be reformulated as:

$$\vdash \varphi \Rightarrow \vdash \psi \text{ if } \not\vdash \varphi$$

and does not this follow from the very meaning of the inference relation $\Rightarrow$? The answer to this question is that although this 'implication interpretation' is clearly intended, it does not exist *a priori*, for it can only be obtained once a completeness correspondence between deductive system and semantics has been established ...and for the latter we need the above clause. Still it may be a bit surprising that some common propositional properties are destroyed now; for example, contraposition does not hold anymore: in the extended system $p \Rightarrow \bot$, but $\top \not\Rightarrow \neg p$. Notice however that this deviation is not caused by either partiality or coherence; precisely the same observation applies to a classical semantics with a total functional valuation.

Theorem 4 *The coherent semantics with absolute falsifiable validity is complete with respect to the (absolute) propositional logic* pL*.

An indirect proof using 'completion' of partial models then suffices once the following properties are established:

Proposition 1 (coherence)
For no $\varphi, S, V, s \in S : \langle S, V\rangle, s \models \varphi$ *and* $\langle S, V\rangle, s \dashv \varphi$.

Proposition 2 (persistence)
For every φ, $M = \langle S, V\rangle$, $M' = \langle S', V'\rangle$, $s \in S$, $s' \in S'$ *such that* $M, s \sqsubseteq M', s'$: *if* $M, s \models \varphi$ *then* $M', s' \models \varphi$, **and** *if* $M, s \dashv \varphi$ *then* $M', s' \dashv \varphi$.

where $\sqsubseteq$ is defined (somewhat more generally than usual) by:

Definition 6 ($\sqsubseteq$)
If V *and* V' *are fixed valuations,* $s \in S$, $s' \in S'$, $M = \langle S, V\rangle, M' = \langle S', V'\rangle$ *and for every atom* $p : V(p,s) = V'(p,s')$ *whenever* $V(p,s)$ *is defined, then* $M, s \sqsubseteq M', s'$.

Now how about *relative* falsifiability, still for coherent partial models? Relative falsifiability demonstrates a peculiar behaviour. As we have seen before, the set of valid formulas coincides with the set of classical tautologies. However, this does not hold for the rules! The *ex falso* principle is now invalid. For let p be undefined in s and q be false,

[5] Although this seems to have been generally overlooked, [Curry63, p.97/8,175/6] is very accurate on this point.

then $s \not\dashv p \wedge \neg p$ while $s \dashv q$. *A fortiori*, the standard (sometimes even the **only**) rule in axiomatizations of pL, viz. *modus ponens* does not hold either. This indicates that valid *formulas* and valid *rules* are independent devices, where a rule that properly deduces tautologies may not qualify as a rule *of* (i.e. within) the same system. The set of FALSIF$_{rel}$ valid rules appears to be dual to the relatively verified ones:

Theorem 5 *On coherent models the set of relatively falsifiably valid rules is completely described by the system* rL*, *which is* rL$^+$ *with* R8 *replaced by its contrapositive:*
(R8*) $\psi \vdash \varphi \vee \neg\varphi$ *(tertium non datur).*

In connection to theorem 1, we are confronted with another striking result: coherent partial semantics does not select an interesting subset from the set of classical tautologies; it either yields the empty set or else the total set of tautologies. So, although this type of model seems to be better motivated for reasons of intuition and efficiency, the outcome is not that much different from classical logic. It may be wise therefore to study the possibility of relaxing the coherence restriction.

2.2 General situation semantics

Without the restriction to coherence, situations may be incoherent with respect to a proposition and a valuation. We will call the intended structure a *(general) situation model*. So formally, $V : Prop \times S \longrightarrow \mathcal{P}(\{0,1\})$, i.e. the valuation function is multiple-valued. The truth and falsity relations are defined in the same way as in section 2.1, with one minor proviso for the basic case:

$$s \models p \Leftrightarrow 1 \in V(p,s) \text{ , and } \; s \dashv p \Leftrightarrow 0 \in V(p,s) \;\; (\text{where } p \in Prop).$$

The definition of *coherent situation* has to be modified accordingly. In addition the notion of *total situation* is of importance in the present setting. Defined with respect to atomic φ both properties generalize to arbitrary propositional formulas:

(totality) if s is total then for all $\varphi : s \models \varphi$ or $s \dashv \varphi$;

(coherence) if s is coherent then for all $\varphi : s \not\models \varphi$ or $s \not\dashv \varphi$.

What kind of logic do general models yield for both definitions of validity?

First, for verification, theorem 1 still holds. Some of the rules permitted by theorem 2, however, are now illegitimate. For example, *modus ponens* is invalid: if $s \models p$ and $s \models p \rightarrow q$, then possibly $s \not\models q$, notably when $V(p,s) = \{0,1\}$ and $1 \notin V(q,s)$. One easily obtains the same result for *modus tollens*.

Next for falsifiable validity, we are confronted with a result similar to theorem 1, with a dual proof: consider a singleton model and a valuation which is overdefined (both true and false) for every atom. Then the model falsifies every formula. So, in general, we obtain for both sorts of validity:

Theorem 6 *There are no valid formulas in general situation semantics.*

Therefore we have to invoke a definition similar in spirit to VERIF$_{rel}$, though now for falsifiability:

Definition 7 (FALSIF$_{rel}$)
φ/ψ is relatively falsifiably valid ($\varphi \not\dashv \psi$) iff for every M, s: if $M, s \not\dashv \varphi$ then $M, s \not\dashv \psi$.

Relative verification and falsification are related as follows:

Proposition 3 $\varphi \not\dashv \psi$ *iff* $\neg\psi \models \neg\varphi$ [6]

Instead of enlarging the interrelation of contraposition and validity concepts, we can establish a much more revealing connection between the two notions of validity by employing a simple transformation of models.

Definition 8 (duality)
For any model $M = \langle S, V\rangle$, its dual $\tilde{M} = \langle S, \tilde{V}\rangle$ is defined by: $1 \in \tilde{V}(p,s)$ *iff* $0 \notin V(p,s)$, *and* $0 \in \tilde{V}(p,s)$ *iff* $1 \notin V(p,s)$.

This property again generalizes to complex formulas (the proof is a straightforward induction):

Proposition 4 (duality)
For every M, s, φ : $M, s \not\models \varphi \Leftrightarrow \tilde{M}, s \dashv \varphi$, and $M, s \not\dashv \varphi \Leftrightarrow \tilde{M}, s \models \varphi$.

We have paved the way for a useful reduction. The following proposition expresses that the validity concepts coincide sort by sort: falsifiability and verification of formulas, absolute verification and absolute falsifiability of rules, etcetera.

Proposition 5 *For general situation models:* $\not\dashv \;=\; \models$.

Propositions 3 and 5 jointly imply that (strong) consequence on general situations is closed under contraposition, and so in this respect the logic is more 'classical' than with coherent situations.

What is the syntactic counterpart of general situation semantics and what is the relation between this rule system and that for verification on coherent models? As we saw before, these two systems are surely different e.g. with respect to *modus ponens*. Now if we inspect the rules of rL^+, $\varphi \wedge \neg\varphi/\psi$ is typically not valid on general models, but the other rules in fact are. More importantly, the set of rules $rL^+ - \{R8\}$ (called rL henceforth) even proves to be complete with respect to general situations.

Theorem 7 *The system* rL *is complete with respect to general consequence.*

One advantage of the more general setting is that it permits useful proof techniques. Furthermore the system of rules happens to coincide with one already existing. One way to reveal this is to translate general situation models into 4-valued total ones. Then the earlier possible sets of truth-values $\emptyset, \{0\}, \{1\}, \{0,1\}$ are identified with the new values

[6] Notice that this fact is independent of the kind of model employed, as long as the truth conditions for negation stay the same. Moreover, the proposition can be easily generalized for consequence between arbitrary sets.

$\frac{1}{2}$, 0, 1 and 2 respectively. So the new value 2 is used if p was overdefined in s, and $\frac{1}{2}$ when p is underdefined. The truth-tables then turn out to be:

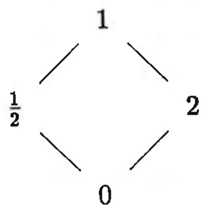
$\neg$	
0	1
$\frac{1}{2}$	$\frac{1}{2}$
1	0
2	2

$\wedge$	0	$\frac{1}{2}$	1	2
0	0	0	0	0
$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0
1	0	$\frac{1}{2}$	1	2
2	0	0	2	2

$\vee$	0	$\frac{1}{2}$	1	2
0	0	$\frac{1}{2}$	1	2
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1
1	1	1	1	1
2	2	1	1	2

$\rightarrow$	0	$\frac{1}{2}$	1	2
0	1	1	1	1
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1
1	0	$\frac{1}{2}$	1	2
2	2	1	1	2

These tables correspond to those given in [Belnap77] for so-called relevance logic.[7] In fact what can be shown is that the logic of general situations provides the rules for his relevance logic! (...and that is why we dubbed the logic rL.) Instead of giving a syntactic proof of the equivalence, we shall relate the semantics of both systems directly. In order to do this, we have to indicate some features of Belnap's approach. The four truth-values are (logically) partially ordered by $0 \leq \frac{1}{2} \leq 1$ and $0 \leq 2 \leq 1$, usually displayed in the (logical) lattice:



Entailment, i.e. relevant consequence between φ and ψ, then holds iff for every model $\langle S, V \rangle$ and every $s \in S$: $\overline{V}(\varphi, s) \leq \overline{V}(\psi, s)$ where $\overline{V}$ extends V to arbitrary formulas. We shall use the notation $\varphi \leq \psi$ for 'φ entails ψ'. Now if we compare $\leq$ to verifiable consequence, at first sight the two notions seem to diverge. If $\overline{V}(\varphi, s)$ and $\overline{V}(\psi, s)$ have the values $\frac{1}{2}$ and 0 (or: $\frac{1}{2}$ and 2, or: 1 and 2) respectively, then s is no counterexample for $\varphi \models \psi$, but *is* a counterexample for $\varphi \leq \psi$. What is at stake here, is that we are comparing the two notions of consequence *locally*, on just one situation, although they deal with ***global*** facts: we have to consider ***all*** models together. From this correct global perspective we can argue as follows: assume one of the three problematic cases to occur, e.g. the first case. Then although s is not a counterexample to $\varphi \models \psi$, it is one for $\neg\psi \models \neg\varphi$, so by propositions 3 and 5, $\varphi \models \psi$ cannot hold. In the other cases, and in the other direction, we argue analogously. So, $\models \Leftrightarrow \leq$.

2.3 Alternative: intuitionistic logic

The systems of 'pure partial logic' studied in this section can be applied and compared with existing proposals. Due to our interest in modal logic we pick one well-known theory with a clear intensional flavour: good-old intuitionistic logic.[8]

As such the semantical approaches of Beth and Kripke are clearly not partial. Still it is said that intuitionism is to be understood partially: the constructive method urges partiality. So let us face the semantics of [Kripke65] in some more detail to settle the case.

[7] In [Belnap77, page 13] the truth table for negation is different in a way that contradicts his monotonicity constraint; so this deviation is likely to be erroneous.

[8] The full paper also deals among other things with the extensional approach of [Blamey86], the early [Humberstone81], supervaluations, and Veltman's work on data semantics.

Kripke proposes using possible worlds semantics; this step was initiated by Gödel's reduction of intuitionistic logic to a fragment of the modal system **S4**. The translation $'$ is defined recursively by:

$$p' = \Box p; \quad (\neg\varphi)' = \Box\neg\varphi'; \quad (\varphi \to \psi)' = \Box(\varphi' \to \psi');$$
$$(\varphi \wedge \psi)' = \varphi' \wedge \psi'; \quad (\varphi \vee \psi)' = \varphi' \vee \psi'.$$

Now let $\langle W, R, V\rangle$ be a fixed possible worlds model; then, for any $w \in W$ intuitionistic and intensional truth should correspond:

$$w \Vdash \varphi \Leftrightarrow w \models \varphi'$$

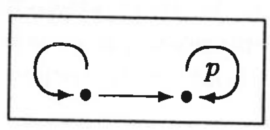
The Kripkean truth condition for $\Vdash$ can then be derived as follows:

$$\underline{w \Vdash p} \Leftrightarrow w \models \Box p \Leftrightarrow \forall v\ wRv : v \models p \Leftrightarrow w \models p \Leftrightarrow \underline{V(p,w) = 1}$$
$$\underline{w \Vdash \neg\varphi} \Leftrightarrow w \models \Box\neg\varphi' \Leftrightarrow \forall v\ wRv : v \not\models \varphi' \Leftrightarrow \underline{\forall v\ wRv : v \not\Vdash \varphi}$$
$$\underline{w \Vdash \varphi \wedge \psi} \Leftrightarrow w \models \varphi' \wedge \psi' \Leftrightarrow w \models \varphi'\ \&\ w \models \psi' \Leftrightarrow \underline{w \Vdash \varphi\ \&\ w \Vdash \psi}$$
$$\underline{w \Vdash \varphi \vee \psi} \Leftrightarrow w \models \varphi' \vee \psi' \Leftrightarrow w \models \varphi'\ or\ w \models \psi' \Leftrightarrow \underline{w \Vdash \varphi\ or\ w \Vdash \psi}$$
$$\underline{w \Vdash \varphi \to \psi} \Leftrightarrow w \models \Box(\varphi' \to \psi') \Leftrightarrow \forall v\ wRv : v \models \varphi' \Rightarrow v \models \psi' \Leftrightarrow$$
$$\underline{\forall v\ wRv : v \Vdash \varphi \Rightarrow v \Vdash \psi}$$

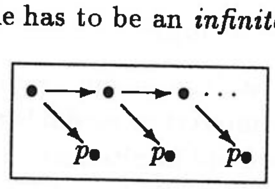
To justify the basis step we need some assumptions:

- R is *reflexive*; but this holds since the modal engine is the system **S4** – so actually R will be *transitive* too;
- V is *positive persistent* with respect to R, i.e. if wRv and $V(p,w) = 1$, then $V(p,v) = 1$.

The modal reduction has some clear advantages: the models have a transparant, concise form, and standard techniques become available. To illustrate the latter point we notice that both Kripke and Beth favour an account where the models are *trees*. To wit, the famous Kripkean counterexample to $p \vee \neg p$ is:



In [Beth56] such a counterexample has to be an *infinite* tree[9]:



Now the interesting point is that, even disregarding the subtleties of Beth's approach, one can see immediately how to derive the infinite tree from Kripke's countermodel: just apply *unravelling* (which already provides a Beth-style counterexample), and then 'prune' it by contracting the non-branching subtrees on which p is always true. Moreover, one can

[9]Beth's actual counterexample is a bit more complicated, but in an inessential way.

obtain a Kripke-style tree model from a structural model by ***bulldozering*** it into a partial order. Finally, a Beth counterexample can be turned into a Kripkean countermodel by *filtration* over a suitable set — in the above case $\Box\neg\Box p$ and its subformulas.

At this point the reader may wonder why we discuss intuitionistic logic at all: the given semantics is clearly total and intensional in character, so the proper place for a discussion would be the next section, or not even there. Although we maintain that formally Beth and Kripke use total models, we have to admit that their motivation is intrinsically partial. The counterexamples given to ***tertium non datur*** show that it is possible that neither a formula nor its negation is true. Furthermore we notice that 'positive persistence' points at implicit partiality. Finally, we notice that the idea of set-theoretic *forcing* may have triggered a partial reinterpretation of intuitionistic logic.

So the problem becomes: can we give an explicitation of Beth's and Kripke's ideas within the partial framework? Actually the solution to this problem is already indicated by one of Kripke's remarks. Kripke notices that $V(p, w) = 0$ should not be read as 'p has been proved *false* at w', but as 'p has not (yet) been proved, verified'. We will implement this idea as follows: let V be a partial function of atom-world pairs to values in $\{1\}$, i.e. a ***partial 1-valued function***. Notice that no stipulation with regards to persistence is needed, once we replace the relation of accessibility by that of extension. So what is really different then from standard accounts of partiality is in the truth conditions, which now become:

$$
\begin{aligned}
&s \models p \Leftrightarrow V(p, s) = 1 \\
&s \models \neg\varphi \Leftrightarrow \forall s' \sqsupseteq s : s' \not\models \varphi \\
&s \models \varphi \wedge \psi \Leftrightarrow s \models \varphi \;\&\; s \models \psi \\
&s \models \varphi \vee \psi \Leftrightarrow s \models \varphi \textit{ or } s \models \psi \\
&s \models \varphi \to \psi \Leftrightarrow \forall s' \sqsupseteq s : s' \models \varphi \Rightarrow s' \models \psi
\end{aligned}
$$

In the next section the combination of partiality and intensionality will be exploited more intensively.

3 Modal logics

Situational model theory for modal logics combines partial propositional valuation and the possible worlds approach to modalities. So a modal situation model is a Kripke model with a partial valuation. More precisely, a partial Kripke model is a triple $\langle S, R, V\rangle$, where $R \subseteq S \times S$ is an accessibility relation and V a partial interpretation of atom-world pairs. A modal model M is *coherent* is all its situations are, i.e. V is a partial function into $\{0,1\}$. Likewise, M is *general* if V is multi-valued.

The truth conditions for the connectives are as stated in section 2.1. In addition the most plausible conditions for the modal operators $\Box$ and $\Diamond$ are: ($M = \langle S, R, V\rangle$ is fixed in the following evaluations)

$$\begin{array}{ll} s \models \Box\varphi \Leftrightarrow & \text{for every } t \text{ such that } sRt : t \models \varphi \\ s \dashv\!\!\mid \Box\varphi \Leftrightarrow & \text{for some } t \text{ such that } sRt : t \dashv\!\!\mid \varphi \\ s \models \Diamond\varphi \Leftrightarrow & \text{for some } t \text{ such that } sRt : t \models \varphi \\ s \dashv\!\!\mid \Diamond\varphi \Leftrightarrow & \text{for every } t \text{ such that } sRt : t \dashv\!\!\mid \varphi \end{array}$$

3.1 Possible worlds revisited

Before we turn to truly partial models we reinspect the possible world semantics. Why do this? Surely, verification and non-falsification amount to the same for classical Kripke models. And the set of valid formulas is consequently the same in both perspectives, viz. the simple normal system **K**. This system is axiomatized by e.g.:

(pL) the axioms and rules (especially *modus ponens*) of pL;

(K) $\vdash \Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$; [10]

(N) if $\vdash \varphi$ then $\vdash \Box\varphi$.

The only mode of verification of rules used to be the *absolute* approach. But even then we have to be careful to add the principle of absolute closure introduced in section 2.1 — otherwise not all absolutely valid modal rules are derivable from **K**). In the sequel we shall use '**K***' to denote the augmented absolute system.

More important is that already for classical possible worlds the relative (or, equivalently, the mixed) approach yields a system of rules different from **K**. The point is that the nature of the rules is essential in modal logic. With respect to N we note that $\varphi \Rightarrow \Box\varphi$ is valid when construed as an absolute rule, but not as a relative rule, i.e.

$$\varphi \vdash \Box\varphi$$

is **not** a rule of the relative system. The reader is confronted with the paradoxical situation that N can be involved in deriving valid formulas, though it does not qualify as a valid rule itself (cf. the similar case expressed by theorem 5).

But what is the characterizing system for relative rules? Apart from pL one needs the following (meta-)rules:

[10] Although the formal language does not contain $\to$ we can reconstruct $\wedge$ and $\vee$ in terms of $\neg$ and $\to$ and derive all valid formulas this way; alternatively, one might prefer to replace (K) directly by $\Box\varphi \vee \Diamond\psi \vee \Diamond(\neg\varphi \wedge \neg\psi)$.

(I_r) if $\varphi \vdash \psi$ then $\Box\varphi \vdash \Box\psi$ [11]

(C_r) $\Box\varphi \wedge \Box\psi \vdash \Box(\varphi \wedge \psi)$

However, valid principles like $p \Rightarrow \Box(p \vee \neg p)$ are not derivable from the combination of pL, I_r and C_r alone. In fact we have to put back N, again construed as a relative rule but now properly:

(N_r) $\top \vdash \Box\top$.

where $\top$ is some tautology or other, e.g. $p \rightarrow p$. Now let $\mathbf{K}_r$=pL + I_r + C_r + N_r. To motivate our nomenclature, notice that axiom K relativized as $\Box\varphi, \Box(\varphi \rightarrow \psi) \vdash \Box\psi$ is derivable from $\mathbf{K}_r$.

This enables us to formulate and prove a completeness theorem for the set of valid relative rules on ordinary Kripke models. Regardless of the type of evaluation (VERIF or FALSIF) we obtain the following state of affairs.

Theorem 8 *For modal languages the possible world semantics with relative or mixed validity is complete with respect to system* $\mathbf{K}_r$.

In all, the resulting logic is pretty much like the old system **K** where absolute rules are circumvented. More changes are to be expected for partial or incoherent models.

3.2 Coherent modal models

Especially for epistemic applications coherent modal models are of interest: we cannot have inconsistent knowledge.

Modal verification

As in the propositional case, the absolute and mixed approaches are hardly interesting for verification on (partial) worlds: the empty set of validities induces for example the total or empty set of rules. So we focus on the relative perspective.

Fortunately the relative system is interesting and certainly nontrivial in its deductive properties. With regard to the problem of completeness, it is clear the rule system should contain rL^+(*vide* section 2.1), and a number of characteristic modal rules (R11–19 below), together forming the system MrL^+.

(R11) $\Diamond\neg\varphi \dashv\vdash \neg\Box\varphi$

(R12) $\Box\neg\varphi \dashv\vdash \neg\Diamond\varphi$

(R13) $\Box\varphi \wedge \Box\psi \vdash \Box(\varphi \wedge \psi)$ (called C_r before)

(R14) $\Diamond(\varphi \vee \psi) \vdash \Diamond\varphi \vee \Diamond\psi$

(R15) if $\varphi \vdash \psi$ then $\Box\varphi \vdash \Box\psi$ (called I_r before)

(R16) if $\varphi \vdash \psi$ then $\Diamond\varphi \vdash \Diamond\psi$

(R17) $\Box\varphi \wedge \Diamond\psi \vdash \Diamond(\varphi \wedge \psi)$

[11] I_r and C_r are the relative counterparts of (R)I: $\vdash \varphi \rightarrow \psi \Rightarrow \vdash \Box\varphi \rightarrow \Box\psi$ and C: $\vdash (\Box\varphi \wedge \Box\psi) \rightarrow \Box(\varphi \wedge \psi)$.

(R18) $\Box(\varphi \vee \psi) \vdash \Diamond\varphi \vee \Box\psi$

(R19) $\Diamond(\varphi \wedge \neg\varphi) \vdash \psi$ (*modal ex falso*).

Comments:

1. Notice that most rules of MrL^+ come in dual pairs; except for the 'ex falso' principles R8 and R19, all displayed rules are subject to *contraposition* in the sense that if $\varphi \vdash \psi$ then also $\neg\psi \vdash \neg\varphi$.
2. We conjecture that the rules of the system are independent, although we do not have a proof of this. The conjecture is based on abortive attempts to reduce the system, as well as on the fact that dual pairs are not interreducible since contraposition does not hold in general. Anyway, independence is of minor importance compared with consistency and completeness.
3. The rules obtained when disjunction and conjunction in R13 and R14 are interchanged, as well as the converses of R13 and R14 also hold (the derivations use R15 and R16). We shall demonstrate the latter:

1	$\varphi \vdash \varphi \vee \psi$	[R5]
2	$\Diamond\varphi \vdash \Diamond(\varphi \vee \psi)$	[1, R16]
3	$\Diamond\psi \vdash \Diamond(\varphi \vee \psi)$	[by analogy]
4	$\Diamond\varphi \vee \Diamond\psi \vdash \Diamond(\varphi \vee \psi)$	[2,3, R6]

4. R18 is equivalent to $\Box(\varphi \to \psi) \vdash \Box\varphi \to \Box\psi$, which again is a relative counterpart of the K-axiom, yet different from the weaker one encountered in subsection 3.1

We arrive at our prime result.

Theorem 9 *On coherent situations* VERIF_{rel} *is sound and complete with respect to the modal system* MrL^+, *i.e.* $A \models \varphi \Leftrightarrow A \vdash_{\mathrm{MrL}^+} \varphi$.

sketch of proof: It is straightforward to check the soundness of the semantics. Completeness can be shown by a Henkin-style proof. First define some relevant syntactic notions (let Σ be an arbitrary set of formulas):

- Σ is *consistent* (w.r.t. MrL^+) iff $\Sigma \nvdash \varphi \wedge \neg\varphi$;
- Σ is *saturated* (w.r.t. $\vee$) iff *if* $\varphi \vee \psi \in \Sigma$ *then:* $\varphi \in \Sigma$ *or* $\psi \in \Sigma$;
- Σ is a *theory* (w.r.t. MrL^+) iff $\Sigma \vdash \varphi$ implies $\varphi \in \Sigma$.

A consistent and saturated theory is called a CST for short. Then the canonical model $\langle \mathcal{S}, \mathcal{R}, \mathcal{V} \rangle$ is defined by: (especially the 'twofolded' definition of $\mathcal{R}$ is crucial)

- $\mathcal{S} = \{\Gamma \mid \Gamma \text{ is a CST}\}$;
- $\Gamma\mathcal{R}\Delta$ iff
  - $\Gamma, \Delta \in \mathcal{S}$,
  - $\Box\varphi \in \Gamma$ implies $\varphi \in \Delta$ for all φ, and
  - $\varphi \in \Delta$ implies $\Diamond\varphi \in \Gamma$ for all φ;

- $\mathcal{V}(p,\Gamma) = 1$ iff $p \in \Gamma$; $\mathcal{V}(p,\Gamma) = 0$ iff $\neg p \in \Gamma$.

Then the usual truth/falsity lemma follows:

Lemma 1 *For all* $\Gamma \in \mathcal{S}$: $\Gamma \models \varphi$ *iff* $\varphi \in \Gamma$; $\Gamma \Dashv \varphi$ *iff* $\neg\varphi \in \Gamma$.

The modal steps of the inductive proof of lemma 1 need some additional lemmas:

Lemma 2 $\Diamond\varphi \in \Gamma$ *iff for some* Δ *such that* $\Gamma\mathcal{R}\Delta$: $\varphi \in \Delta$.

[To show the difficult direction assume that $\Diamond\varphi \in \Gamma$. The required CST Δ is then defined step by step (starting with $\{\delta \mid \Box\delta \in \Gamma\} \cup \{\varphi\}$) such that for all ε: $\Delta \vdash \varepsilon \Rightarrow \Diamond\varepsilon \in \Gamma$. [12] In particular this property implies that $\Gamma\mathcal{R}\Delta$.]

Lemma 3 $\Box\varphi \in \Gamma$ *iff for all* Δ *such that* $\Gamma\mathcal{R}\Delta$: $\varphi \in \Delta$.

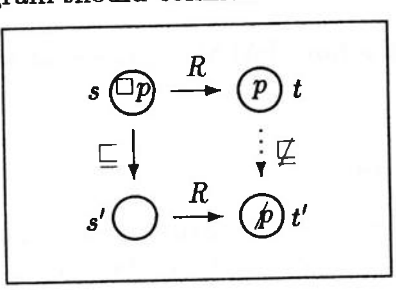
[Here the difficult side is from the right to the left. By contraposition, assume that $\Box\varphi \notin \Gamma$. Next construct a CST Δ, starting with $\{\delta \mid \Box\delta \in \Gamma\}$, such that for all ε: $\Delta \vdash \varepsilon \vee \varphi \Rightarrow \Diamond\varepsilon \in \Gamma$. This subtle choice implies both $\Gamma\mathcal{R}\Delta$ and $\varphi \notin \Delta$.]

The rest of the proof of theorem 9 is rather easy: assume that $A \not\vdash \varphi$ then a similar, yet simpler construction, shows there exists a CST $\Sigma \supseteq A$ such that $\varphi \notin \Sigma$. So by the truth lemma $\Sigma \models A$ and $\Sigma \not\models \varphi$, ergo $A \not\models \varphi$. ■

Modal falsifiability

Since for pure propositions FALSIF on coherent situations leads to classical pL, one expects a similar classical result for modal formulas. One is not disappointed in one's expectations: for absolute falsificational validity the coherent semantics yields system **K***.

A prerequisite for a proof of this fact is to reinspect some properties discussed in sections 2.1 and 2.2. Do *coherence* and *persistence* hold for modal logic? Yes and no! Yes, a coherent **model** will be coherent with respect to arbitrary formulas; in fact, unless we change our truth/falsity conditions for modal operators into highly counterintuitive clauses, modalities will not disturb propositional coherence. No, a coherent **situation** can be incoherent with respect to a modal formula if there exists an accessible incoherent situation. The obvious reason for this deviation is that the modal truth/falsity clauses have a genuine *global* nature, where the definition of *coherent situation* is entirely local. Something quite similar can be said for *totality* and *persistence*. To illustrate violation of general persistence, imagine the situation where $s \models \Box p$, there is only one t such that sRt and $s \sqsubseteq s'$. So $t \models p$. Then for any t' which is R-accessible from s' it should be the case that $t' \models p$, but nothing urges $t \sqsubseteq t'$. In fact there is no compelling reason why the relations in the following diagram should commute.



[12] The essential properties in this and the following subproof were suggested to me by Johan van Benthem.

We seem to be in big trouble here: the proof of theorem 4 depended on coherence and persistence, so even pL would be invalidated by the present semantics. But on closer inspection there is no reason to panic: the relevant models are coherent and a rather **weak** sort of persistence suffices; with respect to *model completion* persistence even holds for the modal case. In fact we can prove in general:

Proposition 6 (modal valuation persistence)
Let $M = \langle S, R, V\rangle$, $M' = \langle S, R, V'\rangle$, *and for every* $s \in S : M, s \sqsubseteq M', s$ *(i.e.* M' *extends the valuation of* M*). Then persistence holds 'pointwise': if* $M, s \models \varphi$ *then* $M', s \models \varphi$ **and** *if* $M, s \dashv \varphi$ *then* $M', s \dashv \varphi$

Theorem 10 *For modal logic the coherent semantics with absolute falsificational validity is complete with respect to modal system* **K***.

In fact this theorem allows for an obvious extension to normal systems:

Theorem 11 *The normal modal systems are captured by the standard Kripkean conditions on accessibility.*

So the tenor of all this is similar to that for the purely propositional case: there is no descriptive difference between standard Kripke semantics and partial semantics employing the 'never false' concept of validity and the *relative* notion of rule. However, there may still be important differences between both approaches. One may be that of the greater intuitive ('realistic') appeal of situations. Another the greater computational possibilities of the partial appoach: the points (worlds or situations) of the partial models can now be specified by considerably smaller sets – moreover a partial model characterising (here in the sense of nonfalsifying) a formula may consequently contain less points than the classical model doing the same job.

In section 3.1 we found that rules can behave very differently for modal languages. Of course this deviation is maintained for FALSIF when we move from possible worlds to (coherent) situations. Especially rule N again does not hold relatively or mixed: $\varphi \not\vdash \Box\varphi$.

Compared with MrL^+ it should be clear that R8 and R19 do not hold anymore. So let MrL^* be the system where R8 and R19 are replaced by R8* and R19* (which result from the *ex falso* rules by contraposition):

(R8*) $\psi \vdash \varphi \vee \neg\varphi$ (*tertium non datur*)

(R19*) $\psi \vdash \Box(\varphi \vee \neg\varphi)$ (*modal tertium non datur*)

Theorem 12 *On coherent situations* $FALSIF_{rel}$ *is sound and complete with respect to the modal system* MrL^*.

3.3 Modal situation models

Releasing the restriction to coherence, the Kripke frame can be supplemented with a partial, possibly incoherent valuation. We arrive at the notion of a (general) modal situation model.

Definition 9 (modal situation model) *A structure $\langle S, R, V\rangle$ is a modal situation model if $R \subseteq S \times S$ and $V : Prop \times S \longrightarrow \mathcal{P}(\{0,1\})$.*

The truth conditions are as before, i.e. the general situation clauses for atoms, the standard ones for the connectives and the partial Kripkean ones for modal operators. Moreover, the definitions for the options with respect to validity and rules are unchanged.

Similar to the propositional case the number of possible systems allowed by the different types of validity and rule definitions is immediately reduced by the following two facts:

- there are still no valid formulas in this semantics. (cf. theorem 6);
- dualization also holds for the modal language (cf. proposition 4); in the revised definition the added accessibility relation is kept fixed.

Consequently, the important reduction of falsifiability to verification expressed earlier in proposition 5 is maintained. Moreover, by their very nature, ***absolute*** and ***mixed*** verification of rules only produce the extreme rule systems **R** and $\emptyset$, respectively. So what remains to be inspected is ***relative verification***. Compared with coherent models we notice that neither the pair R8, R19 nor the pair R8*, R19* hold. Analogous to the propositional case elimination of these rules supplies a complete set of rules. Thus let MrL, the modal counterpart of relevance logic, be triggered by the description of MrL^+ minus {R8, R19}.

Theorem 13 *The modal logic for general relative validity is* MrL.

The completeness proof is a modification of that for theorem 9. ■

With this result another route to our earlier proof for MrL^+ becomes available: show for lemmas 2 and 3 that the addition of axioms R8 and R19 brings about the consistency of Δ, given the consistency of Γ. We avoided such a reformulation for expository reasons: the earlier proof is surely more transparent.

Apart from establishing results such as theorem 13 for the full class of models one can sometimes show facts about subclasses more easily. To prove theorem 12 notice that (relative) falsification on coherent situations amounts to (relative) verification on total ones.

Proof of theorem 12: Using modal dualization we can transfer coherent models into total ones, and simultaneously interchange 'true' and 'not-false'. Then it suffices to show that the canonical modal is total, especially that the Δ's constructed in the lemmas conserve totality. ■

3.4 Alternatives

We will now consider some alternatives to our standard theory. In the literature one can find proposals to vary the truth conditions, the notion of validity and the art of the accessibility relation. Some of these proposals are rather eclectic or even ***ad hoc*** in nature.

3.4.1 Levesque

[Levesque84] provides a challenging logic for explicit belief (B). On the one hand Levesque wants to retain classical propositional logic as well as its modal instantiations, e.g. $B\varphi \vee \neg B\varphi$. On the other hand he wants to get rid of *logical omniscience* — here paradigms are results of rule N and axiom scheme K such as $B(\varphi \vee \neg\varphi)$ and $(Bp \wedge B(p \rightarrow q)) \rightarrow Bq$.

To achieve this Levesque uses a rather intricate semantics. A model in his semantics is essentially[13] of the form $\langle S, \mathcal{B}, V\rangle$, where S is an arbitrary set of (general) situations, $\mathcal{B} \subseteq S$ a set of believed situations, and V a (possibly partial and incoherent) valuation function.

The truth and falsity conditions are standard, except for the case:

$$M, s \mathrel{=\!\!\!|} B\varphi \quad \text{iff} \quad \text{for all } t \in \mathcal{B} : M, t \not\models \varphi.$$

The obvious effect of this stipulation is that with regard to modalized formulae (of the form $B\varphi$) the semantics is bivalent, 'classical' again.

The sort of validity involved is ***absolute verification***, but now ***restricted to possible worlds***. This move ensures that the classically valid propositional (without B) formulae are also valid in this semantics. Still, in the context of partial logic such invocation of (total and coherent) worlds is a rather desperate thing to do. One should desire a less severe restriction here. Our earlier results in sections 2.1 and 3.2 suggest that one might replace 'worlds' by 'coherent situations' here, once VERIF is replaced by FALSIF. A complication for the justification of this change is that the semantics is *non-persistent* (cf. the above clause for the falsity of $B\varphi$); so a mere completion of a countermodel analogous to our earlier proofs will not suffice. But we can do better than that.

Proposition 7 *In the Levesque semantics a formula is always true on possible worlds iff it is never false on coherent situations.*

Proof: From the right to the left this is trivial: every falsifying world is also a coherent situation. To show the other direction, let $M = \langle S, \mathcal{B}, V\rangle$ be a falsifying model, i.e. there is a coherent $s \in S$ such that $M, s \mathrel{=\!\!\!|} \varphi$. Let s^* be a completion of s, and $M^* = \langle S \cup \{s^*\}, \mathcal{B}, V^*\rangle$, where V^* restricted to S equals V. Then, by induction on the structure of φ, $M, s \mathrel{=\!\!\!|} (\models)\psi \Rightarrow M^*, s^* \mathrel{=\!\!\!|} (\models)\psi$.[14] ∎

This rather heterogenous approach eliminates at least some types of logical omniscience: N does not hold since e.g. $p \vee \neg p$ is valid, but in the model $\langle\{s\}, \{s\}, V\rangle$ where $V(p, s) = \emptyset$, it is the case that $s \mathrel{=\!\!\!|} B(p \vee \neg p)$. The counterexample for K is the model $\langle\{s, t\}, \{t\}, V\rangle$ with $V(p, s) = \emptyset$, $V(q, s) = \emptyset$, $V(p, t) = 2$, and $V(q, t) = \emptyset$, then $s \models Bp$, $s \models B(p \rightarrow q)$ but $s \mathrel{=\!\!\!|} Bq$, consequently $s \mathrel{=\!\!\!|} (Bp \wedge B(p \rightarrow q)) \rightarrow Bq$.

The deductive system Levesque proposes has the following features:

- the (modal instantiations) of the pL axioms; and Modus Ponens;

[13] The actual formulation in [Levesque84] makes use of two functions $\mathcal{T}$ (for 'truth') and $\mathcal{F}$ (for 'falsity') from *Prop* to $\mathcal{P}(S)$; the obvious correspondence with our formulation is given by: $s \in \mathcal{T}(p) \Leftrightarrow 1 \in V(p, s)$, and $s \in \mathcal{F}(p) \Leftrightarrow 0 \in V(p, s)$.

[14] Notice that the modal steps work out right because we left $\mathcal{B}$ unchanged.

- $\vdash B\varphi \to B\psi$ iff $\varphi \vdash_{\mathrm{rL}} \psi$

In effect Levesque proceeds by applying the last rule directly to a set of rules ('axioms') for entailment equivalent to our rL and intends to use the derived formulas instead of the last rule. However his list contains the formula $(B\varphi \wedge B\psi) \to B(\varphi \wedge \psi)$, which does not result from the rule. Unfortunately the obvious alternative for Levesque's last rule

$$\vdash \varphi \to \psi \quad \text{iff} \quad \varphi \vdash_{\mathrm{MrL}} \psi$$

does not hold: due to rule R18 of MrL this yields K. So we should add **C** to our list of axioms for the logic of Levesque:[15]

- **(C)** $\vdash (B\varphi \wedge B\psi) \to B(\varphi \wedge \psi)$

However the critical reader might notice at this point that the system described thus far is still incomplete: the semantics validates the *introspection* axioms **4** and **5**:

- **(4)** $\vdash B\varphi \to BB\varphi$;
- **(5)** $\vdash \neg B\varphi \to B\neg B\varphi$.

In fact these axioms (especially **5**) are extremely doubtful for the notion of explicit belief under consideration. Levesque eliminates such validities by an *ad hoc* constraint on the syntax to *noniterative* formulas.

It seems to us that we can improve and at the same time standardize Levesque's account here by replacing $\mathcal{B}$ by an accessibility relation R. Without the restriction to noniterative formulas R obviously has to be *transitive* and *euclidian* to mimic the effect caused by Levesque's $\mathcal{B}$.

Instead of mimicking $\mathcal{B}$ we can also demand more sound restrictions on R, such as that it be *serial*. Different from what Levesque thinks, there seems to be some agreement that believing in contradictions should be impossible. However, in order to verify

- **(D)** $\vdash \neg B(\varphi \wedge \neg\varphi)$

not only seriality but also *coherence* is required (see also [F&H85] and [K&T89]).

3.4.2 Muskens

In [Muskens89] various modal operators connected to verbs such as 'know' and 'believe' are analysed in intensional logic with a translation that formalizes a more usual type of modeltheoretic interpretation. Consequently, accessibility is given the same (partial) treatment as other predicates. Although to us such an approach does not seem fully supported by intuition, we surely must consider this alternative too.

Rephrasing matters in our notation, $s \models \Box\varphi$ amounts to urging $[\![\lambda i \forall j(Rij \to \varphi(j))]\!](s) = 1$ and $s \mathrel{=\!\!|} \Box\varphi$ likewise to $[\![\lambda i \forall j(Rij \to \varphi(j))]\!](s) = 0$. Then by the (partial) interpretation rules for application, λ-abstraction, quantification, predication, the clauses for $\to$ stated

[15] The system given in [Levesque84] contains a number of redundancies. Moreover, I have not been able to trace the announced completeness proof, and postpone such a proof and a thorough discussion of vices and virtues of Levesque's paper to another occasion.

in section 2.1, and the redefinition $R^* = (R^-)^c$ [16], this leads to the following normalized truth/falsity conditions for $\Box$ and $\Diamond$:

$$s \models \Box\varphi \Leftrightarrow \text{ for every } t \text{ such that } sR^*t : t \models \varphi$$
$$s \mathrel{=\!\!|} \Box\varphi \Leftrightarrow \text{ for some } t \text{ such that } sR^+t : t \mathrel{=\!\!|} \varphi$$
$$s \models \Diamond\varphi \Leftrightarrow \text{ for some } t \text{ such that } sR^+t : t \models \varphi$$
$$s \mathrel{=\!\!|} \Diamond\varphi \Leftrightarrow \text{ for every } t \text{ such that } sR^*t : t \mathrel{=\!\!|} \varphi$$

So a model M is now of the form $\langle S, R^*, R^+, V\rangle$. For *coherent* situations we need a restriction on admissible model structures: only models with $R^+ \subseteq R^*$ are allowed. With respect to these models, propositional coherence again entails modal coherence. In fact we can obtain full completeness:

Theorem 14 *With partial accessibility relations,* VERIF_{rel} *on coherent models is sound and complete for the system* $\mathrm{MrL}^+ - \{\mathrm{R18}\}$.

The proof parallels that of theorem 9 but is somewhat easier. The definition of $\mathcal{R}^+$ is still 'double', but that of $\mathcal{R}^*$ is 'single':

- $\Gamma\mathcal{R}^*\Delta$ iff $\Gamma, \Delta \in \mathcal{S}$ and $\Box\varphi \in \Gamma$ implies $\varphi \in \Delta$ for all φ.

The rest of the proof is rather straightforward, so we again omit the details. ∎

Once we allow incoherent situations, as advocated by Muskens, the $\subseteq$-order between the accessibility relations can be entirely dismissed. The proof system is consequently smaller:

Theorem 15 *With partial accessibility relations,* VERIF_{rel} *on general models is sound and complete for the system* MrL $-\{$ R17,R18$\}$.

The canonical $\mathcal{R}^+$ and $\mathcal{R}^*$ are now both defined by 'single' clauses:

- $\Gamma\mathcal{R}^*\Delta$ iff $\Gamma, \Delta \in \mathcal{S}$ and $\Box\varphi \in \Gamma$ implies $\varphi \in \Delta$ for all φ;
- $\Gamma\mathcal{R}^+\Delta$ iff $\Gamma, \Delta \in \mathcal{S}$ and $\varphi \in \Delta$ implies $\Diamond\varphi \in \Gamma$ for all φ.

The separated clauses avoid interference of $\Box$'s with $\Diamond$'s in the sublemmas, which trivializes the proof.

Although the general version of Muskens's system is more elegant and already indicates the feasibility of its being made part of a typed partial logic, we feel that the coherent version, which should be preferable for modelling epistemic attitudes, falls short of being transparent.

[16] R^+ is the so-called *(positive) extension* of R, and R^* the *complement of the anti-extension*.

4 Conclusion and summary

Different values of semantic parameters (such as validity, kind of model, type of rule) give various systems of logic. The results are summarized in tables 1 and 2 at the end of this paper. Space precludes a lengthy evaluation of all this, so we will just mention some features not encountered in the main text.

First, notice we have achieved symmetry in these tables by fully exploring ***total situations***. The duality operation (section 2.2) transforms coherent situations into total ones, and vice versa; meanwhile switching from verification to falsifiability then suffices.

Second, there is one possibility which has been overlooked so far: nothing prohibits an *absolute* type of rule where the validity type is mixed. In this way one can conclude from 'always verified' to 'never falsified', or the other way round. It turns out, however, that this move only provides one new system of a rather pathological nature, where the premises of the conclusion are non-tautologies and the conclusion arbitrary. The default case for relative rules is still the one in which validity is 'straight'; mixed is the exception.

To have the notions of valid formulas and consequence at hand we summarize them: (with shorthand notation in parenthesis)

VERIF $\models \varphi$ iff for all $M, s : M, s \models \varphi$.

VERIF$_{abs}$ $\varphi \Rightarrow \psi$ iff *if* $\models \varphi$ *then* $\models \psi$.

VERIF$_{rel}$ $\varphi \Rightarrow \psi$ iff for all M, s: *if* $M, s \models \varphi$ *then* $M, s \models \psi$. $(\varphi \models \psi)$

VERIF$_{abs,mix}$ $\varphi \Rightarrow \psi$ iff *if* $\not\dashv \varphi$ *then* $\models \psi$.

VERIF$_{rel,mix}$ $\varphi \Rightarrow \psi$ iff for all M, s: *if* $M, s \not\dashv \varphi$ *then* $M, s \models \psi$.

FALSIF $\not\dashv \varphi$ iff for all $M, s : M, s \not\dashv \varphi$.

FALSIF$_{abs}$ $\varphi \Rightarrow \psi$ iff *if* $\not\dashv \varphi$ *then* $\not\dashv \psi$.

FALSIF$_{rel}$ $\varphi \Rightarrow \psi$ iff for all M, s: *if* $M, s \not\dashv \varphi$ *then* $M, s \not\dashv \psi$. $(\varphi \not\dashv \psi)$

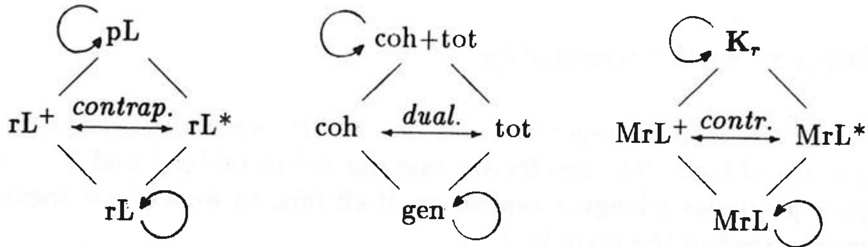
FALSIF$_{abs,mix}$ $\varphi \Rightarrow \psi$ iff *if* $\models \varphi$ *then* $\not\dashv \psi$.

FALSIF$_{rel,mix}$ $\varphi \Rightarrow \psi$ iff for all M, s: *if* $M, s \models \varphi$ *then* $M, s \not\dashv \psi$.

Using (absolute) falsifiability on coherent models we were able to give a partial semantics for the modal system **K**.

Especially relative validity turns out to be interesting. How are the various systems of relative consequence related to each other? Of course restricting the type of situations involved leads to an extended set of valid rules.[17] This state of affairs can be displayed in the following diagram, which contains a triplet of lattice-like structures:

[17] I.e. if $\mathcal{M} \subseteq \mathcal{N}$ then $S_{\mathcal{N}} \subseteq S_{\mathcal{M}}$. For if e.g. $(\varphi \vdash \psi) \in S_{\mathcal{N}}$, $M \in \mathcal{M}$ and $M, s \models \varphi$, then $M \in \mathcal{N}$ and so $M, s \models \psi$.



The logical systems are related by inclusion (lines, upwards) and contraposition (arrows); the types of semantics are also connected by inclusion (lines, downwards) and dualization (arrows). Are these structures really lattices? Yes, although perhaps not in a self-evident way. The problem resides in the lower half of these structures: the intersection of the systems $rL^+$and rL^* (and likewise for MrL^+ and MrL^*) is not the system rL (MrL, respectively) since the following rules are members of the respective intersections, but are not in rL (MrL):

($R8^{+*}$) $\varphi \wedge \neg\varphi \vdash \psi \vee \neg\psi$

($R19^{+*}$) $\Diamond(\varphi \wedge \neg\varphi) \vdash \Box(\psi \vee \neg\psi)$

So if we want the meet operation to correspond with intersection of the full systems, the bottom element of the lattice should be rL^{+*}, which is $rL^+$with R8 replaced by $R8^{+*}$ (and rL could be added below rL^{+*}). [18] We claim that something similar holds for the modal case:

Conjecture 1 *Relative verification for modal models that are either coherent or total is described by* MrL^{+*}*which is* MrL^+ *with rules* R8 *and* R19 *replaced by* $R8^{+*}$ *and* $R19^{+*}$.

Yet the displayed lattices are absolutely correct when not the full systems but their *finite descriptions* (R1–19 and the like) are intended.

Before the lurking danger of self-complacency overtakes us, I have to add hastily that this picture is by no means complete. Although on the dimension of situations we seem to have exhausted the possibilities (but one never knows ...), we surely have not examined every possible notion of validity and consequence, or every possible truth/falsity condition. Perhaps this is intrinsically impossible, since there seems to be no upper bound to the complexity of its definitions. Nevertheless we seem to have succeeded in describing what we consider the standard cases, which should be the gist for any future extension.

Acknowledgement

I am grateful to the responsive audiences of the Amsterdam ITLI 'promovendi colloquium' in November 1987 and the ITK research group on interpretation and representation in Tilburg, May 1989. Especially Johan van Benthem, Frank Veltman and Reinhard Muskens have made very valuable suggestions. Many others ought to be praised for their patience. Because of the limited length of contributions to the present proceedings most of the proofs, applications, and a lot of explanation and elaboration have been omitted from the full paper [Thijsse90]. A modified version of the full paper will be part of my Ph.D. thesis.

[18]cf. rL^{+*} to Blamey's system for 'double-barrelled' consequence on coherent models and the 'mixed' system in [Veltman87].

Table 1: partial propositional logics

	possible worlds	coherent situations	total situations	general situations
VERIF	pL	$\emptyset$	pL	$\emptyset$
VERIF_{abs}	pL*	**R**	pL*	**R**
VERIF_{rel}	pL	rL^+	rL*	rL
$\text{VERIF}_{abs,mix}$	pL*	$\text{pL}^c \times$ *Form*	**R**	**R**
$\text{VERIF}_{rel,mix}$	pL	$\emptyset$	pL	$\emptyset$
FALSIF	pL	pL	$\emptyset$	$\emptyset$
FALSIF_{abs}	pL*	pL*	**R**	**R**
FALSIF_{rel}	pL	rL*	rL^+	rL
$\text{FALSIF}_{abs,mix}$	pL*	**R**	$\text{pL}^c \times$ *Form*	**R**
$\text{FALSIF}_{rel,mix}$	pL	pL	$\emptyset$	$\emptyset$

Table 2: partial modal logics

	possible worlds	coherent situations	total situations	general situations
VERIF	**K**	$\emptyset$	**K**	$\emptyset$
VERIF_{abs}	**K***	**R**	**K***	**R**
VERIF_{rel}	$\mathbf{K}_r$	MrL^+	MrL*	MrL
$\text{VERIF}_{abs,mix}$	**K***	$\mathbf{K}^c \times$ *Form*	**R**	**R**
$\text{VERIF}_{rel,mix}$	$\mathbf{K}_r$	$\emptyset$	$\mathbf{K}_r$	$\emptyset$
FALSIF	**K**	**K**	$\emptyset$	$\emptyset$
FALSIF_{abs}	**K***	**K***	**R**	**R**
FALSIF_{rel}	$\mathbf{K}_r$	MrL*	MrL^+	MrL
$\text{FALSIF}_{abs,mix}$	**K***	**R**	$\mathbf{K}^c \times$ *Form*	**R**
$\text{FALSIF}_{rel,mix}$	$\mathbf{K}_r$	$\mathbf{K}_r$	$\emptyset$	$\emptyset$

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Did the Guns of Navarone Hit Miles Twice?*

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1. The problem

Currently it has become quite fashionable to quantify over events: in the logical representations of sentences like (1) event-variables are included. Thus, (1) is analyzed as (2) or (3) rather than as (4),

(1) Miles hit the ball
(2) ∃e Hit(m, the ball, e)
(3) ∃e (Hit(e) & Agent(m,e), & Patient(the ball, e))
(4) Hit(m, the ball)

The analysis (2) is proposed by Davidson (1967). He treats *hit* as a three-place rather than as a two-place predicate, the latter being standard in atemporal analyses like (4). A variant of (2) is (3). Dowty (1989) calls it neo-Davidsonian: all arguments of the verb *hit* are "spelled out" as conjunctions to the Hit(e)-proposition.

* I would like to thank Johan van Benthem, Jaap van der Does, Michael Moortgat and Remko Scha for the discussions about some of the ideas developed here, Reinhard Muskens for his help in trying to formally express the index-shift in terms of his version of the intensional type-logical language Ty2, and Peter Coopmans for improvements of the prefinal version and for his wish to make certain parts more perspicuous. The present text is an eye bird's view on some of the issues involved. A more detailed treatment of the material discussed here is given in Verkuyl (in preparation.). I also thank Rossana Petrova for her comment on the interpretation of some data.

Parsons (1989) is a neo-Davidsonian. He rejects an interval-based approach and proposes to take events as ontological primitives. Critical to his decision seems to be Bennett's example (5).

(5) Miles was wounded twice (by a bullet) yesterday

Parsons says that "the natural treatment of [(5)] within interval semantics would be to propose something like:

(6) For two distinct intervals, I_1 and I_2, Miles was in the extension of 'be wounded by a bullet yesterday' at both I_1 and I_2" (1989:233)

and says that (6) would entail that the wounds in (5) were not simultaneously inflicted upon Miles, the problem for interval semantics being that there is a natural reading of (5) saying that Miles got his wounds simultaneously. So, for Parsons it seems to be necessary to have a 1:1-relation between an interval and an event.

Another argument against intervals as an essential ingredient for eventhood is the active sentence (7):

(7) Mary and Miles hit the wall

The objection is here that interval semantics cannot distinguish between a situation in which Mary and Miles hit the wall one after the other and the situation in which Mary hit the wall and Miles hit the wall at exactly the same time without their being collectively involved. I heard this sort of objection from Emmon Bach and Remko Scha (pers. comm.): there seems to be no way to distinguish between these two forms of distributivity in terms of intervals.

I do not want to defend interval semantics as it was developed in the literature of the seventies. Here, I simply want to show that there is something dubious about Parsons' argument against (5). His notion of event turns out to be contaminated with agentive germs. To show this, I shall create a simple "Gedankenexperiment" by promoting Miles to the rank of the commanding officer of an American ship sailing in the range of the guns of Navarone during the Second World war. Let *Miles was hit* stand short for *Miles' ship was hit* and let us assume - for the sake of coming very close to (5) - that if a ship is hit by a cannonball, it is wounded.

2. Navarone

According to the well-known movie, the guns of Navarone consisted of two parallel guns, each of which was able to produce a cannonshot on its own, having its own crew. Both crews were commanded by a (German) officer. I will now discuss some hypothetical situations.

Situation 1. The German officer says *Feuer*, the guns fire and Miles is hit by two projectiles. One can speak here of one event involving two hits. So, if one says (8a), it will be interpreted as (8b).

(8a) Miles was hit twice
(8b) Miles got two hits

The use of (8a) is somewhat forced: (8b) is more appropriate. It would be strange to say about the German officer *He is going to hit Miles now for the third time*, whereas it would be quite normal to say *He is going to give Miles his third wound*.

Situation 2. The German officer says *Feuer*, the guns fire at exactly the same moment and hit Miles distinctly one after the other, say half a minute later. What happened for the Germans may still count as one event, so they might even say (the German equivalent of) *We got a hit*, meaning that Miles got two hits at once. For them the Feuer-command means a salvo. What counts for the Germans is the perspective of the commanding officer, not whether the event in question has internal structure: a salvo may count as an event even though its hits do not reach the object at exactly the same time. The hits will be placed in the same time span. Note, however, that some engineer Jones at the lower deck on Miles' ship may have experienced what happened as two separate events, not knowing where the hits came from. If some electronical device on the leading battleship miles away from Miles can register Jones' observations sequentially i.e by registering only one datum after the other, it becomes even more appropriate for someone on this ship to use (8a). He could certainly add: ... *and we are now waiting for the third hit*.

Situation 3. The guns were taken apart and put on a different location, say one mile from each other. They have now different commanders being able to shoot independently from one another according to their own judgment. The two commanders have been given an order by a general to sink Miles. Now, the guns happen to fire one after the other and Miles is hit by two projectiles at exactly the same time. Is it reasonable to speak of one event? It depends on the observer. For Miles (not being able to tell the sound of the two guns apart from that of other guns) it is possible to think of one event consisting of receiving two hits at the same time. For those who do not know about the span of control of the German general, there are two events: they do not take into account coordination of the gun.

Situation 4. This is exactly the same situation as Situation 3, but now the first gun is held by the Germans, the second one is conquered by the English who hit Miles thinking that it was Von Dobschütz they aimed at. It is certainly possible to speak here about two events, even more than in Situation 3, because there is no unifying span of control between the guns. From the point of view of Miles himself it does not really matter who did it.

3. Collective involvement and agency

The four situations make clear that the notion of agency is essential to Parsons' judgment. His interpretation of (5) as pertaining to two events appears to be dependent on whether or not there are two different agents inflicting wounds upon Miles. Indeed, for each individual inflicting a wound upon Miles this can be seen as an event. But from other points of view, e.g. the one taken by Miles, there may be just one event. In other words, Parsons' conclusion that interval semantics cannot account for a situation in which the woundings were simultaneous because (6) is the only means for it to account for (5), is not valid. One and the same situation can be experienced as one event or as more than one event, without there being thematic roles to express whether or not one the speaker wishes to speak about one event or not. Parsons' analysis is based upon secretly taking into account the role of agency. For him, simultaneity cannot go together with two actions involving two different agents with different purposes.

This latter position actually implies that it is not possible to have collective quantification such that the agents involved can act independently from one another. For Parsons, the idea seems to be that if one says

(9) Two men hit the wall

the only way to get a collective reading would be to assume that the two men were involved in the hit-action together, sharing a common goal, in this case to hit the wall. This is the counterpart of the distributive reading in which each of the men is supposed to hit the wall independently from one another. Here again the notion of agency seems to pervade the analysis.

Again there is a difficult problem for Parsons. How to deal with (9)? The only way for him to get a reading in which there are two men hitting the wall at the same time such that it is reasonable to speak about two events, is to assume that the hits are unrelated to one another, in some sense of the word. But how can you do this if the notion of agency is not available any longer? What if the two men are dead bodies? Can there be collective quantification in this case? Yes, Parsons would be ready to say: when the two men happened to hit the wall at the same interval: $I_1 = I_2$. But what about a gust of wind lifting the bodies one after the other against the wall. Here we have two different times, so $I_1 \neq I_2$, and yet we might wish to speak of one event. It is the gust which provides the time span. But note that the wind stays completely out in (8b): there is no thematic role for it in (9). In other words, speakers are able to provide for unifying time spans, if they wish to present something as an event.

This hold also true for a situation described by the Dutch sentence:

(10) Mary and John crossed the border

which may pertain to a situation in which Mary crossed the border at San Diego and John at Buffalo at exactly the same time.[1] If so, (10) can have two interpretations: the first one is in which it expresses one event because Mary and John happen to be part of a master plan directed by someone in New York without any awareness of this on their side. Sentence (10) can also be viewed as pertaining to two events. The same applies to (10) when it is used to report about a border crossing at Buffalo by Mary and John at the same time. Whether or not there are two events also depends on whether or not they belong to a span of control. Even when they sit in one car there might be two events dependent on the perspective one has (John might be a hitch-hiker, for example).

What I would like to point out here is that the internal structure of intervals is far more complicated than was argued for by interval semanticists and that this view together with the view that perspective plays a role in the question of whether or not one speaks about an event, solves a lot of problems allegedly inherent to interval semantics. For example, one can have an event consisting of different subintervals such that one body hit the wall after the other, the notion of event itself being tied up with the sum of the two subintervals. Thus, the apparent counterexample (5) felt by Parsons as crucial and deadly for interval semantics, fails to be convincing: it is possible to analyze (5) such that two hits are assigned to an interval I leaving unspecified whether or not they were simultaneous. In certain situations (5) may be understood as referring to two events, in other situations it may pertain to just one event having two subevents.

In my view, collective quantification should abstract away from agency by dealing with (10) irrespective of the question of whether or not the two men are alive or dead. Then it is possible to analyze (10) as expressing that the two men are assigned to an interval on the basis of a (participancy) equivalence relation which can be circumscribed as 'hit the wall in the same interval I', where I can have *internal temporal structure*. And (5) should be reconsidered such that the concealed agentive analysis cannot apply to it.

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1 In English it is possible to use the plural *the borders* to underscore that there are two borders involved. In the Dutch counterpart *Mary and John passeerden de grens* it would be awkward to do that. In this paper, I abstract from subtle differences like these. My object language is basically Dutch: the English examples all have Dutch counterparts that show the same behaviour as described here.

From the point of view of eventhood it should be irrelevant whether or not the entities fulfilling the role of external argument of *hit* acted as agents and whether or not they cooperated.

4. Events and subevents

Frequency adverbials have an interesting property: in pseudo-clefts they can occur before and after the copula yielding quite different interpretations. Consider the following sentences.

(11) He hit Miles six times
(12) a. What he did was hit Miles six times
b. What he did six times was hit Miles
(13) a. What happened was that he hit Miles six times
b. What happened six times was that he hit Miles

Sentence (11) is ambiguous. The first reading is expressed by (12a) and (13a), the second by (12b) and (13b). In (12a) and (13a), there is one event in which there are subevents (most probably six). So one event can have substructure: it can consist of subevents. In (12b) and (13b), there are six events. During each of these six events Miles can have been hit many times[2]

What we see, is that frequency adverbials like *twice, six times* can operate on events as well as on subevents. Assuming that the intervals out of which events are construed, have filter structure, it is easy to explain why (12) and (13) display the behaviour of frequency adverbials as they do. Technically, this means that an interval is taken as a generalized quantifier, i.e. as a set I of intervals where the internal structure of I is defined such that the interval (0,1) is contained in (0,2), (0,2) in (0,3), and so on. Intuitively, 1, 2, 3, etc. are "counting points", i.e. points at which substructural units (pairs of intervals and hits, in this case) are counted as having been mentally processed. At point 2 a certain number m of hits have been related to time and at point 3 the number is m+k.

```
  1 hit     3 hits        2 hits
0_______1_________2_________3
```

Figure 1

```
  1 hit    10 hits  7 hits  1 hit   2 hits
0_______1______2_____3____4______5____6
```

Figure 2

[2] cf. Verkuyl (1972:165ff.) for the analysis of (11) - (15); cf. also Verkuyl (1980).

Figure 1 represents a possible realization of (13a): in (0,1) he had hit Miles once, in (0,2) 1+3 = 4 times, in (0,3) 4+2 = 6 times. Figure 2 expresses that as far as (13b) is concerned, we need six intervals, each relating to at least one hit. But as said, there can be more hits per interval.

The advantage of taking 1,2,3 etc. as counting points is that this makes it unnecessary to take into account the dense structure of the subintervals. The numbers associated with the intervals belong to the mental organization of the language users: they can treat dense time discretely by using two different ordering systems at once. What we are basically doing is to abstract away from the internal structure of the intervals themselves, because we are interested in properties that are conveyed by the system **N** of natural numbers. This is exactly what makes it possible to map two hits into the same interval with out being forced to assume that the two hits took place at exactly the same time. Moreover, they can follow one another in time without forcing us to say that there are two events involved. At the level of subevents we are simply not interested in too much structure, as shown by (13b) which abstracts away from the possible many hits that he gave Miles each time.

The general conclusion is that intervals do play a decisive rol in the structure of events. The problem raised by Parsons is not a real problem and its solution prevents us from taking into account the interaction between the reals and the naturals involved in the relation between events and subevents, the crucial point being that at the rock bottom of temporal there is always temporal structure modelled on **R** (the reals, or perhaps **Q**, the rationals, cf. Van Benthem 1983), whereas natural numbers give us the possibility to get at higher levels of temporal structure.[3]

5. Intervals as indexes.

There is a promising way of looking at intervals from quite a different point of view than is customary: as "lowest-level" indices. We take indices in whatever form they appear as indicating the way people focus on things they are talking about. In this sense, the study of aspect can be placed in a long tradition: for centuries aspect has been seen in terms of a perspective, a point of view. That intervals can be understood as indices is due to the mapping of an interval to a (mental) counting point: every interval can be assigned a number which serves as its index (we often measure progress in time by simply counting).

Now, let us start out with the Montagovian concept of an index. It is used to break away from the semantically too narrow notion of extension. In

[3] For a more detailed description of the use of two number systems, see Verkuyl (1987), Oversteegen & Verkuyl (1985) and Oversteegen (1989).

sentence (10), a strictly extensional approach to the meaning of the phrase *the border* would be a specific border, say the one at Buffalo (or two borders, one at Buffalo, the other one at San Diego). Of course, this is felt as insufficient, though for many parts of mathematics a purely extensional approach appears to be called for. Montague used indices to define intensions: the meaning of *the border* is certainly not its extension in a certain model but rather a function yielding at every index its extension. The relevant index in (10) is generally taken as a pair consisting of a time and a world, say <i,w>. Thus, if the extension of *the border* is the border at Buffalo at the time i' and world w' when Mary and John crossed it (I take only one reading here), the extension of *the border* at time i'' and world w' could be the Dutch border at Breda, whereas *the border* at time i''' and w'' could be the border of Atlantis. On this view, to know the meaning of a noun phrase like *the border* is to be able to assign to it a proper extension, given an i and a world w.

The preceding sections suggest a different though not unrelated interpretation of the notion of an index. This can be understood if one realizes that the meaning of the VP *to hit six ships* in the sentence is atemporally treated as the set of those entities who hit six ships. But note that this VP is quite vague as to the internal structure of what went on in sentences like (16):

(14) Von Dobschütz hit six ships

This sentence covers a vast range of combinatioral possibilities. One salvo could have hit the six ships, he could have hit them one by one, or in a number of groups whose sum total is six. Note that the number of combinations is fully determined by the cardinality of the direct object NP. It would be very unwise, however, to let the number of readings depend on the cardinality of some NP. Moreover, in view of (11) we also want to be able to speak about one event even though the ships were hit consecutively.

In the theory of generalized quantification determiners like *six* in *six ships* are taken as a functor relating two sets. Thus, a sentence like (15) is interpreted as (16):

(15) Six ships sailed off

(16) $[\![\text{Six}]\!]([\![\text{ships}]\!])\ ([\![\text{sailed off}]\!]) = 1$ iff $|[\![\text{ships}]\!] \cap [\![\text{sailed off}]\!]| = 6$

which says that (15) is true if and only if the intersection of the set of ships and the set of entities that sailed off is six, in the model under consideration. This situation is illustrated in Figure 3a.

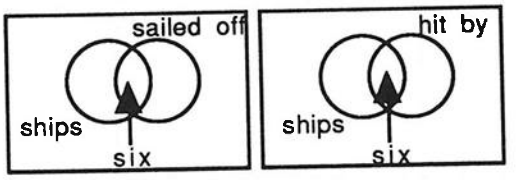

Figure 3 a b

The situation is somewhat more complicated for the analysis of determiners of noun phrases in direct object position, because in (14) the two-place predicate *hit* is involved, though the situation in Figure 3b is quite similar to the one in Figure 3a. The VP *to hit six ships* is atemporally characterized as the intersection of two sets one of which is the set of things hit by entities in the domain, as in (17):

(17) $[\![six]\!]([\![ships]\!])([\![hit]\!]) = \{e \in E: \mid [\![ships]\!] \cap \{d \in E: <e,d> \in [\![hit]\!]\} \mid = 6\}$

Thus, [[to hit six ships]] is the set of all individuals e in E such that the cardinality of the intersection of the set of ships and the things d in E hit by e is six (cf. Van Benthem 1986:60). If (14) is true, Von Dobschütz belongs to the set defined in (17).

Whatever the temporal account of (14) may be, it is clear that the atemporal account of (17) must hold as well, because it can be considered as the *resulting* state of some process that must have taken place: the information associated with the intersection in Figure 3b may contain its history. Let us, for simplicity and concreteness, assume that in a given model Von Dobschütz hit the six ships such that we can distinguish three subsets ordered on the basis of being involved in the predication at the same interval: S_1 (one ship), S_2 (three ships in a salvo), S_3 (two ships) and let us assume that (17) pertains to exactly that situation. Let $\mathbf{S} = \{S_1,S_2,S_3\}$ in this particular model. Note that there are three indices, which may bring about some order, if we want them to do that. In Verkuyl (1987), it was argued that the verb should be interpreted as contributing temporal structure by producing temporal structure comparable to the way in which natural numbers can be produced by the successor's function. Thus, the indices on the sets can be seen as being contributed by the verb, whereas the sum **S** of the three sets S is contributed by the object denotation. In the section 4, I illustrated this machinery pictorially with the help of Figure 1 and Figure 2. We can extend Figure 1 now to Figure 4:

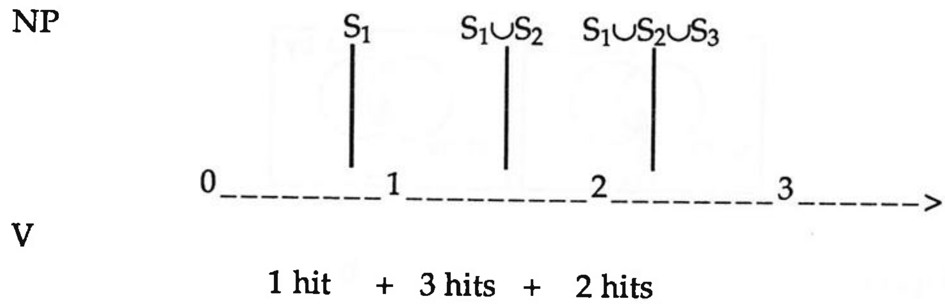

Figure 4

One way to account for the situation we have in mind is to treat the denotation of the VP in a specific model as an indexed family of sets, in general:

(18) $\mathbf{A} = \{A_i\}_{i \in I}$

the index set I being the set of intervals and $A_i \subseteq A$, where A is the denotation of the N which is the head of the direct object NP. Taken in this way, the denotation of VP in a specific model is a function operating on the members of the index set I and yielding sets[4]. On this treatment, if we take J as the set of intervals generated by the verb, in the model under consideration the index set $J = \{i_1, i_2, i_3\}$, so that

(19) $\mathbf{S} = \{S_i\}_{i \in J} = \{S_1, S_2, S_3\}$ where $\bigcup\{S_i\}_{i \in J} = S = \{s1,s2,s3,s4,s5,s6\}$

To say that someone hit s1 first, then s2,s3 and s4, and then s5 and s6, is to say that there is a function f from the set of intervals J into Pow(S) such that $f(J) = \{\langle i_1, S_1\rangle, \langle i_2, S_2\rangle, \langle i_3, S_3\rangle\}$. That is, f(J) is a possible application of this function to the domain, where S is the set of ships in the domain that were hit. Let this be the case. Then it is clear that to say that (19) holds, is to say that f(I) yields a structured temporal entity if we manage to properly define the relationship between the intervals i_1, i_2, and i_3.

We can now reconcile the atemporal treatment of (17) with the temporal approach sketched here. Rather than (17) we obtain (20a) as a more general characterization of the meaning of the VP *to hit six ships*, where **S** is defined as in (20b):

(20) a. $\{e \in E : | \bigcup\mathbf{S} \cap \{d \in E : \langle e,d\rangle \in [\![\text{hit}]\!]\} | = 6\}$
b. $\{S_i\}_{i \in I} = \{S_1, \ldots, S_i, \ldots, S_n\}$ $(1 \leq i \leq n)$

The relation between the [[NP]] and [[V]] illustrated in Figure 4 by lines connecting members of **S** with intervals, can be modeltheoretically defined in terms of an injective function. Let us first take the direction from the NP to

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[4] See Verkuyl, in prep, for a more detailed description.

V. S can be considered the (indexed) quotient set which partitions the set of ships S into subsets (cells) S_i on the basis of the thematic (temporal) equivalence relation Θ. This relation can be described as '(counting as) undergoing predication at the same time interval'. In Figure 4, $S_1 \cup S_2 \cup S_3 = \{s1,\ldots,s6\}$, so $|\bigcup S| = 6$. Thus, S is captured in terms of a well-known standard linguistic concept: 'thematic participancy'. For a more detailed account, see Verkuyl (1987), but the general idea should be clear: the temporal analysis of sentences simply requires that the denotation of an NP is not dealt with holistically. We need temporal substructure to deal with sentences like (5), (10) and (14), and connected with this we need to take into account the way the denotation of the direct object NP is "spread over" this temporal substructure. The inverse way of looking at Figure 4 is to define a mapping from intervals to subsets of S. This follows automatically from taking the denotation of the VP as an indexed family of sets, as pointed out earlier.

An interesting consequence of the present analysis is that Ty2, the intensional two-sorted type-logic, which is both a generalization and a simplification of Montague's IL, can be used to make a point with respect to the perspective taken by speakers of a language. In the version of Ty2 used in Muskens (1989), the derivation *to hit ships* is given in three steps:

(21) a. *six* $\rightsquigarrow \lambda P_1\lambda P_2\lambda i\exists 6y\exists j\subseteq i(P_1(y)(j)\ \&\ P_2(y)(j))$
b. *six ships* $\rightsquigarrow \lambda P\lambda i\exists 6y\exists j\subseteq i(ship'(y)(j)\ \&\ P(y)(j))$
c. *to hit six ships* $\rightsquigarrow \lambda x\lambda i\exists 6y\exists j\subseteq i(ship'(y)(j)\ \&\ hit'(y)(x)(j))$

The VP in (21c) can be read as representing the a property of individuals (x = Von Dobschütz in (14)), or in other words, a function from indices to sets of entities. That is, the VP is of type <e,<s,t>> which is equivalent to <s,<e,t>>. Muskens follows the standard approach of treating members of the type s as worlds. *Six* is a functor of type <<e,<s,t>>,<<e,<s,t>>,<s,t>>> taking properties to yield quantifiers (i.e. denotations of noun phrases). *Three ships* is of type <<e,<s,t>>,<s,t>>.

Now what about interpreting indices as intervals and worlds, dependent on the way we are looking at things? As explained above, (20b) can pertain to the event described in (14). In that case, the indices are intervals. By doing this we construct the denotation of *six ships* stepwise, where the number of steps corresponds with the number of intervals making up temporal substructure. Note also that this also could be the way to handle sentences like (11) *He hit Miles six times*. Taking it as pertaining to one event, we build substructure to accommodate subevents.

What if (18) does not pertain to one event? That is, what if indices are not intervals, but "worlds"? To see what happens, consider the situation in which I am sitting in front of my Apple Macintosh window looking at the set of icons representing the documents of the famous Tarski game invented by J. Barwise and J. Etchemendy. Let us say that we are looking at (the icons of) about twenty documents, as exemplified in Figure 5.

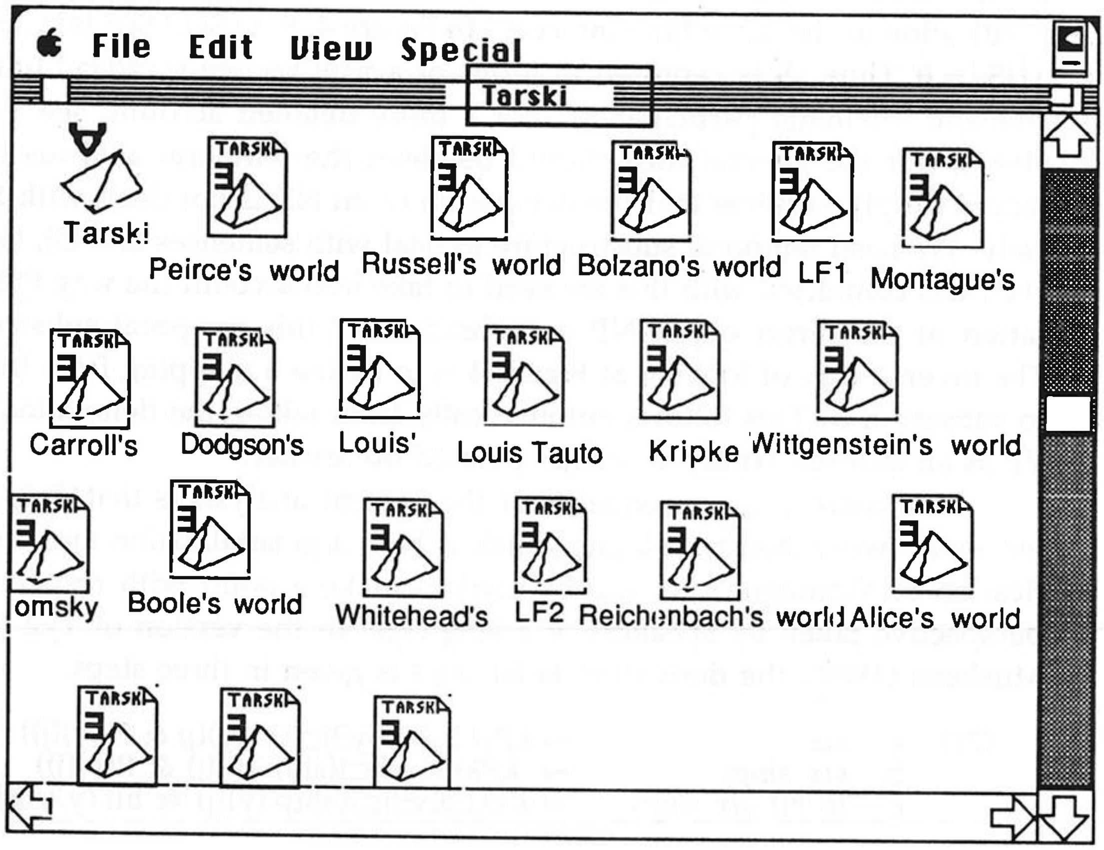

Figure 5

When we open a document (by clicking it twice), we see a world with cubes and other mathematical objects which we are able to introduce, to eliminate and to move. The crux of the game is that we can train ourselves to properly use standard first order logic to speak about the world in question, but this feature of the game does not play a role in the present analysis.

Note that each world (behind a document icon) has a temporal structure in the sense that we can talk about a particular domain in terms of things that happened.Thus I can say about a world w12

(22) I moved the cube to the left of the tetrahedron

This means that I can speak about w12 as if I am part of it. I can report about it. Note that the variable assignment in w12 may differ drastically from the assignments in other worlds. In fact we may assume that all assignments in the set of worlds on the window W = {w1, ... w20} are different from one another.

A sentence like (23)

(23) For half an hour I moved the cube to the left of the tetrahedron.

shows that (22) is terminative if applied to w12: there is a specific cube and a tetrahedron and I moved the cube repeatedly to the left of the tetrahedron.[5] The well-known test with *in a minute* also applies:

(24) I moved the cube to the left of the tetrahedron in a minute.

What happened was that I managed to bring about an event which took place in w12 within the interval of one minute.

Now, it can be observed that I can use (22) to report to someone coming in about a period in which I had opened most of the documents consecutively, having the task to do something with the objects and closing them after I did my job. I am saying (22) now and both of us are looking at the window seeing the twenty icons representing twenty closed documents. What I am doing is to report about my placing the cube to the left of the tetrahedron in those worlds that I opened and in which there was a cube and a tetrahedron.

The important thing to realize is that I am now speaking at a level at which I am bound to abstract away from the individual time structure of particular worlds. Crucial is that (22) is durative as soon as it pertains to my speaking about the icons: the well-known aspectual repetition is absent here. Thus, (23) does not express repetition when interpreted in this way. In short I am reporting about an activity (process) rather than about an event. And the indices I am using now are the worlds w_i rather than the intervals in a specific world.

As said, not every world needs to have a cube or a tetrahedron, so (22) must be analyzed in terms of a paraphrase: whenever I saw a cube and a tetrahedron, I moved the former to the left of the latter. This brings us in a Lewisian-position in which we quantify over occasions (models) rather than over individuals. Thus (23) can be analyzed as (25).

(25) For half an hour it was such that for all pairs x and y, if x the cube and y the tetrahedron, then I moved x to the left of y.

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[5] Rossana Petrova (p.c.) pointed out to me that in Bulgarian (22) when occurring with the durational adverbial assumes two different forms to bring about an interpretation: (a) *As premestvah kubceto naliavo ot piramidata* which together with the durational adverbial conveys the idea of repetition; and (b) *As mestih kubceto naliavo ot piramidata* which together with the durational adverbial expresses a "step-by-step" -process not completed within half an hour. This distinction is not grammaticalized in Dutch, but as is well-known from the aspectual literature a terminative sentence combined with a durational adverbials results either in the interpretation of a forced repetition or in stretching out an event. In Bulgarian, secondary imperfectivization after prefixal perfectivization in the Imperfect tense yields the former effect, whereas the "stretching" is effectuated by imperfective aspect in the Aorist tense.

The new feature with respect to Lewis (1975) is that on top of the universal quantifier over occasions (worlds), there is a durational adverbial that does not operate on the internal structure of the worlds involved. It bears on the temporal structure of the mental model which covers the operations I am describing.

If (22) is used to speak one event in a particular model, we can interpret the index i generated by the verb *move* as an interval as part of the making up of some mental construct which we can interpret as an event. If (22) is used to speak about a situation captured by (25) the index i is of a higher level taking us away from one particular model so that we do not deal with an event at all. What sentences like (22) do in the non-one-model reading is to report about our mental organization of things that happened in "lower level models", which are contracted for the time being. It is exactly this mental jump which makes it profitable to take indices both for intervals at the lowest index-level and for "worlds" (or "occasions", or the like).The question which of them we use as indices depends on the way we want to speak.[6]

If (14) is used to speak about a sum total of ships that were hit by Von Dobschütz, we generalize over several models with different assignments and as a result the aspect will change. Thus, there is a durative reading of (14), the one we obtain if we happen to be interested in the sum total of the ships hit by Von Dobschütz. It is very hard to get this reading with the standard litmus-tests because the introduction of temporal adverbials has the effect of introducing one model. But consider a sentence like (26)

(26) For an hour now Von Dobschütz has hit six ships

and imagine my looking into the archives of the German Navy and reporting between times about the sum total of the hits he made. In this case, (14) must be taken as durative, because *for an hour now* applies to my mental time, and not to the same time structure in which Von Dobschütz made his hits.

As to (5), there is one index i corresponding with a set of two wounds which are mapped to the same interval. They may or may not have hit Miles at exactly the same time. It does not really matter, just as in the interpretation of *Three men crossed the street* there is no point in distinguishing between a situation in which the street is crossed in two groups {a,b} and {c} such that men a happens to reach the other side of the street somewhat earlier than b.This would be too fine-grained. Note, finally, that if we want to

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6 Technically, this means that we do not burden the representational language as Ty2 with a difference between intervals and worlds. This difference is made outside the language itself.

use (5) to report about two distinct events, Miles may have five wounds in sum.

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in prep.*A Theory of Aspect.*

CONDITIONALS, PROBABILITY, AND BELIEF REVISION

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Abstract
A famous result obtained in the mid-seventies by David Lewis shows that a straightforward interpretation of probabilities of conditionals as conditional probabilities runs into serious trouble. In this paper we try to circumvent this trouble by defining extensions of probability functions, called *conditional probability functions*. We further defend the position that rational partial beliefs (about conditionals) are better represented by *sets* of (conditional) probability functions satisfying certain constraints than by a *single* (conditional) probability function. From this position it is possible to distinguish several notions of belief updating, which leads us to question the intimate relation some authors have claimed to exist between the Ramsey test for conditionals and belief revision and to comment on Peter Gärdenfors' recent generalization of Lewis' result.

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1. Introduction

In this paper we adhere to some version of the slogan (*) *probabilities of conditionals are conditional probabilities*. (*) has been proposed by several authors and has played an important role in attempts to give a probabilistic semantics for conditionals. However, as shown by the following famous triviality result of Lewis (1976), a straightforward probabilistic interpretation of conditionals by means of (*) runs into serious trouble:

DEFINITION 1.1 Let P be a probability measure on a σ-algebra Σ of subsets of Ω.

(i) If $P(A) > 0$, then $P(B \mid A) =_{def} P(A \cap B)/P(A)$.

(ii) If $P(A) > 0$, then P_A is the probability measure s.t. $\forall B \in \Sigma\ P_A(B) = P(B \mid A)$.

DEFINITION 1.2 Let Π be a class of probability functions on a σ-algebra Σ of subsets of Ω.

(i) Π is called *closed under conditionalizing* iff $\forall P \in \Pi\ \forall A \in \Sigma$ if $P(A) > 0$, then $P_A \in \Pi$, where P_A is the probability function defined by $\forall B \in \Sigma\ P_A(B) = P(B \mid A)$.

(ii) $\rightarrow$ is called a *probability conditional* for Π iff $\forall P \in \Pi\ \forall A,B \in \Sigma$ if $P(A) > 0$, then $P(A \rightarrow B) = P_A(B)$.

PROPOSITION 1.3 Suppose that $\rightarrow$ is a probability conditional for a class Π of probability measures on a σ-algebra Σ of subsets of Ω and suppose that Π is closed under conditionalizing. Then

(i) there do not exist three pairwise disjoint sets $A,B,C \in \Sigma$ and a $P \in \Pi$ such that $P(A) > 0$, $P(B) > 0$ and $P(C) > 0$.

(ii) every $P \in \Pi$ is at most four-valued.

(Unless indicated otherwise, proofs of propositions can be found in the appendix.)

Adams (1975) concludes from the above that truth-functional compounds of conditionals and iterated conditionals cannot be given probabilistic interpretations by means of (*). This of course seriously limits the range of applicability of the probabilistic analysis of conditionals. In this paper we employ a kind of probability measures, defined in section 3 and called *conditional probability functions*, which do enable us to handle iterated conditionals. In addition, counterfactuals are treated on a par with indicative conditionals.

Although our approach shows some similarities to earlier work like Appiah (1984) and Harper (1976), it differs in a number of essential ways. An important distinctive feature of our approach is that we believe that rational partial beliefs are best described by a *set* of (conditional) probability measures satisfying some constraints and not by a *single* (conditional) probability measure. This view, which is explained and defended in section 2, makes it possible to distinguish between several notions of updating, which results in questioning the intimate relation some authors have claimed to exist between the Ramsey test for conditionals and belief revision. In particular, we argue that the Ramsey test refers to *hypothetical* belief revision, which essentially differs from belief revision as studied in the logic of theory change. In our opinion, this sheds an interesting light on Peter Gärdenfors recent generalization of Lewis' result. This issue will be treated in sections 4 and 5.

2. Generalized probability functions

The traditional (Kolmogorov) set-up for probability consists of a probability space (Ω,Σ,P), where Ω is a sample space (also known as space of elementary events or as frame of discernment), Σ is a σ-field (also known as σ-algebra) of subsets of Ω (i.e. $\Omega \in \Sigma$, $\forall A \in \Sigma\ A^c \in \Sigma$ and if S is a countable set of elements of Σ, then $\cup\{A \mid A \in S\} \in \Sigma$) and P is a probability measure $\Sigma \to [0,1]$ (i.e. P is a function $\Sigma \to [0,1]$ such that $P(\Omega) = 1$ and for every countable set S of pairwise disjoint elements of Σ $P(\cup\{A \mid A \in S\}) = \Sigma\{P(A) \mid A \in S\})$.

For simplicity, we will concentrate on probability spaces with a finite sample space and with the powerset of their sample space as σ-field. Throughout this paper Θ will denote some finite sample space.

DEFINITION 2.1 A *probability function* P on Θ is a probability measure $2^\Theta \to [0,1]$, i.e. P is a function $2^\Theta \to [0,1]$ such that $P(\Theta) = 1$ and $\forall A,B \subseteq \Theta\ (A \cap B = \varnothing \Rightarrow P(A \cup B) = P(A) + P(B))$.

As the following example illustrates, in some situations the available evidence does not completely determine a unique probability function:

EXAMPLE 2.2 A ball is drawn from either urn I or from urn II. Each ball in urn I is either red or green and the balls in urn II are either blue or yellow. The sample space is {red,green,blue,yellow}. All that is further known is that the choice between urn I and II is made by tossing a fair coin. In our opinion this situation is best described not by a single probability function, but by the set of probability functions P on {red,green, blue,yellow} which satisfy the constraint that P({red,green}) = P({blue,yellow}) = 0.5.

The question whether in situations similar to example 2.2 one is allowed to loosen the formalism of probability theory or one should invoke some principle of uniformity to arrive at a completely specified probability measure is highly controversial and will not be treated in detail. But since we did take sides on this issue, some justifying remarks are in order. Let us therefore take a closer look at the Dutch Book theorem, which is central to the best known line of argument (dating back to Ramsey and de Finetti) in favour of the position that a probability measure is the unique right representation of rational partial beliefs:

DEFINITION 2.3 A *Dutch Book* over Θ against an individual X is a bet concerning the occurrence of $A_i \subseteq \Theta$ which would result in a sure loss for X.

Let us assume that the acceptance of a bet is independent of the (positive) stake and that if an individual is both willing to accept bet b_1 and willing to accept bet b_2, then he is willing to accept the combined bet consisting of both b_1 and b_2.

PROPOSITION 2.4 If one takes an individual X's degree of belief in $A \subseteq \Theta$ to be the number $d_X(A)$ such that X is willing to bet on A at odds $d_X(A) : 1\text{-}d_X(A)$ and against A at odds $1\text{-}d_X(A) : d_X(A)$, then it is not possible to construct a Dutch Book over Θ against X iff there exists a probability function P on Θ s.t. $\forall A \subseteq \Theta$ $P(A) = d_X(A)$. (For a proof, see e.g. Kemeny (1955).)

A well-known argument against the above is that in situations with insufficient evidence an individual may not be able to give odds such that he is both willing to bet on A at those odds as willing to bet against A at the reverse odds. In the situation of example 2.2 it seems perfectly rational to be willing to bet against {red} only at odds as least as favourable as 1 : 1, without being willing to bet on {red} at odds 1 : 1. For all one knows, all balls in urn I may be red, but it might also be the case that urn I only contains green balls.

DEFINITION 2.5 Let X be an individual.

(i) $ld_X(A)$, X's *lower degree of belief* in A, is the number d such that d : 1-d are X's lower odds on A (i.e.the least favourable odds at which X is willing to bet on A).

(ii) $ud_X(A)$, X's *upper degree of belief* in A, is the number d such that d : 1-d are X's upper odds on A (i.e. 1-d : d are the least favourable odds at which X is willing to bet against A).

(iii) X is said to satisfy the *negative condition of coherence* (C^-) on Θ iff it is not possible to construct a Dutch Book over Θ against X.

(iv) X is said to satisfy the *positive condition of coherence* (C^+) on Θ iff X accepts every bet concerning $A_i \subseteq \Theta$ which would result in a sure gain for X.

PROPOSITION 2.6 Assume that an individual X accepts a bet on the sure event Θ. X satisfies C^- and C^+ on Θ iff there exists a set F of probability functions on Θ such that $\forall A \subseteq \Theta$ ($ld_X(A) = \inf\{P(A) \mid P \in F\}$ & $ud_X(A) = \sup\{P(A) \mid P \in F\}$). (For a proof, see Williams (1976). The assumption is only necessary to prove that $\forall A \subseteq \Theta$ ($ld_X(A) = \inf\{P(A) \mid P \in F\}$ & $ud_X(A) = \sup\{P(A) \mid P \in F\}$) $\Rightarrow$ X satisfies C^+ on Θ.)

The above generalized Dutch Book theorem suggests to represent rational partial beliefs by means of *sets* of probability functions. On our view, such sets should consist of all probability functions satisfying certain constraints which are inferred from the available evidence.

DEFINITION 2.7 Let S be a set of constraints on a probability function on Θ. Then the *generalized probability function P_S on Θ induced by S* is the function $2^\Theta \rightarrow 2^{[0,1]}$ defined by $P_S(A) = \{P(A) \mid P$ is a probability function on Θ satisfying $S\}$.

Generalized probability functions are of course closely related to the generalizations of probability functions which have been considered by authors as Smith, Good, Suppes, Shafer, Kyburg, Gärdenfors, etc. E.g. if P_S is a generalized probability function on Θ, then S_* (S^*), defined by $\forall A \subseteq \Theta$ $S_*(A) = \inf(P_S(A))$ ($S^*(A) = \sup(P_S(A))$), is a lower (upper) probability measure. Notice, however, that essentially different sets of constraints may thus give rise to the same lower and upper probability measures.

The possibility of conditioning on new evidence is one of the main strengths of (Bayesian) probability theory. However, it is a notorious problem to find a suitable generalization of conditioning in the context of generalized probability.

EXAMPLE 2.8 Suppose we are in the situation of example 2.2 except that we now have no evidence about the way the choice between I and II is made, but we do know that the number of red balls in I equates the number of the green balls in I. Since the probability interval of I being chosen contains 0, it is difficult to define what conditioning on this event might mean. Nevertheless, one would like to say that if it becomes known that the ball is drawn from I, the probability of the ball being red will be 0.5.

In Dempster-Shafer theory (a generalization of probability theory which has recently gained some popularity among Artificial Intelligence researchers) Dempster's rule of conditioning (a special case of Dempster's combination rule) is supposed to solve the problem of conditioning. However, the justification of this rule is highly problematic. (See Voorbraak (1988)). In addition, situations like the one described in example 2.8 cannot straightforwardly be represented in Dempster-Shafer theorie. (Compare Kyburg (1987).)

In the next section we "solve" the conditioning problem by using a particular kind of functions which make it possible to treat conditional information not as necessarily derived from or secondary to unconditional information.

3. Conditional probability functions

DEFINITION 3.1 For $n \geq 0$, $BT_n(\Theta)$, the *set of binary trees of depth n on Θ*, and $BT(\Theta)$, the *set of binary trees on Θ*, are defined as follows:

(i) $BT_0(\Theta) = 2^\Theta$.

(ii) $\forall n \geq 1$ $BT_n(\Theta)$ is the smallest set satisfying (1) $BT_{n-1}(\Theta) \subseteq BT_n(\Theta)$, and (2) if $\alpha, \beta \in BT_{n-1}(\Theta)$, then $<\alpha,\beta> \in BT_n(\Theta)$.

(iii) $BT(\Theta)$ is the smallest set satisfying (1) $\Theta \subseteq BT(\Theta)$, and (2) if $\alpha,\beta \in BT(\Theta)$, then $<\alpha,\beta> \in BT(\Theta)$.

Notice that elements of $BT(\Theta)$ may be viewed as binary trees with endnodes labelled by subsets of Θ and that $BT(\Theta) = \cup\{BT_i(\Theta) \mid i \geq 1\}$. Instead of $<\alpha,\beta>$ we will use the more suggestive notation $\alpha \rightarrow \beta$.

DEFINITION 3.2 For $n \geq 0$, $CP_n(\Theta)$, the *set of n-conditional probability functions of depth n on Θ*, and $CP(\Theta)$, the *set of conditional probability functions on Θ*, are defined as follows:

(i) $P \in CP_0(\Theta)$ iff P is a probability function on Θ.

(ii) $P \in CP_1(\Theta)$ iff P is a function $BT_1(\Theta) \rightarrow [0,1]$ such that

(1.0) $P \upharpoonright BT_0(\Theta) \in CP_0(\Theta)$.

(1.1) $\forall A \subseteq \Theta$ $P(\Theta \rightarrow A) = P(A)$

(1.2) $\forall A,B \subseteq \Theta$ $P(B \rightarrow A) = P(B \rightarrow A \cap B)$

(1.3) $\forall A \subseteq \Theta$ $P(A \rightarrow A) = 1$

(1.4) $\forall A \subseteq \Theta \forall B,C \subseteq A$ $[B \cap C = \varnothing \neq A \Rightarrow P(A \rightarrow B \cup C) = P(A \rightarrow B) + P(A \rightarrow C)]$

(1.5) $\forall A,B,C \subseteq \Theta$ $[A \subseteq B \subseteq C \Rightarrow P(C \rightarrow A) = P(C \rightarrow B)P(B \rightarrow A)]$.

(iii) For $n>1$, $P \in CP_n(\Theta)$ iff P is a function $BT_n(\Theta) \rightarrow [0,1]$ such that

(n.0) $P \upharpoonright BT_{n-1}(\Theta) \in CP_{n-1}(\Theta)$

(n.1) $\forall \alpha,\beta \in BT_{n-1}(\Theta)$ $[P(\alpha) = 1 \Rightarrow P(\alpha \rightarrow \beta) = P(\beta)]$

(n.2) $\forall \alpha,\beta \in BT_{n-1}(\Theta)$ $P(\alpha \rightarrow \beta)P(\alpha) \leq P(\beta)$

(n.3) $\forall \alpha,\beta,\gamma,\delta \in BT_{n-1}(\Theta)$ $[(\forall P' \in CP_{n-1}(\Theta)\ P'(\alpha) = P'(\beta)\ \&\ P'(\gamma) = P'(\delta)) \Rightarrow$ $P(\alpha \rightarrow \gamma) = P(\beta \rightarrow \delta)]$

(n.4) $\forall \alpha,\beta \in BT_{n-1}(\Theta)$ $[(\forall P' \in CP_{n-1}(\Theta)\ P'(\alpha) = 0) \Rightarrow P(\alpha \rightarrow \beta) = 1]$

(iv) $P \in CP(\Theta)$ iff $\forall n \geq 0$ $P \upharpoonright BT_n(\Theta) \in CP_n(\Theta)$.

PROPOSITION 3.3 Let $P \in CP_{(1)}(\Theta)$.

(i) $\forall A \subseteq \Theta$ $[P(A \rightarrow A^c) = 1 \Leftrightarrow A = \varnothing]$.

(ii) Let $\varnothing \neq A \subseteq \Theta$. Define $P_A : 2^\Theta \rightarrow [0,1]$ by $P_A(B) = P(A \rightarrow B)$. Then P_A is a probability function on Θ and $P_A(B)P(A) = P(A \cap B)$.

In some sense, the conditional probability functions generalize the binary probability functions which were introduced in Popper (1959) to allow conditioning on propositions with zero probability and which have become known in the literature as Popper functions:

PROPOSITION 3.4
(i) Let $P \in CP_{(1)}(\Theta)$ and let $p : 2^\Theta \times 2^\Theta \rightarrow [0,1]$ be given by $p(A,B) = P(B \rightarrow A)$. Then p is a Popper probability function on the algebra $(2^\Theta, \cap, ^c)$.
(ii) Let $p : 2^\Theta \times 2^\Theta \rightarrow [0,1]$ be a Popper probability function on $(2^\Theta, \cap, ^c)$ such that $p(A^c, A) = 1$ iff $A = \varnothing$ and let $P : BT_1(\Theta) \rightarrow [0,1]$ be defined by $P(B \rightarrow A) = p(A,B)$, and $P(A) = p(A,\Theta)$. Then $P \in CP_1(\Theta)$.

The following proposition implies that every probability function can be extended to a conditional probability function.

PROPOSITION 3.5 $\forall n \geq 0\ \forall P \in CP_n(\Theta)\ \exists P' \in CP(\Theta)$ such that $P' \upharpoonright BT_n(\Theta) = P$.

(n.1) - (n.4) constitute a rather weak set of conditions on the values of conditional probability functions on elements of $BT(\Theta) \backslash BT_1(\Theta)$. We do not exclude the possibility that some stronger conditions may be reasonable for some (or even all) interpretations of conditional statements. For example, an intuitively appealing strengthening of the notion of conditional probability function is obtained by adding the condition (n.5) $\forall \alpha,\beta \in BT_{n-1}(\Theta)\ [(\exists P' \in CP_{n-1}(\Theta)\ P'(\alpha) \neq 0) \Rightarrow \exists P' \in CP_{n-1}(\Theta)\ P(\alpha \rightarrow \beta) = P'(\beta)]$ to (n.1) - (n.4). The given definition of conditional probability functions may perhaps best be interpreted as a first proposal for the minimal extension of probability functions to a domain including conditionals.

Even more so than in the case of absolute probability, it seems too demanding to insist that rational partial beliefs about conditionals should be represented by a completely specified conditional probability function. Hence it will be no surprise that we again choose to consider *sets* of conditional probability functions satisfying certain constraints. In order to get some grip on the resulting generalized conditional probability functions, we will assume these constraints to be of a particular form.

DEFINITION 3.6 Let $P \in CP(\Theta)$.
(i) $CC(\Theta)$, the *set of conditional constraints on* Θ is the set of expressions of the form $\alpha \rightarrow_x \beta$, where $x \in [0,1]$ and $\alpha,\beta \in BT(\Theta)$.

(ii) $CC_n(\Theta)$, the *set of n-conditional constraints on Θ* is the set of expressions of the form $\alpha \rightarrow_x \beta$, where $x \in [0,1]$ and $\alpha,\beta \in BT_{n-1}(\Theta)$.
(iii) P is said to *satisfy* $\alpha \rightarrow_x \beta$ (notation: $P \models \alpha \rightarrow_x \beta$) iff $P(\alpha \rightarrow \beta) = x$ and P is said to satisfy a set $S \subseteq CC(\Theta)$ (notation: $P \models S$) iff for all $c \in S$ $P \models c$.

Without much difficulty, one could generalize the notion of conditional constraint by also allowing these constraints to be of the form $\alpha \rightarrow_{\neq x} \beta$, $\alpha \rightarrow_{\leq x} \beta$ or $\alpha \rightarrow_{\geq x} \beta$ (with the obvious interpretation).

PROPOSITION 3.7 Let $S \subseteq CC_n(\Theta)$. $\exists P \in CP(\Theta)\ P \models S$ iff $\exists P \in CP_n(\Theta)\ P \models S$.

DEFINITION 3.8 Let $S \subseteq CC(\Theta)$. [S], the *generalized conditional probability function on Θ induced by S* is the function $BT(\Theta) \rightarrow 2^{[0,1]}$ defined by $[S](\alpha) = \{P(x) \mid P \in CP(\Theta)\ \&\ P \in S\}$. We write $[S](\alpha) = x$ for $[S](\alpha) = \{x\}$.

By defining $[S]_\alpha(\beta) = [S](\alpha \rightarrow \beta)$ one can now easily condition a generalized conditional probability function. It is easy to see that this notion of conditioning is conservative over the usual one: if $[S](A \cup B) = x$ and $[S](A) = y > 0$, then $[S]_A(B) = [S](A \rightarrow B) = x/y$.

A preliminary version of this paper contains a rather unsatisfactory attempt to define a notion of conditioning a generalized probability function [S] on Θ to an arbitrary set $S' \subseteq CC(\Theta)$. We still consider such a general notion desirable, but we do not need it to make our main points.

One of these main points is that Lewis' result can be circumvented by not requiring conditionals to be elements of the σ-algebra on which probability is defined. In this manner one admittedly does not really give conditionals a probabilistic semantics, but one can account for the fact that people seem to be able to assign probabilities to conditional statements in a meaningful way. Using the above-defined notion of conditioning one can even defend slogan (*) mentioned in the introduction.

Another main point is that in our opinion conditioning (as defined above) corresponds with hypothetical belief revision, which essentially differs from the notions of belief updating which are studied in the logic of theory change (expansion, revision, contraction). We will return to this point in section 5. The next section contains a brief description of and some comments on Gärdenfors interesting generalization (using notions of the logic of theory change) of Lewis' result.

4. Gärdenfors' triviality result

DEFINITION 4.1 Let L be a languange with a standard consequence operation Cn.

(i) $TH = \{\Sigma \subseteq L \mid \Sigma = Cn(\Sigma)\}$ is the set of *theories* of Cn.

(ii) S is *consistent* (notation $CON(\Sigma)$) iff $Cn(\Sigma) \neq L$.

(iii) $Cn(\Sigma \cup \{A\})$ is called the *expansion* of Σ with A (notation Σ/A).

(iv) An *update-model* is a pair $\langle K,f \rangle$, with $K \subseteq TH$ and $f : K \times L \rightarrow K$. f is called an *update-function* for K. If no confusion is likely to occur, we write κ_A for $f(\kappa,A)$.

Consider the following properties of an update-function for K:

(BP) $CON(\kappa \cup \{A\}) \Rightarrow \kappa/A \subseteq \kappa_A$ (belief preservation)

(CP) $CON(\kappa), CON(\{A\}) \Rightarrow CON(\kappa_A)$ (consistency preservation)

(UM) $\kappa \subseteq \kappa' \Rightarrow \kappa_A \subseteq \kappa'_A$ (update monotony)

(CC) $CON(\kappa \cup \Sigma) \Rightarrow Cn(\kappa \cup \Sigma) \in K$ (closure condition)

(NT) $\exists \kappa \in K\ \exists A,B,C \in L\ [CON(\kappa \cup \{A,B\})\ \&\ CON(\kappa \cup \{A,C\})$
$\&\ CON(\kappa \cup \{B,C\})\ \&\ \neg CON(\{A,B,C\})]$ (nontriviality)

PROPOSITION 4.2 (Gärdenfors)
There exists no update-model satisfying (BP), (CP), (UM), (CC) and (NT).

If one assumes that L contains a binary connective > for expressing conditionals, then the above proposition remains valid after replacing (UM) by the following property:

(RT) $A>B \in \kappa \Leftrightarrow B \in \kappa_A$ (Ramsey Test)

Gärdenfors formulates his theorem in terms of 'belief revision sets' which satisfy (CC) by definition. In Morreau (1990) it is argued that (CC) is in a sense to blame for the impossibility or triviality result, but Langholm (1989) seems to contradict this. Gärdenfors rejects (UM) and hence also (RT), since he believes that (BP) can be defended by referring to Bayesian conditioning. In probability theory the following holds:

(PP) $P(A) > 0\ \&\ P(B) = 1 \Rightarrow P_A(B) = 1$ (probabilistic preservation)

(BP) may be considered to be the generalization of (PP) obtained by associating with every probability function P a belief state t(P) = {A | P(A) = 1} and translating P(A) > 0 by CON{t(P)∪{A}) (and keeping in mind that A ∈ t(P_A)).

In our opinion, this generalization is problematic for at least two reasons. Firstly, (BP) is considered to be applicable even in the presence of a connective > which is given a meaning by (RT), while it is unclear how it that case formulas can be assigned probabilities. Secondly, the correspondence between CON(κ∪{A}) and "A is assigned a positive probability" is only valid if there is *actually* a probability function underlying the belief state. Every belief state can of course be considered to be the "top" t(P) of some probability function P, but it may be the case that there is actually only a *generalized* probability function underlying the belief state.

EXAMPLE 4.3 Consider the situation of example 2.2 and suppose one learns that the ball is drawn from urn I. The resulting knowledge or belief state can be represented by the propositional theory κ of consequences of "red or green". Then CON(κ∪{"red"}), but "red' is not assigned a positive probability.

In the context of generalized probability measures we only have a correspondence between CON{t(P)∪{A}) and 0 ∉ P(A), which is not sufficient to derive the analogue of Gärdenfors result:

DEFINITION 4.4 A *generalized probabilistic update-model* for L with consequence operation Cn is a pair <Π,f>, with Π a collection of generalized probability functions on L and f : Π×L → Π.

PROPOSITION 4.5 For some languange L with a standard consequence operation Cn there exists a generalized probabilistic update-model <Π,f> for L which satisfies

(GBP) $0 \notin P(A) \Rightarrow Cn(t(P)\cup\{A\}) \subseteq t(f(P,A))$

(GCP) $CON(t(P)), CON(\{A\}) \Rightarrow CON(t(f(P,A)))$

(GUM) $t(P) \subseteq t(P') \Rightarrow t(f(P,A)) \subseteq t(f(P',A))$

(GCC) $CON(t(P)\cup\{\Sigma\}) \Rightarrow \exists P' \in \Pi\ t(P') = Cn(t(P)\cup\Sigma)$

(GNT) $\exists P \in \Pi\ \exists A,B,C \in L\ [CON(t(P)\cup\{A,B\})\ \&\ CON(t(P)\cup\{A,C\})$
$\&\ CON(t(P)\cup\{B,C\})\ \&\ \neg CON(\{A,B,C\})]$

Although we have some reservations about Gärdefors' Bayesian justification of (BP), we do agree with Gärdenfors that in a sense (RT) is the cause of the trouble. In fact, with hindsight one could argue that the incompatibility of (RT) with (BP) and (CP) could easily have been foreseen, since (BP) and (CP) are plausible principles for revision, while the Ramsey test refers to *hypothetical* revision or conditioning. The difference between revision and conditioning is the topic of the next section.

5. Conditioning versus belief revision

DEFINITION 5.1 Let P and P' be two generalized belief functions. P' is *at least as specific as* P (notation $P' \leq P$) iff $\forall\alpha\ P'(\alpha) \subseteq P(\alpha)$.

Consider a belief state κ represented by a generalized probability function P based on the evidence E. One can distinguish several possibilities that can arise if κ gets updated by new evidence E':

(1) $E \cup E'$ gives rise to a generalized probability function $P' \leq P$.

(2) $E \cup E'$ gives rise to a generalized conditional probability function P' such that $\neg(P' \leq P)$ but $t(P) \subseteq t(P')$. In this case, some of the old (conditional) constraints are withdrawn in the light of the information E', but E and E' do not conflict in the sense that some of the evidence of E»E' has to be withdrawn. (See example 5.2.)

(3) E and E' are in conflict. If this conflict is resolved by withdrawing some of the evidence of E. then it is in general not the case that the top t(P) of P is a subset of the top of the resulting P'.

(These descriptions suggest that there might be a better representation of a belief states than sets of constraints, namely one which explicitly mentions the evidence underlying the constraints. A general reaction to this is that for the study of belief updating several levels of abstraction and the relation between them may be of interest. (Cf. Gärdenfors (1988) and Lindström & Rabinowicz (1989).))

EXAMPLE 5.2 Suppose a dice has been thrown. Based on the evidence that the dice is fair, the probability that the result was 1 is 1/6. If one gets the additional information that the result was odd, then the probability of 1 becomes 1/3. Hence the conclusion that the probability of 1 is 1/6 is withdrawn although none of the evidence on which this original conclusion was based is withdrawn.

Conditioning a generalized probability function P on an event A such that $1 \in P(A)$ corresponds to possibility (1) above. Conditioning P on an event A such that $1 \notin P(A)$ and $P(A) \neq 0$ corresponds to (2). However, conditioning P on an event A such that $P(A) = 0$ does *not* correspond to (3). The reason is that P_A is defined in terms of P (by $P_A(B) = P(A \rightarrow B)$) and hence no old evidence is withdrawn. One has only (hypothetically) disregarded some of the consequences of the old eviodence which are in conflict with A.

The effect of expansion is the same as that of hypothetical expansion, since in neither case evidence has to be withdrawn. Proper revision (i.e. revision which is not expansion) is not equivalent to its hypothetical variant, since it requires withdrawing some of the old evidence, while hypothetical revision retains all evidence, but (hypothetically) discards some of the conclusions of the old evidence.

EXAMPLE 5.3 (After Stalnaker (1984))
Suppose X believes that (i) Germany did not win the war and (ii) if Hitler had decided to invade England in 1940, Germany would have won the war. Hypothetically adding the belief that Hitler had decided to invade England in 1940 does not require withdrawing any of the evidence underlying (i) and (ii), but nevertheless results in the (hypothetical) belief (based on the above-mentioned hypothesis and discarding (i), which contradicts this hypothesis) that Germany would have won the war. Now suppose that X discovers that (iii) Hitler did in fact decide to invade England in 1940 (although he never carried out his plan). (i) - (iii) are inconsistent, hence a revision is called for and some of the evidence underlying (i) - (iii) has to be given up. In this case the conditional (ii) is the most likely candidate for being withdrawn.

We believe that the Ramsey test for conditionals is essentially correct. However, we do not conclude that if one believes a conditional and comes to believe its antecedent, one should believe its consequent. In our opinion, this conclusion is only justified if coming to believe the antecedent of the conditional is done by *conditioning* on the antecedent. The reason for this is that if one hypothetically adds the antecedent A of a conditional to one's stock of belief - which is required by Ramsey's test -, then conditioning on A seems appropriate, while updating one's belief state with A is often not done by conditioning on A. (See example 5.3.) Conditional beliefs do not describe one's belief revision behaviour. The following two questions are essentially different: If H, would G? If you would discover that H, would G?

In general, finding a contradiction between the new evidence and (some consequences of) the old evidence seems to be a good reason for revising one's belief rather than conditioning on the new evidence. In a small way, this observation contributes to the solution of the interesting and rather difficult problem of characterizing the type of situation in which the different kinds of updating are appropriate. The extended Dutch Book theorems of Teller (1976) and Shafer (1983) are more interesting contributions to the solution of this problem. In our opinion, the results point to the conclusion that conditioning is only appropriate under very special circumstances.

Appendix. Propositions and proofs

PROPOSITION 1.3 Suppose that $\rightarrow$ is a probability conditional for a class Π of probability measures on a σ-algebra Σ of subsets of Ω and suppose that Π is closed under conditionalizing. Then

(i) there do not exist three pairwise disjoint sets $A,B,C \in \Sigma$ and a $P \in \Pi$ such that $P(A) > 0$, $P(B) > 0$ and $P(C) > 0$.

(ii) every $P \in \Pi$ is at most four-valued.

Proof. (After Lewis (1976).)

(i) Let $A,B,C \in \Sigma$ be pairwise disjoint such that $P(A)$, $P(B)$, and $P(C)$ are positive. On the one hand,

$$P(A \mid A{\cup}B) = \frac{P(A{\cap}(A{\cup}B))}{P(A{\cup}B)} = \frac{P(A)}{P(A{\cup}B)} > P(A), \text{ since } P(A{\cup}B) \leq P(\neg C) < 1.$$

On the other hand,

$$\begin{aligned} P(A \mid A{\cup}B) &= P((A{\cup}B) \rightarrow A) \\ &= P(A)P_A((A{\cup}B) \rightarrow A) + P(A^c)P_{A^c}((A{\cup}B) \rightarrow A) \\ &= P(A)P_A(A \mid A{\cup}B) + P(A^c)P_{A^c}(A \mid A{\cup}B) \\ &= P(A). \text{ Contradiction.} \end{aligned}$$

■

(ii) Assume that $P(A)$, $P(B)$, and $P(C)$ are distinct, positive and smaller than 1. Then $P(A) + P(B) \neq P(A) + P(C)$, hence either $P(A) + P(B) \neq 1$ or $P(A) + P(C) \neq 1$. If $P(A) + P(B) \neq 1$, then $P(A^c \cap B^c) = 1 - P(A) - P(B) + P(A{\cap}B) \neq P(A{\cap}B)$. Further, it follows from the assumptions that $P(A{\cap}B^c) \neq P(A^c{\cap}B)$ and that $P(A{\cap}B)+P(A{\cap}B^c)$, $P(A{\cap}B)+P(A^c{\cap}B)$, $P(A^c{\cap}B)+P(A^c{\cap}B^c)$, and $P(A{\cap}B^c)+P(A^c{\cap}B^c)$ are all positive.

Hence at least three of $P(A \cap B)$, $P(A \cap B^c)$, $P(A^c \cap B)$, and $P(A^c \cap B^c)$ are positive. But this is shown to be impossible by (i).
Similarly, $P(A) + P(C) \neq 1$ is impossible. Contradiction. ■

PROPOSITION 3.3 Let $P \in CP_{(1)}(\Theta)$.

(i) $\forall A \subseteq \Theta$ $[P(A \to A^c) = 1 \Leftrightarrow A = \emptyset]$.

(ii) Let $\emptyset \neq A \subseteq \Theta$. Define $P_A : 2^\Theta \to [0,1]$ by $P_A(B) = P(A \to B)$. Then P_A is a probability function on Θ and $P_A(B)P(A) = P(A \cap B)$.

Proof. (i) "$\Leftarrow$" $P(\emptyset \to \Theta) =_{(1.2)} P(\emptyset \to \emptyset) =_{(1.3)} 1$.
"$\Rightarrow$" Suppose $A \neq \emptyset$. Then $P(A \to A) =_{(1.4)} P(A \to A) + P(A \to \emptyset) =_{(1.2)} P(A \to A) + P(A \to A^c)$. Hence $P(A \to A^c) = 0$. ■

(ii) $P_A(\Theta) = P(A \to \Theta) =_{(1.2)} P(A \to A) =_{(1.3)} 1$. Let $B \cap C = \emptyset$. Then $P_A(B \cup C) = P(A \to B \cup C) =_{(1.2)} P(A \to ((B \cap A) \cup (C \cap A)) =_{(1.4)} P(A \to B \cap A) + P(A \to C \cap A) =_{(1.2)} P(A \to B) + P(A \to C) = P_A(B) + P_A(C)$. Hence P_A is a probability function on Θ. Further, $P_A(B)P(A) =_{(1.1)} P(A \to B)P(\Theta \to A) =_{(1.3)} P(A \to A \cap B)P(\Theta \to A) =_{(1.5)} P(\Theta \to A \cap B) = P(A \cap B)$. ■

PROPOSITION 3.4

(i) Let $P \in CP_{(1)}(\Theta)$ and let $p : 2^\Theta \times 2^\Theta \to [0,1]$ be defined by $p(A,B) = P(B \to A)$. Then p is a Popper probability function on the algebra $(2^\Theta, \cap, ^c)$.

(ii) Let $p : 2^\Theta \times 2^\Theta \to [0,1]$ be a Popper probability function on $(2^\Theta, \cap, ^c)$ such that $p(A^c, A) = 1$ iff $A = \emptyset$ and let $P : BT_1(\Theta) \to [0,1]$ be defined by $P(B \to A) = p(A,B)$, and $P(A) = p(A,\Theta)$. Then $P \in CP_1(\Theta)$.

Proof. We use the definition of Popper probability functions of (Harper,1976):
$p : 2^\Theta \times 2^\Theta \to [0,1]$ is a Popper probability function on $(2^\Theta, \cap, ^c)$ iff $\forall A,B,C \subseteq \Theta$:

(p1) $p(B,A) \leq p(A,A) = 1$

(p2) $p(A,B) = 1 = p(B,A) \Rightarrow p(C,A) = p(C,B)$

(p3) $p(C,A) \neq 1 \Rightarrow p(B^c,A) = 1 - p(B,A)$

(p4) $p(A \cap B,C) = p(A,C)p(B,A \cap C)$

(p5) $p(A \cap B,C) \leq p(B,C)$

(p6) $\exists A,B,C,D \subseteq \Theta$ $p(A,B) \neq p(C,D)$.

(i) Let $P \in CP_{(1)}(\Theta)$. Define $p(A,B) = P(B \to A)$.
(p1): $p(B,A) = P(A \to B) \leq 1$. $p(A,A) = P(A \to A) = 1$.

(p2): Assume $p(A,B) = 1 = p(B,A)$. Then $P(B \to A) = 1 = P(A \to B)$. Hence $A = \emptyset$ iff $B = \emptyset$. If $A = \emptyset$ and $B = \emptyset$, then by proposition 3.3(i) $\forall C \subseteq \Theta$ $1 = P(A \to C) = P(B \to C)$.
Assume $A \neq \emptyset$ and $B \neq \emptyset$. Then $p(C,A) =_{(1.2)} P(A \to A \cap C) =_{(1.4)} P(A \to A \cap B \cap C) + P(A \to A \cap B^c \cap C) =_{(1.4\&P(A \to B) = 1)} P(A \to A \cap B \cap C) =_{(1.5)} P(A \to A \cap B)P(A \cap B \to A \cap B \cap C) =_{(P(A \to B) = 1)} P(A \cap B \to A \cap B \cap C)$. Similarly, $p(C,B) = P(A \cap B \to A \cap B \cap C)$.
(p3): Assume $p(C,A) \neq 1$. Then, by prop. 3.3(i), $A \neq \emptyset$. Hence $p(B^c,A) =_{(1.2)} P(A \to B^c \cap A) =_{(1.3,1.4,1.5)} 1 - P(A \to B) = 1 - p(B,A)$.
(p4): $p(A \cap B,C) =_{(1.2)} P(C \to A \cap B \cap C) =_{(1.4)} P(C \to A \cap C)P(A \cap C \to A \cap B \cap C) =_{(1.2)} p(A,C)p(B,A \cap C)$.
(p5): $p(A \cap B,C) =_{(1.2)} P(C \to A \cap B \cap C) \leq P(C \to A \cap B \cap C) + P(C \to A^c \cap B \cap C) =_{(1.4)} P(C \to B \cap C) =_{(1.2)} p(B,C)$.
(p6): Since $\Theta \neq \emptyset$, $p(\emptyset,\Theta) \neq p(\Theta,\Theta)$. ■

(ii) Let p be a Popper probability function on $(2^\Theta,\cap,^c)$ such that $p(A^c,A) = 1$ iff $A = \emptyset$. Define $P : BT_1(\Theta) \to [0,1]$ by $P(B \to A) = p(A,B)$, and $P(A) = p(A,\Theta)$.
(1.0): $P(\Theta) = p(\Theta,\Theta) =_{(1.1)} 1$. Additivity follows from the proof of (1.4).
(1.1): By the definition of P.
(1.2): $P(B \to A \cap B) =_{(p4)} p(A,B)p(B,A \cap B) =_{(*)} P(B \to A)$. (*): $1 =_{(p1)} P(A \cap B,A \cap B) \leq_{(p5)} p(B,A \cap B) \leq_{(p1)} 1$.
(1.3): By (p1).
(1.4): Assume $B \cap C = \emptyset \neq A$. Then $p(A^c,A) \neq 1$. If $(B \cup C) \cap A = \emptyset$, then $B \cap A = C \cap A = \emptyset$ and $p(B \cup C,A) = p(\emptyset,A) =_{(p3)} 1 - P(\Theta,A) =_{(p1,p5)} 0$. Similarly, $p(B,A) = p(C,A) = 0$. Assume $(B \cup C) \cap A \neq \emptyset$.
Then $p(B,A) + p(C,A) = p((B \cup C) \cap B,A) + p(B \cup C) \cap C,A)$
$=_{(p4)} P(B \cup C,A)[p(B,(B \cup C) \cap A) + p(C,(B \cup C) \cap A)]$
$=_{(1.2)} p(B \cup C,A)[p(A \cap B,(B \cap A) \cup (C \cap A)) + p(A \cap C,(B \cap A) \cup (C \cap A)]$
$=_{(p4)} p(B \cup C,A)[p(A \cap B,(B \cap A) \cup (C \cap A)) + p((B \cap C)^c,(B \cap A) \cup (C \cap A))p(A \cap C,A \cap C)]$
$=_{(p1,p3)} p(B \cup C,A)$.
(1.5): Assume $A \subseteq B \subseteq C$. Then $P(C \to A) = p(A \cap B,C) =_{(p4)} p(B,C)p(A,B \cap C) = P(C \to B)P(B \to A)$. ■

PROPOSITION 3.5 $\forall n \geq 0\ \forall P \in CP_n(\Theta)\ \exists P' \in CP(\Theta)$ such that $P' \upharpoonright BT_n(\Theta) = P$.
Proof. Let $P \in CP_n(\Theta)$. Define for $i \geq 0$ $P_{(i)} : BT_i(\Theta) \to [0,1]$ as follows:
If $i \leq n$, then $P_{(i)} = P \upharpoonright BT_i(\Theta)$.

If $1 = i > n$, then $P_{(i)}$ is given by

$\forall A \subseteq \Theta \quad P_{(i)}(A) = P(A).$

$\forall A,B \subseteq \Theta \quad P_{(i)}(A \rightarrow B) = P(A \cap B)/P(A)$ if $P(A) > 0$.

1 if $A = \varnothing$.

$|A \cap B|/|A|$ otherwise.

If $1 \neq i > n$, then $P_{(i)}$ is given by

$P_{(i)} \upharpoonright BT_{i-1}(\Theta) = P_{(i-1)}$

$\forall \alpha,\beta \in BT_{i-1}(\Theta) \quad P_{(i)}(\alpha \rightarrow \beta) = \min(1, P_{(i-1)}(\beta)/P_{(i-1)}(\alpha))$ if $P_{(i-1)}(\alpha) \neq 0$

1 otherwise.

Let P' be the function $BT(\Theta) \rightarrow [0,1]$ such that $\forall i \geq 1\ P' \upharpoonright BT_i(\Theta) = P_{(i)}$. Then $P' \in CP(\Theta)$ and $P' \upharpoonright BT_n(\Theta) = P$. ■

PROPOSITION 3.7 Let $S \subseteq CC_n(\Theta)$. $\exists P \in CP(\Theta)\ P \models S \Leftrightarrow \exists P \in CP_n(\Theta)\ P \models S$.
Proof. "$\Rightarrow$" Trivial. "$\Leftarrow$" Let $P \in CP_n(\Theta)$ such that $P \models S$. By proposition 3.5, $\exists P' \in CP(\Theta)$ such that $P' \upharpoonright CP_n(\Theta) = P$. But then $\exists P' \in CP(\Theta)\ P \models S$. ■

PROPOSITION 4.5 For some languange L with a standard consequence operation Cn there exists a generalized probabilistic update-model $<\Pi,f>$ for L which satisfies

(GBP) $0 \notin P(A) \Rightarrow Cn(t(P) \cup \{A\}) \subseteq t(f(P,A))$

(GCP) $CON(t(P)), CON(\{A\}) \Rightarrow CON(t(f(P,A)))$

(GUM) $t(P) \subseteq t(P') \Rightarrow t(f(P,A)) \subseteq t(f(P',A))$

(GCC) $CON(t(P) \cup \{\Sigma\}) \Rightarrow \exists P' \in \Pi\ t(P') = Cn(t(P) \cup \Sigma)$

(GNT) $\exists P \in \Pi\ \exists A,B,C \in L\ [CON(t(P) \cup \{A,B\})\ \&\ CON(t(P) \cup \{A,C\})$
$\&\ CON(t(P) \cup \{B,C\})\ \&\ \neg CON(\{A,B,C\})]$

Proof. Let L be the propositional languange built up from the atoms p,q and r, let Cn be the usual propositional consequence operation and assume $p \vee q \vee r \in Cn(\varnothing)$ an $Cn(\{p \wedge q\}) = Cn(\{p \wedge r\}) = Cn(\{q \wedge r\}) = L$.

Define $\Pi = \{P \mid P$ is a generalized probability function on L and there exists a non-empty subset S of $\{p,q,r\}$ such that $P(A) = 1$ if $A \in Cn(\vee S)$ and $P(A) = 0$ otherwise.$\}$
Every element P of Π is characterized by a non-empty subset of $\{p,q,r\}$. $c_P \subseteq \{p,q,r\}$ denotes the characteristic set of P and $<S>$ denotes the element of P characterized by S. Similarly, every formula A of L corresponds with the subset c_A of $\{p,q,r\}$ such that A eq. $\vee c_A$, where A eq. B iff $A \in Cn(\{B\})\ \&\ B \in Cn(\{A\})$.

The update-function f is now defined as follows:

$f(P,A) = <c_P \cap c_A>$ if $c_P \subseteq c_A$ or $c_A \subseteq c_P$

$<c_A>$ otherwise.

It is easy to check that (GBP), (GCP), (GUM) an (GCC) hold. To establish nontriviality, we choose P = <{p,q,r}>, A = p∨q, B = p∨r and C = q∨r. ■

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The Syntax of Slavic Aspect

Extended Abstract

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The aspect literature assumes a fundamental difference between the Slavonic aktionsarts and the Germanic aspect. The paper presents a syntactic theory of verb structure which reduces these contrasts. Arguments are given that Polish aspect-formation is inflectional and that aspectual prefixes are SP(VP) and SP($\bar{V}$). These two positions are landing sites for the perfective-forming Affect P, i.e. (i) P-insertion, (ii) movement of directional Ps to SP($\bar{V}$) and (iii) movement of quantificational Ps to SP(VP). $\bar{V}$-aspect rules are formally identical with English verb-forming rules, where, however, the prefixes do not c-command their traces, thus P-doubling is precluded. The formal convergence of Affect P in Polish and English strongly supports the syntactic approach to inflectional and derivational morphology.

Key Words and Phrases: aspect, inflection, derivation, Aktionsart, Affect P, Move α, word structure, word formation, specifier, lexicon, thematic roles, θ-roles, zero derivation, P-incorporation, *Over-cliticization, Particle Movement, Load*-verbs, root, perfective, imperfective, prepositions, c-command, aspect transition, Derived Imperfectives, quantifiers, distributivity, P-doubling, Morphological Transparency, case assignment.
1985 Mathematic subject classification: 68Q50
1987 CR Categories: F.4.2, I.2.7.

Introduction

Polish and English appear to differ diametrically in the morphosyntactic determination of the two categories relevant to aspectual composition: verbs and NPs. English NPs are obligatorily determined for (in)definitness, while verbs carry no morphological marker of their aspectual status as state, process or event. In Polish the situation is reversed: nominal determiners are optional (Polish is a gender language) while verbs are always morphologically marked for (im)perfectivity. This contrast may promote a view of Slavonic aspect as lexical, a view which is backed by the proverbial complexity of Slavonic verbal morphology. In the descriptive literature aspectual morphology is treated as derivational.

The goal of the paper is to present a theory of Slavonic verb structure that overcomes the drawbacks of descriptive and lexicalist approaches to word structure. This theory, I hope, is a step towards a better understanding of the contrasts and similarities between Slavonic and English aspect composition. C.f. Verkuyl 1989.

In Part 1 I give arguments that Polish aspect is an inflectional process. I propose that the Polish VP contains two specifier positions which are crucial for the formation of perfective: SP(VP) and SP($\bar{V}$).

These two positions serve as landing sites for Affect P, a cluster of perfective rules which affect prepositions; Verbal prefixes are prepositions. In addition to the two prefixal positions (always perfective) there is a suffixal SP(V^0) position (lexically always imperfective). Each of these positions carries a feature FOR (imperfective) or IN (perfective). In contrast with English, Polish sentence contains a (possibly abstract) obligatory adverbial operator IN or FOR, which binds the aspect feature in one and only one of the affixes, and thus defines the operating aspect of a projection. The bound affix is always empty. It is obligatorily filled by a syntactic or morphological operation. Polish aspect formation thus consists of two parts: binding and filling. For prefixal positions filling is always a result of Affect P:

	Prefix		Adverb
adverbial binding	Δ_u	....	α_u
	Prefix		XP
Affect P	P_i	....	e_i

Slavonic aspect is intrinsically parasitic, i.e. it may be realized only through an accompanying process (Affect P). The optionality of the adverbial operator in English sentence allows English sentences to be ambiguous with respect to aspect, which is impossible in Polish.

The enormous complexity of Slavonic aspectual forms results from the amazing diversity of constructions that fall under the Affect P part of the Slavonic aspect. Some of these constructions are discussed in Part 2, for SP(VP), and Part 3, for SP($\bar{V}$) positions. There exists a symmetric division of labor between the two positions: SP(VP) attracts P operators: adverbial modifiers of action and prepositional quantifiers of NPs. SP(V) attracts directional Ps, either intransitive, i.e. particles, or transitive governors of argument NPs.

Although the syntactic realization of the rules is particular to Polish, most - if not all of them - find their counterpart in English, either as Logical Form operations (for quantifiers) or various rules of English derivational morphology, such as *Over*-Cliticization, P-Absorption, Particle Movement, Zero Derivation, *Load*-verb construction and others. Aspectual effects of these rules in English deserve systematic investigation.

The present study leads to the conclusion that Polish verbal Roots, represented as V^0 in X-bar notation, are lexically always imperfective (states or processes). Perfective verbs are not Roots, but V projections (morphologically complex verbs) which are formed via Affect P. The derived nature of the (Slavonic) perfective will be a matter of further research.

Part 1. The structure of Polish Verb

1.1 Outline of the analysis: the structure of Polish Verb

Polish has two SPEC(V) positions which provide insertion sites for prepositional prefixes: SPEC(VP) and SPEC($\bar{V}$):

(1)

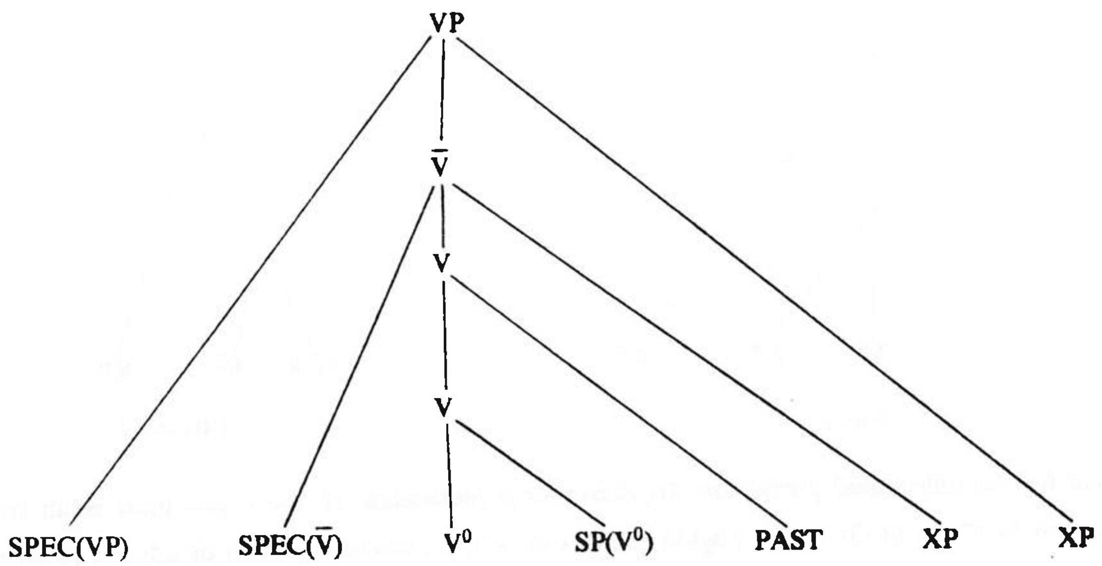


Figure 1

Both positions serve as hosts for perfective prefixation, but they radically differ as semantic categories. SP($\bar{V}$) accepts exlusively directional prepositions. Prepositions in SP(VP) are either pure-perfective (my term) or else they are various operators, cf. modal, distributive, accumulative, completive. Operators may be inserted in situ, cf. pure-perfective, or by movement. Directionals are always inserted by movement from an argument position. The two categories may cooccur under certain conditions. In syntax the prefixes c-command the 'basic V projection' which contains Root V^0, the imperfective suffix SP(V^0) and participial inflection (PAST or PRESENT). Directionals c-command the arguments of V^0, and operators c-command both arguments and adjuncts. Both prefixal specifiers are bound morphemes. In phonological form they cliticize onto the basic V projection[1].

1. Alternatively, the two prefixal positions may be considered complementizers, daughters of VP and $\bar{V}$, respectively. The final analysis will require more comparative data. Arguments for the COMP hypothesis in Polish;

(i) a possible lack of (COMP, $\bar{\Sigma}$,, thus multiple wh-movement;

(ii) the prefixal positions are Ps, while the suffixal position is V; for Emonds 1985 complementizers are Ps. Affect P would be a structure preserving substitution, analogous to structure-preserving analysis of wh-movement.

(iii) Hungarian aspect-forming adverbial preverbs are inserted in COMP, cf. Horvath 1981.

The structure in (1) represents the sole source of Polish verbal prefixation, i.e. there is no other verbal prefixation but perfective. Predictably, inflection and derivation will overlap.

English has no position which corresponds to Polish SPEC(VP) or SPEC($\bar{V}$). Furthermore its basic V projection is Root = V^0. English is a Root = Word language. In Polish and most other Slavic languages V^0 and N^0 Roots are always closely tied to their inflection. The Root-structure of English verb is therefore as in (2)

(2)

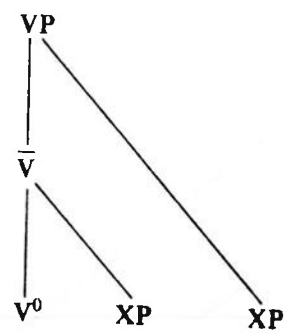


FIGURE 2

(3)

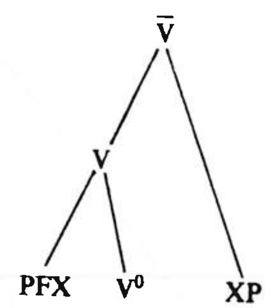


FIGURE 3

English has no inflectional prefixation. Its derivational prefixation, cf. *over-*, *out-* must result from adjunction to V^0, as in (3). Thus English prefixes never c-command argument or adjunct positions. This contrast in c-command distinguishes the two morphologies. C-command is required for P-doubling which is therefore impossible in English, cf. section 3.1.

2. ARGUMENTS THAT POLISH ASPECT IS INFLECTIONAL AND THAT THE PREFIXES ARE PREPOSITIONS

A. There are sixteen aspectual prefixes. Except for *roz-* all have a homonimous, often synonimous, prepositional counterpart. *Roz-* is an intransitive preposition, i.e. particle. In Polish all particles are bound morphemes.

B. New prefixed verbs are easily coined and understood. They are not listed in dictionaries.

C. The number of listed verbs is enormous (tens of thousands, WRÓBEL 1984).

D. Operator prefixes do not lexicalize semantically. Predictably, verbs headed by directional prefixes may lexicalize, as they are in the thematic Complex Verb domain.

E. Aspectual prefixation is a productive process of native Polish syntax. There are three borrowed prefixes, i.e. *dis-*, *de-* and *re-*. They never make a perfective verb, and thus must cooccur with with SP(V) to form aspect. They don't have prepositional counterparts. Their phonology is different. I assume that borrowed prefixed verbs are inserted in the head position of VP, with a + boundary, cf. *de + militaryzować* 'demilitarize'.

F. The Perfective/Imperfective category is a subcategorized category, cf. aspectual verbs such as *być* 'be' select only the imperfective class; past participle *-wszy* 'having V-ed' takes all and only perfective verbs; present participle *-ąc* takes all and only imperfective verbs. Derivational categories, cf. zero-affixed verbs, are never subcategorized for.

G. Perfective prefixes c-command and may be c-commanded by inflectional suffixes:

[[[PFX[PFX[V^0-SP(V^0)-TENSE]]]AUX]

My analysis provides a solution for a well known paradox of Slavic prefixal phonology, cf. PESETSKY 1985, SZPYRA 1987.

H. Perfective prefixes are the only source of verbal prefixation in Polish.

I. Perfective prefixation interferes with case assignment, operator scope, selected features of verbal arguments and other facets of clausal syntax and semantics.

J. All members of category V have an aspectual form.

1.3 Aspect Transition and Derived Imperfectives

Figure 1 shows that Polish verb structure contains three aspectual positions. Prefixes are always perfective, which I will indicate by feature IN, and suffixes are always imperfective, which will be indicated by feature FOR. Since all three affixes may cooccur in one verbal projection it is necessary to formally account for the fact that every verb structure has only one operating aspect. The uniqueness of the operating aspect may be derived from the bijection principle, on the assumption that each sentence contains only one aspectual adverb which binds the aspectual feature on one of the affixes. This is expressed by the coindex (a).

Derivation of aspectual projections and the aspect transition is illustrated in (4 o-iii) with the Roo. '*sτ*' 'send'

(4) 0. sτ-a-τ

IMPERFECT ASSIGNMENT

i. *sτ*- $\text{a}_{\text{FOR(a)}}$ -τ

PERFECT FORMATION

ii. $\text{przy}_{\text{IN(a)}}$-*sτ*- a_{FOR} -τ

DERIVED IMPERFECT

iii. przy_{IN}-syτ- $\text{a}_{\text{FOR(a)}}$ -τ

$

i. Imperfect Assignment $V^0 \rightarrow$ FOR

Polish Roots are always imperfective states or processes. Unlike in English, their imperfectivity is transparently expressed by the morphology of the SP (V^0) position. Formally, this may be represented by the rule of Imperfect Assignment:

$$FOR(a)/V^0\_\_$$

ii. Perfect Formation FOR $\rightarrow$ IN

Structure (i) is mapped into (ii) via Perfect Formation, i.e. a transformation Affect P, which applies obligatorily in the context

$$SP (V),\ \ IN (a), \Delta \ldots\ldots SP(V^0), FOR$$

PF is an insertion or movement of P into an empty SP(V) position, indicated by Δ.

(iii) Derived Imperfect IN $\rightarrow$ FOR

(ii) is mapped into (iii) by Derived Imperfect Formation (DI), which applies obligatorily in the context:

$$SP(V), IN, P_i\ +F \ldots\ldots SP(V^0), FOR(a), \Delta$$

DI applies exclusively to structures where SP(V) is non-empty, which may be achieved solely by PF, i.e. mapping from (i) to (ii). In addition, DI requires that P carry a semantic feature + F, which limits DI to semantic subclasses of categories in SP(V) position and excludes DI from applying to pure-perfectives, which do not carry +F. Morphological realization of DI is strictly local. It is either an insertion of the morpheme into an empty Δ SP(V^0) position or else a phonological operation on the V^0 stem adjoined to the aspect position (GUSSMANN 1984). Comparison of (i) and (iii) indicates that the latter operation applies to *sт* 'send'. The context dependency and nonlocality of the syntactic DI operation is evidenced by the fact that the morphological operation may occur only in the (distant) presence of the prefix. Unprefixed DI forms do not exist:

(5) **sy т-a -т*

The transition of the index (a) plays crucial role in the statements of aspect rules. So far the position of the index has been taken as a primitive, however. The rules indicate that the transition of (a) has the direction in (6).

(6) $0 \rightarrow \underset{FOR}{1} \rightarrow \underset{IN}{2} \rightarrow \underset{FOR}{3}$

I hypothesise that the transition in (6) is universal and the languages may have language particular, possibly morphological means of implementing it. Polish has the following language particular Lexical Redundancy Rules:

(7) IN → SP(V) prefixes

(8) FOR → SP(V^0) suffixes

Thus the transition in (6) is always a chain in (9):

(9) 0→ 1 → 2 → 3
suffixation *prefixation* *suffixation*

In English the aspect transition is not correlated with inflectional morphology. In particular, English does not express 1 by suffixation. Still, should the transition of (a) in (6) hold as a universal, (1) i.e. Imperfect Assignment may be carried on in the Root Lexicon. Part 3 of the paper which contrasts English derivational morphology with Polish aspect rules brings evidence that English has some remnants of perfective prefixation (2), which may also be the basis of an abstract (3).

Part 2 VP-aspect. Operators

2.1.0 Pure-perfective

(10) NA-pisaт listy
SP(VP)-write-SP(V^0)-PAST letter-PL:GEN
He wrote the letters

The sole function of pure-perfective prefixes is aspect. Other prefixes of this class are *z-*, *u-*, *prze-*, *po-*, *wy-*. Directional Ps like *ob-*, *do-*, *roz-* never occur in this structure. The prefixes are inserted in situ. About 250 verbs may form perfective this way. The verbs do not form a semantic class, though they are never verbs of movement. Most borrowed verbs form perfective via this structure.

Pure perfective does not impose any restrictions on the number feature of the arguments and does not affect case. As observed by Verkuyl 1989 for Russian unbounded NPs are not felicitous with perfective and undetermined nouns must be interpreted as definite. This is also true for Polish.

The individual prefixes of this class c.f. *na-* are not categories. The prefixes are merely phonological realizations of the category perfective. There is no semantic selection between the verb and the prefix. The verb selects pure-perfective prefix entirely idiosyncratically. Predicatably, this structure does not allow any semantic lexicalization.

In Polish these structures do not allow Derived Imperfectivization, cf. **napisywaт listy*. The proper generalization is this. The only non-phonological feature on the SP(VP) in this structure is +PERFECT, since the prefixes are not categories. Derived Imperfect may only be formed with specifiers which contain a semantic feature, c.f. +Directional. Forming a Derived Imperfective of pure-perfective does not make any sense. Derived imperfectivity is a result of the parasitic nature of the Slavic aspect.

2.2.0 Operators

2.2.1 Accumulative NA-

(11) NA- padaro śniegu
There fell a lot of snow
(12) NA- obieraliśmy ziemniaków
We peeled quite a number of potatoes.

NA- denotes summation of parts of action. It binds un-accusative subjects (11) and direct objects (12), which must be mass or plural, and assigns them partitive genitive case[2].

OPERATORS AND CASE: (hypothesis) Accusative/Nominative is assigned by the verb in the scope of tense. In Polish an accusative becomes genitive when a closer operator intervenes (cf. negation, partitive, opaque, accumulative). In other languages this may be expressed by indefinite or bare plural objects.

2.2.2 Accumulative NA- .... się-self

(13) NA-chodzir się
He walked a lot
(14) NA-czytar się kryminarówGEN
He read a lot of detective stories
(15) NA-syparo się śniegu-GEN
There fell a lot of snow

Selectional restrictions and case like in 2.2.1. In addition, however, the accumulation of the parts of action affects the Agent, which may be considered a species of experiencer. This explains the presence

2. The semantic classification of prefixes in section 2.2.0 draws from Grzegorczykowa e.a. 1984. Some examples also come from this source.

of się = 'self' in (13) and (14).

2.2.3 Distributive PO-

(16) Goście PO-s-chodzili się

Guests gathered together one by one.

(17) PO-wy-nosił stoły

He carried away the tables one by one.

The distributive PO- emphasises individual portions of action. It binds plural or mass subjects and objects of agentive verbs. Case: nominative/accusative.

2.2.4 Supplementary DO-

(18) DO-słodził herbatę ACC

He sweetened the tea a bit more

(19) DO-uczył się

He studied more

(20) DO-kroił chleba GEN

He cut more bread

DO- means 'in addition to the same previous action'. No selectional restrictions, but mass objects receive partitive genitive, cf. (20).

2.2.5 Completive WY-

(21) naród WY-ginął

The entire nation died out.

(22) WY-łapaliśmy muchy

We caught up all the flies.

WY- is a universal quantifier which binds unacussative objects and subjects. These arguments must be mass, plural or collective. Plurality of a non-unacusative subject is not sufficient:

(23) WY-łapaliśmy *muchę/muchy

we caught up a fly/ the flies

The case of the object is never partitive. This is predictable, since universal quantifiers do not allow partitive genitive.

10

2.2.6 Serial Completive PRZE-

(24) PRZE-badał pacjentów

He examined all the patients one after one

Like WY-, but spanning over the individual objects of a collection.

3.0 The Distributive PO-$\overline{N}$ phrase

Polish has a distributive PO-$\overline{N}$ phrase, whose properties may shed additional light on the syntax of some of the prefixal operators. PO is a preposition which attaches to unaccusative subjects and direct objects, which must be count. PO disambiguates sentences like (26a). (26b) has only distributive reading.

(25) W kurnikach ubyło PO kurze

There disappeared a hen in each coop.

(26) a. Jan i Piotr podnieśli pianino

Jan and Peter lifted a piano

b. Jan i Piotr podnieśli PO pianinie

PO assigns Locative to singular nouns and accusative to plural nouns. The distribution, selectional requirements and case assignment indicate that PO is a nominal quantifier. i.e. like the prefixes it does not bind locations but objects and subjects. In addition, PO must be in the scope of a plural argument:

(27) *W kurniku-SG ubyło PO kurze.

(28) * Piotr podniósł PO pianinie

These sentences are grammatical, however, if the verb is imperfective.

(29) W kurniku-SG ubywało PO kurze

(30) Piotr podnosił PO pianinie.

In imperfective the meaning is iterative. This contrasts PO with the prefixes, which always occur in imperfective structures, and do not form derived perfectives. In English the distributive floating quantifier *each* is ungrammatical with a singular subject:

(31) * Peter lifted a piano each.

* Peter each lifted a piano.

The contrast between (30) and (31) may be attributed to the morphological transparency of Polish aspect, i.e. in Polish the durative aspect itself may satisfy the scope requirements of PO.

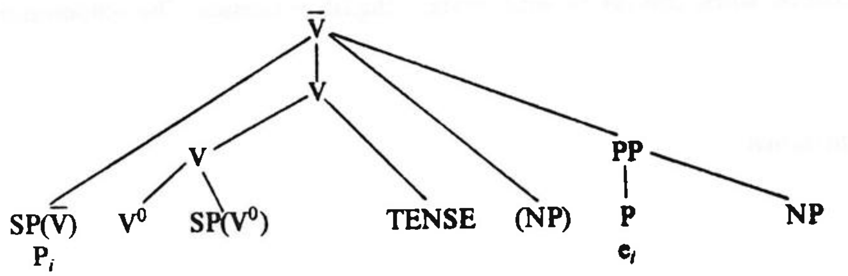
DIRECTION OF THE ANALYSIS: PO and the prefixes belong to the same category of adverbial operators. They differ in their surface position. Since this position matters for case assignment, the formatives must be inserted prior to NP-structure, where case is assigned. The prefixes must c-command the bound NPs at NP-structure, as in Figure 1, on the assumption that c-command is necessary for long distance case assignment. In contrast to directionals, operators do not double as they are not lexical governors of the NPs, c.f. section 3.1.

PART 3. $\bar{V}$ ASPECT. DIRECTIONALS.

3.1 Polish aspectual prefixation and P-doubling

This section of the paper analyses transitive and intransitive structures in Polish, where prefective configuration is formed by reanalysis and movement of transitive Ps, as in (32).

(32)



Analysis: The process consists of two operations:

(i) Insertion of the prefix in the configuration SP($\bar{V}$)$_i$ · · · P_i to which I will refer as P-movement. P-movement is obligatory when the *SP*($\bar{V}$) carries feature +PERFECT, in analogy to wh-movement.

(ii) P-doubling: insertion of the copy of P in the empty P position. I propose the following condition on P-doubling:

(33) C-COMMAND CONDITION ON P-DOUBLING

Insert a copy of P in the empty position e in the configuration

$$P_i...[[e]_{P_i}\ NP]$$

where P_i c-commands $[e]_{P_i}$

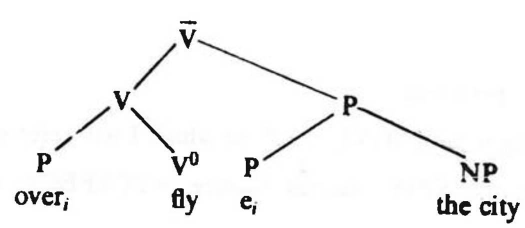
The condition (33) makes appropriate predictions for other Polish structures, and contrasts Polish with English, where P prefixation is an adjunction. In (34) I illustrate doubling with V^0 *biegT* 'he run' and *las* 'forest'

(34)	do-biegT do lasu	TO
	od-biegT od lasu	FROM
	pod-biegT pod las	TO NEAR
	po-biegT do lasu	TOWARD (outside point of reference)
	przy-biegT do lasu	TOWARD (inside point of reference)
	wy-biegT z lasu	OUT OF
	w-biegT {w las / do lasu}	INTO
(35)	prze-biegT (przez) las	THROUGH, ACROSS, ABOUT

In (34)-(35) V^0 assigns Theme to the subjects. I assume that it is the 'orginal' P which assigns the Location role to the NP. The copy P follows only the semantic instructions of the prefix, which is the reason for certain freedom in the copying. The copy P is a case assigner, and as shown in (34)-(35) the P is obligatory, except for the preposition *prze-*. This optionality of doubling with *prze-/przez* may be better understood when contrasted with related English processes: *Over*-cliticization and P-absorption.

3.2 English Over-Cliticization

(36)



English does not allow P-doubling (cf. **to overfly over the city*). The c-command condition (33) is not met in the English structure. In English P-cliticization is not a structure preserving substitution, but an adjunction to head. English does not have the SP(V) position, and the process is restricted to *over* and *under*, often accompanied by semantic lexicalization. From the doubling condition (33) it follows that that English *over*-cliticization may not reanalize PPs governed by transitive verbs:

(37) a. to throw the ball over the fence

b. *to overthrow the ball (over) the fence.

Interestingly, P-cliticization does occur with transitive V^0 Roots when P is intransitive, i.e. a particle:

(38) a. to throw the government over

b. to $over_i$ throw the government e_i

In (38) there is no reanalysis. P may move as as there is no NP which needs case. C-command condition (33) correctly predicts that (37b) is grammatical in Polish:

(39) prze-rzuciт piтkζ przez pтot

He threw the ball over the fence

3.3 P-absorption in English

English has another process which shares many properties with P-cliticization. Consider (40)

(40)a. He strode over the ditch

b. He strode - the ditch

I derive the structure in (40b) by the rule of P-absorption. P-absorption applies to 50 or so English Roots. It affects zero Ps, when certain thematic and semantic conditions are met. The syntactic environment of P-absorption is identical to *Over*-clitization i.e. P may not be absorbed where it is a case assigner, cf. (41)-(42).

(41) a. I walked through the street.

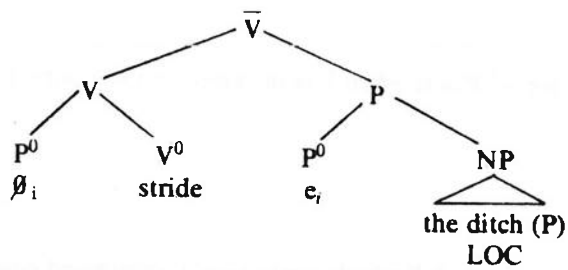
b. I walked - the street.

(42) a. I walked him through the street.

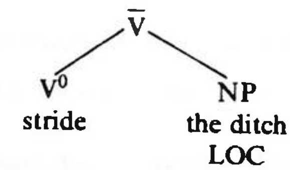
b. *I walked him - the street.

Following the suggestion of Zubizaretta 1988 for *enter* I propose that P-absorption creates a complex verb. In the present system it is expressed in X-bar notation, i.e. the verb in (40a) is V^0, a Root, while the verb in (40b) is V, i.e. a lexical projection. Thus the structure derived by English P-absorption is like in (43), not like in (44).

(43)



(44)



ARGUMENTS FOR THE COMPLEX VERB ANALYSIS IN (43):

A. The introduction of (44) to the Root Lexicon violates Root Identity of intransitive verbs. In principle, any intransitive Root V^0 could then take a direct object, i.e. subcategorize for both NP and PP.

B. Thematic structure: In (43) the empty P assigns the thematic index (P) to Location. Since I assume that V^0 has only one thematic index: for the Theme NP or predicate XP, the NP in (44) is left without a role.

C. Historical: of 50 verbs which allow P-absorption half once occured with the directional prefix *over* (cf. OED)

D. Contrastive: Polish strictly prohibits P-absorption, i.e. P may not be deleted with an unprefixed root:

(45) a. kroczył przez granicę
He strode across the border

b. *kroczył granicę

c. prze-kroczył granicę
cross-strode the border

The contrast is explained as follows: Both English and Polish disallow (44). Thus P-absorption may only arise via constructing a complex verb projection. In Polish the perfective prefixation is the only source of verbal prefixation. Due to the principle of Morphological Transparency, cf. section 3.4., the prefix in Polish must always be phonological. In Polish it is the only marker of the inflectional category Perfective.

3.4 Morphological Transparency of Polish Aspect

I propose that Polish perfective affixes must be phonologically visible due to the principle of Morphological Transparency of a non-paradigmatic Inflection:

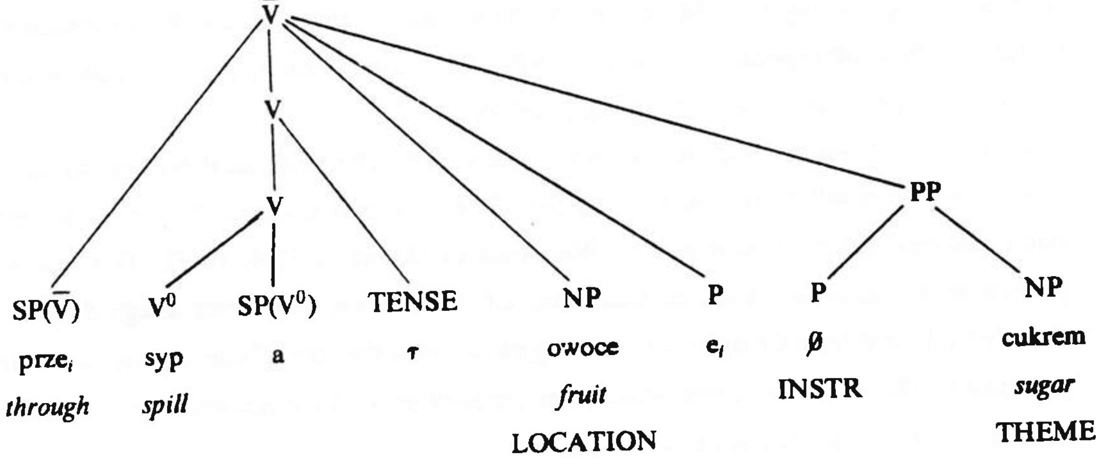
(46) Morphological Transparency: a category α must be phonologically visible if (i) α carries an inflectional feature and (ii) α is not in a paradigm.

Unlike case or tense inflections, aspectual prefixes and suffixes do not enter conjugational paradigms. This very complex and interesting matter will be discussed in detail in the paper. One of the factors is the parasitic nature of Slavic aspectual prefixation: perfective is executed via an accompanying process, save for pure-perfective. The processes are very diverse, cf. syntax of operators or zero derivation. Any verbal root has many aspect forms, depending on its class and semantics.
Principle (46) does not prohibit zero-inflection in paradigms, which is correct, as there are case markers which are phonologically zero in Polish (paradigm 27, sg nom cf. GUSSMANN 1984).

3.5 Load-verbs in Polish

Load-type verbs attracted attention of syntacticians (RAPPAPORT and LEVIN 1987), LEVIN and RAPPAPORT 1986, WALIŃSKA DE HACKBEIL 1986). These verbs are interesting for their case and thematic properties, as well as for the study of aspect. Polish, where they are syntactically productive, offers new evidence as to their nature: the structures arise exclusively as part of Perfective Formation:

(47)



Other prefixes of this class are:

(48) ZA all over location
PO over location

WY	all over location
O/OB	around location
PRZE	through a layer in location
NA	inside location

The most salient feature of Polish *load*-verbs is the fact that the argument structure (49) does not occur without the prefix, which is a problem for affixation theories of word formation:

(49) * V Location-NP-ACC Theme-NP-INSTR

ANALYSIS: The prefixes in (48) are intransitive prepositions, corresponding to English particles. Polish does not have free particles. Since Perfective Formation is obligatory, cf. 1.3, the intransitive Ps move obligatorily to SP($\bar{V}$). This analysis correctly predicts the lack of P-doubling in two of the possible targets. P-doubling occurs only in the configuration:

P_i......$[e]_{P_i}$ NP, where P_i c-commands e_i

In (47) the Location NP is not governed by an $[e]_P$ c.f. (50). The intransitive $[e]_P$ is c-commanded by the prefix, but it does not govern an NP c.f. (51).

(50) prze-sypaт (*przez) owoce cukrem

(51) prze-sypaт owoce (*przez) cukrem

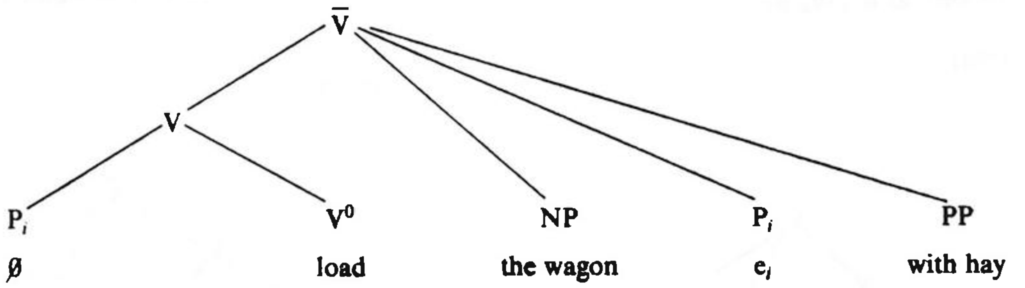
The analysis also correctly predicts the ungrammaticality of the non-prefixed INSTR-Theme structures in Polish, cf. (49). The obligatory movement of P is assured by the presence of the perfective specifier. Under my analysis *load*-verbs are not formed by the reanalysis of a Location PP and incorporation of a governing P to the verb but by movement of an intranstive P. I consider the class of intransitive Ps in (48) specifiers of Location-NPs. They differ semantically from transitive directionals in (34): they specify the mode of covering a surface.

In (47) the ∅ P assigns INSTR case to the Theme NP. The technical details of the thematic structure of *load*-verbs will be studied in the paper. Briefly: Theme is assigned by V^0. The role of Location is assigned under predication (c.f. WALIŃSKA DE HACKBEIL 1986, 1989). This makes important predictions for zero derivation in *load*-verbs, c.f. section 3.8; only roles assigned under case, i.e. directly by V or P form Complex Verb Configuration with the verb. These arguments are thematically governed by the verb. Arguments whose roles are assigned under predication, c.f. the direct object in *load*-verbs may not be incorporated into the verb.

3.6 English load-verbs and aspect

I propose that English *load*-verbs are derived analogically to the Polish *load*-verbs in (47), i.e. they form an abstract complex verb.

(52)



There is a strong historical evidence for the analysis in (52). Many *load*-verbs once occured with particles *be-/by* and *over*:

(53) bespatter, besprinkle, bestrew, bestow, bedabble, bedeck, bedoub...

(54) oversprinkle, overswarm, overspread

Since English has unbound particles, *over* may still occur in the original position.

(55) He sprinkled it over with water.

The historical fate of the specifying *be-* and *over*, i.e. their dissapearance, follows naturally from my analysis: these prefixes are not protected by Morphological Transparency (46) as they are not inflectional in English. The analysis is further supported by the fact that English has unbound aspect particles, c.f. *up*:

(56) He drank the wine up *FOR/IN

Rappaport and Levin 1987 note that *load*-verbs in their WITH-variant entail achievement of state in contrast to the locative variant:

(57) Henry loaded the hay on the wagon

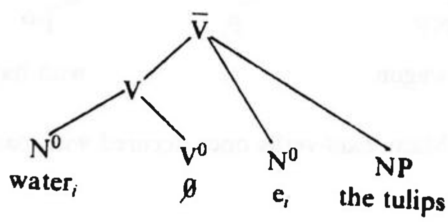
(58) Henry loaded the wagon with hay

They make an attempt of explanation using their Lexical Conceptual Structure representation. In the paper I give arguments against their analysis and defend the absorbed P hypothesis as an explanation of the perfectivity of (58).

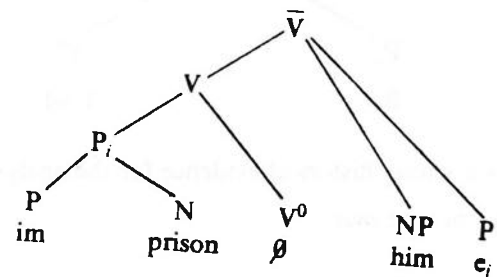
3.7 Zero Derivation and aspect

Zero Derivation is a process of forming verbs from other categories, c.f. *wet NP, water NP, march NP.* In Walińska de Hackbeil 1985, 1986 I argue that Zero Derivation involves incorporation of a thematic argument into verb via move α, where $\alpha =$ A,N,V,P. As a result, a complex verb structure is formed, as in (59a-b).

(59a)



(59b)

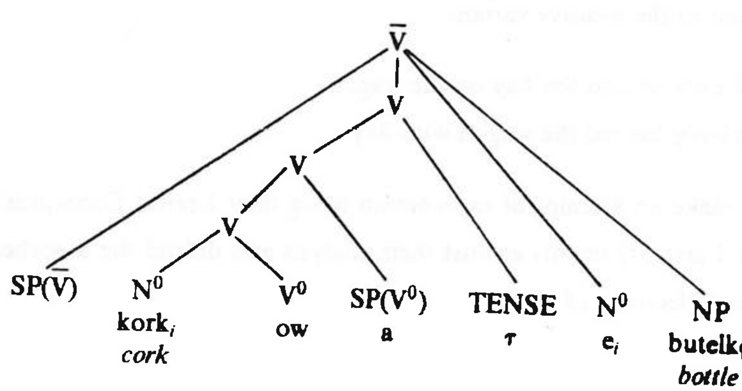


English Zero Derivation is typically independent of prefixation, c.f. (59a), but may be accompanied by it, c.f. (59b). Polish, too, displays these two modes of deriving verbs, and unsurprisingly the prefixed verbs are always perfective, since Polish has no verbal prefixation independent of Perfective Formation. This means that unprefixed structures corresponding to (59b) are not grammatical, which is also true for English:

(60) * prison him

En-prefixation was once very productive in English. Like *over* and *be, en-* is not protected by Morphological Transparency, thus it has massively dropped in several hundreds of zero derived verbs. The structures below are Polish structures which correspond to the English (59a) and (59b) respectively:

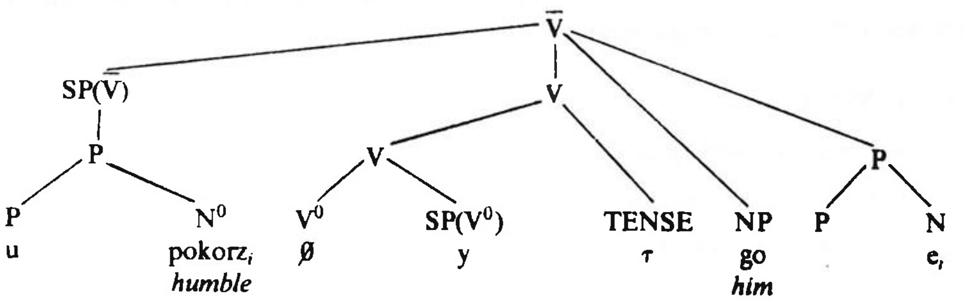
CLASS A (61a)



He corked the bottle FOR/*IN

CLASS B

(61b)



He humbled him IN/*FOR

The two classes differ significantly, and the contrast is predicted by the analysis.
Class A is very free in terms of argument structure of verbal heads. The derived verbs are imperfective, so they form perfective as any other verb and may cooccur with operators and directionals. Class B is zero derived and perfectivized simultaneously. Since the incorporated P phrase occurs in the SP(V̄) position these verbs may cooccur only with operators. I consider the prefixes of class B copulative or predicative prepositions, c.f. English *as* or *en-* in *imbitter NP* (Walińska de Hackbeil 1985, 1986). Most productive one in Polish is *u-*. These prefixes always head predicate phrases, so the structure B is restricted to causatives and inchoatives.

3.8 Zero derivation in load-verbs.

As argued in section 3.5, in *load*-verbs only the Theme argument is coindexed thematically with the verb, while Location is assigned under predication. It follows that only Theme may be incorporated into the verb. Although the zero derived verbs may occur without perfective prefixation and form perfective with other perfective classes, c.f. *bandażować, za-bandażować*, the thematic restriction indicates that incorporation in *load*-verbs constitutes a special case, and the two processes must be correlated. Incorporation of Location in *load*-verbs is strictly ungrammatical:

(62) * O-butelkowaт wino
O-bottled wine
He bottled the wine

CONCLUSIONS

$\bar{V}$ ASPECT: English does not have the morphological aspect, yet the rules which derive Polish perfective are also part of English grammar:

Polish: a. Movement of Directional Ps
b. Optional doubling of *prze-*
c. Prefixal Zero Derivation
d. *Load*-verbs (overt P)

English: a. *Over*-cliticization
b. P-absorption
c. *En*-prefixation as Zero Derivation
d. *Load*-verbs (abstract P)

In both languages the Affect P cluster occurs at the same level representation, i.e. d-structure restricted to V^0 Roots: causative, locational and manner of movement verbs. The language particular contrasts in the application and the effects of the rules are not expressed in the statements of the rules or by classifying the English Affect P as derivational and the Polish one as inflectional. The contrasts follow from the properties of sentence structure independent of Affect P:

(i) The specification of IN/FOR feature of the aspectual operator (Parameter):
obligatory POLISH/optional ENGLISH

(ii) The presence in the Polish syntactic structure of two prefixal SP(V) positions:
POLISH: Affect P is a structure preserving substitution, the prefix c-commands its trace.
ENGLISH: Affect P is an adjunction to the head, the prefix does not c-command its trace.

VP ASPECT: Affect P, where P is a quantificational operator, occurs at the level of unrestricted d-structure, thus the rules are not Root-governed and may apply to structures derived at $\bar{V}$ level. Formal properties of these rules require further research. Since these rules crucially involve the quantification of argument NPs they may represent Polish-particular or Slavonic type of aspect composition.

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Dependency of Belief in Distributed Systems

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ABSTRACT

This paper investigates the problem of dependency of belief in distributed environments, where an agent may take others' beliefs as its own beliefs. Kripke models for dependency of belief in distributed systems are presented, The notion of dependency of belief can be viewed as an intuitive extension to the notion of awareness, which is generally introduced in the systems of knowledge and belief. Synchronous distributed systems with dependency of belief are formalized. The corresponding logic system, S5Dn, is offered.

1. Introduction

A distributed system consists of a collection of agents are connected by a communication network. These agents may be processes, humans, or robots, which generally have limited resources. Reasoning about knowledge in distributed systems has been viewed as an important topic of investigation for analyzing distributed systems. In recent years, reasoning about knowledge in distributed environment has found many applications such as distributed knowledgebases, communication and cooperation for multiagent planning in artificial intelligence and knowledge engineering.[BG88]

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In distributed environments, it is frequently beneficial to enable agents to communicate their knowledges or beliefs among agents, because these agents generally may have limit resources, or may lack computation capability for some specified problems or facts. This means that in such distributed environments some agents may take others' knowledges or beliefs as their own knowledges or beliefs. In real lifes, the phenomenon is well known. We call the processes in which some agents can take others' beliefs as their own beliefs the dependency of belief. In this paper, we investigate the problem of dependency of belief in distributed environments.

Based on the classical possible worlds model for knowledge and belief, we present a general model for distributed systems with dependency of belief, in which some functions of dependency are introduced. We also find that notions of dependency of belief in distributed environments can be more intuitively as an extension to the notion of awareness, which is generally introduced in the logic system of belief and knowledge. That is because awareness of an agent not only includes that the agent can figure out the truth by himself, but also includes that the agent knows from which other agent the truth would be obtained correctly. Whereas the dependency of belief can be implemented only in distributed environments.

For synchronous distributed systems, we offer the corresponding formalism of dependency of belief. The system of logic with dependency of belief, called S5Dn, is provided, which formalizes synchronous distributed systems with dependency of belief in which some special properties hold.

Our work has somewhat close relationship with the work of [HL85], [FV86] and [FH88a]. In [HL85], Halpern and Fagin present a formal model that captures the interaction between knowledge, action, and communication in distributed systems. They extend the standard notion of protocol by defining knowledge-based protocols, ones in which a processor's action may explicitly depends on its knowledge. They also present the notion of honest protocol, where one only sends messages that it knows to be true. In [FV86], Fagin and Vardi characterize knowledge and implicit knowledge that are attainable in distributed systems. Implicit knowledge is the knowledge that can be obtained by pooling together the knowledge of a group. In [FH88a], Fagin and Halpern introduce and study several new logics for belief and knowledge. In these logics, the set of beliefs of an agent dose not necessarily beliefs of agents with limited reasoning capabilities. Especially, in [FH88a], a logic of general awareness is offered.

In distributed environments with dependency of belief, an agent only take others' true beliefs as its own belief. Therefore, the system with dependency of belief also can be viewed to be honest. However, our work is quite different from their work. First of all, our models for dependency of belief are introduced not only based on distributed protocols but also based on the general multi-agents environments. This means that the formalism we offer also can be used in general relevant fields such as Philosophy and Linguistics. Moreover, our systems for dependency of belief allow that nested modalities for knowledge and belief.

The organization of this paper is as follows: The section " General Model for Knowledge in Distributed Systems" provides a general overview of fundamental notions and relevant work concerned with the study for dependency of belief. In the section 3, first, we examine the notion of dependency and its relationship with awareness, then a general Kripke model for dependency of belief is offered. The section 4 examines some properties of the system for dependency of belief. In the section 5, we provide a Kripke model based on synchronous distributed systems with dependency of belief. The section 6 contains the system S5Dn of logic for dependency of belief, which formalizes the synchronous distributed system with direct dependency of belief. In the section 7, we summarize the study of dependency of belief, and offer some future researches.

2. General Model for Knowledge in Distributed Systems

2.1 The Classical Kripke Model for Knowledge and Belief

In this section we briefly review the possible worlds semantics for knowledge and belief. Suppose we consider a distributed system which consists of n agents, say 1,...,n, and we have a set Φ_0 of primitive propositions about which we wish to reason. In order to formalize the reasoning about knowledge in distributed systems, we use a modal propositional logic, which consists of the standard connectives such as $\wedge$, $\vee$, and ~, and some modal operators L1,...,Ln. A formula such as $Li\varphi$ is read "agent i believes φ."

We give semantics to these formulas by means of Kripke structures, which formalize the intuitions behind possible worlds. A Kripke structure for knowledge for n agents is a tuple $(S, \pi, L_1,...,L_n)$, where S is a set of states, $\pi(p,s)$ is a truth assignment to the primitive propositions of Φ_0 for each state $s \in S$, and *Li*, i=1,...,n, are binary relations on S which is serial, transitive, and Euclidean. A relation R is *serial* if for each $s \in S$ there is some $t \in S$ such that $(s,t) \in R$; R is *transitive* if $(s,u) \in R$ whenever $(s,t) \in R$ and $(t,u) \in R$; R is *Euclidean* if $(t,u) \in R$ whenever (s,t) $\in$ R and (s,u) $\in$ R., The relation L_i is intended to capture the possibility relation according to agents i: (s,t) $\in$ L_i if in world s agent i considers t a possible world.

We now assign truth values to formulas at a state in a structure. We write M,s $\models \varphi$ if the formula φ is true at state s in Kripke structure M.

M, s $\models$ **true**,

M, s $\models$ p, where p is a primitive proposition, iff π(p,s)=true

M, s $\models \neg\varphi$ iff M,s $\not\models \varphi$,

M, s $\models \varphi \wedge \psi$ iff M,s $\models \varphi$ and M,s $\models \psi$,

M, s $\models$ Liφ iff M,t $\models \varphi$ for all t such that (s,t) $\in$ L_i.

We say a formula φ is *valid in structure M* if (M,s) $\models \varphi$ for all states s in M; φ is *satisfiable in M* if (M,s) $\models \varphi$ for some states in M. We say φ is *valid* if it is valid in all Kripke structures; φ is *satisfiable* if it is satisfiable in some Kripke structure.

The logic of belief above is characterized by the following axiom system, called weak S5 or KD45.

(L1) All instances of propositional tautologies.

(L2) $L_i\, \varphi \wedge L_i(\varphi \rightarrow \psi) \rightarrow L_i\psi$.

(L3) $\neg L_i(\mathbf{false})$.

(L4) $L_i\varphi \rightarrow L_iL_i\varphi$.

(L5) $\neg L_i\varphi \rightarrow L_i\neg L_i\varphi$.

(R1) $\vdash\varphi, \vdash\varphi\rightarrow\psi \Rightarrow \vdash\psi$.

(R2) $\vdash\varphi \Rightarrow \vdash L_i\varphi$.

If an axiom stronger than (L3),

(L3') $L_i\varphi \rightarrow \varphi$,

is added, the resulting system is called S5 and the corresponding Kripke structure are reflexive, transitive, and symmetric, i.e. the relations become equivalence relations.The classical modal logic of knowledge is S5.

2.2 Distributed Protocols

A distributed system consists of a collection of agents, alternatively called processes.These processes run some protocol.Formally, we can definite a distributed protocol as follows,(cf. [FI86] for the details).

$P=< n,Q,I,\tau >$,

It consists of a number n of processes, a set Q of local states, a set $I \in Q^n$ of initial global states, and a next move relation $\tau \subseteq Q^n \times Q^n$ on global states.

For any protocol P, the τ-reachable global states RP of P is the set of all global states we can reach by starting in I and taking any number of τ steps. Only the reachable global states can occur in a run of P. A *run* of P is a sequence of global states, which describes a possible execution of a system over time. For $p \in Q^n$ a global state and i an agent, we write $[p]_i$ to denote the i-th component of p. We also define a binary relation $p \sim_i q$, for i=1,...,n,and $p,q \in Qn$ iff $[p]_i=[q]_i$, i.e. process i has the same local state at the global states p and q.

The Kripke model for distributed system with distributed protocol P is

$K=<RP,\pi,\sim_1,...,\sim_n>$

where π is a truth assignment to the primitive propositions of Φ_0 at every global states $p \in$ RP.

Based on Kripke models for distributed system,for a language of the system of knowledge and belief, the relation " $\models$" can similarly be defined as that in the subsection 2.1.

The general definition above for distributed protocols can precisely formalize the synchronous distributed environments, However, in which the distributed protocols lack local physical clocks. In [HF85], in order to capture the subtle interaction between knowledge, action and communication in distributed system, Halpern and Fagin assume a distributed system where processors communicate over links and have local physical clocks. A deterministic protocol is defined to be one where the messages sent by a processor depend only on its initial state, its message history, and its physical clock reading. A message history for processor p is a record of all the messages p has sent or received up to a certain point in real time. It seems to be a flexible one for the formalization of asynchronous distributed system with dependency of belief. Inasmuch as we only investigate the case of synchronous distributed systems in this paper, we assume a distributed system without local physical clock.

2.3 Logic of Awareness

Possible worlds semantics for knowledge and belief do not appropriate for modelling human reasoning since they suffer from the problem of logical omniscience. So-called *logical omniscience* means that agents are assumed to be intelligence that they must know all valid formulas, and that their knowledge is closed under implication, so that if an agent knows p, and that p implies q, then the agent must also know q.

However, in real life people are not such ideal reasoners. In order to provide a more realistic representation of human reasoning, various attempts to deal with this problem have been proposed. In [FH88a], Fagin and Halpern pointed out that 'lack of awareness' is one of sources of logical omniscience. They argue that one cannot say he knows p or doesn't know p if p is a concept he is completely unaware of. Another source of logical omniscience is that people are resource-bounded. The problem of resource-bounded means that any agent does not necessarily know all relevant facts , and that agents may lack the complete computational resources to reason all consequences of the facts they know.

In order to solve the problem of awareness, Fagin and Halpern offer a solution in which one can decide on a metalevel what formulas an agent is supposed to be aware of. In [FH88a], they provide a logic of general awareness.

A Kripke structure for general awareness is a tuple $M=(S,\pi,A_1,...,A_n,B_1,...,B_n)$, where S is a set of states, $\pi(s,\cdot)$ is a truth assignment for each state $s \in S$, and B_i is a serial, transitive, Euclidean relation on S for each agent i, and $Ai(s)$ to be an arbitrary set of formulas.

A relation $\models$, is defined inductively as follows:

$M,s \models$ **true**,	
$M,s \models p$,	where p is a primitive proposition, iff $\pi(s,p)=$true,
$M,s \models \neg\varphi$	iff $M,s \not\models \varphi$,
$M,s \models \varphi_1 \wedge \varphi_2$	iff $M,s \models \varphi_1$ and $M,s \models \varphi_2$,
$M,s \models A_i\varphi$	iff $\varphi \in Ai(s)$,
$M,s \models L_i\varphi$	iff $M,t \models \varphi$ for all t such that $(s,t) \in B_i$,
$M,s \models B_i\varphi$	iff $\varphi \in Ai(s)$ and $M,t \models \varphi$ for all t such $(s,t) \in B_i$.

$A_i\varphi$ is read "agent i is aware of φ", it also can be interpreted as agent i is able to figure out the truth of φ. $B_i\varphi$ is read "agent i explicitly believes φ". $B_i\varphi \equiv L_i\varphi \wedge A_i\varphi$ means that no agents can have explicit beliefs about formulas they are not aware of.

In [HMe88],van der Hoek and Meyer introduce a notion of principles, or prejudices, which is as dual of the notion of awareness. The notion of principle can model "reasoning against the facts". While awareness can prevent an agent from believing φ, that he would believe if he were logical omniscience, principles even enable him to believe $\neg\varphi$ in such a case.

3. General Model for Dependency of Belief

3.1 Dependency Functions

It is well known, in real life, people often consult other people and take other people's beliefs as their own beliefs. In a distributed system with n agents (processes or persons), if agent i take a formula φ, which is believed by agent j, as its own belief, that seems to mean that agent i think that the agent j is his adviser about φ. Of course, agent i may think that the agent j is its adviser in a special field of knowledge which consists of φ and other more formulas. It seems to be necessary to classify some specified formulas into some fields of knowledge $\Psi_1,\ldots,\Psi_m$.

On the other hand, agent might have two or more credible advisers about a special formula φ. Unfortunately, those credible advisers' opinion may be conflicting. It seems to require to impose a hierarchical structure on the agents with respects to their advisers and beliefs, even more generally, fields of knowledge, in order to solving the problem of the conflicting of advisers.

However, no matter what field of knowledge a formula φ will belong to, and no matter how many advisers agent i would have about the formula φ, in our opinion, agent i could eventually decide who the most credible adviser would be about the formula φ. We assume that the problems above have been solved in the metalevel, which results in the simplicity of dependency structures. We only introduce a function *Di*, for each agent i, that determines which agent would be the most credible adviser and be consulted by agent i about formula φ in the possible world

s. For example, $Di(\varphi,s)=j$ means that agent j is the most credible adviser for agent i about the formula φ in the possible world s. If j=i, that mean agent i only believe in himself about formula φ. Of course, for most formulas, agent i even does not know who the most credible adviser would be (including himself). Therefore, we introduce a special symbol "λ" in dependency functions.The special symbol "λ" means that nobody. thus, $Di(\varphi,s)=\lambda$ means that agent i has no credible agents about formulaφ, even does not believe in himself about formula φ. Let Ψ be the set of all formulas and S be the set of possible worlds.

$$Di: \Psi\times S \rightarrow \{1,2,\dots,n\}\cup\{\lambda\}.$$

Intuitively, we can give $Di(\varphi,s)=j$ a number of interpretations: "agent i depends on agent j about believing φ in the state s", "agent j is the credible adviser of agent i about φ in the state s", even simplely, "agent i asks agent j about φ in the state s". Specially in distributed process networks, "processor i can obtain the knowledge about φ from processor j in the state s", or "processor i receives a responsive message about φ from processor j in the state s".

3.2 Awareness Functions based on Dependency Functions

In [FH88a], Fagin and Halpern give the formula $A_i\varphi$ a number of interpretations: "i is aware of φ", "i is able to figure out the truth of φ" and when reasoning about knowledge bases "i is able to compute the truth of φ within time T". In order to express intuitively the necessary of awareness function in belief and knowledge of logics, they introduce an example cornerning Bantu tribesman: One can imagine the puzzled from on a Bantu tribesman's face when asked if he knows that personal computer prices are going down!

We find that awareness function can be intuitively introduced based on dependency functions in belief and knowledge of logics. Aiφ can be defined as $Di(\varphi,s)=j$ and $j\neq\lambda$. This means that agent i is aware of φ if and only if agent i believe in himself about φ or agent i could get the truth of formula φ by consulting his credible adviser about φ. As concerns intuitiveness of awareness functions, there seems to be no problem that Aiφ holds if agent i can figure out the truth of formula φ by himself, i.e. $Di(\varphi,s)=i$. In the case of $Di(\varphi,s)=j$ where $j\neq i$ and $j\neq\lambda$, we argue that agent i also can be said to be aware of φ. One can image an example about doctors and patients. When a patient is asked if he knows he is hypothyroid, he can say that he will be aware of hypothyroid because he can ask his doctor about that.

Therefore, we can formally define the awareness function based on dependency function as follows:

$A_i(\varphi,s)$ holds in the possible world s if and only if $Di(\varphi,s)\neq\lambda$ in the Kripke model for dependency of belief .

3.3 Logic of Dependency Beliefs

In this subsection we shall define the language in which we shall express our notion and notations about the logic of belief with dependency function first, then the semantics of the language will be given based on a Kripke structure with extension of dependency functions.

Suppose we will deal with the n agents case, the language **L** is the minimal set of formulas closed under the following rules:

(i) a set Φ_0 of primitive propositions is included in **L**,

(ii) if $\varphi,\psi\in$ **L** then $\varphi\wedge\psi\in$ **L**,

(iii) if $\varphi\in$ **L** then $\neg\varphi\in$ **L**,

(iv) if $\varphi\in$ **L** then $L_i\varphi\in$ **L**,where $i\in\{1,2,\dots,n\}$

(v) if $\varphi\in$ **L** then $D_{i,j}\varphi\in$ **L**,where $i,j\in\{1,2,\dots,n\}$.

We define $\varphi\vee\psi\equiv\neg(\neg\varphi\wedge\neg\psi)$, $(\varphi\rightarrow\psi)\equiv(\neg\varphi\vee\psi)$. $Li\varphi$ can be read as "agent i know formula φ", $D_{i,j}\varphi$ can be said as "agent j is the most credible adviser about φ" or "agent i depends on agent j about the belief of φ", etc.. See the subsection 3.1 for the details.

A Kripke structure for dependency beliefs is a tuple M

$M=(S,\pi,D_1,\dots,D_n,L_1,\dots,L_n)$

where S is a set of possible worlds, π is a truth assignment to the primitive propositions for each possible world $s\in S$, and Li is an equivalence relation on S for i=1,...,n. D_i is a function from each possible world and each formula in the language **L** into the set $\{1,\dots n,\lambda\}$.

The relation $\models$ is defined inductively as follows:

$M,s\models$**true**, where **true** is a special formula included in **L**,

$M,s\models p$, where p is a primitive proposition, iff $\pi(s,p)$=true,

$M,s \models \neg\varphi$ iff $M,s \not\models \varphi$,
$M,s \models \varphi_1 \wedge \varphi_2$ iff $M,s \models \varphi_1$ and $M,s \models \varphi_2$,
$M,s \models L_i\varphi$ iff $M,t \models \varphi$ for all t such that $(s,t) \in L_i$,
$M,s \models D_{i,j}\varphi$ iff $Di(\varphi,s)=j$.

Furthermore, we can define awareness function based on dependency function about belief: $M,s \models A_i\varphi$ iff $Di(\varphi,s) \neq \lambda$. Similar to Fagin and Halpern's logics of general awareness, we can also define explicit beliefs $B_i\varphi \equiv A_i\varphi \wedge L_i\varphi$.

Logical connectives such as $\rightarrow$ and $\vee$ are defined in terms of $\neg$ and $\wedge$ as usual. **false** to be an abbreviation of $\neg$**true**.

In the case of multi-agents, beliefs may be transitive among agents. Therefore, we extend the definition of dependency beliefs into indirect dependency beliefs as follows:

$M,s \models D^*_{i,j}\varphi$ iff for some $j1,\ldots,jm$, $M,s \models D_{i,j1}\varphi \wedge D_{j1,j2}\varphi \wedge \ldots \wedge D_{jm,j}\varphi \wedge D_{j,j}\varphi$.

$D^*_{i,j}\varphi$ is read "agent i indirectly depends on agent j about φ",or "agent i relies on the agent j about φ". The sequence <i,j1,j2,...,jm,j> is said to be a chain of dependency belief. Notice that, in the definition above, the last agent in a chain of dependency belief is one who only believes in himself about φ.

We define also $I^*_{i,j}\varphi \equiv D^*_{i,j}\varphi \wedge L_j\varphi$, where $I^*_{i,j}\varphi$ is read "agent i learns from agent j that φ is true".

From the definitions above, we can easily show the following propositions:

Proposition 3.1
(a) *Confidence*
$D^*_{i,j}\varphi \rightarrow \neg D^*_{j,k}\varphi$ ($k \neq j$).

(b) *Uniqueness*
$D^*_{i,j}\varphi \rightarrow \neg D^*_{i,k}\varphi$ ($k \neq j$).

(c) *Quasi-transitivity*

$D_{i,j}\varphi \wedge D^*_{j,k}\varphi \rightarrow D^*_{i,k}\varphi$.

Proposition 3.2

(a) *Same-source- propagation*

$D^*_{i,k}\varphi \wedge I^*_{j,k}\varphi \rightarrow I^*_{i,k}\varphi$.

(b) *Strong-consistence*

$I^*_{i,j}\neg\varphi \rightarrow (\forall k)(\neg I^*_{k,j}\varphi)$.

(c) *No-same-source-assertion*

$I^*_{i,j}\varphi \wedge \neg I^*_{k,j} \rightarrow \neg D^*_{k,j}\varphi$.

4. Some Properties about Functions of Dependency Belief

In order to examine and express pricisely the characteristics of distributed system with dependency belief, based on the functions of dependency belief in Kripke structure for logic of dependency belief, two kinds of important binary relations would be introduced. One is the *dependency order relation* $Ra[\varphi,s]$ on the set $\{1,2,...,n\}$ for each formulas φ in every state s, which is defined as follows:

$$i \; Ra[\varphi,s] \; j \quad \text{iff} \quad D_i(\varphi,s)=j$$

The another kind of relations is the *formula relation* $Rf[i,s]$ on the set of all formulas in the language L for each agent i in every state s, which is introduced as follows:

$$\varphi \; Rf[i,s] \; \psi \quad \text{iff} \quad D_i(\varphi,s)=D_i(\psi,s).$$

Obviously, the formula relation is an equivalence relation. For each formula relation $Rf[i,s]$ and any formula φ,ψ, formula relation $Rf[i,s]$ is said to be *closed under the logical connective* $\neg$ if $\varphi \; Rf[i,s] \; \neg\varphi$. $Rf[i,s]$ is closed under $\wedge$ if $\varphi\wedge\psi \; Rf[i,s] \; \psi$ whenever $\varphi \; Rf[i,s] \; \psi$. $Rf[i,s]$ is closed under $\rightarrow$ if $\varphi \; Rf[i,s] \; \psi$ whenever $\varphi \; Rf[i,s] \; \varphi\rightarrow\psi$. $Rf[i,s]$ is said to be *closed under modal operators* D, L, if $\varphi \; Rf[i,s] \; D_{i,j}\varphi$ and $\varphi \; Rf[i,s] \; L_j\varphi$ for the unique j and any formula φ.

In the section 3 about the definition about logic of dependency belief, we have placed no restrictions on the set of formulas that an agent depends on the other agents, similar to the situation about Awareness in

[FH88],we may well to add some restrictions to the dependency functions to capture certain properties. Some typical restrictions we may want to add to D_i can be expressed by some close properties of the corresponding formula relation under some logical connectives.In order to capture a notion of dependency generated by a set of primitive propositions, formula relations are closed under the logical connectives $\wedge, \neg$ and modal operator $D_{i,j}$ are necessary.

In the distributed system with dependency belief, since the messages of belief are send and received by a communication network, in order to guarantee the communication, an important property is deadlock freedom. This means that any agent can finally obtain the messages it wants. The property of deadlock freedom can be expressed by the following axiom:

$$(DF) \quad D_{i,j}\varphi \rightarrow (\exists \kappa)\, D^{*}_{i,k}\, \varphi.$$

An order relation R is called to be *reflexive in the limit*,if the following property is hold by R:

$$(\forall t,s)(t\ R\ s \rightarrow (\exists s')(s\ R^*\ s' \wedge s'\ R\ s')) \qquad (1)$$

The axiom of deadlock freedom needs that every dependency order relation ,which is introduced from the functions of dependency belief, is reflexive in the limit, in any state.

Specially, in this paper , we are interested in the special system of dependency belief for which the simplicity property of reflexive in the limit holds. The simplicity property of reflexive in the limit is:

$$(\forall t,s)(t\ R\ s \rightarrow s\ R\ s). \qquad (2)$$

The relation R is called to be *almost reflexive* if the property above holds. The corresponding axiom is:

$$(DF') \quad D_{i,j}\varphi \rightarrow D_{j,j}\, \varphi.$$

The system of logic about dependency belief with the axiom (DF') has close relationship with synchronous distributed system with dependency belief, which is examined in the sections 5 and 6.

In distributed systems with dependency of belief, an interesting property is that $I^{*}_{i,j}\varphi \wedge L_i\neg\varphi$ is satisfiable, This means that an agent may implicitly believe that it originally does not believe. Therefore,it raises an important problem of the system of dependency belief, which can be called the problem of *credibility of consulting*. It requires that in the system never make any agent believed in what he originally does not believe. This property can be expressed by the following axiom:

(CC) $\overset{*}{I}_{i,j}\varphi \rightarrow \neg L_i \neg \varphi$.

5. Kripke Model of Dependency Belief Based on Distributed Protocols

In this section, we examine Kripke models of dependency belief based on distributed protocols, specially, we study the models based on synchronous protocols in which every processor sends a message to every other processor during each round. In [FI86], One way to model the synchronous protocols is given.[1] Each processors' local state consists of an n-tuple, in which the i-th entry of j's state is the value of the message sent from i to j during the previous round. That is done as follows: for all processors,i,j, and for all global states p,q,r,s, if <p,q> and <r,s> are in τ and if processor i has the same state in p as in τ, then the i-th component of processor j's state is the same in q as in s.

As far as dependency beliefs are concerned, in a synchronous distributed system, we expand the synchronous distributed protocols with functions of dependency beliefs. One way to model this is to append a special message which specifies the function of dependency belief to each normal message. Let L be the language of logics of dependency belief for a distributed system. The formulas in the language L can be enumerated. Let Ψ be the set of all formulas in the language L,i.e. $\Psi=\{\varphi_1,\varphi_2,\ldots,\varphi_m,\ldots\}$.

For a synchronous distributed system, in every global state, each processor receives messages which encode the function of dependency belief and the truth of corresponding formulas from any other processors.Since there exist communication chains among processors in a distributed system,some messages may be retarded. Therefore, in encoded messages, we introduce a special value "**waiting**" , which denotes that required information is retarded, and a value "**undefined**", which means that corresponding formula is not depended on. Formally, the set of the encoded messages of dependency functions is a subset of the set of the functions {F|F:$\Psi \rightarrow$ {**true, false, waiting, undefined**}}, where Ψ is the set of all formulas. Let D be the set of encoded messages of dependency functions and their corresponding values,i.e. D$\subseteq${F |F:$\Psi \rightarrow$ {**true, false, waiting, undefined**}}, and Q' ={$m_1,m_2,\ldots,m_k,\ldots$} be the set of normal messages as in the synchronous distributed protocols

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[1] *Halpern and Fagin provide a different definition: R is a synchronous system if for all processes i and points (r, m), (r', m') in R, if (r, m) ~i (r', m'), then m=m'.* [HF89]

without dependency beliefs. Let Q be the set of local states in a synchronous distributed system with dependency belief , thus, Q is a subset of $(Q'\times D)^n$. It means that a local state in a synchronous distributed system consists of n components of normal communication message appended with encoded message which specifies the partial function of dependency beliefs and their corresponding values.

In order to formalize precisely the Kripke model of logic for dependency belief in a synchronous distributed system, we have the following definitions:

For the set Q of local states , and a global state $p \in Q^n$,

$[p]_i$'s definition is as previous,i.e, $[p]_i$ is the i-th component of global state p,

$[p]_i^j$ is the j-th component of local state $[p]_i$,it means that message send by processor j to processor i in the global state,

Let Qc be the set of all components of local states,i.e. $Qc=\{[p]_i^j \mid i,j \in \{1,2,...,n\}$ and $p\in Q^n\}$. For any $r=\langle q',d\rangle \in Qc$ where $q' \in Q'$ and $d \in D$, we definite:

Π_M: $Qc \rightarrow Q'$ $\quad \Pi_M(r)=q'$.

Π_D: $Qc \rightarrow D$ $\quad \Pi_D(r)=d$.

$\Psi([p]_i^j)$=df $\{\varphi \mid \Pi_D([p]_i^j)(\varphi) \in \{\mathbf{true,false,waiting}\}\}$.

$\Psi([p]_i^j)$ denotes the formula set about which agent i depends on agent j in global state p.

$\Sigma M([p]i)$=df $\langle \Pi_M([p]_i^1),...,\Pi_M([p]_i^n)\rangle$, which denotes all of normal messages that agent i receives in state p. For a synchronous distributed system with dependency of belief, we assume that two global states are indiscernible for an agent so long as it receives the same normal messages in the two states, although the encoded dependency mesages may be different.

For a synchronous distributed system with a set Q' of normal communication message and a set D of encoded messages which specify the function of dependency beliefs among n agents and the values of corresponding formulas, and a truth assignment π for each primitive proposition, the synchronous distributed protocols P is a tuple as follows:

$P=\langle n,Q,I,\tau\rangle$

where $Q \subseteq (Q' \times D)^n$ is a set of local states, $\tau \subseteq Q^n \times Q^n$ is a next move relation, $I \subseteq Q^n$ is a set of initial global states, and the following conditions are satisfied for the synchronous distributed protocol P with dependency belief :

(1) *Deadlock freedom.* It requires that no two agents are implicitly depended on each other about believing any formula. In order to satisfy this condition, a naive strategy is to introduce a hierarchy among agents.

(2) *Dependency Uniqueness.* It requires that any agent has only one adviser for each formula.

Formally, the two conditions above can be expressed as follows:

for any $i,j \in \{1,...,n\}$, $p \in Q^n$, $\varphi \in \Psi$,
(1) Deadlock freedom

(1.a) $\Pi_D([p]_i^j)(\varphi)=\mathbf{true} \rightarrow (\exists k)(\Pi_D([p]_j^k(\varphi)=\mathbf{true}))\vee \Pi_D([p]_j^j)(\varphi)=\mathbf{true}$.

(1.b) $\Pi_D([p]_i^j)(\varphi)=\mathbf{false} \rightarrow (\exists k)(\Pi_D([p]_j^k)(\varphi)=\mathbf{false}) \vee \Pi_D([p]_j^j)(\varphi)=\mathbf{false}$.

(1.c) $\Pi_D([p]_i^j)(\varphi)=\mathbf{waiting} \rightarrow (\exists k)(\Pi_D([p]_j^k)(\varphi)=\mathbf{waiting} \vee \Pi_D([p]_j^k)(\varphi)=\mathbf{true} \vee \Pi_D([p]_j^k)(\varphi)=\mathbf{false})$.

(2) Dependency Uniqueness

$\forall j_1,j_2(\Psi([p]_i^{j1}) \cap \Psi([p]_i^{j2}) \neq \varnothing \rightarrow j_1=j_2)$.

Remarks: The conditions (1.a), (1.b) and (1.c) guarantee the property of reflexive in the limit holds in the synchronous distributed system, which will be shown in the theorem 5.3

Based on a synchronous distributed protocol with dependency beliefs, we can introduce an equivalence relation $\sim_i$ and a function Di of dependency belief, for each processor i, in every global state. That can be done as follows:

for any global states p, q, and processor i

$p \sim_i q$ iff $\Sigma M([p]_i)=\Sigma M([q]_i)$.

$$Di(\varphi,p)=\begin{cases} j & \text{if } \varphi \in \Psi([p]_i^j). \\ \lambda & \text{else} \end{cases}$$

Now,based on a synchronous distributed protocol P=< n,Q,I,τ >, we have a Kripke model K for synchronous distributed system with dependency belief as follows:

$K=(S,\pi,D_1,\ldots,D_n,\sim_1,\sim_2,\ldots,\sim n)$

where $S \subseteq Q^n$ is a set of global states,

π is a truth assignment for each primitive proposition in every global state,

D_i are functions of dependency belief introduced based the synchronous distributed protocol,

~i are equivalence relations on global states,which is introduced based the synchronous distributed protocol.

For a Kripke structure K which is introduced based on a synchronous distributed system P, and the language L of logic of dependency belief, relation "|=" is defined that as in the section 3.

Proposition 5.1

For a synchronous distributed protocol P with dependency of belief P=<n,Q,I,τ> with the set Q' of normal messages and the set D of encoded message, and a Kripke structure which is introduced based on P, the following property holds:

$K,p \models L_j\varphi$ if $\Pi_D([p]_j^j)(\varphi)$=**true.**

Proposition 5.2

$(\exists i,j)I_{i,j}^{*}\varphi$ iff $(\exists p)(\exists k)(\Pi_D([p]_k^k)(\varphi)$=**true**).

Theorem 5.3

In a synchronous distributed protocol, let Ra be the dependency order relation introduced from the function of dependency in its corresponding Kripke structure

Ra[φ,p] *is reflexive in the limit , for any formula φ and any global state p.*

Proof.(Sketch)

For any agent i, j,

$$i\ Ra[\varphi,p]\ j \Rightarrow D_i(\varphi,p)=j \Rightarrow \varphi \in \Psi([p]_i^j)$$

$$\Rightarrow \Pi_D([p]_i^j)(\varphi) \in \{\textbf{true, false, waiting}\}$$

$$\Pi_D([p]_i^j)(\varphi)=\textbf{true} \Rightarrow (\exists k_1 k_2 \ldots k_l)(\ \Pi_D([p]_j^{k_1})(\varphi)=\textbf{true} \wedge \ldots \wedge$$

$$\Pi_D([p]_{k_l}^{k_l})(\varphi)=\textbf{true} \qquad (\text{Condition (1.a)})$$

Similarly, based on Condition (1.b), we also can show that the property holds where the value is "**false**". Based on Conditions (1.a), (1.b) and (1.c),the property holds where the value is "**waiting**".

Therefore, $Ra[\varphi,p]$ is reflexive in the limit.

For distributed systems with dependency of belief, an important feature is to allow the communication about nested belief and knowledge. The feature has close relationship with the notion of common knowledge, which generally is introduced in knowledge systems. In [HM84],Halpern and Moses argue that while common knowledge is desirable, it is unattainable in many realistic distributed systems. For example, if communication is not guarantied, then common knowledge is not attainable. In [HM84], they introduce various relaxations of common knowledge that are attainable in many cases of interest. In this paper, we assume that the communication is guarantied. Meanwhile, we argue that common knowledge is attainable by ask and answer among agents in distributed systems with dependency of belief, because in which the conditions of deadlock freedom and some dependence constraints are satisfied. we call the common knowledge which is attained by ask nested belief by ask.

For example, there exist three agents i, j, and k in a distributed system. At the first time, i may ask k if a formula p is true, and j may ask k if a formula q is true, where p and q are formulas that do not concern any belief modalities. Now agent k is able to tell agents i and j about its knowledge. At the second time, i may ask k if L_jq is true. Now agent k is able to tell the agent i that L_jq is true, because agent j has known L_kq, and agent j believe what agent k has told. Therefore, p and q can be considered as the common knowledge among agents i,j and k. Of course, at the first time, i may ask k if L_jq is true, then agent k can tell agent i "waiting" for the answer till the second time. As a matter of fact, no waiting, agent k even can directly answer the question as long as agent k certainly knows that agent j can ask k about q. That suggests a hiearchical modality constraint on dependency function, in order to enable nested beliefs by ask to be attainable. The constraint axiom is expressed as follows:

(C1) $D_{i,j}L_k\varphi \rightarrow D_{k,j}\varphi$. (Dependence constraint)

Notice that in the example above we confuse L_jq and L_kq. As a matter of fact , Although agent j believe in k about q, the meanings L_jq and L_kq still are different. In order to enable common knowledge to be attainable, the condition of credibility of consulting should be satisfied. But, in realistic distributed systems, the condition seems to be impossible. However, we can introduce a weaker condition, *no-doubt agreement,* which says that any agent never doubt what its adviser tells. In the concrete, it means that agents never intend to compare the diffirences between its explicit beliefs and its implicit beliefs. Therefore, based on the no-doubt agreement,we can consider that L_jq and L_kq have the same meanings approximately. The corresponding axiom is as follows:

(C2) $D_{i,j}\varphi \rightarrow (L_i\varphi \leftrightarrow L_j\varphi)$. (no-doubt agreement)

As mentioned above, the dependency of belief means ask and answer among agents in distributed systems. However, in synchronous distributed protocols with dependency belief which are proposed above, there no exist any explicit expression about ask and answer. That is because we assume the dependency function is known by both consulting and consulted agents in each round. For synronous distributed systems, in each round,the communication about dependcy of belief expresses not only the information of ask, but also the information of answer.

In a synchronous distributed system, every processor sends and received a message for every other processor in each round. Therefore, in order to simplify the relationship of communication of dependency belief among processors, it is reasonable to suppose that every processor can directly communicates with any other processor about dependency belief in a synchronous distributed system.This means that there are no any processor need any other processor as median processor for the communication of dependency belief. Under this circumstance, the property of simplified deadlock freedom holds,i.e., $D_{i,j}\varphi \rightarrow D_{j,j}\varphi$. We call the synchronous distributed system with dependency belief in which the property of simplified deadlock freedom holds the synchronous distributed system with directly dependency belief. In the section 6, we introduce a system of logic, called S5Dn, which formalizes the synchronous distributed system with directly dependency of belief.

6. The System of S5Dn

Let $M=(S,\pi,D_1,...,D_n,L_1,...,L_n)$ Be a Kripke structure with dependency of belief. M is an S5Dn structure, If M satisfies the following conditions:

(1) every dependency order relation introduced from Di,i=1,...,n, is almost reflexive.

(2) every formula relation introduced from Di,i=1,...,n, is closed under logical connectives, $\wedge,\neg$, and modal operators D, L.

(3) $Di(\varphi,s)=j \rightarrow Di(\varphi,t)=j \wedge Di(\varphi,t')=j$ for any t and t' such that $(s,t) \in L_i$ and $(s,t') \in L_j$.

In synchronous distributed system with directly dependency belief, because the property $D_{i,j}\varphi \rightarrow D_{j,j}\varphi$ holds, we need the definition of explicit belief with directly dependency belief . This needs to add the following definition on the standard model

$M,s \models I_{i,j}\varphi$ iff $D_i(\varphi,s)=j$ and $M,t \models \varphi$ for all t such that $<s,t> \in L_j$.

As a matter of fact, $I_{i,j}\varphi \equiv D_{i,j}\varphi \wedge L_j\varphi$. Now, In S5Dn system, $I^{*}_{i,j}\varphi$ reduces to $I_{i,j}\varphi$

The formal system S5Dn,which consists of the following axioms and rules:

Axioms:

(L1) All instances of propositional tautologies.

(L2) $L_i\varphi \wedge L_i(\varphi\rightarrow\psi)\rightarrow L_i\psi$.

(L3) $L_i\varphi\rightarrow\varphi$.

(L4) $L_i\varphi\rightarrow L_iL_i\varphi$.

(L5) $\neg L_i\varphi\rightarrow L_i\neg L_i\varphi$.

(D$\neg$) $D_{i,j}\varphi\equiv D_{i,j}\neg\varphi$.

(D$\wedge$) $D_{i,j}(\varphi\wedge\psi) \equiv D_{i,j}\varphi\wedge D_{i,j}\psi$.

(DD) $D_{i,j}\varphi\equiv D_{i,j}D_{i,j}\varphi$.

(DL) $D_{i,j}\varphi\equiv D_{i,j}L_j\varphi$.

(LD) $D_{i,j}\varphi\rightarrow L_iD_{i,j}\varphi$.

(LD') $D_{i,j}\varphi \rightarrow L_j D_{i,j}\varphi$.

(DF') $D_{i,j}\varphi \rightarrow D_{j,j}\varphi$.

(DU) $D_{i,j}\varphi \rightarrow \neg D_{i,k}\varphi$ $(k \neq j)$.

Axioms (L1)-(L5) guarantee that the system about modal operator L_i is a system of logic of Knowledge,i.e., S5 system.Axiom (D¬) expresses that an agent's dependency of beliefs is closed under negation. Axiom (D∧) means that closed under conjunction. Axiom (DD) says that an agent 's consulting should be agreed with the consulted agent. Axiom (DL) means that if an agent consults another agent about a formula, then he also should consult the another agent about his knowledge concerning the formula.Axiom (LD) and Axiom (LD') express that every agent knows their consulting and consulted. Axiom (DF') says that the property of simplified deadlock freedom holds. Axiom (DU) guarantees the uniqueness of dependency.

Rules:

(R1) $\vdash \varphi, \vdash \varphi \rightarrow \psi \Rightarrow \vdash \psi$.

(RI) $\vdash \varphi \Rightarrow \vdash L_i\varphi$.

For distributed systems, the common knowledge play an important role in reasoning about knowledge. Intuitively, common knowledge can be viewed as the state of knowledge where every one knows, everyone knows that everyone knows,etc.. Generally, In order to extend the system of logics so that one can reason about common knowledge, modal operators E_G and C_G are introduced, where G is a subset if {1,...,n} , $E_G\varphi$ is read "everyone in the group G know φ", and C_G is read "φ is common knowledge among the group G".

$(M,s) \models E_G\varphi$ iff $(M,s) \models L_i\varphi$ for all $i \in G$

$(M,s) \models C_G\varphi$ iff $(M,s) \models E_G^k\varphi$ for all $k >= 1$,where $E_G^1\varphi$ is an abbreviation for $E_G\varphi$, and $E_G^{k+1}\varphi$ is an abbreviation for $E_G E_G^k\varphi$.

From axioms (LD), (LD'), (DL), and the definitions about common knowledge above, we have the following propositions:

Proposition 6.1

(1) $\vdash D_{i,j}\varphi \rightarrow E_{\{i,j\}} D_{i,j}\varphi$.

(2) $\vdash D_{i,j}\varphi \rightarrow C_{\{i,j\}}D_{i,j}\varphi$.
(3) $\vdash D_{i,j}\varphi \rightarrow C_{\{i,j\}}D_{i,j}L_j\varphi$.
(4) $\vdash D_{i,j}\varphi \wedge D_{i,j}(\varphi\rightarrow\psi) \rightarrow D_{i,j}\psi$.
(5) $\vdash D_{i,j}\varphi \equiv D_{i,j}I_{i,j}\varphi$.

We introduce the definition of awareness $A_i\varphi \equiv D_{i,1}\varphi \vee D_{i,2}\varphi \vee ... \vee D_{i,n}\varphi$.

Proposition 6.2
(1) $\vdash A_i\varphi \wedge A_i(\varphi\rightarrow\psi) \rightarrow A_i\psi$.
(2) $\vdash A_i(\varphi\wedge\psi) \rightarrow A_i\varphi \wedge A_i\psi$.
(3) $\vdash A_i\varphi \rightarrow L_iA_i\varphi$.

Proposition 6.3
(1) $\vdash I_{i,j}\varphi \wedge I_{i,j}(\varphi\rightarrow\psi) \rightarrow I_{i,j}\psi$.
(2) $\vdash I_{i,j}\varphi \rightarrow I_{i,j}I_{i,j}\varphi$.
(3) $\vdash \neg I_{i,j}\varphi\wedge D_{i,j}\varphi \rightarrow I_{i,j}\neg I_{i,j}\varphi$.
(4) $\vdash I_{i,j}\varphi \rightarrow I_{i,j}L_j\varphi$.

Proof.

(1) $\vdash I_{i,j}\varphi \wedge I_{i,j}(\varphi\rightarrow\psi) \equiv D_{i,j}\varphi \wedge L_j\varphi \wedge D_{i,j}(\varphi\rightarrow\psi) \wedge L_j(\varphi\rightarrow\psi)$
$\rightarrow D_{i,j}\psi \wedge L_j\psi \equiv I_{i,j}\psi$.

(2) $\vdash I_{i,j}\varphi \equiv D_{i,j}\varphi \wedge L_j\varphi \rightarrow D_{i,j}D_{i,j}\varphi \wedge L_j\varphi$
$\rightarrow D_{i,j}D_{i,j}\varphi \wedge D_{i,j}L_j\varphi \wedge L_j\varphi \wedge D_{i,j}\varphi$
$\rightarrow D_{i,j}(D_{i,j}\varphi \wedge L_j\varphi) \wedge L_j(D_{i,j}\varphi \wedge L_j\varphi)$
$\rightarrow I_{i,j}I_{i,j}\varphi$.

(3) $\vdash \neg I_{i,j}\varphi \wedge D_{i,j}\varphi \rightarrow \neg I_{i,j}\varphi \wedge D_{i,j}I_{i,j}\varphi \rightarrow \neg (D_{i,j}\varphi \wedge L_j\varphi) \wedge D_{i,j}\varphi \wedge D_{i,j}L_j\varphi$
$\rightarrow \neg L_j\varphi \wedge D_{i,j}\neg I_{i,j}\varphi \rightarrow L_j\neg I_{i,j}\varphi\wedge D_{i,j}\neg I_{i,j}\varphi \rightarrow I_{i,j}\neg I_{i,j}\varphi$.
(4) evident.

In order to show that soundness and completeness of the system S5Dn, we use the standard techniques (cf. [FH88a],[MH88],[HC68]). First, we need the following definitions: A formula p is *consistent* (with respect to an axiom system) if $\neg p$ is not provable. A finite set $\{p_1,...,p_k\}$ is consistent exactly if the formula $p_1 \wedge ... \wedge p_k$ is consistent. An infinite set of formulas is consistent if every finite subset of it is consistent. A set

F of formulas is a *maximal consistence set* if it is consistent and any strict superset is inconsistent. As pointed out in [FH88a], using standard techniques of propositional reasoning we can show

Lemma 6.4 *In any axiom system that includes (L1) and (R1):*
(1) Any consistent set can be extended to a maximal consistent set.
(2) If F is a maximal consistent set, then for all formulas φ and ψ:
- *(a) either $\varphi \in F$ or $\neg\varphi \in F$,*
- *(b) $\varphi \wedge \psi \in F$ iff $\varphi \in F$ and $\psi \in F$,*
- *(c) if $\varphi \in F$ and $\varphi \rightarrow \psi \in F$, then $\psi \in F$,*
- *(d) if φ is provable, then $\varphi \in F$.*

We also can easily show:
Lemma 6.5 (Uniqueness of Dependency) In any axiom system that includes (L1), (R1) and (DU), if F is a maximal consistent set, then for all formula φ, the following property holds:
if $D_{i,j}\varphi \in F$, then $D_{i,k}\varphi \notin F$, for any k such that $k \neq j$.

Theorem 6.6 (Soundness and Completeness) *The axioms system of S5Dn is sound and complete for any structure of S5Dn.*

Proof. Soundness is evident. For the completeness, we must show every valid formula is provable. Equivalently, we can show that every consistent formula is satisfiable. A canonical structure Mc is constructed as follows:

$$Mc=(S,\pi,D_1,\ldots,D_n,L_1,\ldots,L_n)$$

where

$$S=\{s_V \mid V \text{ is a maximal consistent set}\},$$

$$\pi(s_V,p)= \begin{cases} \textbf{true} & \text{if } p \in V \\ \textbf{false} & \text{if } p \notin V \end{cases}$$

$$Di(\varphi,s_V)= \begin{cases} j & \text{if } D_{i,j}\varphi \in V \\ \lambda & \text{if } D_{i,j}\varphi \notin V \end{cases}$$

Lemma 6.5 guarantees the construct of D_i is valid.

$$Li=\{(s_V,s_W) \mid \{\varphi \mid L_i\varphi \in V\} \subseteq W\}.$$

First, we show that Mc is an S5Dn structure. Axioms (L3), (L4), and (L5) guarantee the *Li* are equivalence relations. Axiom (DF') express that every dependency order relation is directly reflexive. Axioms (D¬), (D∧), (DD), and (DL) enable every formula relation is closed under ¬, ∧, D, and L. Axioms (LD) and (LD') guarantee the condition (3) in the definition of S5Dn structure is satisfied. In order to show every formula φ is satisfiable, we should show $\varphi \in V \rightarrow Mc,s_V \models \varphi$ for any maximal consistent set V. Because of Maximality of V, we can easily show by induction on the structure of formulas that $\varphi \in V$ iff $Mc,s_V \models \varphi$.

7. Conclusions

We have examined the problem of dependency of belief, and presented a general Kripke model for dependency of belief. Also, we have argued that the notion of dependency of belief can be viewed as an intuitive extension to the notion of awareness. We expect the aspects of general awareness can be more intuitively captured by the logics of dependency about beliefs in distributed environments.

Moreover, we have offered a Kripke model for dependency of belief, which is based on the synchronous distributed protocols with dependency of belief. The system of S5Dn we offer formalizes synchronous distributed systems with dependency of belief in which some simplified properties hold. Of course, an interesting topic is to formalize asynchronous distributed system with dependency of belief. Intuitively, we feel that a subtle formalism about asynchronous distributed systems with dependency of belief seems to need the temporal logics as tools, because there exist the physical local clocks in the standard asynchronous distributed systems .

Another interesting direction is to impose some different frames on the functions of dependency, which allow temporal changes of dependency. In such distributed environments actions of agents seems to be more flexible, even more intelligent.

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THE PROPOSITIONAL INTERPRETATION OF NOUN PHRASES[1]

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1 The problem

Derived NPs seem to have different interpretations with different verbs. While sentence (1) may be paraphrased by (2),

(1) John is informed of Mary's arrival
(2) John is informed that Mary arrived

(3) John remembers Mary's arrival
(4) John remembers that Mary arrived,

sentences (3) and (4) differ in their truth-conditions: in order for (3) to be true, John must have witnessed Mary's arrival, but no such condition is required for (4) to be true. The data in (1)-(4) raise thus the following questions:

(a) Why is the NP Mary's arrival appropriately paraphrased by the that-clause that Mary arrived when it occurs in object position of is informed of? Why no such paraphrase is possible when this NP is the object of remember?

2 A Vendlerian account of (1)-(4)

An answer to (a) is suggested by Vendler's theory of semantic selection. Vendler has observed that derived NPs may be paraphrased by that-clauses in some contexts, but not in others:

(5) the collapse of the Germans is unlikely
(6) that the Germans will collapse is unlikely

(7) the collapse of the Germans was gradual,
(8) # that the Germans collapsed was gradual

His account of (5)-(8) rests on the following assumptions:

(A) derived NPs (like **Mary's arrival** or **the collapse of the Germans**) are semantically ambiguous: they may either denote propositional entities or event-like entities;

(B) predicates may differ in their semantic selection properties: some predicates select for event-like entities, but do not select for propositional entities.

According to Vendler, the predicate **is gradual** selects for event-like entities, but does not select for propositional entities. Sentence (8) is thus ill-formed, because **is gradual** cannot combine with an expression denoting a propositional entity. The predicate **is unlikely**, on the other hand, selects for propositional entities. The propositional denotation may then be assigned to the subject NP in (5), thus accounting for the fact that (6) is an admissible paraphrase of (5). We may try to extend this account to (1)-(4) in the following way: (2) is an admissible paraphrase of (1), since **is informed of** selects for propositional entities, and thus the NP **Mary's arrival** in (1) may be assigned the same denotation as the **that**-clause in (2).[2] Sentence (3), on the other hand, does not allow paraphrase (4), since NP-taking **remember** selects for event-like entities, but not for propositional entities, and thus the NP **Mary's arrival** in (3) cannot be assigned the same denotation as the **that**-clause in (4). The semantic selection properties of NP-taking **remember** require instead that the object NP in (3) denote an event. According to this view, (3) and (4) differ in their truth-conditions, because, in order for somebody to remember an event, different conditions must be met from those required for somebody to remember a propositional entity.

3 Some Questions

I am going to refer to the assumption in (A), from now on, as **the ambiguity hypothesis**. In evaluating the Vendlerian account of (1)-(4), one needs then to address the question:

(b) is the ambiguity hypothesis correct?

Now, suppose for a moment that the answer to (b) is yes. Then, we must also explain why NPs such as **Mary's arrival** are unable to appear in some positions in which that-clauses can appear:

(i) John believes that Mary arrived
(ii) # John believes Mary's arrival

Notice, by the way, that, although one might be tempted to take the data in (i)-(ii) as evidence that derived NPs like **Mary's arrival** unambiguously denote events, there are reasons not to reject the ambiguity hypothesis simply on the basis of (i)-(ii). Vendler (1967, 1968, 1975) has argued that gerundive nominals like

(iii) the soprano's performing the song

denote propositional entities. Evidence for Vendler's claim is provided by (iv)-(vi):

(iv) # the soprano's performing the song occurred at noon/ was gradual/ was sudden

(v) the soprano's performing the song surprised us

(vi) (the fact) that the soprano performed the song surprised us

Sentences (v)-(vi) show that the gerundive NP in (iii) can occur with predicates that select for propositional entities. Indeed, (v) appears to be synonymous with (vi). Sentence (iv), on the other hand, shows that (iii) cannot occur with predicates of event-like entities. The data in (iv)-(vi) indicate thus that (iii) denotes a propositional entity. Nonetheless, it may be observed that not all predicates of propositional entities are acceptable with (iii). In particular, (iii) is unable to occur in object position of **believe**:

(vii) # John believes the soprano's performing the song

I'll come back to the problem posed by (vii) later on in the paper. For the time being, I simply want to point out that the ill-formedness of (ii) is not sufficient to establish that the NP **Mary's arrival** cannot denote a propositional entity: the are good reasons to assume that the NP **the soprano's performing the song** denotes a propositional entity, and yet this NP is also unable to occur in object position of **believe**.

In addressing the issues raised in this section, I'll proceed as follows. First, I'll concentrate on question (b). Then, I shall come back to the problem posed by (vii) and by (i)-(ii). Finally, I'll provide a formal implementation of my analysis of the meaning of the nominals **Mary's arrival** and **the soprano's performing the song**.

4 Other Arguments for the Ambiguity Hypothesis

The behavior of derived NPs with the predicate **surprise** has led some linguist (e. g., Lees (1960) and Fraser (1970)) to conclude that derived NPs may denote propositional entities. Consider,

(9) (the fact) that Mary resigned surprised us

(10) Mary's resignation surprised us

(9) and (10) may differ in their truth-conditions. For example, (10) may be truthfully uttered in a situation in which we expected that Mary would resign, but, contrary to our expectations, her resignation was quick. However, in some cases, we may also use (10) to express the claim in (9). Suppose that the NP **Mary's resignation** is ambiguous between the event-denotation and the propositional denotation. This would explain why (10) allows the paraphrase in (9) in some cases, since we may assume that the subject NP in (10), in one interpretation, denotes the same entity the subject NP in (9) denotes. The hypothesis that

Mary's resignation is ambiguous between event-denotation and propositional denotation, moreover, would also make room for the fact that (9) and (10) may not be synonymous, since the conditions required for an event to be surprising may differ from the conditions required for a propositional entity to be surprising.

A different argument for the ambiguity hypothesis is given in Baeuerle (1987). Baeuerle points out that negated nominals like **the non-arrival of the train** are acceptable with the predicate **surprise**, but are not acceptable with predicates of events:

(11) the non-arrival of the train surprised us
(12) # the non-arrival of the train lasted an hour/ was slow/ was sudden
(13) the arrival of the train lasted an hour/ was slow/ was sudden

Suppose that the noun **arrival** is ambiguous: its meaning may either be a property of events or a property of propositional entities. Suppose, moreover, that **non**, from a semantic point of view, takes properties of propositions as arguments. The presence of **non** in an NP will thus force us to assign to the NP a propositional denotation. This explains the contrast in (11)-(13). Sentence (11) is ill-formed, since the predicates **is slow**, **is sudden**, **lasts an hour** select for events, and not for propositional entities. Sentence (12), on the other hand, is well-formed, since **surprised us** can select for propositional entities. Finally, (13) is well-formed, since the subject NP in (13) can denote an event, and is thus able to occur with predicates of events.

5 An Alternative Account of (1)-(4)

In this section, I argue, that the evidence presented so far is insufficient to establish that derived NPs are ambiguous between the propositional denotation and the event-denotation. I suggest that the data that are claimed to support the ambiguity hypothesis may also be accounted for without assuming that derived NPs are ambiguous.

5.1 Surprising Resignations

NP-subjects of **surprise** may be assigned a propositional interpretation also in cases in which it is implausible to assume that they denote propositions. For example, the sentences in (14) allow sometimes the paraphrases in (15):

(14) i. John's false teeth surprised us
ii. John's bruised eye surprised us
iii. John's broken arm surprised us
iv. John's rough skin surprised us

(15) i. (the fact) that John's teeth were false surprised us
ii. (the fact) that John's eye was bruised surprised us
iii. (the fact) that John's arm was broken surprised us
iv. (the fact) that John's skin was rough surprised us

I take it that the subject NPs in (14) cannot be plausibly maintained to denote propositional entities. The existence of the possible paraphrases in (15) should then be accounted for without supposing that the subject NPs in (14) denote propositional entities. It seems to me, moreover, that the same point may be also made for (16):

(16) Last night, we witnessed Mary's resignation$_i$.
That event$_i$ surprised us.

I take it that the NP **that event** in (16) denotes the event of Mary's resignation. Notice, however, that (16) admits the same range of interpretations as (17):

(17) Last night, we witnessed Mary's resignation.
Mary's resignation surprised us

Namely, the sentence **That event surprised us** in (16) may either be interpreted as claiming that (a) it surprised us that Mary resigned, or as claiming that (b) something else about the event of Mary's resignation surprised us. The existence of the paraphrase in (a) must then be accounted for compatibly with the fact that the NP **that event** in (16) denotes an event.

I suggest the following account. Suppose that the predicate **surprise** may be assigned two possible translations:

surprise$_1$

surprise(P)

surprise$_1$ expresses a relation between individuals and propositional entities. **surprise(P)**, on the other hand, expresses a relation that can hold between individuals and events, or simply between individuals. I assume that P is a predicate variable whose meaning is determined by the context of utterance of the predicate. Following Heim (1977), I take it that the set of indices that represent the context of utterance contains a variable assignment whose task is to provide a value for those variables, like P, that are contextually interpreted.[3] The meaning of **surprise(P)** is then specified via the meaning of **surprise$_1$** and the contextually determined property which is the value of P:

x stands in the relation denoted by **surprising(P)** with an event or an individual y iff x stands in the relation denoted by **surprising$_1$** with the proposition that y has the property denoted by P[4]

We are now in a position to explain why (15i.), for example, seems to be a possible paraphrase of (14i.). Whenever the property assigned to P in the context of utterance of (14i.) is the property of being an x such that x is false, we predict indeed that (14i.) is true iff (18) is true:

(18) John's false teeth are such that it surprised us that they were false.

A similar account may be extended to the other pairs in (14)-(15). We may also explain why (9) seems to be a possible paraphrase of (10):

(9) (the fact) that Mary resigned surprised us
(10) Mary's resignation surprised us

Whenever the property assigned to P in the context of utterance of (10) is the property of being an x such that x occurs, (10) will be synonymous with (19):

(19) Mary's resignation$_i$ is such that it surprised us that it$_i$ occurred

In this proposal, the existence of a reading of (10) synonymous with (9) is thus accounted for without appealing to the assumption that derived nominals are ambiguous between the propositional interpretation and the event-interpretation.

Notice, by the way, that this proposal accounts nicely for the fact that the sentences in (14) allow other readings besides those in (15). For example, (14i)

(14)i. John's false teeth surprised us

may be uttered truthfully in a situation in which we knew that John's teeth were false, but we were surprised that his false teeth looked perfectly natural. This reading may arise, according to this proposal, since the property of being an x such that x looked perfectly natural may be contextually salient in the contexts of utterance of (14i). Similarly, we are now in a position to explain why (10) may be uttered truthfully in a situation in which we expected that Mary would resign, but we were surprised that her resignation was quick. This is possible since the property of being an x such that x was quick may be contextually salient in the context of utterance of (10).

5.2 Being informed of Mary's Arrival

The likeness of meaning between (1) and (2) may also be accounted for without assuming that derived NPs are ambiguous.

(1) John is informed of Mary's arrival
(2) John is informed that Mary arrived

Suppose that a possible translation of the predicate **is informed of** is

(20) $\lambda y\ \lambda x$ (x is informed$_1$(occur'(y))),[5]

where **is informed$_1$** in (20) is the translation of the predicate **is informed** in (2). The complex predicate in (20) expresses a relation between individuals and events, as shown by the presence of the formula **occur'(y)**.[6] The existence of the translation in (20) explains why (1) and (2) are similar in meaning compatibly with the assumption that the object NP in (1) denotes an event: according to (20), being informed of an event e is the same as being informed that e occurs. Notice, again, that the likeness of meaning between (21) and (22) provides independent evidence that the translation in (20) is needed anyway:

(21) Last night, Mary arrived.
The police are informed of that event.

(22) Last night, Mary arrived.
The police are informed that Mary arrived.

5.3 Negation

The data in (11)-(13) have been claimed to provide independent support for the ambiguity hypothesis:

(11) the non-arrival of the train surprised us
(12) # the non-arrival of the train lasted an hour/ was slow/ was sudden
(13) the arrival of the train lasted an hour/ was slow/ was sudden

Here, I want to argue that (11)-(13) provide evidence that (a) negated NPs denote propositional entities, but they do not establish that (b) non-negated NPs like **the arrival of the train** or **Mary's arrival** are ambiguous between propositional denotation and event-denotation. The way in which conclusion (b) may be avoided is by now a familiar one. Suppose that the negation **non** is assigned the translation in (23) (where Q is a property of eventualities and **i** creates individual-type expressions out of sentential expressions):

(23) $\lambda Q\ \lambda p\ (p = i(\text{not}\exists y\ (Q(y)))$

The NP **the non-arrival of the train** will then denote the proposition that there is no arrival of the train. The claim that negated NPs denote propositional entities, as Baeuerle observed, makes it possible to explain the ill-formedness of (12). But the account of the role of negation in NPs suggested here is also consistent with the assumption that non-negated nominals unambiguously denote events. I conclude that (11)-(13) do not provide conclusive evidence for the Vendlerian account of (1)-(4), in particular (11)-(13) fail to establish the ambiguity hypothesis in (A).

6 A Problem for the Vendlerian Account

So far, I have argued that the arguments presented in the literature in favor of the ambiguity hypothesis are not conclusive. In this section, I raise a problem for the ambiguity hypothesis. Suppose that derived nominals are semantically ambiguous between the event-denotation and the propositional denotation. If this is correct, the NP **Mary's arrival** should denote the event of Mary's arrival in one reading, and, in the other reading, the proposition that Mary arrived (or some other propositional entity related to this proposition). This predicts that (3) should have a reading synonymous with (4):

(3) John remembers Mary's arrival
(4) John remembers that Mary arrived

It seems to me, however, that this prediction is incorrect. Sentence (4) may be true, for example, in a situation in which John has been told that Mary arrived by a reliable source, but he has not witnessed her arrival. Sentence (3), however, is false in identical circumstances. Notice that the lack of a reading of (3) synonymous with (4) cannot be explained by supposing that NP-taking **remember** semantically selects for events, but not for propositional entities. In fact, (24) shows that NP-taking **remember** selects for propositional entities as well:

(24) John remembers the fact that Mary arrived

The ambiguity hypothesis runs thus into a problem: if this hypothesis were correct, than we should expect (1) to have a reading synonymous with (4). But this is not the case.

7 Some Reasons not to Give up on the Ambiguity Hypothesis

At this point, one might perhaps be satisfied with the conclusion that derived NPs are not ambiguous between the event-denotation and the propositional denotation. Yet, it seems to me that there are some reasons not to give up on the ambiguity hypothesis. Consider the translation of event-taking **is informed of** I proposed:

(20) $\lambda y\ \lambda x$ (x is informed_1(occur′(y))),

In (20) y is an individual variable. Of course, we could modify the translation in (20) by allowing the position of y to be a generalized quantifier's position:

(25) $\lambda Q\ \lambda x$ (x is informed_1(Q(occur′))),

But there are good reasons not to do so. If (25) is a possible translation of **is informed of**, (26) is predicted to have a reading synonymous with (27):

(26) John is informed of only three arrivals of Mary
(27) John is informed that only three arrivals of Mary occurred

This seems to be incorrect, since (27) entails that only three arrivals of Mary occurred, but (26) does not have a reading with this entailment. The interpretation of (26) provides thus evidence that

(20), and not (25), is the translation of event-taking **is informed of**

Now, suppose that derived NPs unambiguously denote events. Then, a problem arises. Given the translation in (20), we predict incorrectly that (28) entails (29), given the plausible assumption that the arrival of Jocasta and the arrival of Oedipus's mother are the same event:

(28) Oedipus is informed of the arrival of Jocasta
(29) Oedipus_i is informed of the arrival of his_i mother

This problem does not arise if derived NPs may also denote propositional entities, since the proposition that Jocasta arrived is not identical to the proposition that Oedipus's mother arrived. There is more. Consider:

(30) the police are informed of the arrival of a mafia boss from Sicily

(31) the police are informed that a mafia boss arrived from Sicily

(30), like (31), can be true in a situation in which there is no particular mafia boss such that the police is informed of his arrival from Sicily. But if the NP **the arrival of a mafia boss from Sicily** unambiguously denotes an event and (20) is the translation of event-taking **is informed of**, we cannot account for the existence of this reading of (30). Again the problem does not arise if derived NPs can denote propositional entities, since in this case the NP **the arrival of a mafia boss from Sicily** may denote the proposition that a mafia boss arrived from Sicily. The hypothesis that derived NPs may denote propositional entities, therefore, seems to do some work in accounting for the interpretation of (28)-(31).

8 A Dilemma

It seems we have reached an impasse. In the last section, I gave some reason for assuming that derived NPs are ambiguous between the event-denotation and the propositional denotation. But, in section 6, I presented evidence that seems to contradict this hypothesis: if derived NPs are ambiguous between the event-denotation and the propositional denotation, why doesn't (3) have a reading synonymous with (4)?

(3) John is informed of Mary's arrival
(4) John remembers that Mary arrived

9 A Solution

In this section, I am going to suggest that the data in (3) and (4) are compatible with the claim that derived NPs are ambiguous between the event-denotation and the propositional denotation. To show how this may be the case, I need to discuss first another puzzle that arises in connection with the ambiguity of lexical items.

Consider the following sentences from Rooth and Partee (1982), Partee and Rooth (1983):

(a) John caught and ate a fish
(b) John hugged and kissed three women

Partee and Rooth have pointed out that "unless the sentences [in (a)-(b)] are given a very marked intonation or the context is heavily loaded, we must interpret (a) as involving just one fish, and (b) as saying that the same three women were hugged and kissed." Partee and

Rooth have observed, moreover, that, if the object positions of extensional transitive verbs are generalized quantifier positions, we should expect instead, given the most natural unified cross-categorial interpretation for **and**, that (a)-(b) should have a reading according to which the fish that is caught is not the same as the fish that is eaten, and the three women that are hugged are not the same as the three women that are kissed. We should conclude, therefore, that the object positions of extensional verbs are not of the type of generalized quantifiers, but of type e (type 1, in the notation I am going to adopt). This conclusion, however, is incorrect: the fact that we need to be able to specify the object positions of extensional transitive verbs as a generalized quantifier-type positions is shown by (c):

(c) John needed and bought a new coat

If the object positions of extensional transitive verbs were only of type e, we should expect (c) to allow only the "quantified in" reading of the NP **a new coat**, but (c) may also be true in a situation in which there is no particular coat John needed. The data in (a)-(c), therefore, raise a problem: on one hand, (a)-(b) seem to show that the object positions of extensional transitive verbs are positions of type e and not of the type of generalized quantifiers; on the other hand, (c) shows that the object positions of extensional transitive verbs may be generalized quantifier-type positions.

What is Partee and Rooth's solution to this puzzle? They assume that extensional transitive verbs are entered in the lexicon with e-type object positions, while intensional verbs are entered in the lexicon with generalized quantifier-type object positions. Lexical rules may then raise the type of the object positions of extensional verbs to the type of generalized quantifiers. The puzzle provided by (a)-(c) may then be solved by supposing there is a processing strategy according to which all expressions are interpreted at the lowest type possible, higher type homonyms being selected only when needed for type coherence. This has the consequence that in (a)-(b) the verb **catch** and **hug** must be translated by predicate constants whose object positions are of type e, thus accounting for the interpretation of (a)-(b). In (c), on the other hand, **buy** must be translated by a predicate constant whose object position is of generalized quantifier-type, since otherwise the type of **buy** would differ from the type of **need** and the translation would be ill-formed. This accounts for the opaque reading of (c).

Now, it may be noticed that the problem with (a)-(c) is, to some extent, parallel to the problem we find with (3) and (30):

(3) John remembers Mary's arrival
(30) the police are informed of the arrival of a mafia boss from Sicily

(a)-(b) seem to show that the object positions of extensional verbs are of type e, and not of the type of generalized quantifiers, but the assumption that the object positions of extensional transitive verbs may be of the type of generalized quantifiers is needed to account for the interpretation of (c). Here, (3) seems to show that derived NPs denote events and not propositional entities; but the assumption that derived NPs may denote propositional entities is needed to account for the interpretation of (30). As for (a)-(c), the solution to the puzzle provided by (1), (30) may then lie in the interaction between the ambiguity of lexical items and the processing strategies to deal with this ambiguity. Suppose that nouns like **arrival** are entered in the lexicon as denoting events and that there are rules that can create nouns denoting propositions out of nouns denoting events.[7] Suppose, moreover that the general processing strategy is to choose the primitive entry over the derived one, unless this leads to ill-formedness. This means that we will interpret nouns like **arrival** as denoting events, the propositional homonym being invoked only when this is needed for semantic well-formedness. Now, we pointed out that **is informed of** and **remember**$_{NP}$ select both for events and for propositional entities. This is shown by (i) and (ii):

(i)a. John is informed of the fact that the Titanic sunk
  b. John is informed of that event

(ii)a. John remembers the fact that the Titanic sunk
  b. John remembers that event

We also noticed, however, that **is informed of** and **remember**$_{NP}$ differ in an important respect. The interpretation of event-taking **is informed of** may be specified in terms of propositional **is informed of** as shown by the fact that (i)b. is synonymous with (i)c.:

(i)b. John is informed of that event
(i)c. John is informed of the fact that that event occurred

The interpretation of event-taking **remember**$_{NP}$, on the other hand, cannot be specified in the same way in terms of proposition-taking **remember**. Remembering an event is not the same as remembering the fact that that event occurred. Take the sinking of the Titanic. I may remember the fact that that event occurred, but this doesn't mean that I remember the sinking of the Titanic, unless I witnessed the sinking. One way we may represent this difference between **remember**$_{NP}$ and **is informed of** is by assuming that, while in the case of **remember**$_{NP}$ both the event-taking predicate and the propositional one are entered as primitive in the lexicon, **is informed of** is entered in the lexicon as a proposition-taking predicate, its event-taking homonym being derived via a lexical rule of the kind in (32):[8]

(32)

if P' is of type <0, 1>, 1> and P'(x)(y) is defined only if x is a proposition, then P_e is of type <<0, 1>, 1>, and P_e = $\lambda x\ \lambda y\ (P'(occur'(x))(y))$

The picture we have got is thus the following:

(I) derived nouns are entered in the lexicon as denoting events, the propositional homonyms are derived by rule from the event-nouns;

(II) predicates like **is informed** of are entered in the lexicon as proposition-taking predicates, the event-taking homonyms are derived by rules of the kind in (32) from the propositional predicates; both propositional **remember**$_{NP}$ and event-taking **remember**$_{NP}$ are entered as primitive in the lexicon;

(III) the general processing strategy is to choose the primitive entry over the derived one, unless this leads to ill-formedness.

Now, let's go back to our examples in (3), (24) and (30)

(3) John remembers Mary's arrival
(24) John remembers the fact that Mary arrived

(30) the police are informed of the arrival of a mafia boss from Sicily

The lack of a reading of (3) synonymous with (24) is now expected. The noun **arrival** is interpreted as denoting an event, unless this leads to ill-formedness. There is no need for the noun to get the propositional interpretation, since event-taking **remember** is a primitive entry in the lexicon. In interpreting (30), on the other hand, a conflict arises, since, according to (II)-(III), **is informed of** should be interpreted as a proposition-taking predicate, but, according to (I), (III), the NP **the arrival of a mafia boss from Sicily** should denote an event. Thus, something must be done to achieve a well-formed translation of (30). Two strategies are available: we can either (a) assume that the predicate is event-taking and let the NP denote an event, or (b) interpret the NP as denoting a proposition and let the predicate select for propositions. We should thus expect (30) to be ambiguous. In particular, the alternative in (b) accounts for the existence of the reading of (30) which is compatible with the police not being informed of the arrival of any particular mafia boss.

This section concludes my discussion of the ambiguity hypothesis. I presented some evidence that derived nominals are ambiguous between event-denotation and propositional denotation. I also tried to explain why this ambiguity does not show up with some predicates. I now turn to the issue of the distributional differences between propositional NPs and that-clauses.

10 The Case for States of Affairs

In section 3, I observed that derived NPs like (33)

(33) the arrival of the soprano

cannot occur with some predicates of propositional entities. For example, (33) cannot be the object of **believe** and **know**, or the subject of **is true**, **is false**:

(34)a. # John believes the arrival of the soprano
b. # John knows the arrival of the soprano
c. # the arrival of the soprano is true
d. # the arrival of the soprano is false

e. John is informed of the arrival of the soprano
f. John is aware of the arrival of the soprano

In section 3, I also pointed out that derived NPs share this feature with other NPs which arguably denote propositional entities, like the gerundive NP in (35):

(35) the soprano's performing the song

The NP in (35), like that in (33), is unable to occur with predicates of propositions like **believe**, **know**, **is true**, **is false**:

(36)a. # John believes the soprano's performing the song
b. # John knows the soprano's performing the song
c. # the soprano's performing the song is true
d. # the soprano's performing the song is false

e. John is informed of the soprano's performing the song
f. John is aware of the soprano's performing the song

Since nothing I have said so far accounts for the ill-formedness of (34a-d) and (36a-d), the question arises:

(37) why are (34a-d) and (36a-d) ill-formed?

There is no obvious syntactic reason why (34a-d), (36a-d) are ill-formed. The predicates **believe** and **know** are able to take proposition-denoting NP-objects and the predicates **is true**, **is false** are able to take proposition-denoting NP-subjects:

(38)a. John believes many things. (He believes that the earth is round, that 2+2=4,...etc.)
b. John believes this proposition
c. John believes everything
d. John believes that

(39)a. John **knows** many things. (He knows that the earth is round, that 2+2=4,...etc.)
b. John knows this proposition
c. John knows everything
d. John knows that

(40)a. this is true
b. many things are true
c. everything is true
d. this proposition is true

(41)a. this is false
b. many things are false
c. everything is false
d. this proposition is false

Sentences (38)-(41), thus, show that the ill-formedness of (34a-d),(36a-d) is not due to the fact that proposition-taking **believe**, **know**, **is true**, **is false** do not subcategorize for NP (since (38)-(41) show there is no such fact).

A possible conjecture concerning (34a-d), (36a-d) is that their ill-formedness is related to the factive character of the NPs **the soprano's performing the song** and **the arrival of the soprano**. Suppose that the NP in (35) denotes, and the NP in (33) can denote, a fact, rather than a proposition.[9] This assumption may then account for the ill-formedness of (34a-d), (36a-d), since the ill-formedness of (42) below suggests that **believe**, **know**, **is true**, **is false** do not semantically select for facts:

(42)a. # John knows the fact that Mary arrived
b. # John believes the fact that Mary arrived
c. # the fact that Mary arrived is true
d. # the fact that Mary arrived is false

Notice, however, that there is evidence that NPs like **the soprano's performing the song** should not be taken to denote facts. The assumption that these NPs denote facts predicts incorrectly that (43) can only have a contradictory interpretation, on a pair with (44)

(43) we prevented his succumbing to the temptation by hiding all the cookies from him

(44) # we prevented the fact that he succumbed to the temptation by hiding all the cookies from him

The existence of a non-contradictory reading of (43) seems thus to provide a reason not to pursue an account of (34), (36) based on the hypothesis that the NP **the soprano's performing the song** denotes a fact.[10]

The ill-formedness of (44) seems to me to suggest that the semantic distinction at stake in the contrasts in (34), (36) is not the distinction between **propositions** and **facts**, but, more generally, the distinction between **propositions**, things that can be true or false and can be objects of belief, and **states of affairs**, things of which one may be aware, may be informed, but which, unlike propositions, cannot properly be said to have the property of truth or falsehood, or be objects of belief.[11] The ill-formedness of (36a-d), according to this view, depends thus on the fact that the NP **the soprano's performing the song** denotes a state of affairs, while the predicates **believe, know, is true, is false** semantically select for propositions, but not for states of affairs. Similarly, sentences (34a-d) are ill-formed, because the NP **the arrival of the soprano** can either denote a state of affairs or an event, but neither states of affairs nor events are selected by **believe, know, is true, is false**. Notice, moreover, that the assumption that **believe, know, is true, is false** do not select for states of affairs may also account for the ill-formedness of (42), since facts, in this proposal, may be regarded as a subset of states of affairs, those states of affairs that **are actual**.

Before I turn to implementing my analysis of the meanings of derived nominals and gerundive nominals in a formal framework, I need to say something more on the relation between states of affairs and propositions. Unlike states of affairs, propositions can be true or false and may have the property of being believed by someone. Unlike propositions, states of affairs are entities of which one may be aware or be informed. If states of affairs are distinguished in this way from propositions, however, in other respects they bear a close relation to propositions. For example, the proposition that the soprano performs the song is true if and only if the state of affairs of the soprano's performing the song is actual. And the proposition that the soprano does not perform the song is true just in case the state of affairs of the soprano's not performing the song is actual, which is the case if and only if the state of affairs of the soprano's performing the song is not actual. Similarly, the proposition that the soprano performs the song and that the tenor is bald is true just in case the state of affairs of the soprano's performing the song and of the tenor's being bald is actual. And so on. Furthermore, states of affairs, like propositions, are finely individuated. For example, the proposition that the soprano performs the song is not identical to the proposition that Maria performed the song. One can believe the latter without believing the former, if he is convinced that Maria is a contralto, and one can believe the former without believing the latter, if he does not know that the Maria is the soprano. Similarly, the state of affairs of the soprano's performing the song is not identical to the state of affairs of Maria's performing the song. One can be informed of the soprano's performing the song without being informed of Maria's performing the song, and vice versa. Again, Oedipus may believe the proposition that he married Jocasta without believing the proposition that he married his mother. Similarly, we may say that Oedipus is informed of his marrying Jocasta, without, however, being informed of his marrying his mother.

11 Implementing the Proposal[12]

I provide an interpretation of (33) and (35) based on Kratzer's (1989) semantics of situations.

(33) the arrival of the soprano
(35) the soprano's performing the song

In this semantics, propositions are represented as sets of possible situations, and situations are ordered by the part of-relation. Each situation and each individual belong to only one world. I assume that situations belonging to different worlds, like individuals belonging to different worlds, may be related to each other by a counterpart relation in Lewis's (1968) sense. The list of ingredients of Kratzer's semantics of situations is given below:

S a set, the set of possible situations

I a set, the set of ordinary individuals

$\leq$ a partial ordering on S such that the following conditions are satisfied:

(i) for every $a \in I$, there is an $s \in S$ such that $a \leq s$
(ii) for no $s \in S$ is there an $a \in I$ such that $s \leq a$
(iii) For all $s \in S \cup I$ there is a unique $s' \in S$ such that $s \leq s'$ and for all $s'' \in S$, if $s' \leq s''$, then $s''=s'$

$\wp(S)$ the power set of S, the set of propositions

W the set of maximal elements with respect to $\leq$, or the set of possible worlds

w_s for all $s \in S$, w_s is the maximal element to which s is related by $\leq$

C is a counterpart relation between members of $S \cup I$

11.1 The Event-Interpretation of the Noun arrival

The event-interpretation of the noun **arrival** is specified as follows: an arrival of x is a situation s such that (a) s is a situation in which x arrives, and (b) there is no situation s' such that x arrives in s' and s' is a proper subpart of s. More formally (assuming that 1 is the type of entities and 0 is the type of propositions), V(arrival′) is the function ω in $D_{<<0, 1>, 1>}$ such that for every a, b, c $\in D_1$, for every s $\in$ S,

(i) $(\omega(a))(b)$ is defined iff b $\in$ S and V(arrive′)(a) is defined

(ii) s $\in (\omega(a))(b)$ iff b $\leq$ s, b $\in$ V(arrive′)(a), and there is no situation s′ such that s′ $\in$ V(arrive′)(a) and s′ < b

11.2 The Interpretation of Gerundive Nominals

In order to provide the interpretation of the NP in (35)

(35) the soprano's performing the song,

I add the following ingredients to Kratzer's list:

A a set, the set of states of affairs

f a function from $\wp(s)$ to A such that, if f(p) = a, then p is true iff a is actual

Intuitively, f relates the proposition that the soprano performs the song to the state of affairs of the soprano's performing the song, the proposition that the soprano does not perform the song to the state of affairs of the soprano's not performing the song, and so on.[13] I also assume that the set of states of affairs A is included in the domain of individuals D_1

$A \subseteq D_1$

Finally, I assume the following syntactic rule and translation rule for gerundive nominals (I take IVP to be the category of Intransitive Verb Phrases in Bach's (1980) sense):

S_{gn}. If b belongs to the category IVP and a is an NP bearing the feature [+genitive], F_k(a, b) belongs to the category NP, where F(a, b)= ab* and b* is the result of suffixing -**ing** on the head of IVP.

T_{gn}. F_k(a, b) translates as i(a′(b′))

The interpretation of i is specified as follows:

V(i) is the function $\omega \in D_{<1, 0>}$ such that, $\forall a \in D_0$,

$\omega(a) = f(a)$, if a $\in \wp(S)$,
otherwise $\omega(a) = a$

The interpretation of i, together with the translation rule T_{gn}, has the effect of assigning to the NP in (35) a state of affairs as a value, the state of affairs that corresponds to the proposition that the soprano performs the song according to the function f.

11.3 The Propositional Interpretation of the Noun arrival

I assume that extensional event-nouns whose translations belongs to the type <<0, 1>, 1> have also higher type translations. In particular, the event-noun **arrival** will have a higher type translation **arrival**^{r} defined as follows:

$$\mathbf{arrival}^r = \lambda Q\ \lambda x\ Q(\lambda y\ (\mathbf{arrival}'(y))(x)))$$

As a second step, I assume the existence of the following syntactic rule and translation rule (I take IN to be the category of intransitive nouns):

S. If $a \in IN$, $F_s(a) \in IN$,
where $F_s(a) = a$
T. $F_s(a)$ translates as s.o.a.(a')

These rules have the function of creating nouns denoting states of affairs out of nouns denoting events. In particular, the operator s.o.a. will carry the semantic burden of this operation. The result of applying the s.o.a. operator to the event-noun **arrival**^{r} is described below (for sake of brevity, I refer to the type of generalized quantifiers <0, <0, 1>> as gq):

$V(\text{s.o.a.}(\mathbf{arrival}^r))$ is the ω in $D_{<<0,\ 1>,\ gq>}$ such that $\forall\ \omega_1 \in D_{gq}$, $\forall a \in D_1$, $\forall s \in S$,

(i) $(\omega(\omega_1))(a)$ is defined only if $a \in A$ (i. e. only if a is a state of affairs)

(ii) $s \in (\omega(\omega_1))(a)$ iff $a = f(p)$ where $\forall s'$, $s' \in p$ iff $\exists s''$ such that $s' \in (V(\mathbf{arrival}^r)(\omega_1))(s'')$

The interpretation of the operator s.o.a is specified as follows:

$V(\text{s.o.a.})$ is the function ω in $D_{<<<0,\ 1>,\ gq>,\ <<0,\ 1>,\ gq>>}$ such that $\forall\omega_1 \in D_{<<0,\ 1>,\ gq>}$, $\forall\omega_2 \in D_{gq}$, $\forall a \in D_1$, $\forall s \in S$,

$s \in ((\omega(\omega_1))(\omega_2))(a)$ iff $a = f(p)$, where $\forall s' \in S$, $s' \in p$ iff $\exists s''$ such that $s' \in (\omega_1(\omega_2))(s'')$

Given these interpretation rules, the NP in (33)

(33) the arrival of the soprano

will denote, in one interpretation, the state of affairs[14] which is the image of the proposition p relative to f, where p is the set of situations that contain as a subpart an arrival of the soprano. This set is identical to the set of situations (i. e., to the proposition) which is the value of the sentence in (45):

(45) the soprano arrives (tenseless)

The interpretation given above for the noun **arrival** predicts that the NP in (33) denotes, in one interpretation, the state of affairs that corresponds to the proposition which is the meaning of (45). According to this way of specifying the interpretation of **arrival**, the NP in (33) and the NP in (46)

(46) the arrival of Maria

may denote different states of affairs. This is the case, since the set of possible situations in which the soprano arrives is different from the set of possible situations in which Maria (or a counterpart of her) arrives.

-- Notes --

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2. In fact, in order to account for the likeness of meaning between (1) and (2), something more needs to be said about the way the temporal interpretation of NPs works, since the **that**-clause in (2) contains a past tense marker, while the derived NP in (1) does not. Here, I am going to ignore this problem because of space limitations and because, as we will see, a similar problem arises also if we account for the similarity of meaning between (1) and (2) without assuming that the object NP in (1) denotes a proposition. This means that the issue of the temporal interpretation of NPs is not likely to yield evidence in favor or against the hypothesis that derived NPs are semantically ambiguous.

3. A similar formal device is adopted in Cooper (1979).

4. In the semantic format I adopt at the end of the paper, the interpretation of **surprise**$_1$ and **surprise(P)** may be specified as follows (see section 11 for a description of the formal system):

V(**surprise**$_1$) is the ω in $D_{<<0,\ 1>,\ 1>}$ such that, $\forall a\ \forall p \in D_1$, $\forall s \in S$,
(i) $\omega(a)(p)$ is defined only if $a \in I$ and $p \in D_0$
(ii) $s \in (\omega(a))(p)$ iff p surprises a

V(**surprise(P)**) is the ω in $D_{<<0,\ 1>,\ 1>}$ such that, $\forall a\ \forall b \in D_1$, $\forall s \in S$,
(i) $\omega(a)(b)$ is defined only if $a, b \in I \cup S$
(ii) $s \in (\omega(a))(b)$ iff it surprises a that b is P (i. e., iff $s \in (V(\textbf{surprise}_1)(a))P(b))$

5. Here, I am ignoring the issue of how the correct temporal interpretation for the predicate **occur** in the translation of **is informed** is achieved. As we saw, the proposal that derived NPs may denote propositions runs into a similar problem.

6. The meaning of **occur'** may be specified as follows in the semantics described in section 11:

$V(\mathbf{occur'})$ is the $\omega \in D_{<0,1>}$ such that $\forall a \in D_1$, $\forall s \in S$,
(i) $\omega(a)$ is defined only if $a \in S$
(ii) $s \in \omega(a)$ iff $a \leq s$

The condition in (i) requires that the argument of **occur'** must denote an event in order for the function denoted by the predicate to be defined.

7. In section 11.3, I provide an explicit formulation of these rules.

8. I take 0 to be the type of propositions and 1 to be the type of entities. Following Cresswell (1973), I assume that propositions are included in the domain of entities.

9. This view was originally suggested by Z. Vendler.

10. Notice that assuming that gerundive NPs are ambiguous between denoting a proposition and denoting a fact, as suggested, for example, by Peterson (1979), would not help here. This proposal, while it would account for the existence of a non-contradictory reading of (43), would lead us to expect that (36a-d) should be well-formed, contrary to the data.

11. In the philosophical literature, a similar distinction has been argued for by Plantinga (1976):

> It is obvious, I think, that there are such things as **states of affairs**: for example, **Quine's being a distinguished philosopher**. Other examples are **Quine's being a distinguished politician**, **9's being a prime number**, and the state of affairs consisting in all men's being mortal... It is also obvious, I believe, that there are such things as **propositions**-the things that are true or false, believed, asserted, denied, entertained, and the like... Are there really two sorts of things, propositions and states of affairs, or only one? I am inclined to the former view on the ground that propositions have a property-truth or falsehood-not had by states of affairs.

Notice that the expressions Plantinga uses to refer to states of affairs are gerundive nominals, a significant fact in the present context. It should also be noticed, on the other hand, that the view I am adopting differs in a number of ways from Plantinga's. In particular, unlike Plantinga, I am assuming the existence of event-like things as entities that are distinct both from propositions and from states of affairs.

12. Due to space limitations, I am unable to motivate here some of the features of the semantics presented in this section. A more detailed discussion of this semantics is given in Zucchi (1989).

13. Given an algebra of states of affairs and an algebra of propositions we could achieve this effect, by making f a homomorphism with respect to the logical connectives.

14. More correctly, it will denote a property of properties of the state of affairs in question.

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Kinds and Generic Terms

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0. Introduction

The focus of the study of generics is the behavior of noun phrases like those in (1) and (2), that involve reference to a kind.

(1) a. The panda is small.
    b. Pandas are small.
    c. A panda is small.

(2) a. The panda is almost extinct.
    b. Pandas are almost extinct.
    c.?A panda is almost extinct.

In this article, I will concentrate on only two of these generic noun phrases, namely the ones in the a- and b-sentences of (1) and (2), that I will refer to as the DEFINITE GENERIC and the BARE GENERIC, respectively. The special thing about these two generics (which distinguishes them from the indefinite singular generic in (1c) and (2c)) is their capacity to refer to kinds in two ways. In (1) they are used in a DERIVED KIND PREDICATION, which means that a property is assigned to each individual member of the kind; in (2) they are used in a PROPER KIND PREDICATION, which means that a property is assigned to the whole kind[1].

I will be concerned with two interrelated questions about these generics. First: how do these two types of generic noun phrases refer to kinds? are they like proper names or definite descriptions? Second: is a unified approach to the bare plural possible?

This last question requires some explanation. In Carlson's theory of generics (see Carlson(1977, 1978, 1979, 1982)), the bare plural is uniformly treated as a proper name for a kind. Recently, however, an alternative treatment has been proposed, in Wilkinson(1986), Krifka(1987), Diesing(1988) and Kratzer(1989). Assuming the framework of Lewis(1975), Kamp(1981) and Heim(1982) of unselective binding, they treat the bare plural as a variable with a predicate, and the interpretation of the bare plural depends on how this variable is bound, by an existential or a generic quantifier[2]. It is not clear whether this variable-analysis can cover all uses of the bare plural, i.e. whether it can provide a unified approach to the bare plural.

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[1] These terms are borrowed from Link(1988) and Ter Meulen(1988).

[2] This is only a very rough characterization of this approach. In the next section I will give a more comprehensible exposition.

My claims in this article will be first, that the definite generic is essentially a proper name for a kind and the bare generic a variable over plural objects, and second, that a unified approach of the bare plural within the 'variable-analysis' is indeed possible.

In order to show this I will start with a review of some differences and similarities of bare generics and definite generics, then I will sketch the approach to bare generics in Diesing(1988) and Kratzer(1989) and finally, I will describe how definite and bare generics can be treated in a DRT-framework, incorporating both the Diesing-Kratzer proposal and Link's lattice-theoretical approach to the logic of plurals.

1. A Comparison of the Bare Plural and the Definite Generic

I will first discuss three aspects in which bare plurals and definite generics are alike and then three in which they differ.

Definite and bare generics are set apart from indefinite singular generics like a whale because they can be subject of so-called KIND PREDICATES like widespread, rare, numerous, die out, as shown in (3) and (4)[3].

(3) a. The whale is rare.
b. Whales are rare.
c.?A whale is rare.

(4) a. The dinosaur is extinct.
b. Dinosaurs are extinct.
c.?A dinosaur is extinct.

Bare and definite generics also show the same behavior in a construction that Carlson used, to support his claim that bare plurals are like proper names: the construction X is so called because Y where X is a proper name and Y is the reason why X has the name that he has.

(5) a. Slim is so-called because of his slender build.
b. Cardinals are so-called because of their color.
c. The cardinal is so-called because of its color.
d. A cardinal is so-called because of its color.

As (5a-c) show, ordinary proper names and bare and definite generics are allowed in this construction. This might be taken as evidence that both bare generics and definite generics are proper names for kinds, but (5d) shows, however, that a generic NP with the indefinite article is also possible. This throws doubt upon the use of this

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[3] It is not impossible, however, to interpret (3c) and (4c). (3c) can be interpreted as something like: Meeting a whale is something that rarely happens. (4c) can be read as: there is a subvariety of the dinosaur kind that is extinct. Both of these readings are not the kind of generic readings we are talking about. I will assume that these readings can be explained under a different theory.

construction as a test for proper name-ness of generics, unless one is willing to treat the generic in (5d) as a proper name for a kind too.

The next similarity is taken from Lawler(1973). Lawler makes a distinction between accidental and essential properties. Accidental properties, like popular in (6) can be attributed to bare generics and definite generics, but not to indefinite singular generics.

(6) a. The madrigal is popular.
    b. Madrigals are popular.
    c.?A madrigal is popular.

Essential properties like polyphonic in (7) can be applied to all three generic NPs.

(7) a. The madrigal is polyphonic.
    b. Madrigals are polyphonic.
    c. A madrigal is polyphonic.

However, it is uncertain on the basis of these examples whether this should really be regarded as a difference between essential and accidental predication. An equally viable point of view is that popular applies to the whole genre of the madrigal, while polyphonic distributes over the individual members of the genre. This assimilates the difference between (6) and (7) to that between Proper Kind Predication and Derived Kind Predication in (1) and (2).

The conclusion that can be drawn is that there is only one clear common property of bare and definite generics: their ability to be the subject of a kind predicate. I will now turn to three aspects in which definite and bare generics differ. As I hope to show, the differences between bare and definite generics are more impressive and revealing than their similarities.

Several authors have pointed out that definite generics obey a very strict restriction on the sort of CN they contain. (Examples (8a,b,c) and (9b,c) are from Krifka(1987,1989).)

(8) a.?The lion with three legs is ferocious.
    Lions with three legs are ferocious.
    b.?The green bottle has a narrow neck.
    Green bottles have a narrow neck.
    c.?The German fly is a lazy insect.
    German flies are lazy insects.
    d.?The big computer is expensive.
    Big computers are expensive.

(9) a. The Siberian tiger is ferocious.
    Siberian tigers are ferocious.
    b. The Coke bottle has a narrow neck.
    Coke bottles have a narrow neck.
    c. The German shepherd is a faithful dog.
    German herders are faithful dogs.

d. The personal computer is cheap.
   Personal computers are cheap.

Krifka states that a definite generic must denote a kind that is WELL-ESTABLISHED. There is not a particular kind, that we are all familiar with, that is called the lion with three legs or the dutch dog. But we can somehow refer to those kinds with bare generics. The Siberian tiger, the Coke bottle and the personal computer on the other hand are well-established kinds that can also be denoted by definite generics as (9) shows.

Another difference is that definite generics can be used as objects of dynamic verbs (verbs expressing change) whereas bare generics cannot. (The examples are again adapted from Krifka(1989).)

(10) a. The Sumerians invented the pottery wheel.
     b.?The Sumerians invented pottery wheels.

(11) a. The Dutch settlers in Mauritius exterminated the dodo.
     b.?The Dutch settlers in Mauritius exterminated dodos.

A final difference, that seems to have gone unnoticed, shows up when we apply collective predicates like meet or gather to bare and definite generics.

(12) a. Spies meet in the dark.
     b.?The spy meets in the dark.

(13) a. Teenagers gather on the streets.
     b.?The teenager gathers on the streets.

According to my judgments a collective predicate, i.e. one that applies to plural objects (or I-SUMS in Link's terminology) can only be used with bare generics, not with definite generics. (12a) can mean that whenever a group of spies meets, it meets in the dark. This reading is not available for (12b). Similarly in (13).

What can this comparison learn us about definite and bare generics? We can safely conclude that (a) both can refer to a kind as a whole, (b) the bare generic is not possible on all positions, whereas the definite generic is possible on all positions (where it is licensed by the selectional restrictions imposed by the predicate) and (c) the definite generic can denote only well-established kinds, the bare generic every possible[4] kind.

The comparison shows, however, that there is no evidence that the bare generic should be treated as a proper name for a kind. The so-called-test does not work

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[4] Possible here means that every bare plural noun phrase that the syntax generates can denote a corresponding kind.

because it applies to all generics and the ability to denote a kind should not be considered as evidence either, because a noun phrase can denote an entity without having to be a proper name for that entity.

In the rest of this article I will work out an account of definite generics as uniformly proper names for kinds and bare plurals as uniformly variables over plural objects. This will account for the similarities and the differences of the two types of generics in a principled way.

Before showing this, I will first explain how the bare plural is treated in Diesing(1988) and Kratzer(1989), because my proposal in section 3 makes use of their theory.

2. The Diesing/Kratzer-approach to Bare Plurals

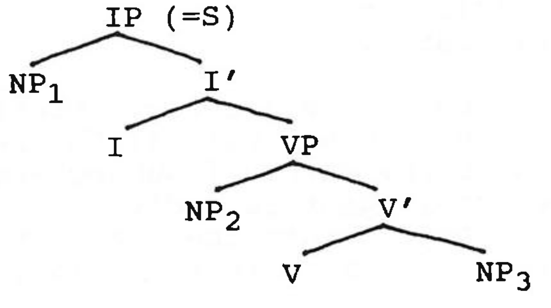
The main tenet of Diesing(1988) is that the syntactic position of a bare plural is crucial for its interpretation. She assumes, following Krifka(1987) and Wilkinson(1986), that the semantic representations of generic and non-generic sentences can be schematically given as in (14).

(14) a. generic : $G[_R \ldots]\ \exists[_S \ldots]$
b. non-generic: $\exists[_S \ldots]$

Generic sentences have a tripartite structure that consists of a generic operator **G**, a restriction **R** and a nuclear scope **S**, which is closed off by an existential quantifier (the so-called existential closure). A non-generic sentence consists of nothing but an existentially closed nuclear scope. Both **G** and ∃ unselectively bind the free variables in their scope. Formulated in DRT-terms: generics involve a DRS-split, non-generic sentences don't.

As for the syntactic part, she assumes that sentences have the schematic syntactic structure in (15)[5]:

(15)



There are potentially two subject-positions in this sentence structure: NP_1 is called the EXTERNAL SUBJECT position (external means outside VP), NP_2 is called the INTERNAL SUBJECT

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[5] In this analysis, proposed in this form in Chomsky(1986), the sentence is a projection of the inflection of the verb, I or Infl, which also carries Tense and Agreement marking. This I takes the subject and the predicate (VP) as a kind of arguments.

position. NP_3 is the object of the verb. If a verb has a subject, this subject is either realized internally or externally, as NP_2 or NP_1.

Kratzer(1989) shows that the position on which the subject is realized can be made to depend on whether the verb has a Davidsonian event-argument or not, which is her way of distinguishing whether it is a stage-level or individual-level predicate. It would go to far to explain exactly how this is accomplished. Kratzer shows that it can be done quite elegantly, making use of general principles that determining how the arguments of a predicate are realized in syntactic structure. Stage-level predicates (like sick or available) project their subject internally to VP (on the position NP_2) and this subject can optionally be moved up to the higher subject-position NP_1. Individual-level predicates (like intelligent or tall) directly project their subject externally (on the position NP_1).

Diesing makes the following proposal as for the mapping between the syntactic structure in (15) and the logical forms in (14). All material inside VP maps into the nuclear scope of a quantification, all material outside VP maps into the restrictive term of the quantification. This means essentially that the variable (reference marker) for a bare plural on position NP_2 or NP_3 is in the nuclear scope (right-hand box) of (14a) or (14b), but that the variable (reference marker) of a bare plural on position NP_1 is in the restrictive term (left-hand box) of (14a). As a result, bare plurals on the external subject position are interpreted generically, but bare plurals inside the VP are interpreted existentially.

Diesing showed that Carlson's theory (bare plurals as proper names for kinds) predicts too few readings for (16) and the theory of Krifka and Wilkinson (bare plurals as variables) too many. The Diesing/Kratzer-proposal however, predicts exactly which readings are possible for these sentences.

(16) a. Firemen are intelligent.
     b. Firemen are available.

(16a) has only one reading: it is a generic statement about all firemen. This is predicted as follows: the subject NP firemen is projected onto the external subject-position because the predicate intelligent is individual-level. Being outside VP, it is mapped into the restrictive term and therefore receives a generic interpretation, shown in (17a).

(17) a. **G[fireman(x)] ∃[intelligent(x)]**
     b. (i) **∃[fireman(x)]**
     (ii) **G[...] ∃[fireman(x)]**
     (iii) **G[fireman(x)] [available(x)]**

(16b) has three readings, represented in (17b): one that says that there are some firemen available now ((i)), another one that says that usually, there are some firemen available ((ii)) and finally, one that says that it is a generic property of firemen to be available ((iii)). They are derived in the following way. In (16b) the NP firemen is generated in deep structure on the internal subject-positions (because available is a stage-level predicate) and can be moved to the external subject-position. If it stays on the VP-internal subject position, it is mapped in the nuclear scope of a non-generic sentence (in (i)) or a generic sentence (in (ii)). In both cases the subject is existentially interpreted, because of unselective binding by existential closure. If it is moved to the external subject position, it is mapped into the restrictive term and it receives the generic reading in (iii).

Notice however, that this proposal doesn't say much about Proper Kind Predications (predications about the kind as a whole), only about Derived Kind Predications (predications about the individual members of a kind). The reason for this is that Proper Kind Predications cannot be treated as a generic quantification, with a restriction and a nuclear scope, as in (14a). (18b) does not seem to be the right semantic representation for (18a), because the property of being widespread should not be assigned to each individual rat (which is what happens in (18b)), but only to the kind as a whole.

(18) a. Rats are widespread.
 b. G_x**[rat(x)] widespread(x)**

Moreover, the proposal of Diesing and Kratzer doesn't say much about definite generics either. Suppose we treat a definite generic like the rat as a variable, subject to the same interpretation mechanism as indefinites. This seems to make the correct prediction that a definite generic can be mapped into the restrictive term like in (19), yielding a Derived Kind Predication - just like bare plurals.

(19) a. The rat is intelligent.
 b. **G[rat(x)]** $\exists$**[intelligent(x)]**

But it also predicts, incorrectly, that the definite noun phrase can be mapped into the nuclear scope of a non generic sentence, yielding an existential reading. In (20), the definite noun phrase the sandwich occupies a position inside the VP. According to Diesing's proposal, it is mapped into the nuclear scope and this means that its variable is bound by the existential closure.

(20) a. I ate the sandwich.
 b. $\exists_x$ **[sandwich(x) & ate(I,x)]**

However, (20b) does not seem to be the correct logical form for (20a). This makes it implausible that we can treat definite noun phrases as variables that can be bound in either part of a generic sentence.

Clearly, the theory of Diesing and Kratzer needs to be extended to account for the kind-interpretation of the bare plural in (2b) and the uses of the definite generic in (1a) and (2a).

(1) a. The panda is small.
    b. Pandas are small.

(2) a. The panda is almost extinct.
    b. Pandas are almost extinct.

This is what I will try to do in the remainder of this article. I will start with the definite generics and then turn to the bare generics.

3. The Treatment of Generic Terms

3.1. The Definite Generic

The definite generic can be used in two ways: in a Proper Kind Predication in (21a), or in a Derived Kind Predication in (21b).

(21) a. The whale has almost died out.
     b. The whale is a mammal.

In this section I will show that treating definite generics as proper names for kinds explains the properties of definite generics that I mentioned in section 1: (a) the ability to figure in Proper Kind Predications, (b) the restriction to well-established kinds, (c) the rather free distribution and (d) the impossibility of distribution over plural objects.

Following Krifka(1989), I will treat definite generic noun phrases - or rather their common nouns[6] - as lexical entries. The common nouns of the definite generics the panda, the Siberian tiger and the Coke bottle are stored in the lexicon as a kind of idiomatic expression. Semantically, they are constants that denote a KIND in the domain of interpretation. The meaning of a definite generic like the Siberian tiger, for example, is not a compositional function of the meaning of the predicates Siberian, tiger and the determiner the. Rather, it behaves like a constant for a kind in the domain. I will make the quite common assumption that the domain is enriched with kinds, just like in Carlson's semantics of bare plurals.

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[6] I think that, in general, the difference between NPs that are proper names and those that are not, is made at the CN-level. The determiner does not play a role. There are three kinds of linguistic evidence for this. Proper names (both for objects and kinds) can sometimes do without an article: John and Man. The CN of a proper name can have (non-restrictive) adjectives: that poor Bill and the rare panda (meaning the panda, which is rare). Finally, the determiner can show some variation: our Mary, this Mr.Smith and that poor panda, this rare Siberian tiger (talking about the species).

Assuming this, definite generics can be incorporated in DRT quite easily, by treating them exactly like ordinary proper names. For a definite generic like the panda, for example, there is a reference marker **x** in the main DR with the identity condition **x=panda**.

This immediately explains the condition on well-established kind-reference that definite generics are subject to. We cannot say something like the dutch dog because dutch dog is simply no lexical entry; we don't have it in our dictionary. Moreover, we cannot add dutch as a restrictive modifiers to dog either, because restrictive modification of a CN is only possible with a predicate that has a free variable. The CN of a definite generic is not treated as a predicate, but as a constant[7]. The possibility of definite generics to figure in Proper Kind Predications follows in a trivial way.

A Proper Kind Predications with a definite generic subject can be represented in DRT as follows:

(22) The personal computer is widespread.

```
x
.................................
x = personal-computer

widespread(x)
```

This DRS can be embedded in a model, in order to determine its truth with respect to that model, in the usual way. The only real change is the introduction of kinds in the domain, but this does not change anything in the formulation of the semantics of DRT. A sentence with a definite generic object is treated similarly:

(23) Edison invented the lightbulb.

```
x  y
............................
x = edison
y = lightbulb

x invented y
```

In this DR, **edison** is a constant for a normal object and **lightbulb** for a kind. We can also understand now why definite generics can be used on every position in the

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[7] Notice however that a definite generic like the dutch dog becomes possible by accomodation in the DRS. Hearing someone talk about The Dutch Dog as a particular kind of dog, we merely add it to our DRS.

sentence. The definite generic is treated as a proper name; its reference marker is always in the main DRS, identified with a particular kind and outside the scope of operators. The distribution of a definite generic is only restricted by the selectional restrictions of a predicate. Notice the contrasts between the a- and b-sentences in (24) and (25).

(24) a.?Edison ate the lightbulb.
 b. Edison created the lightbulb.

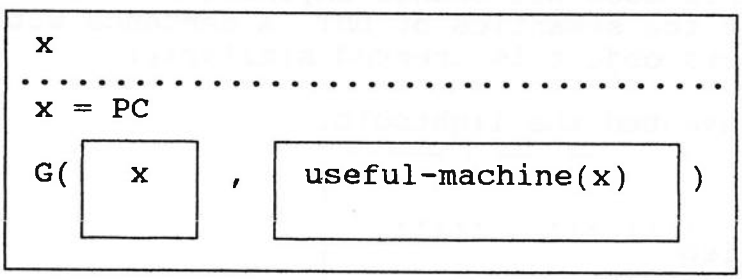
(25) a.?The personal computer broke down at midnight.
 b. The personal computer made its appearance around 1980.

(24a) and (25a) are strange - if the definite noun phrases are read generically - because kinds of things cannot be eaten and cannot break down at midnight.

This much for the use of definite generics in Proper Kind Predications. I will now turn to their use in Derived Kind Predications, in which a property is assigned to individual members of the kind.

Recall from section 2 that Derived Kind Predications with bare plural subjects are assumed to have a tripartite logical structure with a generic operator, a restrictive term and a nuclear scope. The bare plural subject is mapped into the restrictive term of this quantificational structure. We might try to treat Derived Kind Predications with definite generic subjects in the same way[8]. The reference marker for the definite generic is introduced in the main DR, because it is a proper name.

(26) The PC is a useful machine.



However, the DRS in (26) amounts to a Proper Kind Predication, because there is only one value available for the **x** in the left box. The property of being a useful machine, however, must be attributed to each individual personal computer and not to the kind itself. We have to shift somehow from the kind to its individual members.

I think that this shift in perspective from the kind-level to the individual-level can be implemented in a very

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[8] Instead of DR-branching, I use the relation G between two boxes as the representation of generic quantification. This is just a notational variant of Kamp's original box-splitting.

natural and insightful way as a replacement of the **x** in the left-hand box in (26) by a complete box that introduces a new reference marker **y** and contains the condition that **y** is a personal computer. In other words: a replacement of the reference marker of the kind with the box of the corresponding predicate. The DRS that results is given in (27).

(27) The PC is a useful machine.

```
+--------------------------------------------------------------+
| x                                                            |
|..............................................................|
| x = PC                                                       |
|                                                              |
|    +------------------+   +-------------------------+        |
| G( | y                | , |                         | )      |
|    |..................|   | useful-machine(y)       |        |
|    |                  |   |                         |        |
|    | pers-comp(y)     |   |                         |        |
|    +------------------+   +-------------------------+        |
+--------------------------------------------------------------+
```

This DRS correctly represents the Derived Kind Predication that the sentence <u>The PC is a useful machine</u> expresses.

There are some aspects about this substitution that have to be cleared up. I will assume that the box that replaces the reference marker **x** is a so-called PREDICATE-DRS. A predicate-DRS is a box that can be used as a predicate. One reference marker in it is distinguished as the individual to which the predicate applies. The use of predicate-boxes was proposed in a paper by Kamp and elaborated in Klein(1986). It can be seen as a kind of lambda-abstraction within DRT.

The right-box of the generic quantification should also be a predicate-DRS and the generic quantifier **G** a relation between two predicate-DRSs. Klein(1986) shows that a quantifier like <u>every</u> can be treated as a relation between two predicate-DRSs and he makes the remark that this treatment 'is strikingly reminiscent of the work on generalized quantifiers which treats determiners as binary relations on sets'. In the same way, the generic quantifier G and the adverbs of quantification can be seen as relations between predicate-boxes. Assuming this, the DRS of the sentence under consideration should be as given in (28).

(28) The PC is a useful machine.

```
+------------------------------------------------------+
| x                                                    |
|......................................................|
| x = PC                                               |
|                                                      |
|      +------------------+   +----------------+      |
|      | [y]              |   | [z]            |      |
|  G(  |..................| , |................|  )   |
|      |                  |   | useful-        |      |
|      | pers-comp(y)     |   | machine(z)     |      |
|      +------------------+   +----------------+      |
|                                                      |
+------------------------------------------------------+
```

In the semantics of the quantifier **G**, the distinguished reference markers **[y]** and **[z]** of the predicate-boxes are related in such a way that (28) is equivalent to (27). I won't give a formal definition of this semantics. The reader is referred to Klein(1986). The semantics of **G** will be similar to the semantics he gives for universal quantification, with one important difference: generic quantification is universal quantification with non-monotonic properties, in order to account for exceptions to the generic generalization (see Krifka(1987)).

The replacement of a predicate box for a reference marker must be licensed somehow. We do not want to allow the possibility that every reference marker can be replaced by a predicate-box; the operation should be possible only if there is a special rule or principle that allows it.

Somewhere in the semantics, a kind name like **ELEPHANT** has to be related to the singular predicate **elephant**, because **ELEPHANT** is not the proper name for an arbitrary kind, it is the proper name for the kind that corresponds to the property of being an elephant. Suppose this connection is part of the DRS as a presupposition or something like that, formulated as in (29). It says that the kind name **ELEPHANT** is a **Name** (name in a special sense) for the predicate-DRS of the singular CN <u>elephant</u>.

(29)

```
                          +----------------+
                          | [x]            |
    Name( ELEPHANT ,      |................|  )
                          | elephant(x)    |
                          +----------------+
```

This **Name**-relation in the DRS corresponds to the relation that exists in the domain between the elephant-kind and the set of its members, the individual elephants, or between the kind and the property.

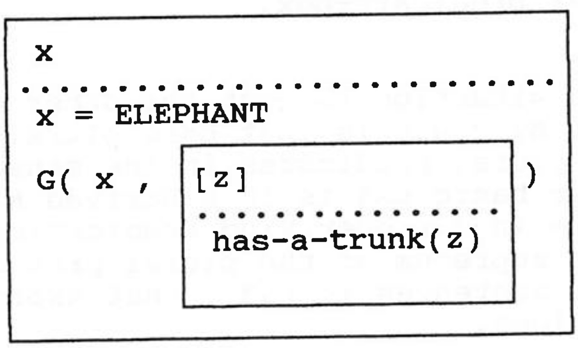
Substitution of a predicate-box for a reference marker is only possible if the reference marker is identified with a kind name and this kind name is related by **Name** to the predicate-box. Moreover, this shift in perspective seems to be possible only if the reference marker is in the restriction of the generic operator, an adverb of quantification or a modal verb like *can*. In all of these cases the left-hand box contains nothing but the kind-marker. It seems natural to say that a box with one reference marker and no conditions is semantically equivalent to just that reference marker alone. The contexts that allow replacement of a kind-marker with a predicate-box are exactly the cases where we have a quantificational relation between a kind-marker as restrictive term and a predicate-box as nuclear scope.

To make clear how the DRSs of (30) are derived - and summarizing in that way the results of this section- I will briefly describe the process of semantic interpretation, from the syntactic structure to the ultimate DRS.

(30) a. The elephant has a trunk.
     b. The dog is widespread.
     c. God created the woman.

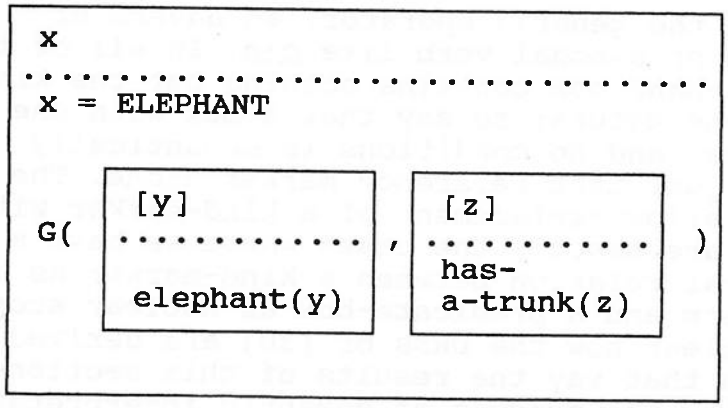
The definite generic subject of (30a) is projected on the external subject-position because the predicate *have a trunk* is an individual-level predicate. The definite generic is a proper name for the elephant-kind, so its reference marker **x** is introduced in the main DR with the condition **x** = **ELEPHANT**. Because of this, the restrictive term of the generic operator remains empty in some sense, though related by the occurrence of **x** to constant **ELEPHANT**. The VP *has a trunk* corresponds to the predicate-box that constitutes the nuclear scope of the generic operator.

(31)



After consultation of the **Name**-relation in (29), the restrictive term **x** in (31) can be replaced by the predicate-box that corresponds to **ELEPHANT**. The DRS that results is given in (32):

(32)



The interpretation of this DRS can roughly be described as: every 'normal' elephant has a trunk, which is the right reading for (30a), a Derived Kind Predication.

The other two sentences in (30) however, do not offer a possibility for the substitution of a kind-marker with a predicate-box, because there is no instance of such a marker in a restrictive term.

In this section I have shown how definite generic noun phrases can be treated in DRT. They are proper names for kinds. Proper Kind Predications have the most straightforward representation, with the marker for the kind occurring directly as argument of a kind-predicate. Derived Kind Predications with definite generics are really derived by substitution of the kind marker in the restriction of the generic quantification with the corresponding singular predicate-box.

3.2. The Bare Generic

For bare generics the situation is just the other way round, in some sense. My claim is that bare plurals are uniformly treated as plural predicates in the sense of Link(1983,1984). Their basic use is in a Derived Kind Predication. Their use in a Proper Kind Predication is derived by taking the supremum of the plural predicate.

Consider the two sentences in (33), that express Derived Kind Predications.

(33) a. Spies are afraid of the dark.
     b. Spies meet in the dark.

In the syntactic structure of (33a) and (33b), the indefinite plural NP <u>spies</u> is the external subject[9]. I will assume, in accordance with the Diesing-Kratzer proposal, that this NP is mapped into the restrictive term of the generic quantifier. For the DRS this results in the

---

[9] The bare plural (33b) can also have an existential reading, when it is the internal subject and not the external subject of the sentence. This is not the kind of reading that I have in mind now.

introduction of the reference marker **x** in the left box with the plural predicate ***spy**. The extension of a plural predicate ***P** is the set of all I-SUMS of the individuals in the extension of **P**; an i-sum of some individuals is the plural object of those individuals taken together. More in line with the preceding section, however, I will take the bare plural to corresponds to a plural predicate-DRS that constitutes the restrictive term of **G**. The predicate in the right-hand box can be of two different kinds: it can be distributive ((33a)) or collective ((33b)). For the distributive use of the predicate, I use Link's distribution-operator ^{D} that takes care that the property **afraid-of-the-dark** will only apply to 'atomic spies' and not to i-sums of spies. The DRS of (33a) is given in (34).

(34) Spies are afraid of the dark.

```
G( [x]          ,  [y]              )
   ............    .............
   *spy(x)         Dafraid-of-
                   the-dark(y)
```

Notice that both the bare plural subject NP spies and the predicate VP are afraid of the dark are treated as predicate-DRSs. The generic operator **G** relates these two boxes.

If there is no distribution operator working on the predicate in the right-hand box, then the predicate is true for i-sums of spies. This is the situation in (35).

(35) Spies meet in the dark.

```
G( [x]          ,  [y]              )
   ............    .............
   *spy(x)         meet-in-
                   the-dark(y)
```

This generic quantification over i-sums is possible with bare plural generics, but not with definite generics. This follows directly from the way I treat Derived Kind Predications with definite generics.

(36) ?The spy meets in the dark.

```
x
.................................
x = SPY

     [y]             [z]
G(   ...........  ,  ............  )
                     meet-in-
     spy(y)          the-dark(z)
```

The predicate-DRS that replaces the reference marker x in the generic quantification contains the singular predicate **spy** that only has atomic spies in its extension. The collective predicate **meet-in-the-dark**, however, only has i-sums of spies in its extension. This makes these two predicate-boxes incompatible for the same reason that *meet* is incompatible with *John* in *John meets*.

Bare plurals that are internal to VP are mapped in the right-hand box and hence get an existential reading, whatever the selectional restrictions of the verb may be. The definite generics *the pottery wheel* and *the dodo*, being proper names, are introduced outside the generic quantification in the main DR and get a simple kind-reading.

(10) a. The Sumerians invented the pottery wheel.
     b.?The Sumerians invented pottery wheels.

(11) a. The Dutch settlers exterminated the dodo.
     b.?The Dutch settlers exterminated dodos.

This explains why definite and bare generics behave differently in the object position of (10) and (11).

Let's now turn, finally to the Proper Kind Predication with bare plurals as it occurs in (37).

(37) a. Rats are widespread.
     b. Dinosaurs are extinct.
     c. Madrigals are popular.

As I already showed in section 1, this kind of sentences cannot be analyzed with a generic quantification. The bare plural in subject-position denotes the kind as a whole and there is no clear reference to individual members.

I wish to show that this kind-denotation can be derived from the basic meaning of the bare plural as a variable over i-sums. The extension of a plural predicate in Link's logic of plurals is a join-semi-lattice. The property of this algebraic structure is that each subset of it has a supremum. For a plural predicate like *dogs* this means that we can take any set of individual or plural dogs and put them together in a plural object, a sum, by applying the sum-operator. The lattice as a whole has a unique sum too, the supremum of the lattice. In Link's logic there is an operator, the operator σ, that gives you the supremum of a plural predicate. The sum of all dutch dogs can be denoted in this way by the term $\sigma x[\text{dutch}(x)\ \&\ {}^{*}\text{dog}(x)]$.

In his 1984 article, Link uses this supremum-operator to represent the meaning of the bare generic. When the supremum-operator σ operates on a plural predicate it yields the kind corresponding to that predicate. Formulated more carefully: the supremum of the predicate can be taken to be identical to the kind corresponding to that predicate. To give an example: the kind denoted by the bare generic <u>dutch dogs</u> is $\sigma x[\ \text{dutch}(x)\ \&\ {}^{*}\text{dog}(x)]$.

This gives us an immediate explanation why the CN of bare generics is not restricted to well-established kinds. The essential property of a join-semi-lattice is that every subset has a supremum. This means that we can add as many restrictive modifiers as we want. The supremum-operator will still yield a kind for it.

Of course, identifying the kind with the supremum of a predicate, raises many interesting and important questions about the nature of kinds. Can we simply identify a kind with the sum of its members? What results does this have for identity of kinds? What sort of intensionality do we need for a kind? Should the logic of plurals be embedded in a modal logic of properties to yield the right results?

However interesting these questions may be, I will leave them aside in this article and simply accept Link's idea that the bare plural can denote a kind if we take the supremum of its predicate.

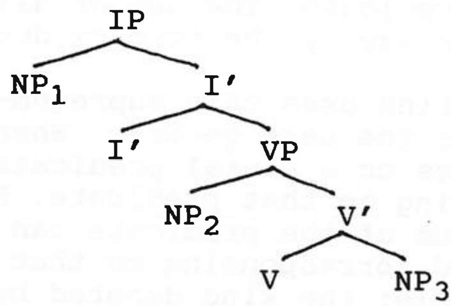
The question that should still be answered is: how does the bare plural actually get to denote the supremum? Again, the syntactic position of the bare plural seems to play an important part. Consider the following sentences:

(38) a. Rats are widespread.
b. Rats reached Australia in 1770. (Krifka(1989))
c. The Sumerians invented pottery wheels.

In (38a) the bare plural <u>rats</u> can only be interpreted as referring to the whole kind. The bare plural in (38b) is ambiguous between an existential reading (some rats) and a generic reading (the kind of the rats as a whole). In (38c) the bare plural <u>pottery wheels</u> can only be read existentially.

The difference in interpretation correlates with a difference in position. The generic bare plural in (38a) is external subject (NP_1), because <u>widespread</u> is an individual-level kind-predicate[10]. The bare plural in (38b) can occupy either the internal (NP_2) or external (NP_1) subject-position, because the predicate is a stage-level predicate; hence the ambiguity. In (38c) the bare plural object (on position NP_3) is existential.

(39)



It seems clear that bare plurals can denote a kind if they occupy position NP_1, but not NP_2 or NP_3. What's the difference between the external argument and the internal arguments?

A quite common idea about the external subject is that it is related to the VP by means of a predication-relation. The internal arguments are **part** of the predicate, and cannot be predicated by it, unless they are moved out of it. This syntactic idea can be related to the DRS of the sentence in the following way. The predication-relation between external subjects and the VP **always** gives rise to a relation between predicate-boxes in the DRS. The left-hand box is filled with information from the external subject and the right-hand box is constituted by the VP. This DR-split takes place, whatever the nature of the external subject NP and whatever the nature of the sentence. The only thing that differs is the type of relationship between the boxes and the way they are embedded in the model. The relation between the boxes can be specified in one of four ways, as given in (40):

(40) 1. The Determiner of the NP (each, all, most etc.)
2. Adverbs of Quantification (always, usually etc.)
3. The Generic Quantifier, G
4. The Predication Relation, Pred

If there is no overt determiner (in the NP) or an adverb of quantification (in the VP) to specify the relationship, we can choose between the other two options: a generic relation between the boxes or a simple predication relation. This predication-relation **Pred** differs from the other three in being not a quantificational relation between predicate-boxes. It is a relation between a unique

---

[10] Notice that the distinctions between stage-level and individual-level predicates and kind-predicates and object-predicates are independent. The first distinction is aspectual (having a Davidsonian event-argument or not), the second distinction pertains to the selectional restrictions of the predicate. Carlson claimed that these distinction were all of the same kind.

entity and a property. For example, it relates the occurrences of the markers of pronouns and proper names to the predicate.

(41) Jill is intelligent.

```
+----------------------------------------------+
| x                                            |
| .............................................|
| x = jill                                     |
|                                              |
| Pred( x ,  +--------------------+ )          |
|            | [y]                |            |
|            | .................. |            |
|            | intelligent(y)     |            |
|            +--------------------+            |
|                                              |
+----------------------------------------------+
```

In (41), **Pred** relates the **x** to the predicate-box of <u>intelligent</u>. In this case, there is an obvious equivalence with the traditional way of writing predication in DRT, so we could apply a kind of 'conversion' here to arrive at a simpler DRS. This conversion is not possible, however, if the first argument of **Pred** is a predicate-box, for example, the predicate-box corresponding to a bare plural. The DRS of such a sentence is given in (42).

(42) Rats are widespread.

```
+-------------------------------------------------------+
|                                                       |
|        +-----------+    +-------------------+         |
|        | [x]       |    | [y]               |         |
| Pred(  | ......... |  , |                   | )       |
|        | *rat(x)   |    | widespread(y)     |         |
|        +-----------+    +-------------------+         |
|                                                       |
+-------------------------------------------------------+
```

Suppose however that the semantics of the **Pred**-relation demands that the left-hand argument is a single reference marker, because **Pred** relates individuals to a property. In other words: conversion of the **Pred**-relation should always be possible. For the DRS in (42) this means that the predicate-DRS of the bare plural must be replaced by a single reference marker. This reference marker has to be related to the predicate-box in some way. We cannot simply identify them, neither can we use the **Name**-relation, because the generic bare plural is not a proper name for a plural predicate.

I think that the reference marker that is used as the left-hand argument of the **Pred**-relation should be identified with the supremum of the predicate-box. In this way it will denote the kind that corresponds to the bare plural. Assuming that **Sigma** is the supremum-operator that yields the supremum of a predicate or predicate-box, the DRS looks as follows:

(43)

```
+------------------------------------+
| z                                  |
| .................................. |
|                                    |
|              +------------+        |
|              | [x]        |        |
| z = Sigma(   | .......... |  )     |
|              | *rat(x)    |        |
|              +------------+        |
|                                    |
|             +------------------+   |
|             | [y]              |   |
| Pred( z ,   | ................ |   |
|             | widespread(y)    |   |
|             +------------------+   |
+------------------------------------+
```

The identification of z with the supremum of the plural predicate-box, gives us the Proper Kind Predication for this sentence.

This result would not be very impressive if **Sigma** was only used to derive one specific reading of the bare generic. I suspect, however, that the **Sigma**-operator has a much wider use in DRT. Consider the following sentence:

(44) The inhabitants of California fear the earthquake.

The definite subject does not denote a familiar object, I think, but it denotes the supremum of the predicate *inhabitants of California*. The DRS of this should look like (45):

(45)

```
+----------------------------------------------------+
| z                                                  |
| .................................................. |
|                                                    |
|              +---------------------------+         |
|              | [x]                       |         |
| z = Sigma(   | .........                 |  )      |
|              | *inhabitants-of-C(x)      |         |
|              +---------------------------+         |
|                                                    |
|             +----------------------------------+   |
|             | [y]                              |   |
| Pred( z ,   | ................................ |   |
|             | Dfear-the-earthquake(y)          |   |
|             +----------------------------------+   |
+----------------------------------------------------+
```

In this DRS, the predicate-box corresponding to the VP *fear the earthquake* is applied distributively to the atoms in the supremum of the CN *inhabitants of California*.

The **Sigma**-operator can be used for all definite noun phrases that do not denote a familiar object (i.e. their reference marker cannot be identified with a reference marker already present in the DRS), but denote the supremum of a predicate. Notice that definite NPs like this have a generic flavor about them.

4. Conclusion

In this article I have presented a treatment of definite and bare generics within the framework of DRT. I showed that definite generics should be treated as proper names for kinds. Their basic use is in Proper Kind Predications; in Derived Kind Predications there is a shift from kind-reference to reference of the individual members. This shift is implemented in the DRS by a replacement of a marker for a kind by the corresponding predicate-box. This substitution is licensed by the fact that a definite generic can be regarded as the name of a singular predicate. Bare plurals were treated as quantifier-free plural predicates. The Derived Kind Predication is their basic use; the Proper Kind Predication is derived by taking the supremum of the plural predicate, which is identical to the kind.

Treating definite and bare generics like this, accounts for the differences and similarities that I discussed in section 2. It supports the approach to bare plurals that is familiar from the work of Wilkinson, Krifka, Diesing and Kratzer, and it provides a treatment of definite generics.

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